METHOD FOR DETERMINATION OF CONFIGURATION FACTORS IN RADIATION HEAT TRANSFER

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Abstract: The current paper presents a method for estimating the configuration factor in heat transfer by radiation in buildings. The numerical procedure can be used to determine exactly this quantity in any enclosure with complex geometry and strong gradient of surfaces temperature. A verification of the method is also reported in this work. The numerical results from the presented method, implemented as a computer program, are compared with the analytical solution. A very good agreement has been found between these two procedures for determination of the configuration factor. The total error of the method, for the recommended density of surface division, is about 1%.

Keywords: configuration factor, radiation heat transfer, numerical integration.

1 Introduction

In engineering practice and research, we often deal with closed spaces that can be characterized by an asymmetry of the temperature distribution on surfaces (for example: rooms with floor heating or cooling ceiling, exhibition rooms with a large area of glazing). The long-wave radiation heat transfer frequently represents a great part of the heat balance in these types of enclosures. It is well known that the radiative flux emitted from arbitrary surface *i* to surface *j* can be approximated by the following equation:

$$q_{i,j} = \sigma\varepsilon_{i,j} F_{i,j}\left(\theta_i^4 - \theta_j^4\right), \tag{1}$$

where: σ = 5.67 x 10^{-8} W/m^2/K^4 – the Stefan-Boltzmann constant, ε – emissivity of surfaces, $F_{i,j}$ – configuration factor, θ – absolute temperature of emitting surfaces.

A correct determination of $F_{i,j}$ factor is difficult and time consuming problem for the complex room geometry. The configuration factors can be calculated between a person and the surrounding surfaces or only between surfaces. The first issue was clearly referred by Rizzo and coauthors in [1], Ozeki et al. in [2], and Kubaha et al. in [3]. In this paper only the second problem is considered. The next chapter presents a method for determination of the configuration factor using numerical integration.

2 Description of the method

The fundamental formula for a configuration factor between isothermal and black-body surfaces *i* and *j* is given by:

$$F_{i,j} = A_i^{-1} \int_{A_i}\int_{A_j} \frac{\cos\beta_i \cos\beta_j}{\pi R_{i,j}^2} dA_i dA_j \ , \tag{2}$$

where: *A* – area of the radiative surface, *R* – length of the vector (see Fig. 1), β – polar angle between vector *R* and unit normal to the surface.

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We can approximate Eq. (2) for the numerical integration form Eq. (3) by dividing i and j surfaces into finite m cells A_a.

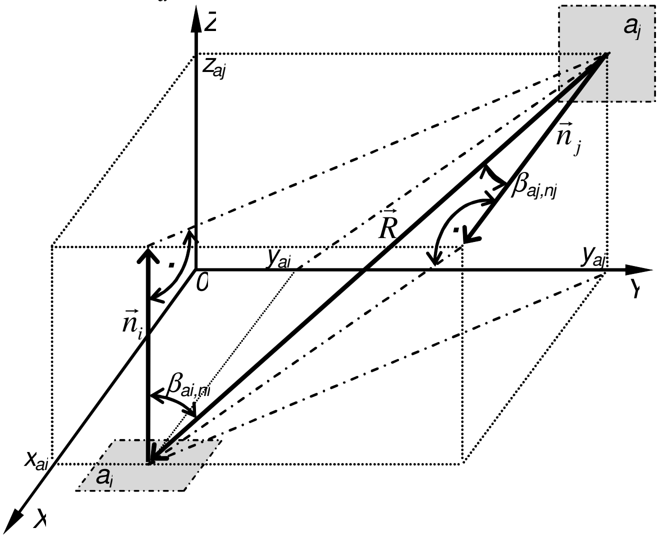


Fig. 1: Geometric relations for determining of the configuration factor.

$$\boldsymbol{F}_{i,j} = \boldsymbol{A}_i^{-1} \boldsymbol{v}_{i,j} \sum_{ai=1}^{m_i} \sum_{aj=1}^{m_j} \left(\frac{\cos \boldsymbol{\beta}_{ai,ni} \cos \boldsymbol{\beta}_{aj,nj}}{\pi \boldsymbol{R}_{ai,aj}^2} \right) \boldsymbol{A}_{ai} \boldsymbol{A}_{aj} \,, \tag{3}$$

where: $v_{i,j}$ – visibility factor ($v_{i,j} = 1$ if dA_i is visible to dA_j and $v_{i,j} = 0$ otherwise).

The first step of the procedure includes: identifying isothermal parts of the enclosure and then cutting these planes into small identical sub-surfaces. Next, the cosine of β angle for each pair of cells is determined through automatic generation of the integration points coordinates. The center point of the Cartesian coordinate system can be placed in any corner of the tested space. Fig. 1 shows the geometrical relations for evaluating $\angle(\overrightarrow{\boldsymbol{a}_i\boldsymbol{a}_j} - \overrightarrow{\boldsymbol{n}_i})$ and $\angle(\overrightarrow{\boldsymbol{a}_j\boldsymbol{a}_i} - \overrightarrow{\boldsymbol{n}_j})$ angles if the first sub-surface is situated on *XOY* plane and the second on *YOZ* plane. Tab. 1 contains the algebraic relations for calculating of $cos\beta$ that can be employed in Eq. 3.

Table 1. The relations for determining $\cos\beta_{ai,ni}$ and $\cos\beta_{aj,nj}$

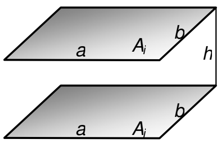
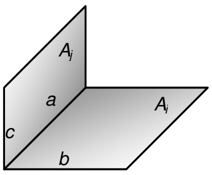
Position ai	aj	$\cos\beta_{ai,ni}$	$\cos\beta_{aj,nj}$
xOy	*yOz*	$\frac{\lvert z_{aj}-z_{ai}\rvert}{\sqrt[2]{(x_{aj}-x_{ai})^2+(y_{aj}-y_{ai})^2+(z_{aj}-z_{ai})^2}}$	$\frac{\lvert x_{aj}-x_{ai}\rvert}{\sqrt[2]{(x_{aj}-x_{ai})^2+(y_{aj}-y_{ai})^2+(z_{aj}-z_{ai})^2}}$
xOz	*yOz*	$\frac{\lvert y_{aj}-y_{ai}\rvert}{\sqrt[2]{(x_{aj}-x_{ai})^2+(y_{aj}-y_{ai})^2+(z_{aj}-z_{ai})^2}}$	$\frac{\lvert x_{aj}-x_{ai}\rvert}{\sqrt[2]{(x_{aj}-x_{ai})^2+(y_{aj}-y_{ai})^2+(z_{aj}-z_{ai})^2}}$
xOz	‖ *xOz*	$\frac{\lvert y_{aj}-y_{ai}\rvert}{\sqrt[2]{(x_{aj}-x_{ai})^2+(y_{aj}-y_{ai})^2+(z_{aj}-z_{ai})^2}}$	$\frac{\lvert y_{aj}-y_{ai}\rvert}{\sqrt[2]{(x_{aj}-x_{ai})^2+(y_{aj}-y_{ai})^2+(z_{aj}-z_{ai})^2}}$
xOy	‖ *xOy*	$\frac{\lvert z_{aj}-z_{ai}\rvert}{\sqrt[2]{(x_{aj}-x_{ai})^2+(y_{aj}-y_{ai})^2+(z_{aj}-z_{ai})^2}}$	$\frac{\lvert z_{aj}-z_{ai}\rvert}{\sqrt[2]{(x_{aj}-x_{ai})^2+(y_{aj}-y_{ai})^2+(z_{aj}-z_{ai})^2}}$
yOz	‖ *yOz*	$\frac{\lvert x_{aj}-x_{ai}\rvert}{\sqrt[2]{(x_{aj}-x_{ai})^2+(y_{aj}-y_{ai})^2+(z_{aj}-z_{ai})^2}}$	$\frac{\lvert x_{aj}-x_{ai}\rvert}{\sqrt[2]{(x_{aj}-x_{ai})^2+(y_{aj}-y_{ai})^2+(z_{aj}-z_{ai})^2}}$

The last step for calculating configuration factor of surface i includes summing of the particular configuration factors. These procedures are implemented as a computer code.

3 Verification of the method

The accuracy of developed method is checked by comparison with calculation results from analytical formulas, which are presented in Tab. 2 [4, pp. 372-373]. The other relations for calculating configuration factors for complex arrangement of surfaces have taken from [5, p. 199].

Table 2. The configuration factor for basic arrangement of isothermal surfaces [4].

Symbols	$F_{i,j}$
	$\frac{1}{\pi}\left[\frac{h^2}{ab}\ln\frac{(a^2+h^2)(b^2+h^2)}{(a^2+b^2+h^2)h^2}-\frac{2h}{a}arctg\frac{b}{h}-\frac{2h}{b}arctg\frac{a}{h}\right]+$ $+\frac{1}{\pi}\left[\frac{2\sqrt[2]{b^2+h^2}}{b}arctg\frac{a}{\sqrt[2]{b^2+h^2}}+\frac{2\sqrt[2]{a^2+h^2}}{a}arctg\frac{b}{\sqrt[2]{a^2+h^2}}\right]$
	$\frac{1}{\pi b}\left(b\,arctg\frac{a}{b}+c\,arctg\frac{a}{c}-\sqrt[2]{b^2+c^2}\,arctg\frac{a}{\sqrt[2]{b^2+c^2}}\right)+\frac{1}{4\pi ab}\ln(d_1d_2d_3)$ $d_1=\left[\frac{(a^2+b^2)(a^2+c^2)}{(a^2+b^2+c^2)a^2}\right]^{a^2}$ $d_2=\left[\frac{(a^2+b^2+c^2)b^2}{(a^2+b^2)(a^2+c^2)}\right]^{b^2}$ $d_3=\left[\frac{(a^2+b^2+c^2)c^2}{(a^2+c^2)(b^2+c^2)}\right]^{c^2}$

It is considered a typical room with floor heating system with a surface temperature gradient, shown in Fig. 2, with n=20 planes. The isothermal surfaces are divided on $m_i \times m_j$ cells.

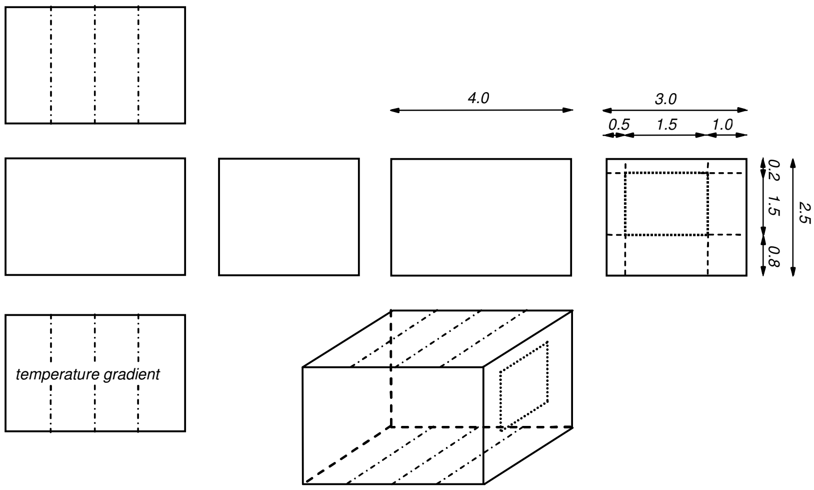


Fig. 2: The detailed shape of the tested room.

The number of sub-surfaces is ranging between 4 and 400 per one plane. The results of calculations are accurate, if the following relation (called summation rule) is equal to 1:

$$\sum_{i=1}^{n}\sum_{j=1}^{n}\boldsymbol{F}_{i,j}=1. \tag{4}$$

It is introduced $\boldsymbol{f_x}$ and $\boldsymbol{f_y}$ factors, which are described by below equations. The accuracy increases if these factors (average value calculated for the all 20 planes) tend to 0 (Fig. 3).

$$\boldsymbol{f_x}=\left|\sum_{i=1}^{n}\left(\sum_{j=1}^{n}\boldsymbol{F}_{i,j}\boldsymbol{n}^{-1}\right)\boldsymbol{n}^{-1}-1\right|,\ \boldsymbol{f_y}=\sum_{i=1}^{n}\left(\sum_{j=1}^{n}\left|\boldsymbol{F}_{i,j}^{a}-\boldsymbol{F}_{i,j}^{n}\right|\boldsymbol{n}^{-1}\right)\boldsymbol{n}^{-1}. \tag{5}$$

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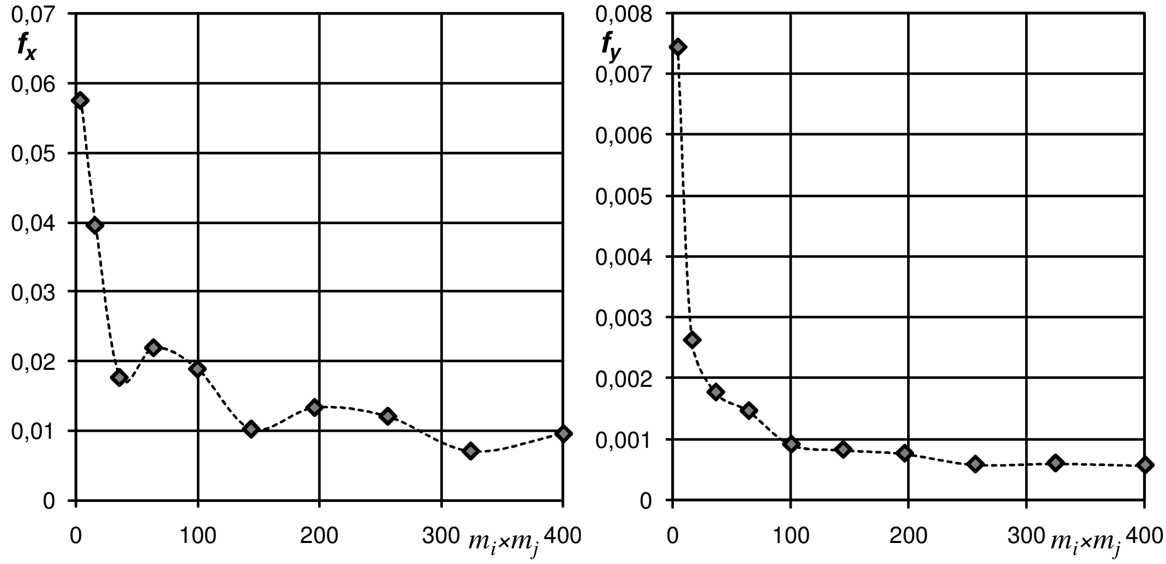

Fig. 3: The dependence of f_x and f_y factors on the number of cells.

From Fig. 3, we can be concluded, that the optimal number of sub-surfaces is 150. The total error of numerical procedure, for considering case, oscillates about 1%. Increasing the number of $m_i \times m_j$ cells above 150 is not recommended by the author for the typical cases.

4 Concluding remarks

The calculations, based on traditional equations, give a very good accuracy for determination of the configuration factors. But analytical method is complicated and time consuming for more complex geometry. The above disadvantage practically eliminates this procedure for analyzing 3-D objects with strong temperature gradients of surfaces.

The method, which is presented in the current paper, allows to automatically generate configuration factors for any enclosure geometry. We can obtain a good accuracy if the number of sub-surfaces is 150. The total error of this numerical estimation is equal about 1% for the optimal density of surface subdivision.

Acknowledgements

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METODA OBLICZANIA STOSUNKÓW KONFIGURACJI PŁASZCZYZN IZOTERMICZNYCH STOSOWANA W RADIACYJNEJ WYMIANIE CIEPŁA

W praktyce inżynierskiej mamy często do czynienia z pomieszczeniami charakteryzującymi się dużą asymetrią temperatury powierzchni przegród (np. w przypadku zastosowania ogrzewania podłogowego lub chłodzenie sufitowe, salonów wystawowych z dużym przeszkleniem). Chcąc w sposób dokładny dokonać analizy komfortu cieplnego oraz uwzględnić promieniowanie długofalowe w całkowitym bilansie ciepła, musimy obliczyć stosunki konfiguracji płaszczyzn izotermicznych zwanych również współczynnikami konfiguracji. Można to uczynić na podstawie wzorów lub nomogramów, gdy rozpatrujemy proste układy geometryczne. Natomiast zastosowanie metody analitycznej lub graficznej do analizy wymiany ciepła na drodze promieniowania długofalowego w pomieszczeniach o złożonym kształcie jest skomplikowane i bardzo pracochłonne. W referacie zaproponowano metodę obliczania wartości współczynnika konfiguracji bazującą na numerycznym rozwiązaniu tradycyjnego równania całkowego. W celu określenia dokładności zaprezentowanej metody porównano rezultaty symulacji komputerowej z wynikami dokładnymi otrzymanymi na podstawie obliczeń z wykorzystaniem wzorów analitycznych. Przy zastosowaniu optymalnej wartości gęstości podziału płaszczyzny maksymalny błąd numerycznego oszacowania wynosi około 1%.

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