JAISCR, 2011, Vol.1, No.4, pp. 259–267

A NEW MULTI-OBJECTIVE OPTIMIZATION ALGORITHM BASED ON DIFFERENTIAL EVOLUTION AND NEIGHBORHOOD EXPLORING EVOLUTION STRATEGY

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Abstract

In this paper a new optimization algorithm based on Differential Evolution, non-dominated sorting strategy and neighborhood exploration strategy for guaranteeing convergence and diversity through the generation of neighborhoods of different sizes to potential candidates in the population is presented. The performance of the algorithm proposed is validated by using standard test functions and metrics commonly adopted in the specialized literature. The sensitivity analysis of some relevant parameters of the algorithm is performed and compared with the classical DE algorithm without the strategy of neighborhood exploration and with other state-of-the-art evolutionary algorithms.

1 Introduction

The Multi-objective Optimization Problem (MOOP) is very common in different areas such as mathematics, engineering and sciences in general. This optimization problem is different from that of a single-objective optimization since MOOP usually has not only one but a set of noninferior optimal solutions, known as Pareto Curves. Besides, the optimality concept for this kind of problem is different of the one used in single optimization problems since in multi-objective optimization it is not possible to find a single optimal solution that satisfies all the goals, simultaneously [1, 2].

Traditionally, there are several methods available in the literature for solving MOOP problems [1, 3]. These methods follow a preference-based approach, in which a relative preference vector is used to scalarize multiple objectives. As classical optimization methods use a point to point approach, in which a solution found for a given iteration is modified to obtain a new solution, the outcome of using a classical optimization method is a single optimized solution. On the other hand, the evolutionary algorithms (EAs) can find multiple optimal solutions in one single simulation run due to their population-based search approach, being this way well adapted for multi-objective optimization problems. More details regarding the methodologies can be found in the literature [1, 3].

Nowadays, one of the most performing techniques used for the solution of single objective optimization problems is the so-called Differential Evolution Algorithm (DE) as proposed by Storn and Price [4]. Due to the success of this technique, several authors have extended its basic ideas to the context of multi-objective optimization obtaining good results as compared with those from other well known evolutionary algorithms [3, 5, 6, 7, 8].

In this sense, the main goal of this paper is to introduce a systematic methodology for the solution of multi-objective optimization problems by using the DE algorithm. This work is organized as follows. Section 2 presents the general aspects regarding the formulation of MOOP. Section 3 shows the classical heuristic approach for MOOP. A review about DE and its extension to deal with multi-criteria optimization are presented in Sections 4 and 5 respectively. In Section 6, the proposed methodology is discussed. Mathematical applications and sensitivity analysis for relevant parameters of the proposed algorithm are presented in Section 7. Finally, the conclusions are outlined in Section 8.

2 Basic Concepts for Multi-Objective Optimization Problems

The notion of optimality for MOOP is different that the one used for single optimization problems. The most common idea about multi-objective optimization found in the literature was originally proposed by Francis Ysidro Edgeworth [9] and later generalized by Vilfredo Pareto [10]. This idea can be described as follows: "*a solution is optimal if it is dominated by no other feasible solution, which means that there exists no other solution that is superior at least in case of one objective function value, and equal or superior with respect to the other objective functions values*" [10]. This definition leads us to find a set of solutions that is called the Pareto optimal set, whose corresponding elements are called non-dominated or non-inferior. The concept of optimality for single objective problems is not directly applicable in MOOPs. For this reason a classification of the solutions is introduced in terms of Pareto optimality, according to the following definitions [1]:

Multi-objective Optimization Problem (MOOP):

$$\text{Minimize } \; [f_1(x)\; f_2(x)\; ...\; f_k(x)] \tag{1}$$

subject to m inequality and l equality constraints

$$g_i(x) \leq 0, \;\; i = 1,..,m \tag{2}$$

$$h_j(x) = 0, \;\; j = 1,..,l \tag{3}$$

and design space defined according to the decision (or design) variables

$$x_n^U \leq x_n \leq x_n^L, \;\; n = 1,...,N. \tag{4}$$

where k is the number of objective functions f_i: $\mathbb{R}^N \rightarrow \mathbb{R}$.

Non-dominated Set: Among a set of solutions P, the non-dominated set of solutions P' are those that are not dominated by any member of the set P.

3 Heuristic Approach for Multi-Objective Optimization Problems

In the last years, several algorithms based on evolutionary mechanisms have been proposed to find approximations to the Pareto optimal solutions. As reported in the literature, the first multi-objective evolutionary algorithm (MOEA) was the VEGA - Vector Evaluated Genetic Algorithm [11], ever after countless efforts have been dispensed in the development of new evolutionary algorithms. The existing MOEAs are classified in two groups according to their characteristics and efficiency [1, 12]. On one hand there is a first group known as the first-generation, which includes all the early MOEAs: VEGA [11], NPGA [13], NSGA [14]. On the other hand, there is a second group named the second-generation MOEAs, which comprises very efficient optimizers like SPEA [12] and NSGA II [15], among others. Basically, the main features that distinguish the second-generation MOEAs from the first-generation group is the mechanism of adaptation assignment in terms of dominance and the incorporation of elitism [15].

4 Differential Evolution — A Review

Differential Evolution is a structural algorithm proposed initially by Storn and Price [4] for single-objective optimization problems. According to the authors, DE has as main advantages: simple structure, easiness of use, speed, and robustness. Basically, the idea of DE is to adapt the search during the evolutionary process generating trial parameter vectors by adding the weighted difference between two population vectors to a third vector. The control parameters in DE are the following: N, the population size, CR, the crossover constant, and, F, the weight applied to random differential (perturbation rate).

DE has been successfully applied in various fields such as digital filter design [17], parameter estimation in fed-batch fermentation process [18], engineering system design applied in a multi-objective context [19], apparent thermal diffusivity estimation of fruits during drying [20], estimation of drying parameters in rotary dryers [21], apparent thermal diffusivity estimation of the drying of fruits [22], Gibbs free energy minimization in a real system [23], estimation of space-dependent single scattering albedo in radiative transfer problems [24, 25, 26], and other applications [16].

5 Multi-Objective Algorithms based on Differential Evolution

Recently, several attempts to extend the DE to solve multi-objective problems have been proposed. The most representative of them is the so-called PDE - Pareto Differential Evolution [27], where only one (main) population is used. The reproduction mechanism is undertaken only among non-dominated solutions, and offspring are placed into the population if they dominate the main parent. In this case, a metric associated to the distance between two points in the Pareto front is used to maintain diversity. Other alternatives are the following: PDEA - Pareto Differential Evolution Algorithm [5], which combines DE with key elements from the NSGA II [15] such as its non-dominated sorting and ranking selection procedure; MODE - Multi-objective Differential Evolution [6], which uses a variant of the original DE so that the best individual is adopted to create the offspring. Also, these authors adopt Pareto ranking and crowding distance in order to produce and maintain well-distributed solutions; VEDE - Vector Evaluated Differential Evolution [28], which is a parallel, multi-population DE approach as based on the Vector Evaluated Genetic Algorithm (VEGA) [11]; NSDE - Non-dominated Sorting Differential Evolution [29] that consists in a simple modification in the NSGA II [15], where the real-coded crossover and mutation operators of the NSGA II are replaced by the DE scheme; DEMO - Differential Evolution Multi-objective Optimization [30], which combines the advantages of DE with the mechanisms of Pareto-based ranking and crowding distances sorting.

6 Multi-Objective Differential Evolution

In this section, we introduce the MODE program (Multi-objective Optimization Differential Evolution), which differs of the algorithms presented previously by the incorporation of two operators to the original algorithm, namely the mechanisms of rank ordering [1, 15] and exploration of the neighborhood potential solution candidates [31]. The general structure of the proposed algorithm for MOOP using DE is briefly described in the following. An initial population of size N is randomly generated. All dominated solutions are removed from the population through the operator Fast Non-Dominated Sorting [1]. In this way, the population is sorted into non-dominated fronts F_j (sets of vectors that are non-dominated with respect to each other). This procedure is repeated until each vector is member of a front. Three parents are selected at random in the population. A child is generated from the three parents (this process continues until N children are generated). Starting from population P_1 of size $2N$, neighbors are generated to each one of the individuals of the population in the following way [31]:

$$\chi(x) = [x - D_k(g)/2, x + D_k(g)/2] \qquad (5)$$

where

$$D_k(g) = \frac{k}{R}[U - L] \qquad (6)$$

$D_k(g)$ is a vector in $\mathbb{R}^n$ and a function of the generation counter g. R is the number of pseudo fronts defined by the user and the initial maximum neighborhood size in a population is $D_k(0)$=[U-L], where L and U represent the lower and upper bounds of the variables. The pre-defined number of individuals in each pseudo front is given by [31]:

$$n_k = r n_{k-1} \qquad k = 2, ..., R \qquad (7)$$

where n_k is the number of individuals in the k-th front and r (< 1) is the reduction rate. For a given population with N individuals, n_k can be calculated as:

$$n_k = N\frac{1-r}{1-r^R}r^{k-1} \qquad (8)$$

According to Hu et al. [31], if $r < 1$, the number of individuals in the first pseudo front is the highest and each pseudo front has an exponentially reducing number of solutions, this emphasizing a local search. On the contrary, a greater value for r results in more solutions in the last pseudo front and hence emphasizes the global search.

In this way, the neighbors generated are classified according to the dominance criterion and only the non-dominated neighbors (P_2) will be put together with P_1 to form P_3. The population P_3 is then classified according to the dominance criterion. If the number of individuals of the population P_3 is larger than a number defined by the user, it is truncated according to the criterion of the Crowding Distance [1]. The crowding distance describes the density of solutions surrounding a vector. To compute the crowding distance for a set of population members the vectors are sorted according to their objective function values for each objective function. To the vectors with the smallest or largest values an infinite crowding distance (or an arbitrarily large number for practical purposes) is assigned. For all other vectors the crowding distance is calculated according to:

$$dist_{x_i} = \sum_{j=0}^{m-1} \frac{f_{j,i+1} - f_{j,i-1}}{|f_{j,max} - f_{j,min}|} \quad (9)$$

where f_j corresponds to the j-th objective function and m equals the number of objective functions.

7 Results and Discussions

To test the performance of the proposed algorithm, a number of benchmark problems designed to represent different families of difficulties to multi-objective evolutionary algorithms - the ZTD functions [32] - will be solved:

$$\text{ZDT} = \begin{cases} \min f_1(x) \\ \min f_2(x) \equiv g(x)h(f_1(x), g(x)) \end{cases} \quad (10)$$

where the corresponding g and h functions are given by:

– ZDT1:

$$f_1(x) = x_1 \quad (11)$$

$$g(x_2, \ldots, x_m) = 1 + 9\sum_{i=2}^{m} \frac{x_i}{(m-1)} \quad (12)$$

$$h(f_1(x), g(x)) = 1 - (f_1/g)^{0.5} \quad (13)$$

– ZDT2

$$f_1(x) = x_1 \quad (14)$$

$$g(x_2, \ldots, x_m) = 1 + 9\sum_{i=2}^{m} \frac{x_i}{(m-1)} \quad (15)$$

$$h(f_1(x), g(x)) = 1 - (f_1/g)^2 \quad (16)$$

– ZDT3

$$f_1(x) = x_1 \quad (17)$$

$$g(x_2, \ldots, x_m) = 1 + 9\sum_{i=2}^{m} \frac{x_i}{(m-1)} \quad (18)$$

$$h(f_1(x), g(x)) = 1 - (f_1/g)^{0.5} - (f_1/g)\sin(10\pi f_1) \quad (19)$$

– ZDT4

$$f_1(x) = 1 - \exp(-4x_1)\sin^6(6\pi x_1) \quad (20)$$

$$g(x_2, \ldots, x_m) = 1 + 9\left(\sum_{i=2}^{m} \frac{x_i}{(m-1)}\right)^{0.25} \quad (21)$$

$$h(f_1(x), g(x)) = 1 - (f_1/g)^2 \quad (22)$$

Test function ZDT1 has a convex Pareto optimal front, while ZDT2 has the non-convex counterpart of the function ZDT1. Test function ZDT3 represents the discreteness features, whose Pareto optimal front consists of several disjointed continuous convex parts. Function ZDT4 contains 21^9 local Pareto optimal fronts.

For all problems considered, a set of 1000 uniformly spaced Pareto optimal solutions are chosen to compare the convergence (Υ) and diversity metric (Δ) [1], as given by:

$$\Upsilon = \frac{\sum_{i=1}^{|Q|} d_i}{|Q|} \quad (23)$$

$$\Delta = \frac{d_f + d_l + \sum_{i=1}^{|Q|-1} |d_i - \overline{d}|}{d_f + d_l + (|Q|-1)\overline{d}} \quad (24)$$

where d_i is Euclidean Distance calculated between the Q non-dominated solution obtained by the algorithm MODE and the Pareto Optimal solution, $\overline{d}$ is the average distance, d_l and d_f are the distances between the extreme solutions of the Pareto Optimal.

Parameters used in the ZTD functions: m=30, generation number (N_{gen}=100 - both algorithms), population size (N=100 - NSGA II and N=50 -

MODE), crossover rate (ρ_c=0.8 - both algorithms), mutation rate (p_m=0.01 - NSGA II), perturbation rate (F=0.8 - MODE), pseudo-curve number (R=10 - MODE), reduction rate (r=0.9 - MODE), and the number of objective functions evaluations (OF) calculated for each algorithm tested.

Figures 1 to 4 shows the computed and exact global Pareto optimal fronts in the parameter space as obtained by NSGA II and MODE.

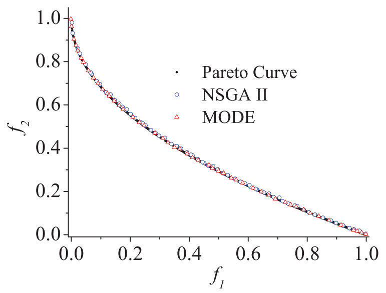

Figure 1. ZDT1 Function.

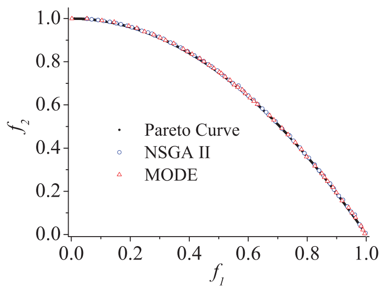

Figure 2. ZDT2 Function.

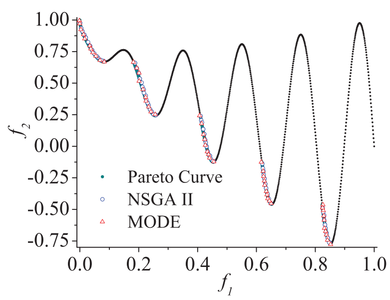

Figure 3. ZDT3 Function.

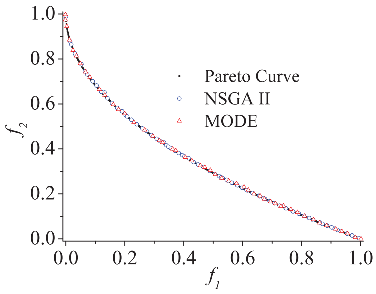

Figure 4. ZDT4 Function.

In these figures above it is possible to observe that both the algorithms lead to a good approximation of the Pareto Curve. However, the solutions obtained by MODE were obtained by using a smaller number of individuals and generations, consequently, a smaller number of objective function evaluations were performed.

The metrics values after 30 runs for each function tested are summarized in Table 1.

	NSGA II			
	Υ	$\sigma^2(\Upsilon)$	Δ	$\sigma^2(\Delta)$
ZDT1	0.0338	0.0047	0.3903	0.0018
ZDT2	0.0723	0.0316	0.4307	0.0047
ZDT3	0.1145	0.0079	0.7385	0.0197
ZDT4	0.5130	0.1184	0.7026	0.0646
	MODE			
	Υ	$\sigma^2(\Upsilon)$	Δ	$\sigma^2(\Delta)$
ZDT1	0.0301	0.0014	0.4155	0.0022
ZDT2	0.0614	0.0011	0.4114	0.0055
ZDT3	0.1111	0.0099	0.7477	0.0201
ZDT4	0.5547	0.1478	0.6644	0.0587

Table 1. Average (Υ, Δ) and variance (σ^2) values obtained by NSGA II and MODE.

In this table, the same performance is observed for both the algorithms NSGA II and MODE.

Now the influence of incorporating the Neighborhood Exploring Operator (NEO) in MODE will be analyzed by using the ZDT1 function. Soon after, the influence of relevant MODE parameters using the ZDT1, ZDT2 and ZDT3 functions will be analyzed.

– *Analysis of the Incorporation of the NEO*

To this analysis three different algorithms are considered: MODE, NSGA II and DE (without using the neighborhood exploring operator). In this analysis the following parameters were used: F=0.6 (MODE and DE), R=3 (MODE), r=0.8 (MODE), ρ_c=0.58 (both algorithms) and p_m=0.01 (NSGA II).

Figures 5 to 8 show the metrics of convergence and diversity by considering the effects of population size and generation number, respectively.

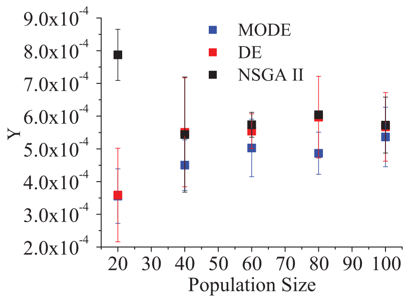

Figure 5. Convergence metric (200 generations).

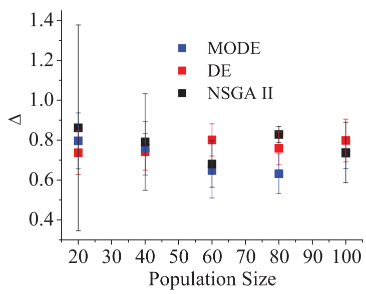

Figure 6. Diversity metric (200 generations).

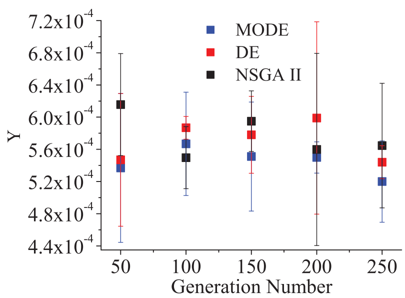

Figure 7. Convergence metric (100 individuals).

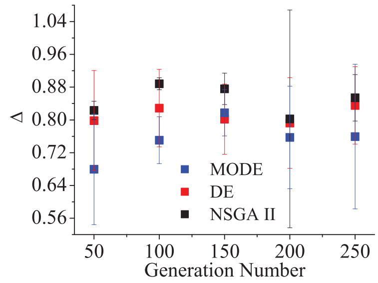

Figure 8. Diversity metric (100 individuals).

In the figures above it is interesting to observe that by increasing the population size or the generation number do not necessarily imply better results for the metric. The incorporation of a refinement strategy for neighborhood exploration leads to better results as compared with those obtained by NSGA II and by DE, thus justifying the use of this operator.

– *Analysis of relevant parameters - MODE*

✓ **Perturbation Rate**

For this analysis the following parameters were used: N_{gen}=150, N=50, ρ_c=0.8, R=10 and r=0.9.

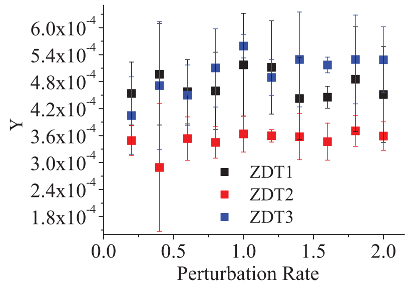

Figure 9. Convergence metric - influence of the perturbation rate (F).

In general, the results obtained are satisfactory as compared with NSGA II (convergence metric equal to 8×10^{-4} and diversity equal to 0.46; [1]). For each one of the ZDT functions different values were obtained for F (leading to good metric values). For example, 1.4 and 1.6 are good values for both the convergence and diversity metrics as far as the ZDT1 function is concerned. However, these values are not the best choice for F when dealing with the functions ZDT2 and ZDT3. In spite of the variation of F, it can be considered that any value between 0.2 and 2 is a good choice for that parameter, except for F equal to 0.2 for the case of the diversity metric.

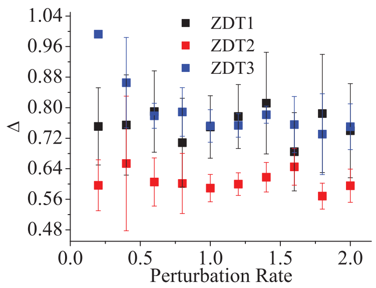

Figure 10. Diversity metric - influence of the perturbation rate (F).

✓ **Crossover Rate**

In this analysis the following parameters were used: N_{gen}=150, N=50, F=0.8, R=10 and r=0.9.

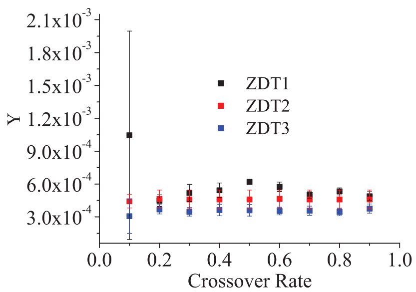

Figure 11. Convergence metric - influence of crossover rate (ρ_c).

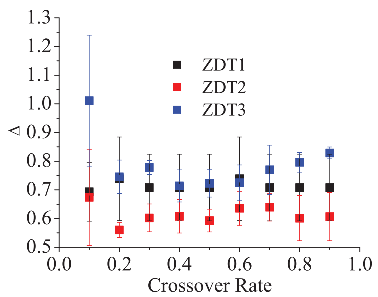

Figure 12. Diversity metric - influence of crossover rate (ρ_c).

Figures 11 and 12 shows that $\rho_c > 0.1$ is a good choice for this parameter. This same result was obtained by Storn and Price [4] and Storn et al. [16]; however, their works were dedicated to single-objective optimization.

✓ **Reduction Rate**

In this analysis the following parameters were used: N_{gen}=150, N=50, ρ_c=0.8, R=10 and F=0.8.

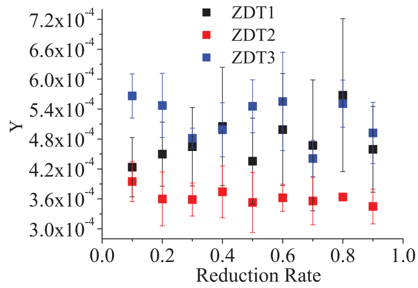

Figure 13. Convergence metric - influence of the reduction rate (r).

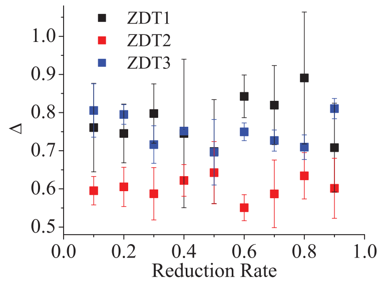

Figure 14. Diversity metric - influence of the reduction rate (r).

As observed in figures 13 and 14, it is not possible to choose a good value for r that is equally recommended for all the analyzed problems. Consequently, a final conclusion regarding the influence of this parameter is not presented.

8 Conclusions

In this paper, a new DE approach is presented for dealing with multi-objective optimization problems. This methodology permits the extension of the algorithm of DE to optimization problems with multiple objectives, through the incorporation of two operators to the original algorithm, namely the mechanisms of rank ordering and neighborhood potential solution candidates exploring. The proposed algorithm is applied to various classical problems, the so-called ZDT functions. Regarding all the obtained comparisons it is possible to observe that the metrics used for this purpose lead to results that are at least equivalent to those obtained by NSGA II. However, it is worth mentioning that a smaller number of objective function evaluations are necessary for MODE. The sensitivity analysis showed that despite the increase of the number of objective function evaluations due to the new operators included in the MODE algorithm, this disadvantage can be softened by using either a smaller number of individuals in the population or a smaller generation number. Finally, the results show that the proposed algorithm represents an interesting alternative for the treatment of multi-objective optimization problems even in the case of conflicting objectives.

Acknowledgment

Dr. Lobato acknowledges the financial support provided by FAPEMIG, Fundação de Amparo à Pesquisa de Minas Gerais. Dr. Steffen is thankful to the financial support provided by the agencies FAPEMIG and CNPq (INCT-EIE).

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