Analysis of Eigenfrequencies of the Foot Prosthesis with Auxetic Component Layer

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Abstract

In this paper, natural frequencies of a three-layered foot prosthesis are investigated. The model of foot prosthesis consisted of a three-layered base, which substitutes a human foot and an element in the shape of an arc that represents a shank of human. The base consists of three layers made of carbon fiber. In the lower part of the prosthesis, the auxetic layer is used as the inner layer. Numerical analysis is made for different parameters of the central layer: the thickness and the value of Poisson’s ratio. The simulations are used to investigate the influence of an auxetic layer on prosthesis vibrations and compare the impact of different parameters on results. Calculations are made using the finite element method implemented in Autodesk Fusion 360. The results show that the auxetic layer has a great impact on tolerance to vibrations and mobility.

Keywords: natural frequencies, mode shapes, finite elements method, foot prosthesis

1. Introduction

It is well known that a designer understands the natural vibration frequencies of a system to ensure that they are not the same as excitation frequencies, thus ensuring safety standards. This is a key component in many fields like civil, aerospace, automotive or medical and biomedical engineering, where the loss of life and property is a major concern. Starting with hand calculations in the 1980s, computer simulations have made great breakthroughs to help improve the quality and robustness of design processes or constructions.

Used materials as components of structures have a great influence on the properties of the whole structure. It is really important to use the proper ones. It is also crucial if we want to build structures with special properties. Engineers and researchers use typical materials as structural components. For many years they also investigate properties of structures composed with metamaterial or smart materials. There are materials with “negative properties” such as negative Poisson’s ratio, negative coefficient of thermal expansion, negative compressibility, negative reflective index, negative magnetic permeability, or negative permittivity, etc.

Modern foot prosthesis is classified into three categories: passive, active, and hybrid prosthesis. To control the process of movement, adaptive foot prostheses are tools up in hardware constructions to achieve different functions. For example, to regenerate energy series elastic actuators, coil spring and clutch motors are applied. Energy-saving mechanisms have been introduced to reduce the power requirement. Furthermore, researchers investigate the properties of currently used materials and tend to find ones that will improve energy stores. The most important requirement for modern foot prosthesis is light, elastic, and stable construction that ensures accumulation, storage, and release of energy during walking. Energy storing prosthetic feet (ESPE) is marked by these properties. They are made by composites with carbon and glass fibers, carbon fiber combined with epoxy resin, glass fiber combined with polyester. Significantly, manipulation of fiber's type, the number of layers, the way of laying method, or combining properties of the same material determine the efficiency and dynamic behavior of foot prosthesis.

In this paper, eigenfrequencies of foot prosthesis with the viscoelastic auxetic layer are studied. The auxetic layer is the core of the foot prosthesis base and lays between two layers made of carbon fiber. Different values of Poisson's ratio (PR) and two values of thickness for the auxetic layer are compared. Calculations are made using the finite element method with Fusion 360 software.

2. Cellular and auxetic materials

Researchers were inspired by old structural material which is wood. It is the only significant building material that is grown. They wondered about their extraordinary properties and think how can convey them to other construction materials. It was the purpose of investigations in the thesis made between 1978 and 1981. Papers aimed to investigate why cellular material can be stiff and light, is a great insulator, are capable of accommodating large elastic deformations. Lora Jane Gibson [1] focused on the physical mechanism and used beam theory. It was admitted that material properties depend on shape and density. Moreover, deformations depend on elastic moduli E, G, and bending stiffness. This research began the next investigations. In 1982 authors [2] analyzed mechanisms that involve deformation of three-dimensional cellular solids or foams. The research compared to tests on rubber and metal models.

A few years later Almgren [3] publishes his investigation about three-dimensional structure with Poisson's ratio equals -1. Auxetic material was associated with negative PR (NPR). It means that such a material expands laterally when stretched. Common materials reduce the size after stretched load. In 1987 Lakes [4] presents foam structure, which exhibits a negative Poisson's ratio. This property isn't commonly observed in real materials. In isotropic material (materials, which don't have identical values of a property in all directions) theoretically, the permissible range of Poisson's ratio is from -1 to 0,5. Foams used in the research were produced from conventional low-density open-cell polymer foams. Lipsett and Beltzera [5] examined NPR materials and take to this purpose extreme value. Scarpa et. al. [6-10] investigated dynamic and acoustic performances of for example sandwich panels, auxetic polymers, and cellular materials. They admit that auxetic honeycombs are a good example of cellular materials with NPR behavior. Moreover, calculations gave a conclusion, that it is possible to obtain in sandwich beams enhanced stiffness per unit weight values and modal loss factors using two-phase cellular solids. Geometry plays a significant role in research. Qin's [11] research implies that the honeycomb structure is distinct from other known auxetic materials.

In the past year, researchers focused on analyses of dynamic response and amplitude vibration auxetic structures [12-14]. Strek et. al. [18] presented the impact of the NPR in selected materials of the contact pressure values and deformations. Their research involved two cases: homogenous plate and layered plate with different PR values. For positive PR isn't observed any dependence. In the first case for more negative values of PR, the contact pressure increases and decreases the length of the contact boundary. In a two-layered plate for greater thickness of the auxetic layer, there is greater contact pressure and lower displacement. Later Strek et. al. [19] investigated the influence of mechanical impedance on a sandwich beam with auxetic metalcore. Additionally, harmonic loads and different parameters were applied. Calculations imply that values of mechanical impedance are greater for higher values of density and the more negative value of PR of the core, the lower mobility. Other properties of auxetic materials were investigated by Airoldi et al. [20]. The authors proposed an innovative concept for energy absorption in case of localized impacts. The concept is based on the contraction of cellular structures with NPR under the impact, which is combined with the energy absorption capability of a foam filler. Composites in energy-storing prosthetic feet were investigated by Dziaduszewska et al [21].

3. Mathematical model

Eigenfrequencies or natural frequencies are certain discrete frequencies, at which a system is prone to vibrate. Natural frequencies appear in many types of systems, for example, as standing waves in a musical instrument or an electrical RLC circuit.

When vibrating at a certain eigenfrequency, a structure deforms into a corresponding shape mode, the eigenmode. An eigenfrequency analysis can only provide the shape of the model, not the amplitude of any physical vibration. The true size of the deformation can only be determined if an actual excitation is known together with damping properties. Determining the eigenfrequencies of a structure is an important part of structural engineering.

Hooke's law for linear elasticity is usually written like [22]

$$\mathbf{S} = \mathbf{C} : \boldsymbol{\varepsilon}, \tag{1}$$

where the stress tensor **S** and the strain tensor **ε** are second-order tensors, while the constitutive tensor **C** is a fourth-order tensor. The ':' symbol means a contraction over two indices.

The elasticity matrix for isotropic materials can be written in terms of Lamé parameters λ and μ, as follows

$$\mathbf{C} = \begin{bmatrix} \lambda + 2\mu & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & \lambda + 2\mu & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & \lambda + 2\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \end{bmatrix}, \tag{2}$$

where Lamé parameters can be expressed in terms of two elastic constants as $\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}$ and $\mu = \frac{E}{2(1+\nu)}$, where E is the Young modulus and ν is the PR.

The Navier's equation of motion with zero volume force can be written as

$$\rho \frac{\partial^2 \mathbf{u}}{\partial t^2} - \nabla \cdot \mathbf{S} = \mathbf{0}, \tag{3}$$

where ρ is the density, $\mathbf{u}$ is the vector of displacements and $\mathbf{S}$ is the stress tensor.
The equation of motion with the linear constitutive relation between stresses and deformations can be written as

$$\rho \frac{\partial^2 \mathbf{u}}{\partial t^2} - (\mu \nabla^2 \mathbf{u} + (\lambda + \mu)\nabla\nabla \cdot \mathbf{u}) = \mathbf{0}. \tag{4}$$

A harmonic displacement is defined by an equation as

$$\frac{\partial^2 \mathbf{u}}{\partial t^2} = -\omega^2 \mathbf{u}, \tag{5}$$

where ω is forcing frequency. The displacement vector has a complex form and is defined as

$$\mathbf{u}(\mathbf{x}) = \mathbf{u}_1(\mathbf{x}) + i\mathbf{u}_2(\mathbf{x}) \tag{6}$$

and the harmonic displacement is a real part of the complex form

$$\mathbf{u}(\mathbf{x}, t) = Re[\mathbf{u}(\mathbf{x})\mathrm{e}^{-i\omega t}], \tag{7}$$

According to aforementioned equations, the harmonic equation of motion of linear elastic material fulfills the formula

$$-\rho\omega^2 \mathbf{u} - (\mu \nabla^2 \mathbf{u} + (\lambda + \mu)\nabla\nabla \cdot \mathbf{u}) = \mathbf{0}. \tag{8}$$

The harmonic equation may be viewed as the eigenvalue equation.

4. Model of the foot prosthesis

In this paper, the modal frequencies study is performed to determine the eigenfrequencies of foot prosthesis. The model of the prosthesis was created for this test by the use of Inventor 2019 software. It consists of a three-layered base, which substitutes a human foot and an element in the shape of an arc that represents a shank of human. The base of prosthesis dimensions are L x H x W, where: L is the length, H is the height, and W is the width of the analyzed part of the prosthesis. The top and bottom layers thicknesses are H_1 and H_3, respectively. The thickness of the central layer (core) of the base of the artificial foot is H_2 (see Figure 1).

The element in the shape of the arc is built of two arcs: an outer arc and an inner arc. The radius of the outer arc is 210 mm and for the inner arc, it is 180 mm, whereas the distance between both these arcs is 24 mm. There is also a distance from the top of the base to the end of the inner arc and to the end of the outer arc. The first one is 396 mm, whereas the second one is 420 mm. The part of the foot prosthesis which connects the base and the element in the shape of the arc is a cleat. A longer height of the cleat which means a distance from the top of the base to the beginning of the inner arc is 35 mm and shorter height of the cleat, i.e. a distance from the top of the base to the beginning of the outer arc totals 20 mm. A distance between those two heights of the cleat is 40 mm. The width of both the cleat and the element in the shape of the arc is the same as the width of the base and totals W = 80 mm.

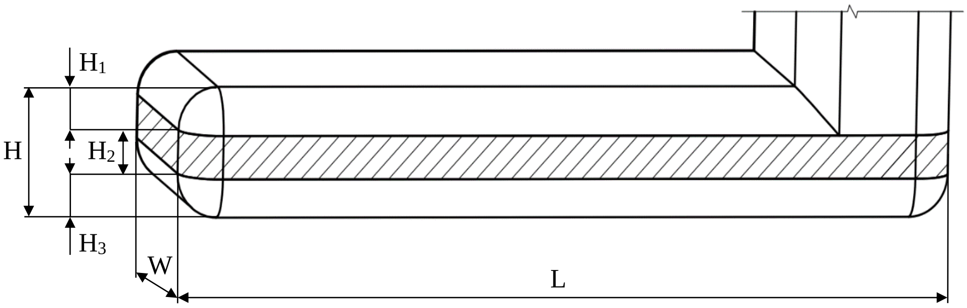

Figure 1. The scheme of the considered base of the foot prosthesis. The auxetic core is an element with lines in this scheme.

In numerical analysis, the model of foot prosthesis was fixed on a curved surface that is located on the bottom layer of the base. This curved surface is in front of the base in a place, where the end of toes would be located on a human foot (see Figure 2).

Figure 2. The model of foot prosthesis fixed on the curved surface of the bottom layer of the base (this curved surface is marked on blue).

5. Numerical results

The material of the analyzed prosthesis is carbon fiber reinforced polymer (CFRP), which is most popular in constructions of this type. The core of the base is computational auxetic CFRP material with NPR from -0.9999 to 0.4999. The numerical values of the materials parameters are presented in Table 1. The values of properties of CFRP are taken from a repertory of materials, which is available in Fusion 360 software. This repertory is some sort of library, in which major properties and parameters of materials are defined.

Table 1. Material properties of foot prosthesis layers and core.

The base of the foot prosthesis	Material	Young's modulus, E [GPa]	Poisson's ratio, ν	Density, ρ [kg/m^3]
Layers – top, bottom	CFRP	133	0.39	1430
Core	Auxetic CFRP	133	from –0.9999 to 0.4999	1430

In this study, two parameters of the core are changed: PR and its thickness. The PR of the core ν made of viscoelastic auxetic CFRP material is changed in the range of –0.9 to 0.49. Additionally, the analysis was performed also for extreme values of the ν of the core i.e., $\nu = -0.9999$ and $\nu = 0.4999$. All the PR values of the auxetic core used in this study are pictured in Table 2.

Table 2. The values of the Poisson's ratio of the auxetic core for 11 experiments.

Experiment No.	1	2	3	4	5	6	7	8	9	10	11
PR, ν	–0.9999	-0.9	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.49	0.4999

Dimensions of the base of the prosthesis for all cases are $L = 200$ mm, $H = 30$ mm, and W = 80 mm. All the curvatures of the base for all cases are $R = 10$ mm. The base of the artificial leg is built of two outer layers made of CFRP and the core made of auxetic CFRP material. There are two variants of construction of the base that was used in the performed study. The first one includes the base in which thicknesses of the top and bottom layers are $H_1 = H_3 = 12.5$ mm and the thickness of the core is $H_2 = 5$ mm. In the second variant of construction, thicknesses of the top and bottom layers are $H_1 = H_3 = 7.5$ mm and the thickness of the core is $H_2 = 15$ mm.

Finite element analysis was applied with the use of Fusion 360 software. The modal frequencies module of software was chosen to execute the study. The mesh used in the analysis for all the simulations is built of 111 427 elements in the shape of a tetrahedron.

Results obtained in the performed study confirm that the value of the PR and thickness of the auxetic core can modify the dynamic behavior of the structure. The basic conclusion from the executed numerical analysis is that increase of the value of the PR results in a decrease of the values of the eigenfrequencies for both selected thicknesses of the auxetic layer. The values of the eigenfrequencies in the range of 0–1000 Hz are pictured in Tables 3. and 4.

The greatest fall of the value of the eigenfrequency appears for change of the PR from –0.9999 to –0.9. In the structure, in which the thickness of the core totals $H_2 = 5$ mm, the value of the first eigenfrequency changes from 140.1 Hz to 107.6 Hz for the relevant change of Poisson's ratio. Respectively, for $H_2 = 15$ mm, this value shifts from 154.2 Hz to 122.8 Hz. Further increase of the Poisson's ratio also causes a decrease of the values of the eigenfrequencies, but it is not so significant. This decrease advances until the Poisson's ratio receives the value of 0.4 and then the values of the eigenfrequencies begin to grow. In the case of the thickness of the core $H_2 = 15$ mm for $v = 0.4$, the first eigenfrequency obtains 105.4 Hz, 109.5 Hz for $v = 0.49$, and 136.8 Hz for $v = 0.4999$.

Table 3. Values of first fifth smallest eigenfrequencies of the foot prosthesis with an auxetic layer with thickness $H_2 = 5$ mm.

	Eigenfrequencies [Hz]				
v	1st	2nd	3rd	4th	5th
-0.9999	140.1	236.4	388.4	779.6	1140
-0.9	107.6	167.4	299.5	660.7	1011
-0.8	106.6	165.1	297.5	648.5	1006
-0.6	105.8	163.4	295.4	635.8	1001
-0.4	105.2	162.5	294.1	628.8	997.1
-0.2	104.9	161.8	293.1	624.5	994.4
0	104.6	161.2	292.4	621.6	992.4
0.2	104.4	160.7	292	619.8	991
0.4	104.4	160.4	292	619.1	990.7
0.49	105	161	293.7	619.8	994.5
0.4999	109.9	166.8	306.7	623.3	1023

Table 4. Values of first fifth smallest eigenfrequencies of the foot prosthesis with an auxetic layer with thickness $H_2 = 15$ mm.

	Eigenfrequencies [Hz]				
v	1st	2nd	3rd	4th	5th
-0.9999	154.2	242.9	555.6	944.9	1871
-0.9	122.8	202.8	333.3	706	1078
-0.8	116.9	189.7	316.7	675.8	1046
-0.6	111.3	178	303.4	648.2	1020
-0.4	108.4	172.3	297	633.8	1006
-0.2	106.5	168.6	293.3	624.7	997.2
0	105.4	166.1	291.1	618.8	991.8
0.2	104.9	164.2	290.2	615	989.2
0.4	105.4	163.3	291.4	613.8	991.6
0.49	109.5	167.1	300.4	617.4	1012
0.4999	136.8	211.4	385.2	651.9	1153

Results obtained in this study also acknowledge that the thicker the auxetic core is, the higher the eigenfrequencies are. All the values of the eigenfrequencies are greater for the structure with the thicker auxetic core. The greatest difference occurs for extreme values of the v of the core, i.e., $v = –0.9999$ and $v = 0.4999$. In the structure in which the thickness of the core totals $H_2 = 15$ mm, for $v = –0.9999$ the value of the first eigenfrequency receives 154.2 Hz, and for $v = 0.4999$ it is 136.8 Hz. For $H_2 = 5$ mm, in the case of $v = –0.9999$ the value totals 140.1 Hz and for $v = 0.4999$ it is 109.9 Hz. Results obtained for both thicknesses of the auxetic core confirm that the more the value of the PR increases, the less significant the differences between the values of the eigenfrequencies are.

In the examined range of frequencies (0–1000 Hz), the number of resonance frequencies changes from four in the case of auxetic core characterized by the v in the range from –0.9999 to –0.6 and for $v = 0.4999$ to five resonance frequencies for v in the range from –0.4 to 0.49 in the case of the thickness of the core $H_2 = 5$ mm. In the case of the thickness of the core $H_2 = 15$ mm, there are some differences, i.e., four resonance frequencies occur for v in the range from –0.9999 to –0.4 and also for $v = 0.4999$ and $v = 0.49$, whereas for v in the range from –0.2 to 0.4 the number of resonance frequencies receives five.

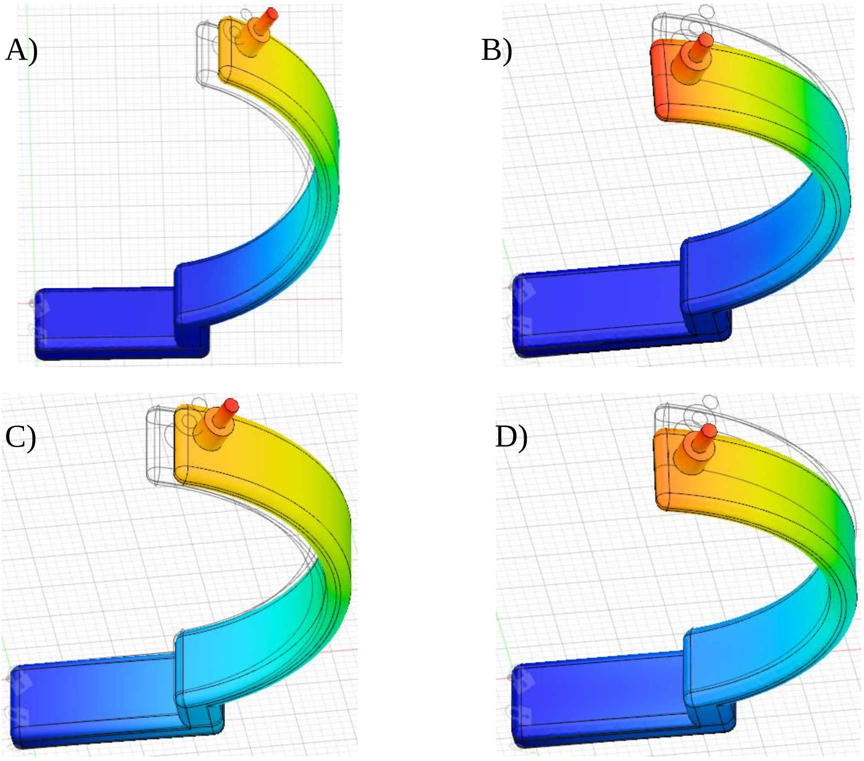

Figure 3. Mode shapes for $H_2 = 15$ mm: A) 1st eigenfrequency for $v = –0.9999$; B) 2nd eigenfrequency for $v = –0.9999$; C) 1st eigenfrequency for $v = 0.4$; D) 2nd eigenfrequency for $v = 0.4$.

Mode shapes are different for each number of eigenfrequencies. Simulations visualize total mode displacement, to wit mode shapes for results. In Figure 3 there is a comparison of mode shapes for first and second eigenfrequencies in a model of foot prosthesis, where the height of the auxetic layer is 15 mm and Poisson's ratio equals -0.9999 and 0.4. For 1st eigenfrequencies (see Figure 3A and 3C) prevails displacement in the x-axis, but for 2nd eigenfrequencies (see Figure 3B and 3D) the most displacement is in the z-axis. In the next numbers of eigenfrequencies, displacements are more significant and mode shapes are more complex.

6. Conclusions

In this paper, eigenfrequencies of foot prosthesis with the viscoelastic auxetic layer were investigated. The material of the analyzed prosthesis is carbon fiber reinforced polymer. The model of foot prosthesis consists of an element in the shape of an arc and a three-layered base, in which the central layer is made of an auxetic CFRP material. This material is often used for the reduction of vibration. Various values of PR and two values of thickness for the auxetic layer are compared. Calculations were made using the finite element method with Fusion 360 software.

Simulation results in the conclusion that using an auxetic layer in the core of foot prosthesis has a great impact on the dynamic behavior of prosthesis. Calculations show that values of eigenfrequencies are greater for more negative PR and thicker auxetic layer in the core. The frequencies that arise in the foot prosthesis as a consequence of using it have relatively small values, hence the eigenfrequencies of the foot prosthesis should be as high as possible. The greatest values of the eigenfrequencies occur for the thicker auxetic layer with the most negative PR value. This combination of properties has the most beneficial effect because the greater value of eigenfrequency is, the less exposed to vibrations the foot prosthesis is.

Acknowledgments

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