Dąbrowska Ewa

Kołowrocki Krzysztof
Gdynia Maritime University, Gdynia, Poland

Modelling, identification and prediction of oil spill domains at port and sea water areas

Keywords

port accident, sea accident, oil spill, oil spill drift, oil spill domain, stochastic modelling. statistical identification, stochastic prediction, Monte Carlo prediction

Abstract

Methods of oil spill domains determination are reviewed and a new method based on a probabilistic approach to the solution of this problem is recommended. A semi-Markov model of the process of changing hydro-meteorological conditions is constructed. To describe the oil spill domain central point position a two-dimensional stochastic process is used. Parametric equations of oil spill domain central point drift trend curve for different kinds of hydro-meteorological conditions are determined. The general model of oil spill domain determination for various hydro-meteorological conditions is proposed. Moreover, statistical methods of this general model unknown parameters estimation are proposed. These methods are presented in the form of algorithms giving successive steps which should be done to evaluate these unknown model parameters on the base of statistical data coming from experiments performed at the sea. Moreover, approximate expected stochastic prediction and Monte Carlo Simulation in real time prediction of the oil spill domain movement are proposed.

1. Introduction

One of the important duties in port activities and shipping is the prevention of oil release from port installations and ships and the spread of oil spills that often have dangerous consequences for port and sea water areas [1]-[5], [8], [12]-[14], [19]-[23]. Thus, as the first step, there is a need for methods of oil spill domain movement modelling based on determination of the oil spill central point drift curve determination and the oil spill domain probable placement at any moment after the accident that could be the tools for increasing the shipping safety and effective port and sea environment protection. Even if, the real trajectory of the oil spill central point and the oil spill domain movement are different from those determined by the proposed methods, they can be useful in the port and sea environment protection.

The oil spill central point drift trend, the oil spill domain shape and its random position distribution fixed for different hydro-meteorological conditions allow us to construct the model of determination of the area in which, with the in advance fixed probability, the oil spill domain is placed [2], [12]. This way, the area determined for oil spill allow us to mark the domain where the actions of mitigating the oil realise consequences should be performed. This approach is proposed to make oil releases at the sea prevention and mitigation actions more effective.

The general model of the oil spill domain determination based on the probabilistic approach may be practically applied in the oil spill consequences mitigation actions at the sea after its statistical identification. Statistical experiments should be performed according to the methods of the model unknown parameters estimation. Thus, the methods of evaluation of unknown parameters of the oil spill central point drift curve and the joint density function should be proposed. Moreover, the procedures of their practical evaluations should be done as well.

2. Modelling process of changing hydro-meteorological conditions at oil spill area

We denote by $A(t)$ the process of changing hydro-meteorological conditions at the sea water areas where the oil spill happened and distinguish m its states from the set $A = \{1,2,...,m\}$ in which it may stay at the moment t, $t \in <0,T>$, where $T > 0$. Further, we assume a semi-Markov model of the process $A(t)$ and denote by θ_{ij} its conditional sojourn time in the state i while its next transition will be done to the state j, where $i, j = 1,2,...,m$, $i \neq j$ [17]-[18], [24]-[25]. Under these assumptions, the process of changing hydro-meteorological conditions $A(t)$ is completely described by the following parameters [15]-[16]:

- the vector of probabilities of its initial states at the moment $t = 0$

$$[p(0)] = [p_1(0), p_2(0),..., p_m(0)], \quad (1)$$

- the matrix of probabilities of its transitions between the particular states

$$[p_{ij}] = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1m} \\ p_{21} & p_{22} & \cdots & p_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ p_{m1} & p_{m2} & \cdots & p_{mm} \end{bmatrix}, \ \forall\, i = 1,2,...,m,$$
$$p_{ii} = 0, \quad (2)$$

- the matrix of distribution functions of its conditional sojourn times θ_{ij} at the particular states

$$[W_{ij}(t)] = \begin{bmatrix} W_{11}(t) & W_{12}(t) & \cdots & W_{1m}(t) \\ W_{21}(t) & W_{22}(t) & \cdots & W_{2m}(t) \\ \vdots & \vdots & \ddots & \vdots \\ W_{m1}(t) & W_{m2}(t) & \cdots & W_{mm}(t) \end{bmatrix},$$
$$\forall\, i = 1,2,...,m, \quad W_{ii}(t) = 0, \quad (3)$$

- the expected values (mean values) of its conditional sojourn times θ_{ij} at the particular states

$$M_{ij} = E[\theta_{ij}] = \int_0^\infty t\, dW_{ij}(t), \ i, j = 1,2,...,m, \ i \neq j, \quad (4)$$

- the variances of its conditional sojourn times θ_{ij} at the particular states

$$V_{ij} = D[\theta_{ij}] = \int_0^\infty (t - E[\theta_{ij}])^2 dW_{ij}(t), \ i, j = 1,2,...,m, \ i \neq j. \quad (5)$$

Having the above parameters of the process of changing hydro-meteorological conditions $A(t)$, $t \in <0,T>$, $T > 0$, this process following characteristics can be determined [9]-[11]:

- the distribution functions of the unconditional sojourn time θ_i of the process of changing hydro-meteorological conditions at the particular states i, $i = 1,2,...,m$,

$$W_i(t) = \sum_{j=1}^{m} p_{ij} W_{ij}(t), \ i = 1,2,...,m; \quad (6)$$

- the mean values of the unconditional sojourn time θ_i of the process of changing hydro-meteorological conditions at the particular states i, $i = 1,2,...,m$,

$$M_i = E[\theta_i] = \sum_{j=1}^{m} p_{ij} E[\theta_{ij}], \ i = 1,2,...,m; \quad (7)$$

- the variances of the unconditional sojourn time θ_i of the process of changing hydro-meteorological conditions at the states i, $i = 1,2,...,m$,

$$V_i = D[\theta_i] = \sum_{j=1}^{m} p_{ij} D[\theta_{ij}], \ i = 1,2,...,m; \quad (8)$$

- the limit values of the process of changing hydro-meteorological conditions transient probabilities at the particular operation states

$$p_i(t) = P(W(t) = i), \ t \in <0,+\infty), \ i = 1,2,...,m, \quad (9)$$

given by

$$p_i = \lim_{t \to \infty} p_i(t) = \frac{\pi_i M_i}{\sum_{j=1}^{m} \pi_j M_j}, \ i = 1,2,...,m, \quad (10)$$

where M_i, $i = 1,2,...,m$, are given by (7), while the steady probabilities π_i of the vector $[\pi_i]_{1xm}$ satisfy the system of equations

$$\begin{cases} [\pi_i][p_{ij}] = [\pi_i] \\ \sum_{i=1}^{m} \pi_i = 1. \end{cases} \quad (11)$$

(In the case of a periodic process of changing hydro-meteorological conditions, the limit transient probabilities p_i, $i = 1,2,...,m$, at the particular states given by (10), are the long term proportions of this process sojourn times at the particular operation states i, i = 1,2,...,m);

- the total sojourn times $\hat{\theta}_i$ of the process of changing hydro-meteorological conditions at the particular operation states i, i = 1,2,...,m, during the fixed time θ, that have approximately normal distributions with the expected value given by

$$\hat{M}_i = E[\hat{\theta}_i] = p_i\theta, \ i = 1,2,...,m. \qquad (12)$$

where p_i, i = 1,2,...,m, are given by (10).

3. Modelling trend of oil spill central point drift

The approach and the results concerned with the survivor search domain at the sea restricted areas determination considered in [2] can be modified, developed and applied to oil spill drift trend determination.

First, for each fixed state k, k = 1,2,…,m, of the process $A(t)$ and time $t \in$ <0,T>, where T is time we are going to model the behaviour of the oil spill domain $\overline{D}^k(t)$, we define the central point of this oil spill domain as a point $(x^k(t), y^k(t))$, $t \in$ <0,T>, k = 1,2,…,m, on the plane Oxy that is the centre of the smallest circle, with the radius $r^k(t)$, $t \in$ <0,T>, k = 1,2,…,m, covering this domain (*Figure 1*). Thus, for the fixed oil spill domain $\overline{D}^k(t)$, we have

$$x^k(t) = \frac{x_1^k(t) + x_2^k(t)}{2}, \ y^k(t) = \frac{y_1^k(t) + y_2^k(t)}{2},$$
$$t \in <0,T>, \ k = 1,2,\ldots,m, \qquad (13)$$

where the $P_1(x_1^k(t), y_1^k(t))$ and $P_2(x_2^k(t), y_2^k(t))$ are the most distant points of the oil spill domain $\overline{D}^k(t)$, $t \in$ <0,T>, k = 1,2,…,m, and the radius $r^k(t)$, called the radius of the oil spill domain $\overline{D}^k(t)$, is given by

$$r^k(t) = \frac{1}{2}\sqrt{[x_1^k(t) - x_2^k(t)]^2 + [y_1^k(t) - y_2^k(t)]^2},$$
$$t \in <0,T>, \ k = 1,2,\ldots,m. \qquad (14)$$

Further, for each fixed state k, k = 1,2,…,m, of the process $A(t)$ and time t, $t \in$ <0,T>, we define a two-dimensional stochastic process

$$(X^k(t), Y^k(t)), \ t \in <0,T>,$$

such that

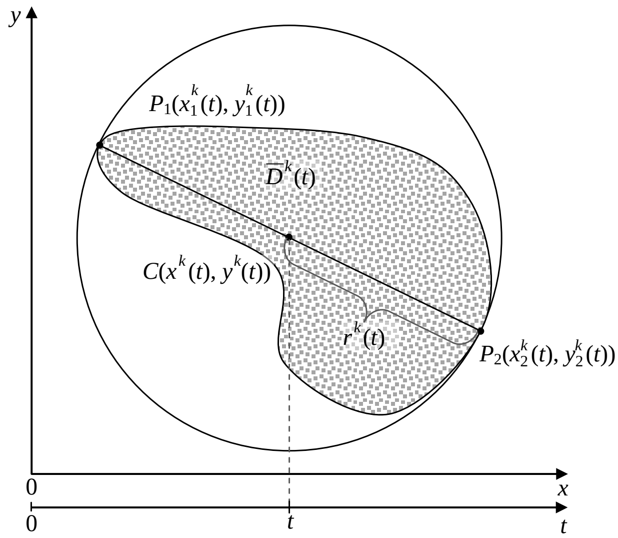

Figure 1. Interpretation of central point of oil spill definition

$$(X^k, Y^k): <0,T> \ \to R^2,$$

where $X^k(t)$, $Y^k(t)$ respectively are an abscissa and an ordinate of the plane *Oxy* point, in which the oil spill central point is placed at the moment t while the process $A(t)$, $t \in$ <0,T>, is at the state k. We set <u>deterministically</u> the central point of oil spill domain in the area in which an accident has happened and an oil release was placed in the water as the origin $O(0,0)$ of the co-ordinate system *Oxy*. The value of a parameter t at the moment of accident we assume equal to 0. It means that the process $(X^k(t), Y^k(t))$, is a random two-dimensional co-ordinate (a random position) of the oil spill central point after the time t from the accident moment and that at the accident moment t = 0 the oil spill central point is at the point $O(0,0)$, i.e.

$$(X^k(0), Y^k(0)) = (0,0).$$

After some time the central point of the oil spill starts its drift along a curve called a drift curve.

In further analysis, we assume that processes

$$(X^k(t), Y^k(t)), \ t \in <0,T>, \ k = 1,2,\ldots,m,$$

are two-dimensional normal processes

$$N(m_X^k(t), m_Y^k(t), \rho_{XY}^k(t), \sigma_X^k(t), \sigma_Y^k(t)), \ t \in <0,T>,$$
$$k = 1,2,\ldots,m,$$

with varying in time expected values

$m_X^k(t) = E[X^k(t)], \ m_Y^k(t) = E[Y^k(t)],$

standard deviations

$\sigma_X^k(t), \ \sigma_Y^k(t)$, $t \in <0,T>$, $k = 1,2,\ldots,m,$

and correlation coefficients $\rho_{XY}^k(t)$, i.e. with the joint density functions

$$\varphi_t^k(x,y) = \frac{1}{2\pi\sigma_X^k(t)\sigma_Y^k(t)\sqrt{1-(\rho_{XY}^k(t))^2}} \exp\{-\frac{1}{2(1-(\rho_{XY}^k(t))^2)}[\frac{(x-m_X^k(t))^2}{(\sigma_X^k(t))^2} - 2\rho_{XY}^k(t)\frac{(x-m_X^k(t))(y-m_Y^k(t))}{\sigma_X^k(t)\sigma_Y^k(t)} + \frac{(y-m_Y^k(t))^2}{(\sigma_Y^k(t))^2}]\},$$

$$(x,y) \in R^2, \ t \in <0,T>, \ k = 1,2,\ldots,m. \quad (15)$$

Thus, the points

$(m_X^k(t), m_Y^k(t))$, $t \in <0,T>$, $k = 1,2,\ldots,m,$

create a curve K^k called an oil spill central point drift trend (*Figure 2*) which may be described in the parametric form

$$K^k: \begin{cases} x^k = x^k(t) \\ y^k = y^k(t), t \in <0,T>. \end{cases} \quad (16)$$

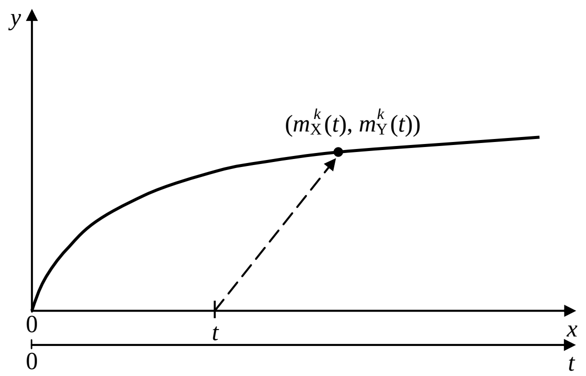

Figure 2. Oil spill central point drift trend

4. Modelling oil spill domain

4.1. Probabilistic approach

We are interested in finding the search domain $D^k(t)$, $t \in <0,T>$, $k = 1,2,\ldots,m,$ such that the central point of oil spill domain is placed in it with a fixed probability p. More exactly, we are looking for c such that

$$P((X^k(t), Y^k(t)) \in D^k(t)) = \iint_{D^k(t)} \varphi_t^k(x,y)dxdy = p,$$

$$t \in <0,T>, \ k = 1,2,\ldots,m, \quad (17)$$

where

$$D^k(t) = \{(x,y): \frac{1}{1-(\rho_{XY}^k(t))^2}[\frac{(x-m_X^k(t))^2}{(\sigma_X^k(t))^2} - 2\rho_{XY}^k(t)\frac{(x-m_X^k(t))(y-m_Y^k(t))}{\sigma_X^k(t)\sigma_Y^k(t)} + \frac{(y-m_Y^k(t))^2}{(\sigma_Y^k(t))^2}] \le c^2\}, \ t \in <0,T>,$$

$$k = 1,2,\ldots,m, \quad (18)$$

is the domain bounded by an ellipse being the projection on the plane $0xy$ of the curve rising as the result of intersection (*Figure 3*) of the density function surface

$$\pi_1^k = \{(x,y,z): z = \varphi_t^k(x,y), (x,y) \in R^2\}, \quad (19)$$

and the plane

$$\pi_2^k = \{(x,y,z): z = \frac{1}{2\pi\sigma_X^k(t)\sigma_Y^k(t)\sqrt{1-(\rho_{XY}^k(t))^2}}\exp[-\frac{1}{2}c^2],$$

$$(x,y) \in R^2\}, \ t \in <0,T>, \ k = 1,2,\ldots,m. \quad (20)$$

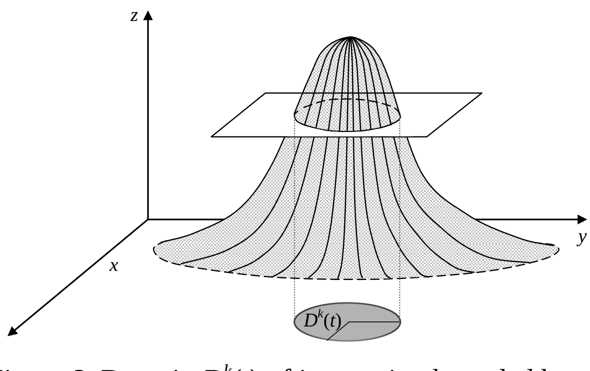

Figure 3. Domain $D^k(t)$ of integration bounded by an ellipse

The graph of the domain $D^k(t)$ is given in *Figure 4*.

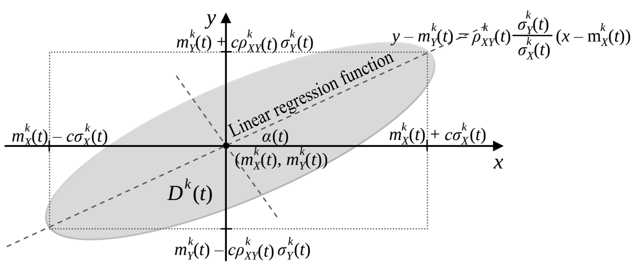

Figure 4. Domain $D^k(t)$ covering oil spill central point with probability p

Since

$$P((X^k(t), Y^k(t)) \in D^k(t)) = 1 - \exp[-\frac{1}{2}c^2],$$
$$t \in <0,T>, \ k = 1,2,\dots,m, \quad (21)$$

then for a fixed probability p, the equality

$$p = P((X^k(t), Y^k(t)) \in D^k), \ t \in <0,T>,$$
$$k = 1,2,\dots,m, \quad (22)$$

holds if $c^2 = -2\ln(1-p)$.

Thus, the domain in which at the moment t the central point of oil spill is placed with the fixed probability p is given by

$$D^k(t) = \{(x,y) : \frac{1}{1-(\rho_{XY}^k(t))^2}[\frac{(x-m_X^k(t))^2}{(\sigma_X^k(t))^2}$$
$$- 2\rho_{XY}^k(t)\frac{(x-m_X^k(t))(y-m_Y^k(t))}{\sigma_X^k(t)\sigma_Y^k(t)} + \frac{(y-m_Y^k(t))^2}{(\sigma_Y^k(t))^2}]$$
$$\le -2\ln(1-p)\}, \ t \in <0,T>, \ k = 1,2,\dots,m. \quad (23)$$

Considering the above and the assumed in Section 3 definition of the central point of oil spill, for each fixed state k, $k = 1,2,\dots,m$, of the process $A(t)$ and time $t \in <0,T>$, we define the oil spill domain

$$\overline{D}^k(t) = \{(x,y) : \frac{1}{1-(\rho_{XY}^k(t))^2}[\frac{(x-m_X^k(t))^2}{(\overline{\sigma}_X^k(t))^2}$$
$$- 2\rho_{XY}^k(t)\frac{(x-m_X^k(t))(y-m_Y^k(t))}{\overline{\sigma}_X^k(t)\overline{\sigma}_Y^k(t)} + \frac{(y-m_Y^k(t))^2}{(\overline{\sigma}_Y^k(t))^2}]$$
$$\le -2\ln(1-p)\}, \ t \in <0,T>, \ k = 1,2,\dots,m, \quad (24)$$

where

$$\overline{\sigma}_X^k(t) = \sigma_X^k(t) + r^k(t), \ \overline{\sigma}_Y^k(t) = \sigma_Y^k(t) + r^k(t),$$
$$t \in <0,T>, \ k = 1,2,\dots,m, \quad (25)$$

and

$$r^k(t), \ t \in <0,T>, \ k = 1,2,\dots,m, \quad (26)$$

is the radius of the oil spill domain $\overline{D}^k(t)$, $t \in <0,T>$, $k = 1,2,\dots,m$.

The graph of the oil spill domain $\overline{D}^k(t)$ is given in *Figure 5*.

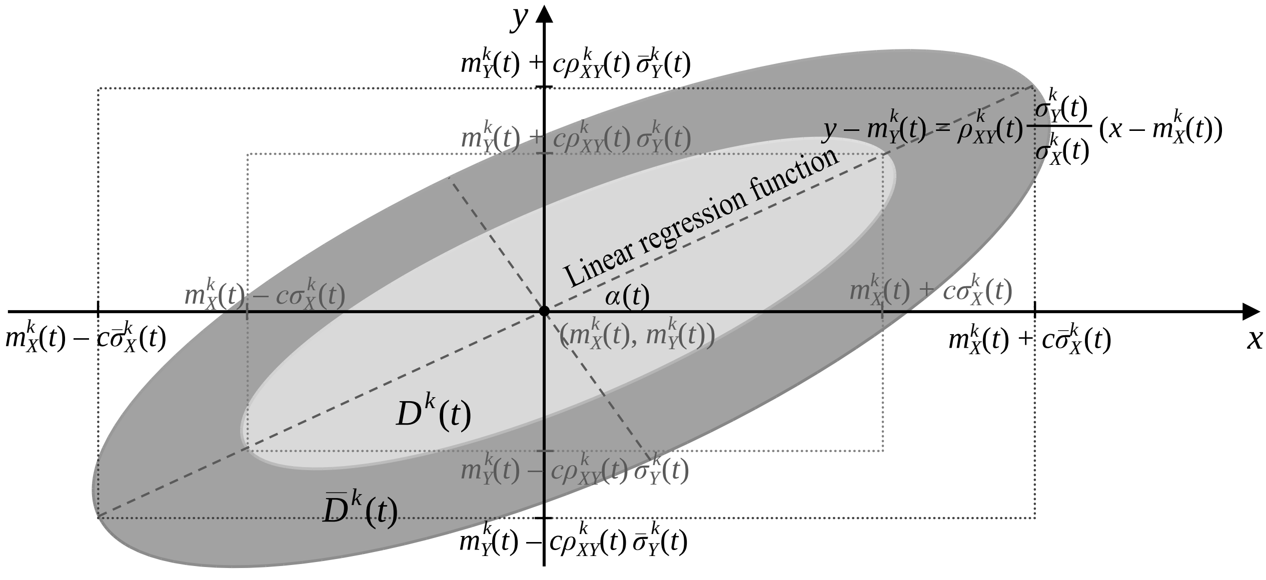

Figure 5. Oil spill domain $\overline{D}^k(t)$

4.2. Oil spill domain for fixed hydro-meteorological conditions

We suppose that the process $A(t)$ for all $t \in <0,T>$, is at the fixed state k, $k = 1,2,\dots,m$. Assuming a time step Δt and a number of steps s, $s \ge 1$, such that

$$(s-1)\Delta t < M_k \le s\Delta t, \ s\Delta t \le T, \quad (27)$$

where

$$M_k = E[\theta_k],\ k = 1,2...,m, \tag{28}$$

are the expected value of the process $A(t)$, $t \in <0,T>$, sojourn times θ_k, $k = 1,2...,m$, at the state k determined in Section 2, after multiple applying sequentially the procedure from Section 4.1, for

$$t = 1\Delta t, 2\Delta t, \cdots, s\Delta t, \tag{29}$$

we receive the following sequence of oil spill domains (*Figure 6*)

$$\overline{D}^k(\Delta t), \overline{D}^k(2\Delta t), \ldots, \overline{D}^k(s\Delta t). \tag{30}$$

Hence, the oil spill domain $\overline{D}^k$, $k = 1,2,...,m$, is described by the sum of determined domains of the sequence (30)

$$\overline{D}^k = \bigcup_{i=1}^{s} \overline{D}^k(i\Delta t) = \overline{D}^k(1\Delta t) \cup \overline{D}^k(2\Delta t) \cup ... \cup \overline{D}^k(s\Delta t),$$
$$k = 1,2,...,m, \tag{31}$$

and illustrated in *Figure 6*.

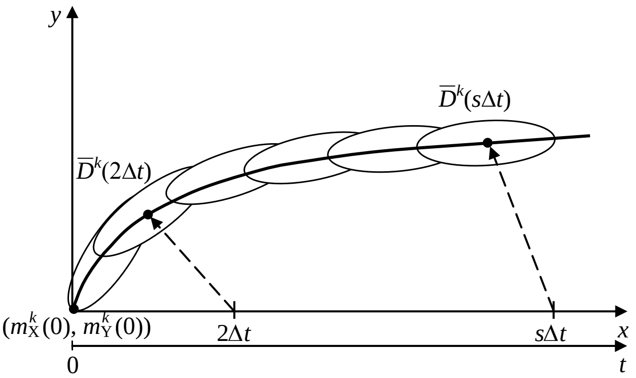

Figure 6. Oil spill domain for fixed hydro-meteorological conditions

Remark 1. The oil spill domain $\overline{D}^k$ defined by (31) and illustrated in Figure 6 is determined for constant radius $r^k(t) = r^k$, $t \in <0,T>$, $k = 1,2,\ldots,m$. If the radius is not constant, we define the sequence of domains

$$\overline{\overline{\boldsymbol{D}}}^k(b\Delta t) = \bigcup_{a=1}^{b} \overline{\overline{D}}^k(a\Delta t) = \overline{\overline{D}}^k(1\Delta t) \cup \overline{\overline{D}}^k(2\Delta t) \cup ...$$
$$\cup \overline{\overline{D}}^k(b\Delta t),\ b = 1,2,...,s,\ k = 1,2,...,m,$$

where

$$\overline{\overline{D}}^k(a\Delta t) := \overline{D}^k(a\Delta t),\ a = 1,2,...,b,\ b = 1,2,...,s,$$
$$k = 1,2,...,m,$$

defined by (24) with

$$\overline{\sigma}_X^k(a\Delta t) := \overline{\overline{\sigma}}_X^k(a\Delta t) = \sigma_X^k(a\Delta t) + r^k(\sum_{c=1}^{a} c\Delta t),$$
$$\overline{\sigma}_Y^k(a\Delta t) := \overline{\overline{\sigma}}_Y^k(a\Delta t) = \sigma_Y^k(a\Delta t) + r^k(\sum_{c=1}^{a} c\Delta t),$$
$$a = 1,2,...,b,\ b = 1,2,...,s,\ k = 1,2,\ldots,m.$$

4.3. Oil spill domain in varying hydro-meteorological conditions

We assume that the process of changing hydro-meteorological conditions in succession takes the states $k_1, k_2, ..., k_n$, $k_i \in \{1,2,...,m\}$, $i = 1,2,...,n$. For a fixed time step Δt, after multiple applying sequentially the procedure from Section 4.1:

- for

$$t = 1\Delta t, 2\Delta t, \cdots, s_1\Delta t, \tag{32}$$

at the process $A(t)$ state k_1;

- for

$$t = (s_1 + 1)\Delta t, (s_1 + 2)\Delta t, \cdots, s_2\Delta t, \tag{33}$$

at the process $A(t)$ state k_2;

…

- for

$$t = (s_{n-1} + 1)\Delta t, (s_{n-1} + 2)\Delta t, \cdots, s_n\Delta t, \tag{34}$$

at the process $A(t)$ state k_n;

we receive the following sequence of oil spill domains (*Figure 7*):

$$\overline{D}^{k_1}(1\Delta t), \overline{D}^{k_1}(2\Delta t), \ldots, \overline{D}^{k_1}(s_1\Delta t), \tag{35}$$

$$\overline{D}^{k_2}((s_1+1)\Delta t), \overline{D}^{k_2}((s_1+2)\Delta t), \ldots, \overline{D}^{k_2}(s_2\Delta t), \tag{36}$$

…

$$\overline{D}^{k_n}((s_{n-1}+1)\Delta t), \overline{D}^{k_n}((s_{n-1}+2)\Delta t), \ldots, \overline{D}^{k_n}(s_n\Delta t), \tag{37}$$

where s_i, $i = 1,2,...,n$, are such that

$$(s_i\text{-}1)\Delta t < \sum_{j=1}^{i} M_{k_j k_{j+1}} \le s_i\Delta t,\ i = 1,2,...,n,$$
$$s_n\Delta t \le T, \tag{38}$$

and

$$M_{k_j k_{j+1}} = E[\theta_{k_j k_{j+1}}],\ j = 1,2...,n\text{-}1, \tag{39}$$

are the expected value of the process $A(t)$, $t \in <0,T>$, conditional sojourn times $\theta_{k_j\, k_{j+1}}$, $j = 1,2...,n-1$ at the states k_j, upon the next state is k_{j+1}, $j = 1,2...,n-1$, k_j, k_{j+1}, $\in \{1,2,...,m\}$, $j = 1,2...,n-1$, determined in Section 2.

Hence, the oil spill domain $\overline{D}^{k_1,k_2,...,k_n}$, k_1, k_2, ..., $k_n \in \{1,2,...,m\}$, is described by the sum of determined domains of the sequences (35)-(37), given by

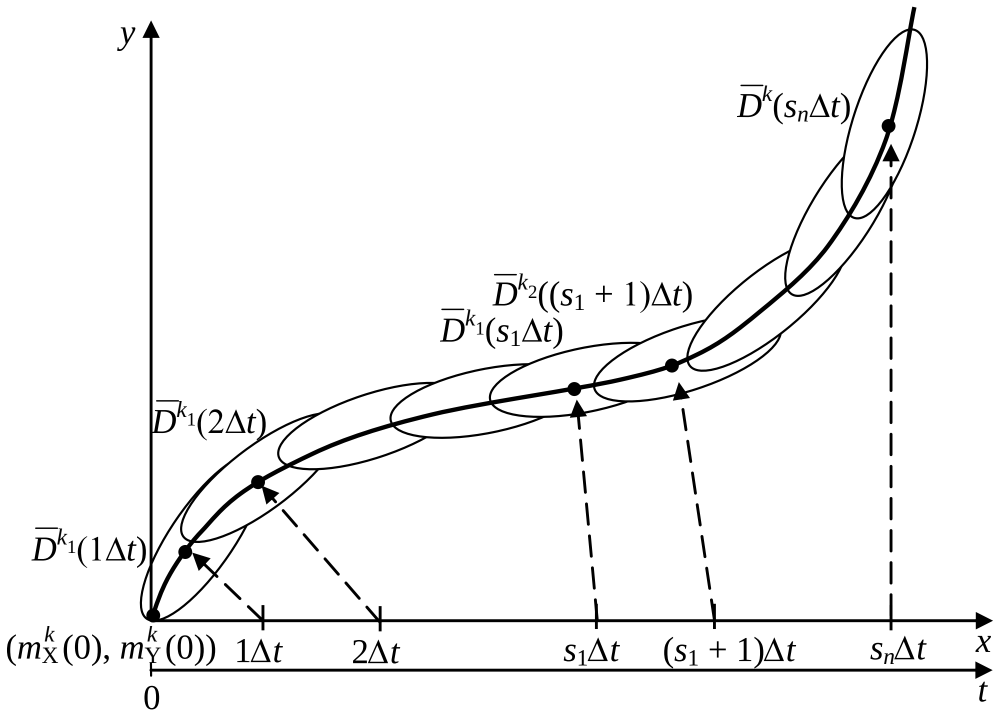

Figure 7. Oil spill domain for changing hydro-meteorological conditions

$$\overline{D}^{k_1,k_2,...,k_n} = \bigcup_{i=1}^{n}\bigcup_{j=1}^{s_i} \overline{D}^{k_i}\left((s_{i-1} + j)\Delta t\right)$$

$$= [\overline{D}^{k_1}(1\Delta t) \cup \overline{D}^{k_1}(2\Delta t) \cup ... \cup \overline{D}^{k_1}(s_1\Delta t)]$$

$$+ [\overline{D}^{k_2}\left((s_1+1)\Delta t\right) \cup \overline{D}^{k_2}\left((s_1+2)\Delta t\right) \cup ... \cup \overline{D}^{k_2}(s_2\Delta t)]$$

$$...+ [\overline{D}^{k_n}\left((s_{n-1}+1)\Delta t\right) \cup \overline{D}^{k_n}\left((s_{n-1}+2)\Delta t\right) \cup ... \cup \overline{D}^{k_n}(s_n\Delta t)],$$

$$k_1, k_2, ..., k_n \in \{1,2,...,m\}, \qquad (40)$$

and illustrated in *Figure 7*.

Remark 2. The oil spill domain $\overline{D}^{k_1,k_2,...,k_n}$ defined by (40) and illustrated in Figure 7 is determined for constant radiuses $r^{k_i}(t) = r^{k_i}$, $t \in <0,T>$, $k_i \in \{1,2,...,m\}$, $i = 1,2,...,n$. If the radiuses are not constant, we define the sequence of domains for each sate k_i, $k_i \in \{1,2,...,m\}$, $i = 1,2,...,n$, in a similar way as that described in Remark 1 in Section 4.2.

5. Monte Carlo simulation prediction of the oil spill domain

The general model of the process of changing hydro-meteorological conditions at oil spill area is proposed in Section 2 and defined by the initial probabilities at its states, the probabilities of transitions between these states and the distributions of the conditional sojourn times at these states. Moreover, its main characteristics, i.e. the mean values and variances of the unconditional sojourn times at particular states, the limit values of transient probabilities at particular states and the unconditional mean values of total sojourn times at the particular states for the fixed time T are determined. In this section, a Monte Carlo simulation approach is applied to the process of changing hydro-meteorological conditions at oil spill area prediction analysis and its characteristics determination. Finally, the way of oil spill domain in varying hydro-meteorological conditions Monte Carlo simulation prediction is proposed.

5.1. Generating process of changing hydro-meteorological conditions at oil spill area

We denote by $k_i = k_i(q)$, $i \in \{1,2,...,m\}$, the realization of the process' $A(t)$ initial state at the moment $t = 0$. Further, we select this initial state by generating realizations from the distribution defined by the vector $[p(0)]_{1\times m}$, according to the formula

$$k_i(q) = k_\xi, \quad \sum_{\xi=1}^{\psi} p_{\xi-1}(0) \le q < \sum_{\xi=1}^{\psi} p_\xi(0), \qquad (41)$$

$$\psi \in \{1,2,...,m\},$$

where q is a randomly generated number from the uniform distribution on the interval $\langle 0,1)$ and $p(0)$ for $\xi = 0$ equals 0.

After selecting the initial state k_i, $i \in \{1,2,...,m\}$, we can fix the next operation state of the process of changing hydro-meteorological conditions at oil spill area. We denote by $k_j = k_j(g)$, $j \in \{1,2,...,m\}$, $i \ne j$, the sequence of the realizations of the operation process' consecutive states generated from the distribution defined by the matrix $[p_{ij}]_{m\times m}$. Those realizations are generated for a fixed i, $i \in \{1,2,...,m\}$, according to the formula

$$k_j(g) = k_\xi, \quad \sum_{\xi=1}^{\psi} p_{i\,\xi-1} \le g < \sum_{\xi=1}^{\psi} p_{i\,\xi}, \qquad (42)$$

$$\psi \in \{1,2,...,m\},\ \psi \ne i,$$

where g is a randomly generated number from the uniform distribution on the interval $\langle 0,1)$ and $p_{i\,0} = 0$.

We can use several methods generating draws from a given probability distribution, e.g. an *inverse transform method*, a *Box-Muller transform method*, *Marsaglia and Tsang's* rejection sampling method [6]. The *inverse transform method* (also known as *inversion sampling method*) is convenient if it is possible to determine the inverse distribution function [7]. This section will consider only this one sampling method, but the other methods are discussed in [6]-[7].

We denote by $t_{ij}^{(\nu)}$, $i,j \in \{1,2,\ldots,m\}$, $i \ne j$, $\nu = 1,2,\ldots,n_{ij}$, the realization of the conditional sojourn times θ_{ij} of the process $A(t)$, $t \in <0,T>$, generated from the distribution function $W_{ij}(t)$, where n_{ij} is the number of those sojourn time realizations during the experiment time T. Thus, using the inverse transform method, the realization $t_{ij}^{(\nu)}$ is generated from

$$t_{ij} = W_{ij}^{-1}(h),\ i,j \in \{1,2,\ldots,m\},\ i \ne j, \qquad (43)$$

where $W_{ij}^{-1}(h)$ is the inverse function of the conditional distribution function $W_{ij}(t)$ and h is a randomly generated number from the interval $\langle 0,1)$;

Having the realizations $t_{ij}^{(\nu)}$ of the process $A(t)$, it is possible to determine approximately the entire sojourn time at the state k_i during the experiment time T, applying the formula

$$\tau_i = \sum_{\substack{j=1\\ j\ne i}}^{m} \sum_{\nu=1}^{n_{ij}} t_{ij}^{(\nu)},\ i \in \{1,2,...,m\}. \qquad (44)$$

Further, the limit transient probabilities defined by (10) can be approximately obtained using the formula

$$p_i = \tau_i / T,\ \ T = \sum_{i=1}^{m} \tau_i,\ i \in \{1,2,...,m\}. \qquad (45)$$

The mean values and variances of the process' unconditional sojourn times at the particular states are given respectively by

$$M_i = E[\theta_i] = \tau_i / n_i,\ \ n_i = \sum_{j=1}^{m} n_{ij},\ i \in \{1,2,...,m\}. \qquad (46)$$

$$V_i = E[(\Xi_b)^2] - (M_i)^2,\ i \in \{1,2,...,m\}. \qquad (47)$$

Other, possible to obtain, interesting characteristics of the process $A(t)$ are its total sojourn times $\hat{\theta}_i$ at the particular states k_i, during the fixed time θ. According to the formula (12) and [11], [15]-[18], the process' total sojourn time $\hat{\theta}_i$ at the state k_i, $i \in \{1,2,...,m\}$, for sufficiently large time has approximately normal distribution with the expected value given as follows

$$\hat{M}_i = E[\hat{\theta}_i] = p_i\theta,\ i \in \{1,2,...,m\}. \qquad (48)$$

5.2. General procedure of Monte Carlo simulation application to characteristics determination of a process of changing hydro-meteorological conditions at oil spill area

The procedure of estimating process' of changing hydro-meteorological conditions at oil spill area characteristics is formed as follows.

First, we have to draw a randomly generated number g from the uniform distribution on the interval $\langle 0,1)$. Next, we can select the initial state k_i, $i \in \{1,2,...,m\}$, according to (41). Further, we draw another randomly generated number g from the uniform distribution on the interval $\langle 0,1)$. For the fixed i, $i \in \{1,2,...,m\}$, we select the next state k_j, $j \in \{1,2,...,m\}$, $j \ne i$, according to (42). Subsequently, we draw a randomly generated number h from the uniform distribution on the interval $\langle 0,1)$. For the fixed i and j, we generate a realization t_{ij}, of the conditional sojourn time θ_{ij} from a given probability distribution, according to (43).

Further, we substitute $i := j$ and repeat drawing another randomly generated numbers g and h (selecting the states k_j and generating realizations t_{ij}, of the conditional sojourn time), until the sum τ_i of all generated realisations t_{ij} reach a fixed experiment time T. Consequently, we calculate the entire sojourn times at the states k_i, $i = 1,2,...,m$, according to (44), limit transient probabilities at the particular states, according to (45), unconditional mean sojourn times at the states, according to (46), variances at the states, according to (47) and mean values of the total sojourn times at the states during the fixed time θ, according to (48).

The general Monte Carlo simulation flowchart for determination of a process of changing hydro-meteorological conditions at oil spill area is illustrated in *Figure 8*.

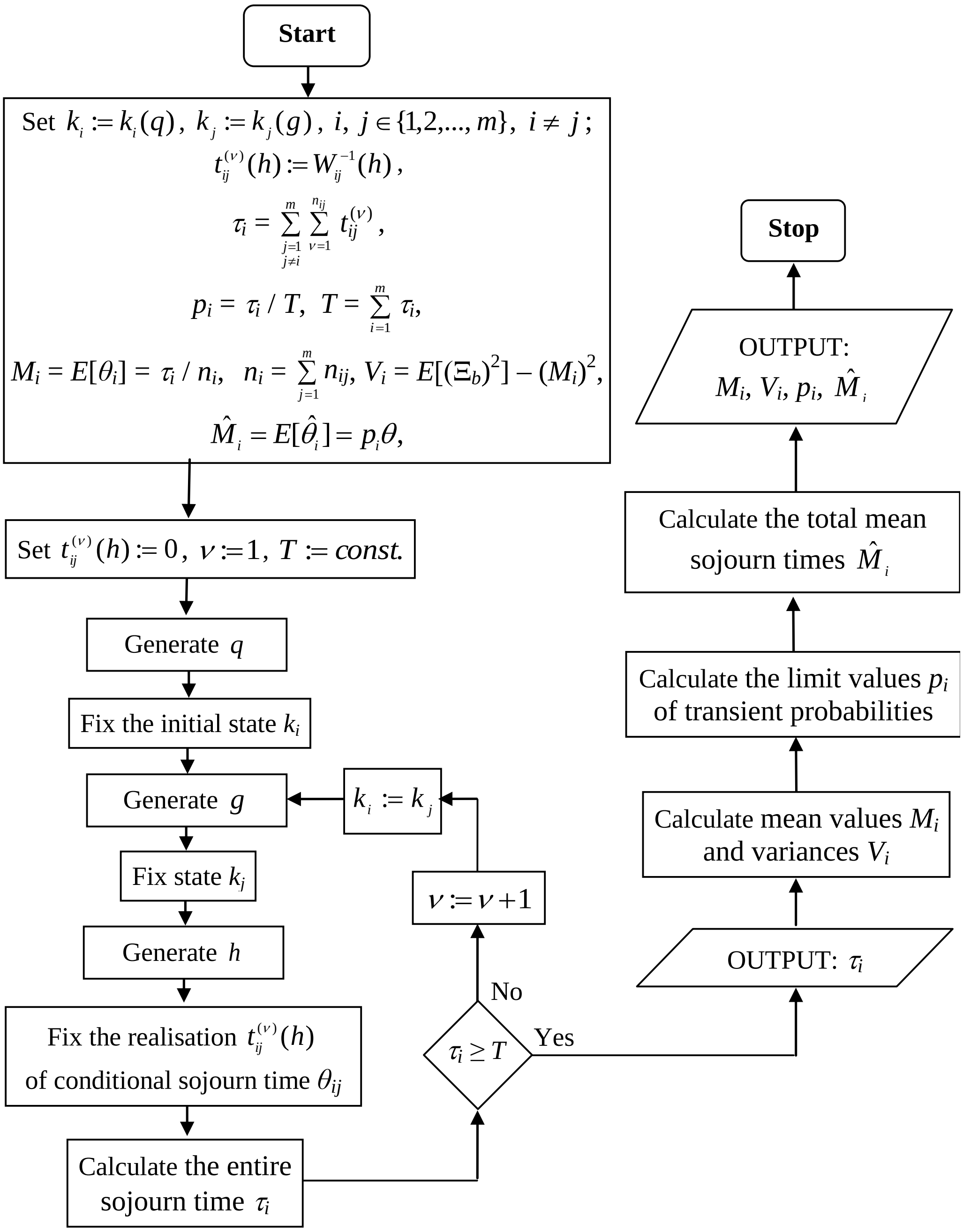

Figure 8. General Monte Carlo flowchart for prediction of a process of changing hydro-meteorological conditions at oil spill area

5.3. Oil spill domain in varying hydro-meteorological conditions Monte Carlo simulation prediction

Using the procedures of the process of changing hydro-meteorological conditions at oil spill area prediction described in Sections 5.1-5.2 and the

modified method of the domain of oil spill determination presented in Section 4.3 the Monte Carlo simulation oil spill domain prediction can be done.
The modified method of the domain of oil spill determination presented in Section 4.3 depends on changing the procedure (32)-(40) by replacing the conditions (38)-(40) by the conditions:
The s_i, i = 1,2,...,n, existing in (32)-(37) are such that

$$(s_i - 1)\Delta t < \sum_{j=1}^{i} t_{k_j k_{j+1}} = s_i \Delta t,\ i = 1,2,...,n,$$
$$s_n \Delta t \le T, \quad (49)$$

and

$$t_{k_j k_{j+1}},\ j = 1,2...,n-1, \quad (50)$$

are the realizations of the process $A(t)$, $t \in$ <0,T>, conditional sojourn times $\theta_{k_j k_{j+1}}$, j = 1,2...,n-1, at the states k_j, upon the next state is k_{j+1}, j = 1,2...,n-1, k_j, k_{j+1}, $\in \{1,2,...,m\}$, j = 1,2...,n-1, defined in Section 2.

6. Identification of process of changing hydro-meteorological conditions

In order to estimate parameters of the process of hydro-meteorological conditions changing $A(t)$, $t \in$ <0,T>, the following step should be done:

- to fix the number of states m of the process $A(t)$;

- to define the states of the process $A(t)$;

- to fix the vector

$$[n(0)] = [n_1(0), n_2(0), ..., n_m(0)], \quad (51)$$

of realisations $n_i(0)$, $i = 1,2,...,m,$ of the numbers of the process $A(t)$ transients at the particular states i at the initial moment t = 0;

- to fix the vector

$$[p(0)] = [p_1(0),\ p_2(0), \ldots,\ p_m(0)], \quad (52)$$

of realisations $p_i(0)$, $i = 1,2,...,m,$ of the initial probabilities of the process $A(t)$ transients in the particular states i at the moment t = 0, applying the formula

$$p_i(0) = \frac{n_i(0)}{n(0)} \text{ for } i = 1,2,..., m, \quad (53)$$

where

$$n(0) = \sum_{i=1}^{m} n_i(0), \quad (54)$$

is the total number of the process $A(t)$ realisations at t = 0;

- to fix the matrix

$$[n_{ij}] = \begin{bmatrix} n_{11}\ n_{12} \ldots n_{1m} \\ n_{21}\ n_{22} \ldots n_{2m} \\ \ldots \\ n_{m1}\ n_{m2} \ldots n_{mm} \end{bmatrix}, \quad (55)$$

of realisations n_{ij}, $i, j = 1,2,..., m,$ of the numbers of the process $A(t)$ transitions from the state i, $i = 1,2,..., m,$ into the state j, $j = 1,2,..., m,$ during the experiment time;

- to fix the matrix

$$[p_{ij}] = \begin{bmatrix} p_{11}\ p_{12} \ldots p_{1m} \\ p_{21}\ p_{22} \ldots p_{2m} \\ \ldots \\ p_{m1}\ p_{m2} \ldots p_{mm} \end{bmatrix}, \quad (56)$$

of realisations p_{ij}, $i, j = 1,2,..., m,$ of the transition probabilities of the process $A(t)$ from the state i into the state j, applying the formula

$$p_{ij} = \frac{n_{ij}}{n_i},\ \text{ for } i,j = 1,2,\ldots,m,\ i \ne j,\ p_{ii} = 0, \text{ for } i = 1,2,\ldots,m, \quad (57)$$

where

$$n_i = \sum_{j \ne i}^{m} n_{ij},\ i = 1,2,...,m, \quad (58)$$

is the realisation of the total number of the process $A(t)$ transitions from the state i, i = 1, 2, ..., m, during the experiment time;

- to collect the realizations

$$\theta_{ij}^{\nu},\quad \nu = 1, 2, ..., n_{ij}, \quad (59)$$

of the process $A(t)$ conditional lifetimes θ_{ij}, $i, j = 1, 2, ..., m, i \neq j$, at the states $i, i = 1, 2, ..., m$, while the next transition is the state $j, j = 1, 2, ..., m$;

- to formulate and to verify the hypotheses about the conditional distribution functions of the process $A(t)$ lifetimes

$$\theta_{ij},\ i, j = 1, 2, ..., m,\ i \neq j, \tag{60}$$

at the states $i, i = 1, 2, ..., m$, while the next transition is the state $j, j = 1, 2, ..., m$, on the base of their realisations θ_{ij}^{ν}, $\nu = 1, 2, ..., n_{ij}$.

7. Identification of oil spill domain drift trend

Having in disposal empirical mean positions of the oil spill domain central point

$$(m'^k_X(t_1), m'^k_Y(t_1)),\ (m'^k_X(t_2), m'^k_Y(t_2)), ...,$$
$$(m'^k_X(t_{N^k}), m'^k_Y(t_{N^k})),\ k = 1,2,\dots,m, \tag{61}$$

at the moments $t_1, t_2, ..., t_{N^k}$, it is possible to fix the equation of the central point of oil spill drift trend curve in the following parametric form

$$K^k: \begin{cases} x = m_X{}^k(t) = \alpha_0^k + \alpha_1^k t^1 + \dots + \alpha_a^k t^a \\ y = m_Y{}^k(t) = \beta_0^k + \beta_1^k t^1 + \dots + \beta_b^k t^b, \end{cases}$$
$$t \in < 0, T >, k = 1,2,..., m, \tag{62}$$

where a and b are natural numbers while

$$\alpha_j^k,\ j = 0, 1, ..., a,\ k = 1,2,\dots,m, \tag{63}$$

and

$$\beta_j^k,\ j = 0, 1, \dots, b,\ k = 1,2,\dots,m, \tag{64}$$

are unknown real coefficients of the assumed curve model.

According to the least squares method the unknown coefficients of the central point of oil spill drift trend are determined from the condition of the minimising the sum of squares of the differences between the realisations (61) and values calculated from the curve $\bar{K}^k$ equations (62), i.e. for each fixed k, $k = 1,2,\dots,m$, the following condition should be satisfied

$$\Delta^k = \sum_{\nu=1}^{N^k}[((m'_X{}^k(t_\nu) - m_X{}^k(t_\nu))^2 + (m_Y{}'^k(t_\nu) - m_Y{}^k(t_\nu))^2]$$
$$= \text{minimum},\ k = 1,2,\dots,m. \tag{65}$$

Hence and from the necessary condition for extremes existence for each fixed k, after substituting

$$\mathrm{A}_{uv}^k = \sum_{\nu=1}^{N^k} t_\nu^{u+v},\ \ u, v = 0,1,\dots,a,\ \mathrm{E}_u^k = \sum_{\nu=1}^{N^k} t_\nu^u m'_X{}^k(t_\nu),$$
$$u = 0,1,\dots,a,\ k = 1,2,\dots,m, \tag{66}$$

$$\mathrm{B}_{uv}^k = \sum_{\nu=1}^{N^k} t_\nu^{u+v},\ u, v = 0,1,\dots,b,\ F_u^k = \sum_{\nu=1}^{N^k} t_\nu^u m'_Y{}^k(t_\nu),$$
$$u = 0,1,\dots,b,\ k = 1,2,\dots,m, \tag{67}$$

and introducing the matrices

$$\mathrm{A}^k = \begin{bmatrix} \mathrm{A}_{00}^k & \mathrm{A}_{01}^k & . & . & . & \mathrm{A}_{0a}^k \\ \mathrm{A}_{10}^k & \mathrm{A}_{11}^k & . & . & . & \mathrm{A}_{1a}^k \\ . & . & . & . & . & . \\ \mathrm{A}_{a0}^k & \mathrm{A}_{a1}^k & . & . & . & \mathrm{A}_{aa}^k \end{bmatrix},$$

$$\alpha^k = \begin{bmatrix} \alpha_0^k \\ \alpha_1^k \\ \dots \\ \alpha_a^k \end{bmatrix},\ \mathrm{E}^k = \begin{bmatrix} \mathrm{E}_0^k \\ \mathrm{E}_1^k \\ \dots \\ \mathrm{E}_a^k \end{bmatrix},\ k = 1,2,\dots,m, \tag{68}$$

$$\mathrm{B}^k = \begin{bmatrix} \mathrm{B}_{00}^k & \mathrm{B}_{01}^k & . & . & . & \mathrm{B}_{0b}^k \\ \mathrm{B}_{10}^k & \mathrm{B}_{11}^k & . & . & . & \mathrm{B}_{1b}^k \\ . & . & . & . & . & . \\ \mathrm{B}_{b0}^k & \mathrm{B}_{b1}^k & . & . & . & \mathrm{B}_{bb}^k \end{bmatrix},$$

$$\beta^k = \begin{bmatrix} \beta_0^k \\ \beta_1^k \\ \dots \\ \beta_b^k \end{bmatrix},\ F^k = \begin{bmatrix} F_0^k \\ F_1^k \\ \dots \\ F_b^k \end{bmatrix},\ k = 1,2,\dots,m, \tag{69}$$

we get

$$\begin{cases} \mathrm{A}^k \alpha^k = \mathrm{E}^k \\ \mathrm{B}^k \beta^k = F^k,\quad k = 1,2,..., m. \end{cases} \tag{70}$$

Hence under the assumption that determinants of the matrices A^k and B^k, defined by (68)-(69), are different from zero we get the unknown coefficient of the curve models from the system of equations

$$\begin{cases} \alpha^k = (\mathrm{A}^k)^{-1}\mathrm{E}^k \\ \beta^k = (\mathrm{B}^k)^{-1}F^k, \quad k = 1,2,\ldots,m, \end{cases} \tag{72}$$

where $(\mathrm{A}^k)^{-1}$ and $(\mathrm{B}^k)^{-1}$ are the inverse matrices of the matrices A^k and B^k, defined by (68)-(69).

The equations of the remaining dependent on time t functional parameters of the join density functions $\varphi_t^k(x,y)$, $t \in$ <0,T>, k = 1,2,…,m, may be found in an analogous way.

Having in disposal empirical values of standard deviations of the oil spill domain central point positions

$$\sigma'_X{}^k(t_1),\ \sigma'_X{}^k(t_2),\ \ldots,\ \sigma'_X{}^k(t_{N^k}),$$
$$k = 1,2,\ldots,m, \tag{72}$$

at the moments $t_1, t_2, \ldots, t_{N^k}$ it is possible to evaluate the coefficients of functional standard deviations of random variables $X^k(t)$, $t \in$ <0,T>, k = 1,2,…,m, given by the following curve in the parametric form

$$\sigma_X{}^k(t) = \gamma_0^k + \gamma_1^k t^1 + \ldots + \gamma_c^k t^c,\ t \in \langle 0,T\rangle,$$
$$k = 1,2,\ldots,m, \tag{73}$$

determining them from the system of equations

$$\gamma^k = (A^k)^{-1}G^k,\ k = 1,2,\ldots m, \tag{74}$$

where

$$\gamma^k = \begin{bmatrix} \gamma_0^k \\ \gamma_1^k \\ \cdots \\ \gamma_c^k \end{bmatrix},\ G^k = \begin{bmatrix} G_0^k \\ G_1^k \\ \cdots \\ G_c^k \end{bmatrix},\ G_u^k = \sum_{\nu=1}^{N^k} t_\nu^u \sigma'_X{}^k(t_\nu),$$
$$u = 0,1,\ldots,c,\ k = 1,2,\ldots,m. \tag{75}$$

Having in disposal empirical values of standard deviations of the oil spill domain central point positions

$$\sigma'_Y{}^k(t_1),\ \sigma'_Y{}^k(t_2),\ \ldots,\ \sigma'_Y{}^k(t_{N^k}),\ k = 1,2,\ldots,m, \tag{76}$$

at the moments $t_1, t_2, \ldots, t_{N^k}$, it is possible to evaluate coefficients of the functional standard deviations of random variables $Y^k(t)$, k = 1,2,…,m, given by the following curve in the parametric form

$$\sigma_Y{}^k(t) = \eta_0^k + \eta_1^k t^1 + \ldots + \eta_d^k t^d,\ t \in \langle 0,T\rangle,$$
$$k = 1,2,\ldots,m, \tag{77}$$

determining them from the system of equations

$$\eta^k = (A^k)^{-1}H^k,\ k = 1,2,\ldots m, \tag{78}$$

where

$$\eta^k = \begin{bmatrix} \eta_0^k \\ \eta_1^k \\ \cdots \\ \eta_d^k \end{bmatrix},\ H^k = \begin{bmatrix} H_0^k \\ H_1^k \\ \cdots \\ H_d^k \end{bmatrix},\ H_u^k(u) = \sum_{\nu=1}^{N^k} t_\nu^u \sigma'_Y{}^k(t_\nu),$$
$$u = 0,1,\ldots,d,\ k = 1,2,\ldots,m, \tag{79}$$

Having in disposal empirical values of correlation coefficients

$$\rho'_{XY}{}^k(t_1),\ \rho'_{XY}{}^k(t_2),\ \ldots,\ \rho'_{XY}{}^k(t_{N^k}),$$
$$k = 1,2,\ldots,m, \tag{80}$$

at the moments $t_1, t_2, \ldots, t_{N^k}$, it is possible to evaluate coefficients of functional correlation coefficients of random variables $X^k(t)$ and $Y^k(t)$, k = 1,2,…,m, given by the following curve in the parametric form

$$\rho_{XY}{}^k(t) = \varsigma_0^k + \varsigma_1^k t^1 + \ldots + \varsigma_e^k t^e,\ t \in \langle 0,T\rangle, \tag{81}$$

determining them from the system of equations

$$\varsigma^k = (A^k)^{-1}I^k,\ k = 1,2,\ldots m, \tag{82}$$

where

$$\varsigma^k = \begin{bmatrix} \varsigma_0^k \\ \varsigma_1^k \\ \cdots \\ \varsigma_e^k \end{bmatrix},\ I^k = \begin{bmatrix} I_0^k \\ I_1^k \\ \cdots \\ I_e^k \end{bmatrix},\ I_u^k = \sum_{\nu=1}^{N^k} t_\nu^u \rho'_{XY}{}^k(t_\nu),$$
$$u = 0,1,\ldots,e,\ k = 1,2,\ldots,m. \tag{83}$$

Having in disposal empirical values of the radius of the oil spill domain

$$r'^k(t_1),\ r'^k(t_2),\ \ldots,\ r'^k(t_{N^k}),\ k = 1,2,\ldots,m, \tag{84}$$

at the moments $t_1, t_2, ..., t_{N^k}$, it is possible to evaluate the coefficients of the radius of the oil spill domain $D^k(t)$, $t \in$ <0,*T*>, k = 1,2,…,*m*, given by the following curve in the parametric form

$$r^k(t) = \zeta_0^k + \zeta_1^k t^1 + \ldots + \zeta_e^k t^f, \ t \in <0,T>, \quad (85)$$

determining them from the system of equations

$$\zeta^k = (A^k)^{-1} J^k, \ k = 1,2,\ldots m, \quad (86)$$

where

$$\zeta^k = \begin{bmatrix} \zeta_0^k \\ \zeta_1^k \\ \ldots \\ \zeta_f^k \end{bmatrix}, \ I^k = \begin{bmatrix} J_0^k \\ J_1^k \\ \ldots \\ J_f^k \end{bmatrix}, \ I_u^k = \sum_{\nu=1}^{N^k} t_\nu^u r'^k(t_\nu),$$
$$u = 0,1,\ldots, f, \ k = 1,2,\ldots,m. \quad (87)$$

Thus, to determine the evaluations of the central point of oil spill drift trend curves (62) and parameters of joint density functions $\varphi_t^k(x,y)$, $t \in$ <0,*T*>, k = 1,2,…*m*, given by (15), it is necessary to perform the following steps:

- to fix the numbers

$$N^k, \ k = 1,2,\ldots,m, \quad (88)$$

of observations of the central point of oil spill positions;

- to fix the moments of observations

$$t_1, t_2, \ldots, t_{N^k}, \ k = 1,2,\ldots,m, \quad (89)$$

of the central point of oil spill positions;

- to fix the numbers of the process $(X^k(t), Y^k(t))$, $t \in$ <0,*T*>, k = 1,2,…,*m*, realisations

$$n^k(1), \ n^k(2), \ \ldots, \ n^k(N^k), \ k = 1,2,\ldots,m, \quad (90)$$

at the moments $t_1, t_2, \ldots, t_{N^k}$, k = 1,2,…,*m*;

- to fix the central point of oil spill positions

$$(x'^k_1(t_\nu), y'^k_1(t_\nu)), \ (x'^k_2(t_\nu), y'^k_2(t_\nu)), \ \ldots,$$
$$(x'^k_{n^k(\nu)}(t_\nu), y'^k_{n^k(\nu)}(t_\nu)), \ k = 1,2,\ldots,m, \quad (91)$$

at each moment t_ν, $\nu = 1,2,...,N^k$, k = 1,2,…,*m*;

- to fix the most distant points of the oil spill domain

$$P_1 : (x'^k_{11}(t_\nu), y'^k_{11}(t_\nu)), \ (x'^k_{12}(t_\nu), y'^k_{12}(t_\nu)), \ \ldots,$$
$$(x'^k_{1n^k(\nu)}(t_\nu), y'^k_{1n^k(\nu)}(t_\nu)), \ k = 1,2,\ldots,m, \quad (92)$$

$$P_2 : (x'^k_{21}(t_\nu), y'^k_{21}(t_\nu)), \ (x'^k_{22}(t_\nu), y'^k_{22}(t_\nu)), \ \ldots,$$
$$(x'^k_{2n^k(\nu)}(t_\nu), y'^k_{2n^k(\nu)}(t_\nu)), \ k = 1,2,\ldots,m, \quad (93)$$

at each moment t_ν, $\nu = 1,2,...,N^k$, k = 1,2,…,*m*;

- to evaluate the central point of oil spill mean positions, according to the formulae

$$m'^k_X(t_\nu) = \frac{1}{n^k(\nu)} \sum_{w=1}^{n^k(\nu)} x'^k_w(t_\nu), \ \nu = 1.2,...,N^k,$$
$$k = 1,2,\ldots,m, \quad (94)$$

$$m'^k_Y(t_\nu) = \frac{1}{n^k(\nu)} \sum_{w=1}^{n^k(\nu)} y'^k_w(t_\nu), \ \nu = 1.2,...,N^k,$$
$$k = 1,2,\ldots,m; \quad (95)$$

- to evaluate the central point of oil spill position standard deviations according to the formulae

$$\sigma'^k_X(t_\nu) = \sqrt{\frac{1}{n^k(\nu)} \sum_{w=1}^{n^k(\nu)} [x'^k_w(t_\nu)]^2 - [m_X^k(t_\nu)]^2},$$
$$\nu = 1.2,...,N^k, \ k = 1,2,\ldots,m, \quad (96)$$

$$\sigma'^k_Y(t_\nu) = \sqrt{\frac{1}{n^k(\nu)} \sum_{w=1}^{n^k(\nu)} [y'^k_w(t_\nu)]^2 - [m_Y^k(t_\nu)]^2},$$
$$\nu = 1.2,...,N^k, \ k = 1,2,\ldots,m; \quad (97)$$

- to evaluate the central point of oil spill position correlation coefficients according to the formula

$$\rho'^k_{XY}(t_\nu) = \frac{\frac{1}{n^k(\nu)} \sum_{w=1}^{n^k(\nu)} x'^k_w(t_\nu) y'^k_w(t_\nu) - m_X^k(t_\nu) m_Y^k(t)}{\sigma_{X_i}^k(t_\nu) \sigma_{Y_i}^k(t_\nu)},$$
$$\nu = 1,2,...,N^k, \ k = 1,2,\ldots,m; \quad (98)$$

- to calculate the radius of the oil spill domain

$$r'^{k}(t_{\nu}) = \frac{1}{2n^{k}(\nu)}$$
$$\cdot \sum_{w=1}^{n^{k}(\nu)} \sqrt{[x'^{k}_{1w}(t_{\nu}) - x'^{k}_{2w}(t_{\nu})]^{2} + [y'^{k}_{1w}(t_{\nu}) - y'^{k}_{2w}(t_{\nu})]^{2}},$$
$$\nu = 1.2,\ldots,N^{k},\ k = 1,2,\ldots,m; \qquad (99)$$

- to find parametric forms (62) of the central point of oil spill drift trend and remaining parameters (73), (77), (81) and (85) of survivor position distributions, applying the formulae (71), (74), (78), (82) and (86) respectively.

8. Conclusions

The improvement of the methods of the oil spill domains determination is the main real possibility of the identifying the pollution size and the reduction of time of its consequences elimination. Therefore, it seems to be necessary to start with the new and better methods of the oil spill domains at port and sea waters determination for different hydro-meteorological conditions. The most important criterion of new methods should be the time of the oil spill consequences elimination minimising. One of the essential factors that could ensure these criteria fulfilment is the accuracy of methods of the oil spill domain determination. Those methods should be the basic parts of the general problem of different kinds of pollution identification, their consequences reduction and elimination at the port and sea water areas to elaborate a complete information system assisting people and objects in the protection against the hazardous contamination of the environment. One of the new efficient methods of more precise determination of the oil spill domains determination could be a probabilistic approach to this problem presented in this report and preliminarily in [12].

The oil spill domains determined for different hydro-meteorological conditions can be also done for other kind of spills, dangerous for the environment. The proposed probabilistic approach to oil spill domains determination would surely improve the efficiency of people activities in the environment protection.

A weak point of the method is the time and cost of the experiments necessary to perform at the port and sea water areas in order to identify statistically particular components of the proposed models. Especially experiments needed to evaluate drift trends and parameters of the central point of oil spill position distributions can consume much time and be costly as they have to be done for different kind of spills and different hydro-meteorological conditions in various areas. A strong and positive point of the method is the fact that the experiments for the fixed port and sea water areas and fixed hydro-meteorological conditions have to be done only once and the identified models may be used for all environment protection actions at this regions.

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