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A DECISION DIAGRAM BASED RELIABILITY EVALUATION METHOD FOR MULTIPLE PHASED-MISSION SYSTEMS

METODA OCENY NIEZAWODNOŚCI SYSTEMÓW WIELOFAZOWYCH W OPARCIU O DIAGRAMY DECYZYJNE

The multiple phased-mission system (MPMS) exists widely in practical engineering, such as aviation, spaceflight and navigation fields. Its distinct characteristic is that the system usually performs multiple missions and each mission consists of different phases. In this paper, we mainly focus on the reliability analysis for MPMS when the components have to accomplish different missions successively. A new modeling method is proposed for MPMS analysis based on the binary decision diagram (BDD) and multi-state multi-valued decision diagram (MMDD). Through this method, different phases of missions are combined with in the whole system by certain merging rules according to the operating time of a common component. Then, the system reliability can be calculated by the common calculation methods of decision diagrams by generating the through. Finally, two case studies are implemented to demonstrate the generation of BDD/MMDD models and the evaluation of system reliability. The experiment results verified the efficiency and accuracy of the proposed modeling methods.

Keywords: *multiple phased-mission systems, binary decision diagram, multi-state multi-valued decision diagram, reliability evaluation.*

Systemy wielofazowe (Multiple Phased-Mission Systems, MPMS), t.j. systemy o wielu zadaniach okresowych są powszechnie stosowane w praktyce inżynieryjnej, np. w lotnictwie, lotach kosmicznych czy nawigacji. Cechą wyróżniającą tego typu systemy jest to, że zazwyczaj wykonują one wiele zadań, z których każde składa się z różnych faz. Głównym tematem poniższej pracy jest analiza niezawodności MPMS dla przypadków, kiedy elementy składowe muszą wykonywać różne misje jedna po drugiej. W artykule zaproponowano nową metodę modelowania dla celów analizy MPMS opartą na koncepcji binarnego diagramu decyzyjnego (binary decision diagram, BDD) oraz wielostanowego wielowartościowego diagramu decyzyjnego (multi-state multi-valued decision diagram, MMDD). Metoda ta polega na łączeniu różnych faz misji w obrębie systemu za pomocą pewnych reguł łączenia wedle czasu pracy wspólnego elementu składowego. Pozwala to na obliczanie niezawodności systemu za pomocą powszechnie stosowanych metod diagramów decyzyjnych poprzez generowanie drzew błędów. W pracy zaprezentowano dwa studia przypadku, które pokazują, w jaki sposób generuje się modele BDD/MMDD oraz ocenia niezawodność systemu. Wyniki eksperymentów wykazały wydajność oraz trafność proponowanych metod modelowania.

Słowa kluczowe: *systemy wielofazowe, binarny diagram decyzyjny, wielostanowy wielowartościowy diagram decyzyjny, ocena niezawodności.*

Notations

a, b, c, A_r	the component in MPMS
r	the ID of a component
w	the total number of components in the system
x_{A_r}	state variable of component A_r
m,n	state of component
i, j	phase of the system
$x_{A_{ri}}$	state variable of component A_r in phase i
P,Q	mission of the system
P_i , Q_i	phases of the mission P and mission Q
t_{A_r}	the component A_r which works on the two missions' time nodes
G,H	Boolean functions
$\text{index}(x_{A_r})$	position of x_{A_r} in the propagation order of all BDD variables
F_i	logical expression of phase i

Acronyms

PMS	phased-mission system
MPMS	multiple phased-mission system
BDD	binary decision diagram
MFTA	multi-state fault tree analysis
MMDD	multi-state multi-valued decision diagram
MSS	multi-state system
DAG	directed acyclic graph
PDO	phase-dependent operation
ite	if-then-else

1. Introduction

Phased-mission systems (PMS) are very common in practical engineering, where the mission of system usually consists of multiple, consecutive, and non-overlapping phases in operation [12, 20, 21]. A simple example is that the phases of car-driving mission include start, acceleration, deceleration, and stop. During each phase, the system has to complete the specific task and may be subject to different stresses and environmental conditions as well as different reliability requirements [12]. Moreover, the system's functioning principle of different phases may change, and hence it is necessary to establish distinct models for each phase.

Accurate reliability analysis of PMS must consider the statistical dependencies of components across different phases, as well as the dynamics of system configurations, success criteria, and component behavior. In the previous study, researchers mainly focused on binary reliability models for PMS. Park and Yoo [11] introduced an iterative Lagrange technique to maximize the mission reliability of PMS by apportioning subsystem reliabilities according to multiple resource constraints. Dugan [2] proposed an automated analysis method of PMS based on the discrete-state continuous-time Markov model. Kim and Park [4] put forward three cases, whose phase durations are deterministic, random variables exponential distribution, to compute the mission reliability based on Markov model. Somani and Trivedi [13] proposed a Boolean algebraic method to analyze PMS reliability, and the failure criterion in each phase can be expressed as a fault tree. Ma and Trivedi [8] described an efficient Boolean algebraic algorithm which combines the fault trees of all the phases into a single fault tree with repeated events. Zang et al [24] established a method based on binary decision diagram (BDD) to analyze the reliability of PMS. Jung et al [3] proposed a BDD algorithm for coherent fault tree, where the truncated if-then-else (ite) connectives and subsuming could be performed in the progress of the BDD structure construction.

Recently, more and more researchers have been concentrated on multi-state systems (MSS) and multi-state PMS. Tang and Dugan [18] built the dependence-BDD for reliability analysis of PMS with multi-mode failures by applying dependence algebra. Xing and Dai [22] proposed a new modeling approach called multi-state multi-valued decision diagrams (MMDD) for the analysis of multi-state systems. Shrestha and Xing [14-16] introduced reliability analysis of multi-state PMS with unordered and ordered states, and used MMDD to analyze the importance of components. Levitin and Xing [5,6] introduced a recursive algorithm based on conditional probability and an efficient recursive formula based on the branch and bound method for reliability evaluation of non-repairable PMS. Xing and Amari [23] put forward an efficient method to evaluate the reliability of k-out-of-n systems with identical components subject to phased-mission requirements and imperfect fault coverage. Wang and Xing [19] established an algorithm for competing failure analysis in PMS with functional dependence in one of the phases. Zang and Bai [25] proposed a mathematical model for success probability analysis of PMS based on minimal path set and system state analysis methods. Mo and Xing [9,10] built a new analytical method based on multi-valued decision diagrams for reliability analysis of non-repairable PMS with multi-mode failures. Li and Tao [7] combined the Bayesian networks with event tree and fault tree analysis to analyze PMS based on conditional probability by giving expression of the phase-dependency.

Multiple phased-mission systems (MPMS) have been applied in a wide range of engineering fields, where a system consists of multiple missions. The state of the component at the end of a mission will be the beginning state of the same component in the next mission. In MPMS, each mission also consists of multiple, consecutive, and non-overlapping phases which are accomplished in sequence. For example, the operational process of landing gear involves two missions: take-off and landing. The take-off mission involves speed skating, lifting, and climbing phases. And the landing mission involves landing gear drop-down, level flight, drift down, and skating phases. The landing gear system needs to complete both two missions for success flight. Compared with PMS, the analysis of MPMS is more difficult because a component may work during two missions in sequence. In PMS, for the component working in different phases, all the phases can be merged as one by existing algorithms. But in the MPMS, it is usually assumed that a component have to work in two missions in sequence. System structure and the environmental conditions will make the state of components more complex. So we need generate some new phases for the common component which works on the two missions' time nodes, and then combine all the phases.

The remainder of this paper is organized as follows. Section 2 presents the basic concept and phase-dependent operation algorithm of BDD and MMDD respectively. Section 3 describes the reliability evaluation methods of MPMS based on BDD and MMDD. Two examples are illustrated in Section 4 to show the efficiency and accuracy of the proposed modeling methods. Section 5 gives conclusion and points out the future work.

2. Methodologies

2.1. Basic concept of BDD

BDD is a rooted, directed acyclic graph representation of a Boolean expression based on Shannon decomposition rule [1]. It has two sink nodes (outputs), labeled as '1' and '0', which represent a binary-state system being either operational or failed. Let $A_r (r = 1,2,...,w)$ be the component. Let w denote the total number of components in the system. The two states of component A_r represented by a Boolean variable, denoted by x_{A_r} . Each Boolean variable x_{A_r} can be represented using the if-then-else (ite) format as $ite(x_{A_r},1,0)$. In general, the (ite) format for expressing Boolean expressions F (representing the system state structure function) in variable x_{A_r} based on Shannon's decomposition is: $F = x_{A_r} \cdot F_{x_{A_r}=1} + \overline{x_{A_r}} \cdot F_{x_{A_r}=0} = ite(x_{A_r}, F_{x_{A_r}=1}, F_{x_{A_r}=0})$. In practical engineering, non-sink node usually corresponds to the component's state. By traversing the BDD's all paths with each path pointing to sink node '1', the probability of occurrence of the system can be calculated.

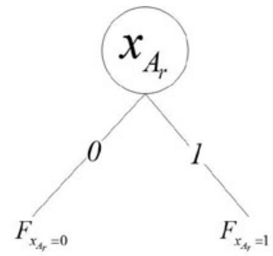


Fig. 1. Binary decision diagram

Each non-sink node in BDD usually has two outgoing edges, called 0-edge and 1-edge, respectively. Supposing there are two sub-BDD models G and H, then they could be encoded with the Boolean expression in the ite format, as:

$$G = x_{A_r} \cdot G_{x_{A_r}=1} + \overline{x_{A_r}} \cdot G_{x_{A_r}=0} = ite(x_{A_r}, G_{x_{A_r}=1}, G_{x_{A_r}=0}) = ite(x_{A_r}, G_1, G_0),$$

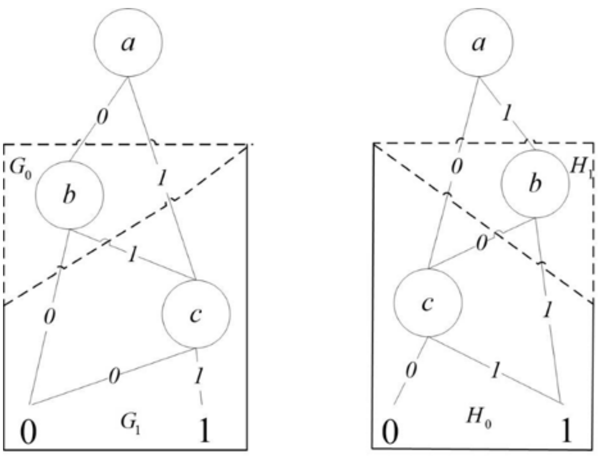
$$H = y_{A_r} \cdot H_{y_{A_r}=1} + \overline{y_{A_r}} \cdot H_{y_{A_r}=0} = ite(y_{A_r}, H_{y_{A_r}=1}, H_{y_{A_r}=0}) = ite(y_{A_r}, H_1, H_0).$$

Phased-mission systems (PMS) are systems in which multiple non-overlapping phases of tasks are accomplished in sequence for a successful mission. To combine different phases, the operation rules for combing two sub-BDD models G and H are as:

$$G \Diamond H = \mathrm{ite}(x_{A_r}, G_1, G_0) \Diamond \mathrm{ite}(y_{A_r}, H_1, H_0)$$
$$= \begin{cases} \mathrm{ite}(x_{A_r}, G_1 \Diamond H_1, G_0 \Diamond H_0) & \mathrm{index}(x_{A_r}) = \mathrm{index}(y_{A_r}) \\ \mathrm{ite}(x_{A_r}, G_1 \Diamond H, G_0 \Diamond H) & \mathrm{index}(x_{A_r}) < \mathrm{index}(y_{A_r}) \\ \mathrm{ite}(y_{A_r}, G \Diamond H_1, G \Diamond H_0) & \mathrm{index}(x_{A_r}) > \mathrm{index}(y_{A_r}) \end{cases}, \quad (1)$$

where the symbol $\Diamond$ represents a logic operation (AND or OR) between two sub-BDD models, the index( ) is assigned to each variable to indicate its position in the propagation order of all BDD variables. For example, $\text{index}(x_{A_r}) < \text{index}(y_{A_r})$ implies that the position of the y_{A_r} is behind the position of the x_{A_r} in the order.

To clearly explain the operation rules in equation (1), the detailed examples of two sub-BDD models G and H are shown in Fig.2[24]. For sub-BDD models G in Fig.2 (a), we know that $G = a \cdot G_1 + \bar{a} \cdot G_0$. Since $G_1 = c \cdot 1 + \bar{c} \cdot 0 = c$ and $G_0 = b \cdot G_1 + \bar{b} \cdot 0 = b \cdot c$, then we can get $G = a \cdot c + \bar{a} \cdot b \cdot c = a \cdot c + b \cdot c$. For sub-BDD models H in Fig.2 (b), we know that $H = a \cdot H_1 + \bar{a} \cdot H_0$. Since $H_0 = c \cdot 1 + \bar{c} \cdot 0 = c$ and $H_1 = b \cdot 1 + \bar{b} \cdot H_0 = b + \bar{b} \cdot c = b + c$, then we can get $H = a \cdot (b + c) + \bar{a} \cdot c = a \cdot b + a \cdot c + \bar{a} \cdot c = a \cdot b + c$.



(a) $G = a \cdot c + b \cdot c$ (b) $H = a \cdot b + c$

Fig. 2. The detailed examples of two sub-BDD models

If G and H are connected with "OR" operator, then the combination process of two sub-BDD models in Fig.2 is shown in Fig.3.

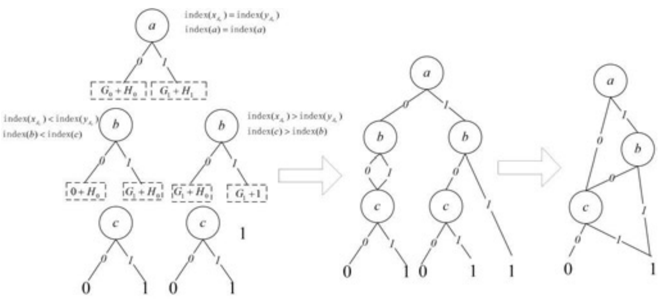


Fig. 3. Combination of two sub-BDD G and H with "OR" operation

Generally, the combination process of sub-BDD models could be concluded as follows:

(1) Compare the two sub-BDD models, it is clear that $\text{index}(a) = \text{index}(a)$. According to the rules of equation (1), we have $\text{ite}(a, G_1 \Diamond H_1, G_0 \Diamond H_0)$.

(2) Compare the G_0 and H_0, we know that $\text{index}(b) < \text{index}(c)$. According to the rules of equation (1), we can get $\text{ite}(b, G_1 \Diamond H_0, 0 \Diamond H_0)$.

(3) Compare the G_1 and H_1, we have $\text{index}(c) > \text{index}(b)$. Accroding to the rules of equation (1), we can get $\text{ite}(b, G_1 \Diamond 1, G_1 \Diamond H_0)$

Simplify the process is as follows:

(1) Because the results of $0 + H_0$ and $G_1 + H_0$ are the same, the node b 0-edge and 1-edge all point to the node c. So the node b at left can be removed.

(2) Because the 0-edge of node a is point to the same sub-tree as the 0-edge of node b at right. One of the two same sub-tree can be reduced.

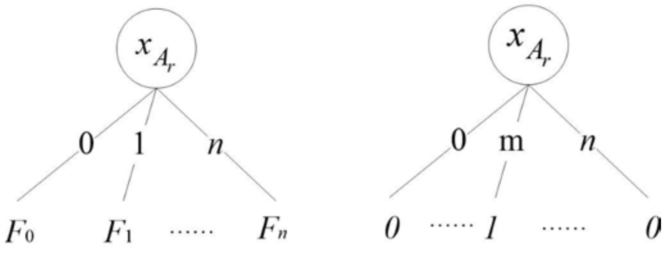
2.2. Basic concept of MMDD

The MMDD is a multi-state extended form of BDD [16]. It is a multi-valued logic structure for the natural representation of the MSS and is widely used in MSS reliability analysis [17]. The nodes MMDD are also divided into two types: sink nodes and non-sink nodes. MMDD only has two sink nodes, labeled '1' and '0', which indicate that the system is either in state '1' or in state '0'. Non-sink node in MMDD can have more than two edges where each edge represents a possible state of the components.

According to [17], Logical expression F in MMDD can be represented as follows:

$$
\begin{aligned}
F &= A_0 \cdot F_{x_{A_0}=0} + A_1 \cdot F_{x_{A_1}=1} + \ldots + A_n \cdot F_{x_{A_n}=n} \\
&= \text{case}(A_r, F_{x_{A_0}=0}, F_{x_{A_1}=1}, \ldots F_{x_{A_n}=n}) \\
&= \text{case}(A_r, F_0, F_1, \ldots F_n)
\end{aligned}
\quad , \qquad (2)
$$

Each non-sink node is associated with a multi valued state variable x_{A_r}, and $x_{A_r} = m$ means that the component A_r is in state m. The F_m can take one of two values: "1" or "0", indicating that F is in or not in state m(m=0,1,2,…,n) respectively. The non-sink note x_{A_r} has $(n+1)$ possible states and can be in a particular state at a specific time. So the logical expression of F has $(n+1)$ possible values. For example, when $x_{A_r} = m$, $F_0 = 0, F_1 = 0, \ldots, F_m = 1, \ldots, F_n = 0$. When sink node x_{A_r} is in state m, the value of F is '1'; otherwise the value is '0', as shown in Fig. 4.



(a) Conventional form of MMDD (b) Basic event of MMDD

Fig. 4. MMDD of node x_{A_r}.

2.3. Phase-dependent operation of decision diagram

In 1999, Zang et al [24] published a paper about the application of BDD for phased-mission systems and derived a special phased-dependent operation (PDO) as in equations (3) and (4). Let component A_r be used in both phase i and j, $i < j$. Using ite format, F_i, F_j express the Boolean expressions of F is in phase i and j, while $x_{A_{ri}}$ denoted the state variable of component A_r in phase i. Then we have: $G_1 = (F_i)_{x_{A_{ri}}=1}, G_0 = (F_i)_{x_{A_{ri}}=0}, H_1 = (F_j)_{x_{A_{rj}}=1}, H_0 = (F_j)_{x_{A_{rj}}=0}$.

$$
\begin{aligned}
F_i &= ite\left[x_{A_{ri}}, (F_i)_{x_{A_{ri}}=1}, (F_i)_{x_{A_{ri}}=0}\right] = ite\left(x_{A_{ri}}, G_1, G_0\right) \\
F_j &= ite\left[x_{A_{rj}}, (F_j)_{x_{A_{rj}}=1}, (F_j)_{x_{A_{rj}}=0}\right] = ite\left(x_{A_{rj}}, H_1, H_0\right)
\end{aligned}
\qquad (3)
$$

$$
F_i \Diamond F_j = \left\{
\begin{aligned}
& ite\left(x_{A_{ri}}, G_1 \Diamond F, G_0 \Diamond F_0\right) ForwardPDO \\
& ite\left(x_{A_{rj}}, G_1 \Diamond F_1, G \Diamond F_0\right) BackwardPDO
\end{aligned}
\right\}
\qquad (4)
$$

Because BDD modeling process depends on the order of the variables, there are two types of ordering methods: forward PDO and backward PDO. For the forward PDO, the variable order is the same as the phase order $(x_{A_{r1}}, x_{A_{r2}} ..., x_{A_{rs}})$. In the backward PDO, the variable order is the reverse of the phase order $(x_{A_{rs}}, x_{A_{r(s-1)}} ..., x_{A_{r1}})$. For combined operations, the same component belongs to two sub-BDDs but in different phases.

To deal with the MPMS problems, we derive a new MMDD operation for Phase Algebra in this paper based on the results of [24]. Similarly, two types of ordering methods are considered: forward PDO and backward PDO. Let component A appear in both phase i and j, $i < j$, then we have:

$$F_i = case\left[x_{A_{ri}}, (F_i)_{x_{A_{ri}}=0}, (F_i)_{x_{A_{ri}}=1} ... (F_i)_{x_{A_{ri}}=n}\right] = case\left(x_{A_{ri}}, G_0, G_1, ... G_n\right) = G$$
$$F_j = case\left[x_{A_{rj}}, (F_j)_{x_{A_{rj}}=0}, (F_j)_{x_{A_{rj}}=1} ... (F_j)_{x_{A_{rj}}=n}\right] = case\left(x_{A_{rj}}, H_0, H_1, ... H_n\right) = H, \quad (5)$$

$$F_i \Diamond F_j = \begin{cases} case\left(x_{A_{ri}}, G_0 \Diamond H_0, G_1 \Diamond H, G_2 \Diamond H, ... G_n \Diamond H\right) ForwardPDO \\ case\left(x_{A_{rj}}, G \Diamond H_0, G \Diamond H_1, G \Diamond H_2, ... G_n \Diamond H_n\right) BackwardPDO \end{cases}. \quad (6)$$

(i) For the forward PDO,

If x_{A_r} is failed in phase i and further it is irreparable, then it keeps failed in phase j, i.e, $x_{A_{ri}} = 0$ implies $x_{A_{rj}} = 0$.

$$\begin{aligned} F_i \Diamond F_j &= case\left(x_{A_{ri}}, G_0, G_1, ... G_n\right) \Diamond case\left(x_{A_{rj}}, H_0, H_1, ... H_n\right) \\ &= case\left[x_{A_{ri}}, (F_i \Diamond F_j)_{x_{A_{ri}}=0}, (F_i \Diamond F_j)_{x_{A_{ri}}=1} ... (F_i \Diamond F_j)_{x_{A_{ri}}=n}\right] \\ &= case\left[x_{A_{ri}}, (F_i)_{x_{A_{ri}}=0} \Diamond (F_j)_{x_{A_{ri}}=0}, (F_i)_{x_{A_{ri}}=m} \Diamond (F_j)_{x_{A_{ri}}=m}, (F_i)_{x_{A_{ri}}=n} \Diamond (F_j)_{x_{A_{ri}}=n}\right] \\ &= case\left(x_{A_{ri}}, G_0 \Diamond H_0, G_1 \Diamond H, G_2 \Diamond H, ... G_n \Diamond H\right) \end{aligned} \quad (7)$$

This derivation uses the equation:

$$(F_j)_{x_{A_{ri}}=m} = F_j = H \quad (8)$$

Since $x_{A_{ri}} = m$ is not relevant to H.

(ii) For the backward PDO,

If x_{A_r} is operational in phase j, then it must be operational in phase i, i.e, $x_{A_{rj}} = n$ implies $x_{A_{ri}} = n$.

$$\begin{aligned} F_i \Diamond F_j &= case\left(x_{A_{ri}}, G_0, G_1, ... G_n\right) \Diamond case\left(x_{A_{rj}}, H_0, H_1, ... H_n\right) \\ &= case\left[x_{A_{rj}}, (F_i \Diamond F_j)_{x_{A_{rj}}=0}, (F_i \Diamond F_j)_{x_{A_{rj}}=1} ... (F_i \Diamond F_j)_{x_{A_{rj}}=n}\right] \\ &= case\left[x_{A_{rj}}, (F_i)_{x_{A_{rj}}=0} \Diamond (F_j)_{x_{A_{rj}}=0}, (F_i)_{x_{A_{rj}}=m} \Diamond (F_j)_{x_{A_{rj}}=m}, (F_i)_{x_{A_{rj}}=n} \Diamond (F_j)_{x_{A_{rj}}=n}\right] \\ &= case\left(x_{A_{rj}}, G \Diamond H_0, G \Diamond H_1, G \Diamond H_2, ... G_n \Diamond H_n\right) \end{aligned} \quad (9)$$

This derivation uses the equation:

$$(F_i)_{x_{A_{rj}}=m} = F_i = G \quad (10)$$

Since $x_{A_{rj}} = m$ is not relevant to G.

In the forward PDO, index($x_{A_{ri}}$)<index($x_{A_{rj}}$) when phase $i < j$. The new MMDD node of the combined sub-MMDD is $x_{A_{ri}}$. The 0-edge of node $x_{A_{ri}}$ is generated, and the operation is applied to G_0 and H_0. In order to generate the m-edge of node $x_{A_{ri}}$ in a combined MMDD, the operation is applied to $G_m\,(m = 1, 2, ... n)$ and the other sub-MMDD model H together. In the backward PDO, index($x_{A_{rj}}$)<index($x_{A_{ri}}$) when phase $i < j$. The new MMDD node of the combined sub-MMDDs is $x_{A_{rj}}$. The n-edge of node $x_{A_{rj}}$ is generated, and the operation is applied to G_n and H_n. In order to generate the m-edge of node $x_{A_{rj}}$ in a combined MMDD, the operation is applied to $H_m\,(m = 0, 1, ... n-1)$ and the other sub-MMDD model G together.

3. Reliability evaluation methods based on decision diagram

The proposed BDD and MMDD methods for MPMS are established according to the following assumptions: (1) Each mission consists of multiple non-overlapping phases; (2) The component completes its missions in sequence. In binary-state MPMS, both a system and its components have two and only two states: functioning or failed, which are labeled as '1' and '0', respectively. In multi-state MPMS, a system has two and only two states, while the components may have more than two states.

In order to evaluate the reliability of MPMS by the BDD and MMDD methods, we need to build the system structure function for each phase. The logical expression (representing the failure of the system in the phase) can be obtained by complementing the system structure function. The graphical representation of the logic expressions in terms of logic AND/OR gates gives the fault tree model for each phase. Based on the generated fault tree models, the proposed BDD-based analysis and the MMDD-based analysis can be performed in the following four steps:

Step 1: New Phase Generation. One component participates in multiple missions consecutively. We can see that a new system consists of two missions and there's a common component which works on two missions sequentially. The new system is divided into many phases by the common component which works on the two missions' time nodes.

Step 2: Single-Mission BDD/MMDD Generation. Traditional method can be used for the generation of BDD model for each phase. In particular, equation (1) is applied to generate BDD based on the fault tree. The MMDD model is generated according to equation (2).

Step 3: Multiple Missions of BDD/MMDD Merged for the Same Phase. Based on the result from the Step 1, we need to merge the two missions of BDD/MMDD in the same phase. Each phase of the BDD/MMDD is generated by performing the logic OR operation of the single phase BDDs/MMDDs generated in Step 2. In a binary-system, equation (4) is applied when operation is performed on two variables of different elements. In a multi-state system, equation (6) is applied when operation is performed on two variables of different elements.

Step 4: Generation of BDD/MMDD for MPMS. In this step, the entire MPMS is generated by performing the logic OR operation on all the merged BDD/MMDDs generated in Step 3. In a binary-system, equation (4) is applied when operation is performed on two nodes which belong to the same component but in different phases. In a multi-state system, equation (6) is applied when operation is performed on two nodes that belong to the same component but in different phases.

4. Illustrative examples

4.1 BDD example

An example is presented to illustrate the application of modeling method based on BDD for a system with one component being engaged in two missions: mission P and mission Q. Mission P needs two components, A_1 and A_2. Mission Q needs three components, A_1, A_3, and A_4. During different time periods, component A_1 participates in mission P and mission Q.

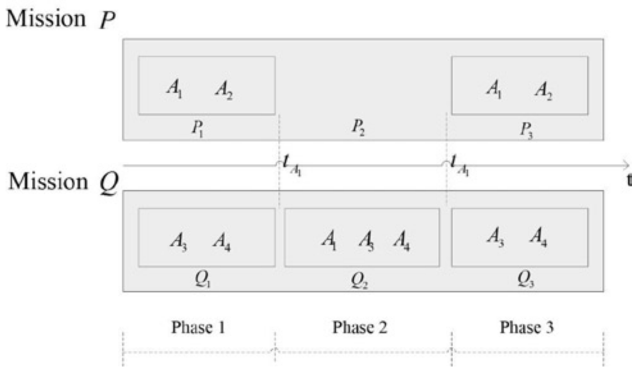


Fig. 5. System structure of BDD example

Step 1: In the system consisting of mission P and mission Q, the common component A_1 works in both missions sequentially. The new system is divided into three phases by the common component A_1 which works on the two missions' time nodes. The P_i and Q_i denoted the phases of the mission P and mission Q. The first phase consists of P_1 and Q_1. The second phase consists of Q_2. The third phase consists of P_3 and Q_3.

The input parameters of each component are shown in Table 1, which shows the computed conditional reliability for each component at each phase, and all the components fail exponentially with constant failure rate.

Table 1. Input parameters about each component.

component	Phase 1	Phase 2	Phase 3
A_1	0.968507	0.980199	0.973215
A_2	0.984127	0.980172	0.965432
A_3	0.974321	0.983543	0.981240
A_4	0.995432	0.984201	0.971205

Step 2: A single-mission single-phase BDD is generated. In a binary-state system, both the system and its components have only two states: '1' and '0', which represents the binary-state system and its components' state: either operational or failed.

In mission P of the phase 1, there are two components: A_1 and A_2. Each component has two states (0, 1). When A_1 and A_2 are in state '1', the system is normal. Fig. 6 shows the BDD model about P_1, where A_{ri} denoted the component A_r in phase i.

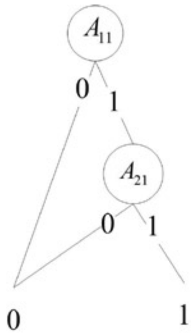


Fig. 6. The BDD model of the phase 1 of mission P

In mission Q of the phase 1, there are two components: A_3 and A_4. Each component has two states (0, 1). When both A_3 and A_4 are in state '1', the system is normal. Fig. 7 shows the BDD model about Q_1.

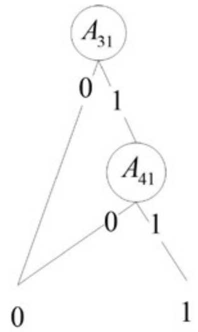


Fig. 7. The BDD model of the phase 1 of the mission Q

In mission Q of the phase 2, three are three components: A_1, A_3, A_4 and each component has two states (0, 1). When A_1 is in state '0', A_4 in state '1', the system is normal. When A_1 is in state '1', A_3 in state '0', A_4 in state '1', the system is normal. When A_1 is in state '1', A_3 in the state '1', the system is also normal. Fig. 8 shows the BDD model about Q_2.

In mission P of the phase 3, there are two components: A_1 and A_2, and each component has two states (0, 1) . When A_1 is in state '1'; or A_1 in state '0', A_2 in state '1', the system is normal. Fig. 9 shows the BDD model about P_3.

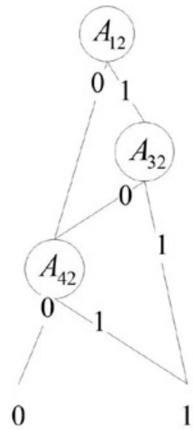


Fig. 8. The BDD model of the phase 2 of the mission Q.

In mission Q of the phase 3, there are two components: A_3 and A_4, and each component has two states (0, 1). When both component A_3 and A_4 are in state '1', the system is normal. Fig. 10 shows BDD model about Q_3.

Step 3: BDDs of two sub-missions merged for the same phase. By applying equation (4) with the order of $index(A_{11}) < index(A_{21}) < index(A_{31}) < index(A_{41}) < index(A_{12}) < index(A_{32}) < index(A_{42}) < index(A_{13}) < index(A_{23}) < index(A_{33}) < index(A_{43})$, the new BDD for each phase is presented in Fig.11.

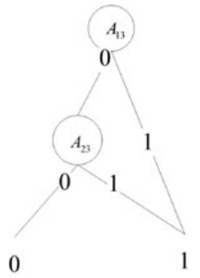


Fig. 9. The BDD model of the phase 3 of the mission P

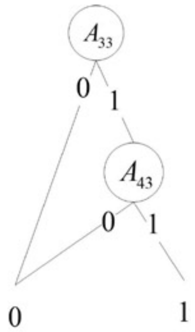


Fig. 10. The BDD model of the phase 3 of the mission Q

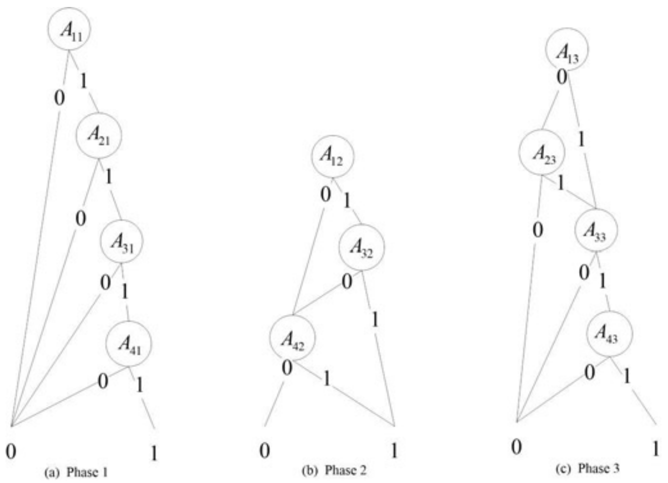


Fig. 11. Merged BDD models of each phase

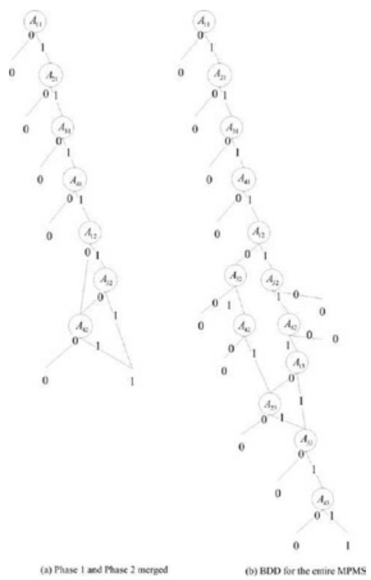


Fig. 12. Merged BDD model

Step 4: Generation of BDD for MPMS. Perform logic OR operation to combine the merged BDDs of three phases by applying equation (4) as shown in Fig.12.

Finally, according to the built BDD for entire MPMS, the overall system reliability is 0.829358.

4.2. MMDD example

We assume that a multi-state system is composed of two missions: mission P and mission Q. The step-by-step analysis of the multi-state MPMS is given as follows.

Step 1: New phase generation. Depending on the mission time

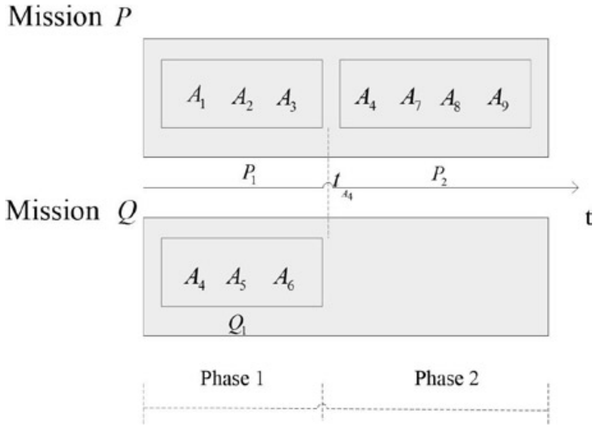


Fig. 13. System structure of MMDD example

Table 2. Input parameters of components.

component	Phase 1	Phase 2
A_1	0.0101(1), 0.982541(2)	-
A_2	0.984127	-
A_3	0.980789	-
A_4	0.986243	0.978568
A_5	0.980741	-
A_6	0.0101(1), 0.9804(2)	-
A_7	-	0.0131(1), 0.9769(2)
A_8	-	0.987562
A_9	-	0.986524

node of component A_4, mission Q has one phase: Q_1. Mission P can be divided into two phases: P_1 and P_2. In mission Q, phase Q_1 consists of three components (A_4, A_5, A_6) and is finished at time $t_{A_4}(Q)$. In mission P, phase P_1 consists of three components (A_1, A_2, A_3). At time $t_{A_4}(Q)$, component A_4 completes its task in mission Q and starts its task in mission P, meaning that phase P_2 consists of four components (A_7, A_8, A_9, A_4).The new system will be divided into two phases: 1) Phase 1 includes P_1 and Q_1; 2) phase 2 includes P_2. For the entire system, component A_4 involves in two missions during different time periods. Table 2 shows the computed conditional reliability for each element at each phase.

Step 2: Built MMDD for each phase. In phase 1 of mission P, the Component A_1 has three states (0, 1, and 2); component A_2 and A_3 have two states (0, 1). When component A_1 is in state '1', A_3 is in state '1', the system is normal. When component A_1 is in state '2' and A_2 in state '1', the system is normal. When component A_1 is in

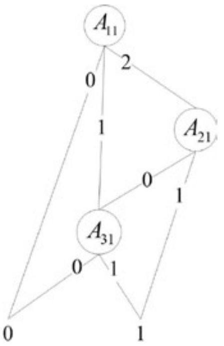


Fig. 14. The MMDD model of the phase 1 of the mission P

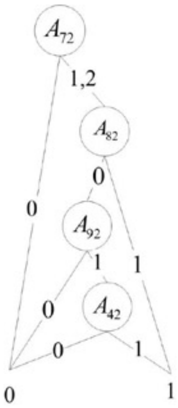


Fig. 15. The MMDD model of the phase 2 of the mission P

state '2' and A_3 in state '1', the system is also normal. Fig.14 shows the MMDD model for P_1.

In phase 2 of mission P, there are four components, when component A_4 is in state '1', A_9 is in state '1', and A_7 is in state '1' or '2', the system is normal. If component A_8 is in state '1', and A_7 is in state '1' or '2', the system is also normal. Fig.15 shows the MMDD model for P_2.

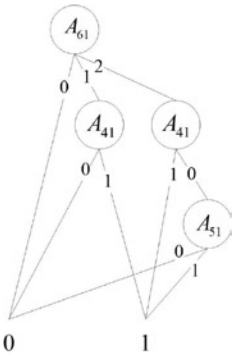


Fig. 16. The MMDD model of the phase 1 of the mission Q

In phase 1 of mission Q, there are three components: A_4, A_5, and A_6. Component A_4 and A_5 have two states (0, 1), and component A_6 has three states (0, 1, and 2). When component A_6 is in state '1' or '2', A_4 is in state '1', the system is normal. When component A_6 is in state '2' and A_5 in state '1', the system is also normal. Fig.16 shows the MMDD model for Q_1.

Step 3: Two missions MMDD merged for the same phase. Applying equation (6), and using the order

$$index(A_{11}) < index(A_{21}) < index(A_{31}) < index(A_{61}) < index(A_{41})$$
$$< index(A_{51}) < index(A_{72}) < index(A_{82}) < index(A_{92}) < index(A_{42})\quad,$$

the new MMDD for each phase is presented in Figs.17- 18.

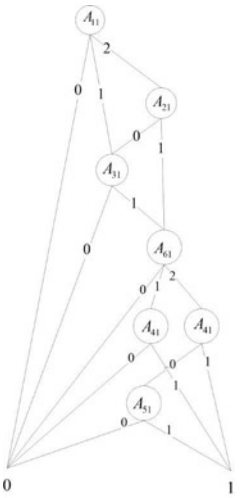


Fig. 17. Merged MMDD models of the phase 1

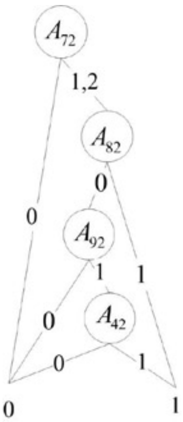


Fig. 18. Merged MMDD models of the phase 2

Step 4: Generate MMDD of entire MPMS. By performing logic OR operation to combine the MMDD of first and second phases, we obtain the final MMDD of MPMS. This is achieved by applying equation (6) on the example system, as shown in Fig.19.

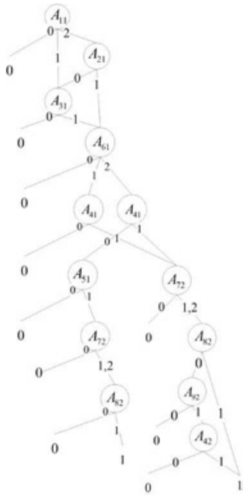


Fig. 19. MMDD for the entire MPMS

Finally, according to the merged MMDD for the entire MPMS, the overall system reliability is 0.971929.

5. Conclusions

In this paper, we proposed an analytical method based on BDD and MMDD to analyze the reliability for MPMS, where the components are engaged in multiple phased-missions sequentially. A four-step procedure is proposed to generate the BDD/MMDD model for obtaining the reliability value of the MPMS. In the MMDD modeling process, the merger regulation was proved. And two examples are implemented to prove the feasibility of the proposed methods.

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