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Performance-increasing method of a wireless system based on determination of time-frequency localization properties of OFDM signal

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A b s t r a c t . In this paper we have analyzed the properties of energy localization in time-frequency plane for rectangular pulse shape. The technique of space analysis of time-frequency localization (TFL) properties for pulse shapes was designed. We provide the method of OFDM signal synthesis with proper TFL based on the criteria of compactness and orthogonality. The comparative analysis of LTE system efficiency indexes for rectangular and compact pulses was conducted. The advantage of interference immunity and spectral efficiency for well-localized signals was proved.

K e y w o r d s : LTE, OFDM, Time-Frequency Localization, window function, Dolph-Chebyshev function.

INTRODUCTION

An important and integral step in the development of mobile networks is the research and designs in the field of improving methods of transmitting discrete information via radio channel [1]. Further development of global telecommunication technologies in this area is the development and implementation of fourth generation (4G) standard for mobile networks. 4G provide higher data rates and service qualities increase and simultaneously reduce the overall operating costs of telecommunications equipment. One technology designed to address these challenges in modern telecommunications is Long Term Evolution (LTE) technology [2], which uses orthogonal frequency division multiplexing – OFDM [3] as a radio access technology. OFDM meets the requirements of high-speed transmission of discrete information over the channels, since there is a number of features in the signal structure that allows to deal successfully with the specific barriers that arise in radio channels. A large number of orthogonal subcarrier signals transmitted in parallel and overlapping in the spectrum are used for OFDM synthesis. A large set of subcarrier frequency in the signal structure determines its properties such as resistance to frequency-selective fading and narrowband interference caused by multipath propagation [4]. The structure and properties of OFDM signal is determined as a linear combination of basic functions [5]. This basis can be obtained by uniform frequency shift rectangular pulse within a given bandwidth. The rectangular shape of the window function is not optimal in terms of resistance to interference, since localization formed on the basis of the basic functions in the frequency domain is the worst. For this reason, in such OFDM systems band radiation levels are too high. In [6] a number of measures to eliminate interference were suggested. However, these methods lead either to a very significant loss of spectral and energy efficiency, or to violation of orthogonality. The aim of this work is to study the possibilities of using localized window function as an alternative to a rectangular pulse.

ANALYSIS OF SIGNAL ENERGY DISTRIBUTION PROPERTIES IN TIME-FREQUENCY DOMAIN

Every signal *s(t)* is characterized by energy, the value of which is determined by the following [7]:

$$E_s = \int_{-\infty}^{\infty} \left| s(t) \right|^2 dt. \tag{1}$$

Signal energy distribution in the time-frequency plane can be represented by a rectangle with sides Δt to time axis and frequency by $\Delta\omega$, which contains 90% of its energy (Fig1).

Consider the signal $s_1(t)$, which is formed by scaling s (t) with a coefficient $\gamma < 1$:

$$s_1(t) = s\left(\frac{t}{\gamma}\right). \tag{2}$$

Then (1) can be written as:

$$E_{s_1} = \int_{-\infty}^{\infty} \left| s(\gamma \cdot t) \right|^2 dt = \frac{1}{\gamma} E_s. \tag{3}$$

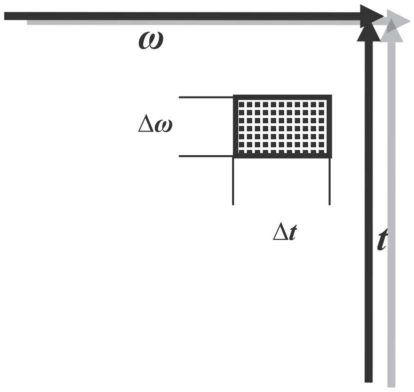

Fig 1. Characteristic of time-frequency signal localization *s(t)*

According to the scaling properties of the Fourier transform [8], the signal spectrum narrows:

$$\Delta\omega_1 = \gamma\Delta\omega . \quad (4)$$

Scaling influence on the signal position in the time-frequency plane is shown in Fig. 2.

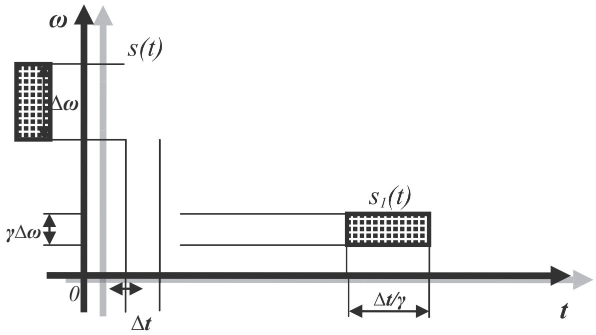

Fig. 2. Function *s(t)* position on the time-frequency plane by scaling with a coefficient γ<1

Significant loss of signal energy is caused by poor localization properties of basic functions. The effectiveness of representation in the time-frequency plane is determined by Dirac and Fourier basic functions. It is known that the Dirac δ-function is an ideal basis for time signal analysis so it has good time localization but uniform spectrum at all frequencies. Basic functions of Fourier analysis $e^{j\omega t}$ have good frequency localization and infinite length in the time domain.

The Fourier transformation location can be obtained by limiting the test signal using a moving time window. This signal is information packets sequence which are formed as a linear combination of Weyl-Heisenberg basic functions [9].

The result of this conversion is a function of two variables - window position τ and frequency ω:

$$F(\omega,\tau) = \int_{-\infty}^{\infty} w(t-\tau)\, s(t)\, e^{-j\omega t} dt. \quad (5)$$

As can be seen in (5), except frequency, the time is introduced as another option during the effective window Fourier transformation. The choice of window function *w(t)* determines the properties of the signal localization in frequency and time domain. As it was studied in [10], Gaussian function should be chosen to achieve the ideal location of transformation (5). This transformation is called Gabor transform.

THE METHOD OF OFDM-SIGNAL SYNTHESIS WITH OPTIMAL TIME-FREQUENCY LOCATION

During the Fourier transformation, modern wireless systems collide with the problem of uncertainty of frequeation, the widths of basis function in time and frequency domains are interrelated. The expansion of function in the time domain leads to its narrowing in frequency and vice-versa.

The regularity that connects these two values is called the uncertainty principle [11]. The product of another central moment function *s(t)* and its spectrum *F(ω)* are taken as a measure of energy concentration of any function of time and frequency:

$$\mu_t^2 = \int_{-\infty}^{\infty} t^2 \left|s(t)\right|^2 dt, \quad (6.a)$$

$$\mu_\omega^2 = \int_{-\infty}^{\infty} \omega^2 \left|F(\omega)\right|^2 d\omega . \quad (6.b)$$

Uncertainty principle states that in order to achieve the perfect energy location in the frequency and time domain, the following condition must be satisfied:

$$\mu_t^2 \mu_\omega^2 \geq \frac{\pi}{2} \text{ in case } \frac{ds}{dt} \leq -\frac{1}{\sqrt{t}}. \quad (7)$$

Inequalities (7) are valid only for Gaussian functions:

$$s(t) = \sqrt{\frac{\gamma}{\pi}} e^{-\gamma t^2}. \quad (8)$$

As it was proved [12], orthogonal basis cannot be synthesized on the basis of Gaussian functions; in practice, a rectangular window function is used instead. It has an extremely large effective width of the spectrum, which does not allow its location in the frequency-time domain. In a number of studies [13,14] the Isotropic Orthogonal Transform Algorithm (IOTA) is proposed to be used. Transformation function is called IOTA function. However, this algorithm is rather difficult to implement in the modern components, so it is advisable to study the simplest ways of forming spatially localized orthogonal basis.

There is a need to form an orthogonal basis in conjunction with compact prototype functions in order to create spatially localized OFDM-signal. Signal basis which is based on compact prototypes is called Gabor basis. Orthogonal Gabor basis (Weyl-Heisenberg basis) is formed by discrete shift window function *w(t)*, in time and frequency:

$$w_{m,n} = e^{jm\omega t} w(t - nT). \quad (9)$$

Basis (9) is called orthogonal if the scalar product of two arbitrary basis functions is zero. Orthogonality condition of the signals at different subcarrier frequencies is written as follows [15]:

$$\left\langle w_m(t), w_{m+1}(t) \right\rangle = \int_0^T w_m(t) w_{m+1}(t) dt = 0. \tag{10}$$

Any synthesized basis must satisfy condition (10) in order to generate OFDM. We propose optimality estimation criteria of time-frequency location based on formulas (6) and uncertainties (7):

$$\lim_{t \to \pm\infty} \frac{\pi}{2\mu_t^2 \mu_\omega^2} = 1. \tag{11}$$

If *w(t)* satisfies the condition (11) and is synthesized on the Weyl-Heisenberg basis, it does not contradict the condition (10), the window function is optimal for use in OFDM systems.

SIMULATION AND COMPARATIVE ANALYSIS OF LTE PERFORMANCE

The technique of 3D-analysis of frequency and time properties is designed in order to determine the location characteristics of window functions. The aim of this technique is to construct a two-dimensional correlation function surface and determine variation parameters of window functions. Two-dimensional correlation function is written [16]:

$$C(t,\omega) = \int_{\mathbb{R}} w\left(t + \frac{T}{2}\right) w^*\left(t - \frac{T}{2}\right) e^{j\omega t} dt, \tag{12}$$

Where: the symbol ‘*’ denotes the operation of complex conjugation.

Two-dimensional correlation function maximum depends on the agreement between *w(t)* and $w^*(t)$, so does the similarity of temporal and frequency pulse shape. The function is used as an indicator of similarity between window function and its frequency transformed version. Fig. 3 shows indicative surface of the two-dimensional correlation function for a rectangular pulse and its projection on the time-frequency plane.

We find peak energy pulse from the indicative surface (Fig.3,a) and its distribution in the time-frequency plane is shown in Fig. 2,b. For comparison, we consider Dolph-Chebyshev function [17], which minimizes norm of the side-lobes for a given main lobe width and satisfies conditions (10) and (11). The Dolph-Chebyshev function is defined as:

$$W(k) = -1^k \cdot \frac{\cos\left[N \cdot \arccos\left[ch\left(\frac{1}{N} ch^{-1}(10^\alpha)\right)\left(\cos\frac{\pi k}{N}\right)\right]\right]}{ch^{-1}\left[N \cdot ch\left[ch\left(\frac{1}{N} ch^{-1}(10^\alpha)\right)\right]\right]}. \tag{13}$$

The indicative surface of the two-dimensional correlation function for a function and its projection on the time-frequency plane is shown in Fig.4.

As noted above, localization properties improvement of OFDM signal allows improvement of wireless system properties, in particular spectral efficiency and robustness. Let us construct a spatial representation of the resource unit of LTE system for rectangular and spatially localized window functions. Figure 5 shows blocks of 3 subcarrier frequencies and three time intervals for the above functions.

As it can be seen from the Fig.5, energy location is much better for the compact window function than for the rectangular one. Let us depict the projection of a given sequence on the time-frequency plane for more accurate image representation (Fig.6). Draw a segment between the extreme points of two adjacent pulses, which correspond to 10% of the maximum level pulse length T_{int}. The resulting time interval shows the minimum delay of the next symbol, during which inter-symbol interference arises.

The greater the value of T_{int}, the more it allows for reducing the inter-symbol interference. Based on the projections, it was determined that for the Dolph-Chebyshev function this value is 2.8 times greater than for the rectangular function [18]. We can construct the OFDM signal without guard interval (cyclic prefix) after each symbol as their duration is approximately equal to 20% of the length of a symbol, so:

$$T_{\text{int}}^{IOTA} > T_{\text{int}}^{прям} + CP. \tag{14}$$

Thus we can predict that the use of compact window functions in OFDM systems theoretically should give gain in robustness compared to rectangular one. That is why appropriate studies were conducted on the basis of simulation model [19]. We have found comparative dependence of the relative occurrence of bit errors on the Eb/No ratio for M-QAM modulation (Fig. 7).

We can see wireless system gain with effective energy location in the time-frequency plane in Fig.7. As shown in Fig. 6 for a similar occurrence rate of bit errors, the system with a compact window function needs less value “signal / noise” than a system with a rectangular function. Calculate the robustness gain in such a system:

$$1 - \left(\left(\text{Eb}/\text{No}\right)_{16\text{QAM}(rect)} / \left(\text{Eb}/\text{No}\right)_{16\ \text{QAM}(TFL)}\right) \\ *100\% = 1 - (15.65/14.12)*100\% = 11\%. \tag{15}$$

If we assume that the ratio of occurrence of bit errors remains the same, we can provide the same quality of transmission under the worst radio channels, or increase the range of the radio channel. Besides robustness gain, effective signal energy localization can dispense with protective interval (cyclic prefix) in the OFDM, as seen from the (14). Let us compare spectral efficiency for systems with cyclic prefix and without it [20]. The formula for determining the spectral efficiency of OFDM/QAM systems with cyclic prefix is:

$$\frac{C}{\Delta F} = log_2 M\left(1 - \frac{T_g}{T_s}\right), \frac{bps}{Hz}, \qquad (16)$$

where: *Tg* - protective interval duration, *Ts* - OFDM symbol duration, *M* - number of positions for QAM modulation. According to QAM modulation, without protective formula (16) without protective interval can be written [10]:

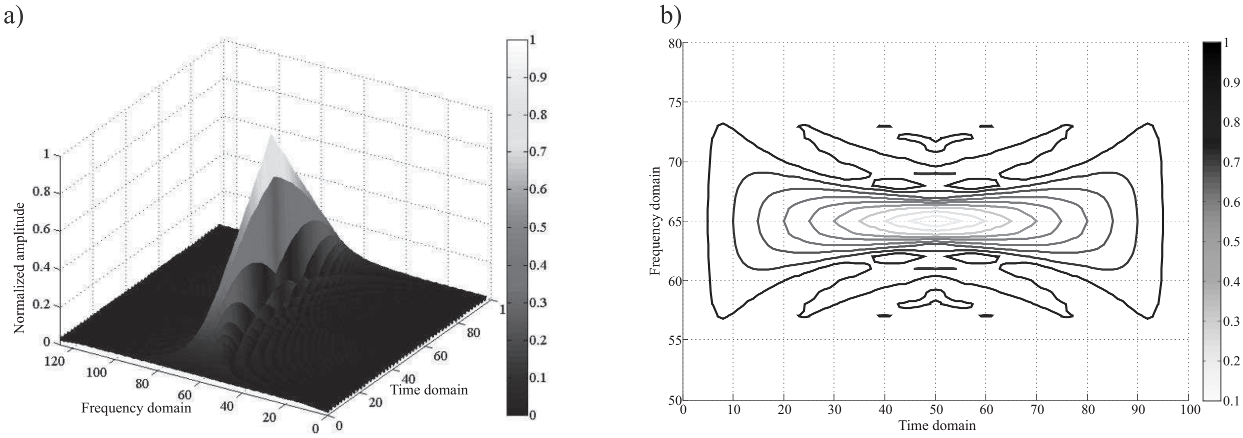

Fig. 3. Indicative surface of the two-dimensional correlation function for a rectangular pulse – a) and its projection on the time-frequency plane – b)

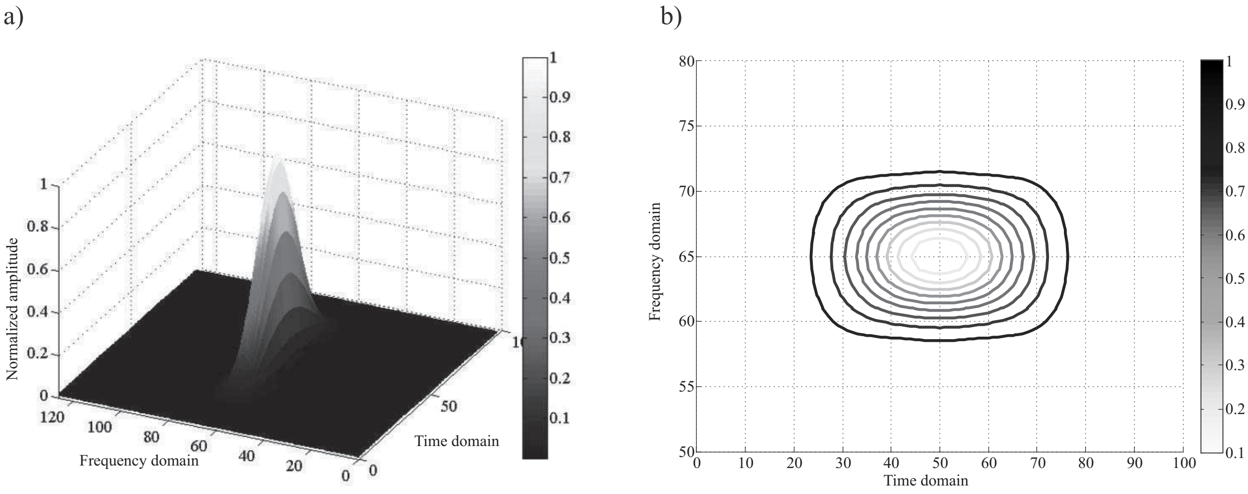

Fig. 4. Indicative surface of the two-dimensional correlation function Dolph-Chebyshev window – a) and its projection on the time-frequency plane - b)

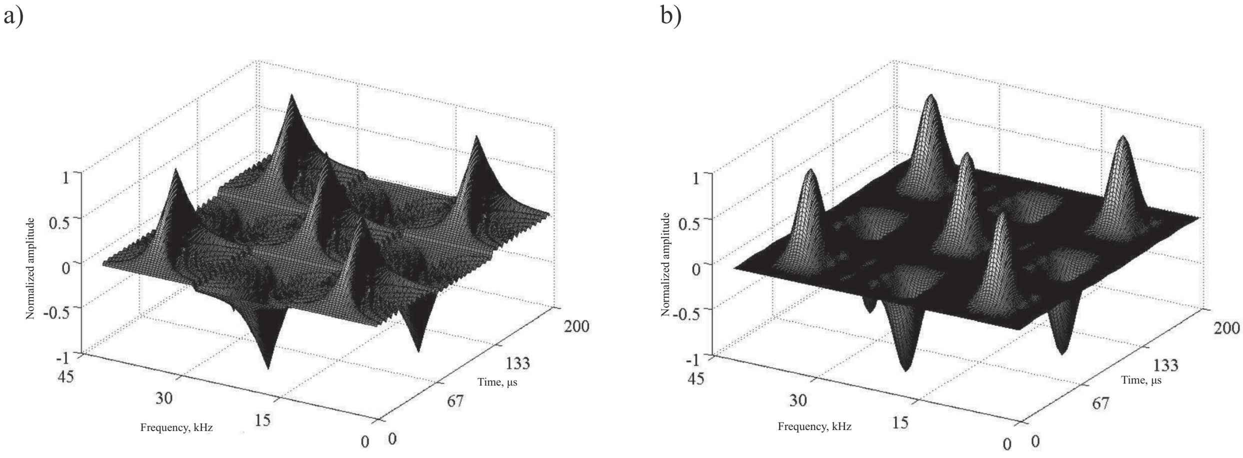

Fig. 5. Spatial representation of the sequence of OFDM symbols for rectangular function - a) and the Dolph-Chebyshev function- b)

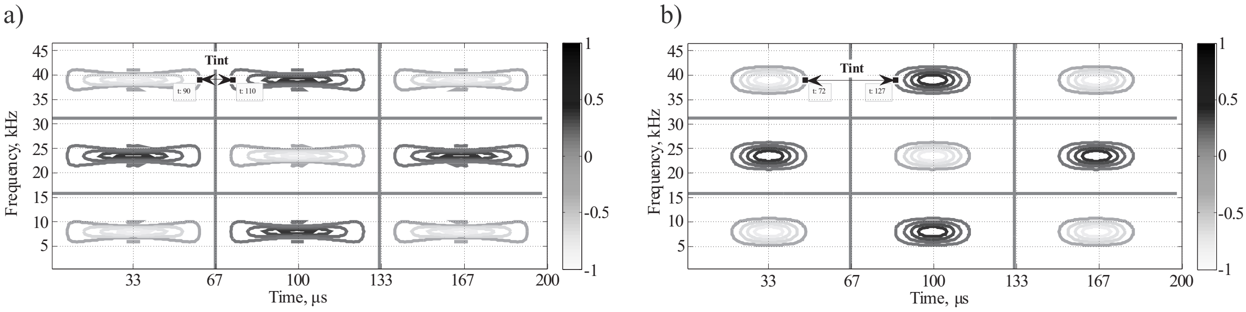

Fig. 6. Sequence of OFDM symbols projection for rectangular function - a) and the Dolph-Chebyshev function - b).

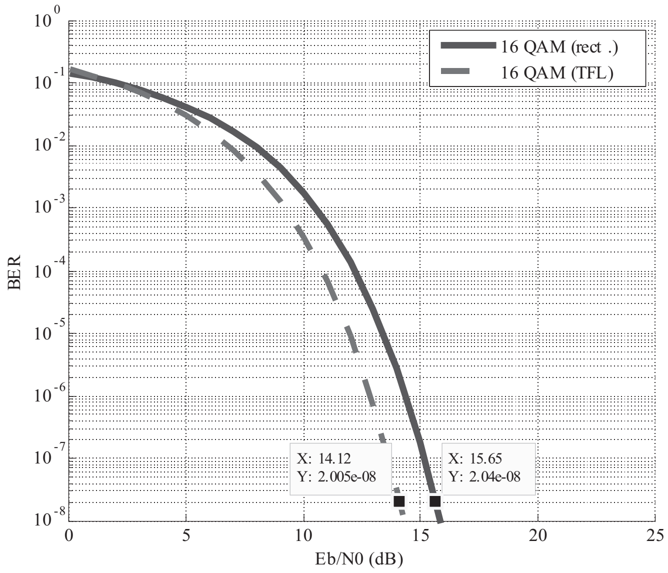

Fig. 7. Dependence of BER on Eb/No for 16 QAM modulations

Table 1. Calculation of peak transmission rates in downlink channel LTE

Bandwidth	1.4 MHz		3 MHz		5 MHz		10 MHz		15 MHz		20 MHz	
N subcarriers	72		180		300		600		900		1200	
Cyclic prefix	+	-	+	-	+	-	+	-	+	-	+	-
MIMO 2x2	10,3	12,7	24,1	30,3	40,4	50,5	80,6	102,7	120,5	153,6	161,8	206,1
MIMO 4x4	16,9	21,3	49,3	56,6	76,1	96,3	156,4	193,9	230,2	292,5	320,3	390,1

$$\frac{C}{\Delta F} = log_2 M, \frac{bps}{Hz}. \qquad (17)$$

Accordingly, information rate in systems with compact window functions will be:

$$C_{TFL} = C_{rect} \cdot \left[\frac{log_2 M}{log_2 M \left(1 - \frac{T_g}{T_s}\right)} \right], bps. \qquad (18)$$

The comparison of the peak transfer rate in downlink channel LTE [21] are presented in Table 1, for the case of 64 QAM modulation for all variations of the radio interface.

CONCLUSIONS

The studies have confirmed that the signal localization properties have a significant impact on the wireless systems performance. We showed the advantages of using window functions with optimal time-frequency localization on the example of Dolph-Chebyshev function. The method of spatial analysis of localization properties of window functions was proposed and the method of synthesis of OFDM-signal based on the criteria of compactness and orthogonality was developed.

The analysis of interference threats by building a spatial image resource block LTE and their projections on the plane was conducted. It is determined that the spacing between symbols using compact window functions is over the interval of a rectangular function, even when you add guard interval. It allows to provide the required BER value at a lower ratio of “Eb/No” without using guard time intervals after each symbol. So we obtain spectral efficiency gain of the system by eliminating the cyclic prefix.

Simulation results confirm the theoretical calculations. It was determined that the use of window functions with well TFL reduces the required value of the “Eb/No” ratio at 11%. Simulation radio interface LTE was conducted in order to determine the peak transfer rate in downlink channel. The results showed that the use of compact window function allows for improvement of spectral efficiency by 20%. Accordingly, using the same frequency band can increase the transmission speed in the downlink channel.

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