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On a general standby system and its optimization

Keywords

warm standby, accelerated life testing, virtual age, switching time, mean lifetime.

Abstract

Redundancy or standby is a technique that has been widely applied to improving system reliability and availability in the stage of system design. In this paper, we consider a standby system with two units in which the first unit (unit 1) starts its operation under active state and the other unit (unit 2) is under cold standby state at the starting point. After a specified time s (switching time), the state of unit 2 is changed to warm standby state and, as soon as the operating unit 1 fails, the state of unit 2 is changed to active state. If unit 1 fails before time s, the system fails. Units can fail at both active and warm standby states. A general method for modeling the standby system is adopted and, based on it, system performance measures (system reliability and mean life) are derived. We consider the problem of determining optimal switching time which maximizes the expected system life. Some numerical examples are studied.

1. Introduction

Standby redundancy is a technique widely used to improve system reliability and availability. In general, there are three types in standby, i.e. cold, hot and warm standby. Cold standby implies that the inactive component has a zero failure rate and cannot fail while in standby state. Hot standby implies that an inactive component undergoes the same operational environment as when it is in active state. This means that the lifetime of an inactive component is stochastically equal to that of the same component in active state. Warm standby is an intermediate case and an inactive component undergoes milder operational environment than that of the same component in active state. Thus, in this case, the lifetime of an inactive component may be stochastically larger than that of the same component in active state.

In practice, warm standby needs to be adopted when the state change from cold standby to active state is not smooth and continuous. By practical reasons, there could be an interruption during the state change, which results in stopping of operation of the system. Then this may cause critical and heavy loss especially when production systems are considered. For a smooth and continuous change of state, the standby unit starts its 'warm' operation (which means the standby unit is under warm standby state) from time $t=0$ and it starts its 'hot' operation as soon as the main operating unit fails. Generally, in almost of all research on warm standby systems, the standby unit starts its warm operation from time $t=0$. However, as standby unit can fail during warm standby period, it would be better to keep it in cold standby state at time $t=0$ and then let it start its warm operation after pre-specified time, e.g., $t=s$ (in what follows it will be called switching time) for optimizing the performance measure of the system. In this paper, a warm standby system with two units, whose operating rule is defined by a pre-specified switching time s , will be studied. The state diagram for this standby system is presented in *Figure 1*.

In most of the research on standby systems, only exponential distribution has been considered for the distributions of the units composing standby systems and the Markov methods are used to obtain performance measures of the system. See, for example, [1], [2], [4], [8]-[10], and [11]. In this paper, we consider general distributions for the lifetimes of the units in a standby system. For modelling lifetimes under different stress levels, as in [3], the basic statistical property commonly used in accelerated life tests will be employed.

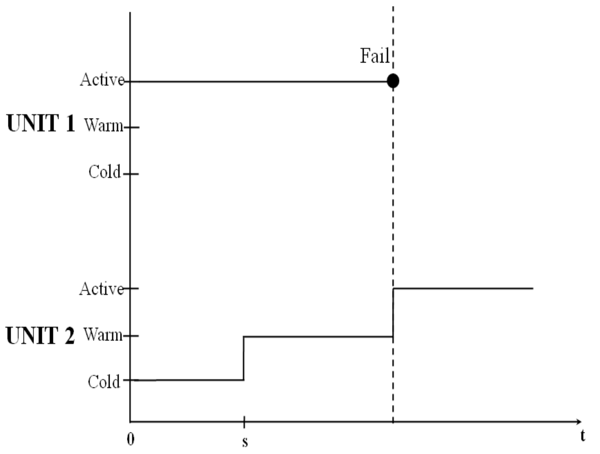

Figure 1. The State Diagram

This paper is constructed as follows. In Section 2, the probabilistic frame for modelling the lifetime distribution of the standby unit is introduced. The accelerated life model and the concept of virtual age are used for modelling the stochastic failure of a standby unit. Some discussions on the relationship between the concepts used in this paper and those proposed in [5] and [6] are made. In Section 3, considering general standby system with switching time, system reliability function and mean lifetime are derived. The problem of determining optimal switching time which maximizes the expected life of the system will be studied. Some numerical examples are also included in the section. Finally, in Section 4, some concluding remarks are discussed.

2. Modelling the lifetime distribution of warm standby unit

[3] developed a stochastic model for lifetime distribution of a standby unit. In this paper, the model developed in [3] will be adopted for a standby unit in the system. This section introduces the probabilistic model for the lifetime distribution of a standby unit under different environments. The methodology used in this section is based on the basic statistical property commonly used in accelerated life tests (ALT). More detailed background and motivation can be found in [3]. [5] introduced two models for modelling lifetimes of wearing-out components operated in different environments in view of accumulated wear process and considered the change points in environment. In [6], the statistical virtual age of a system was defined based on a model introduced in [5]. The approach introduced in this section and that given in [5] and [6] are quite different, but, basically, there are many similarities in the adopted stochastic methodologies.

In order to incorporate the basic statistical property commonly used in ALT, it is necessary to interpret the mechanism by which the accelerated lifetime is modelled. Denote random variable X the lifetime of a component used in the usual level of environment and $F(t), f(t), r(t)$, the distribution, probability density and failure rate functions of X. Also denote random variable X_A the lifetime of a component operated in the accelerate level of environment and $F_A(t), f_A(t), r_A(t)$ the corresponding distribution, probability density and failure rate functions, respectively. The 'Accelerated Failure Time'(AFT) regression model is the most widely used parametric failure time regression model in ALT. Under this model higher stress has the effect of shrinking time through a scale factor. This can be expressed

$$F_A(t) = F(\rho t), \forall t \geq 0, \tag{1}$$

where ρ is a constant called the 'acceleration factor' and it depends on the accelerated stresses. As given in Section 3 of [7], a more general model can be expressed as

$$F_A(t) = F(\rho(t)), \forall t \geq 0, \tag{2}$$

where $\rho(t)$ depends on the accelerated environment. Since the accelerated environment gives rise to higher stresses than usual environment, reasonable assumptions are $\rho \geq 1$ for the model (1) and $\rho(t) \geq t$ for all t and $\rho(0) = 0$ for the model (2). Furthermore it should be assume that $\rho(t)$ is a non-decreasing function. Then the model given in (2) implies that the lifetime of a component in the usual level of environment is larger than that in the accelerated environment in the sense that

$$\overline{F}(t) \geq \overline{F}_A(t), \forall t \geq 0,$$

that is, X is greater than X_A in the usual stochastic order, denoted by $X_A \leq_{st} X$.

In standby systems, the standby unit in warm standby state can be considered to be operated in an environment which is milder than the usual level of environment. Thus, if we let X_M be the lifetime of a standby unit in warm standby state, then the distrition function can be expressed,

$$F_M(t) = F(\gamma(t)), \forall t \geq 0, \quad (3)$$

where $\gamma(t) \leq t$ for all t, $\gamma(0) = 0$, and $\gamma(t)$ is a non-decreasing function. Then the model given in (3) implies that $X \leq_{st} X_M$.

In ALT, an important issue is related to the failure process after the change of the stress level on a unit. Similar problem can arise when there is a change of states in a standby unit. In standby system, after the starting point of warm operation there are two stress levels, i.e. the stress levels under warm standby state and active state, and we introduce the virtual age concept as a simple model. Suppose that a standby unit has been operated during [0, u] without failure under warm standby state and it is just activated at u. Then we assume that the failure distribution function of the unit is related to $F(t)$ but the age of the unit at time u is $w(u)$ which is not greater than u. Thus, under this assumption, the distribution function of the residual lifetime of the standby unit which has been just activated at u is given by

$$\frac{\overline{F}(w(u)+t)}{\overline{F}(w(u))} = \exp\{-\int_{w(u)}^{w(u)+t} r(t)dt\} = \exp\{-\int_0^t r(w(u)+t)dt\}, t \geq 0 \quad (4)$$

where $w(u) \leq u$ for all u, and $w(0) = 0$ is assumed to be a non-decreasing function. Then the equation (4) implies that the deteriorated level of the unit which has been operated under warm standby state during [0,u] is the same as that of a unit which has been operated in the usual level of environment during [0,w(u)].

Remark: In cumulative exposure model (refer Nelson (1990)), if the following relationship is assumed,

$$F_M(u) = F(w(u))$$

then it obviously holds that

$$w(u) = F^{-1}F(\gamma(u)) = \gamma(u).$$

3. System performance and optimal switching time

In this section the system performance measures are derived under the model described in the previous section and the optimal switching time will be considered.

Notation

$F_i(t), \overline{F}_i(t), f_i(t)$: Cumulative distribution function, survival function and probability density function of unit i at active state

$F_{m2}(t), \overline{F}_{m2}(t), f_{m2}(t)$: Cumulative distribution function, survival function and probability density function of unit 2 at warm state, $i = 1,2$.

$w(t)$: Virtual age of a unit which has been operated during $[0,t]$ without failure under warm standby state and is just activated at t.

s : switching time to warm standby state of unit 2.

In this paper, the optimal switching time is defined in the following Definition 1.

Definition 1.

A non-negative real value s^* which satisfies

$$ET_{s^*} = \max_{s \geq 0} ET_s,$$

where ET_s is the mean lifetime of the system as a function of switching time s, is called the optimal switching time.

Assumptions

1. Unit 1 and 2 have three states: active, warm, cold.
2. At starting point, the first unit is operated in active state and the second unit is switched to warm state after specified time, s.
3. Switching from warm state to active state is perfect, i.e. instantaneous and failure-free.

To obtain the system reliability, we consider two exclusive cases when the system survives time t.
Case 1: the first unit does not fail until t
Case 2: the first unit fails before t and the second unit is ready in warm state at failure time of the first unit. The second unit survives the remaining time.
From two exclusive events, we can obtain the system reliability as follows;
If t is less than s, the system reliability is given by

$$R_s(t) = \overline{F}_1(t).$$

Otherwise,

$$R_s(t) = \overline{F}_1(t) + \int_s^t \overline{F}_{m2}(u-s)\frac{\overline{F}_2(w(u-s)+t-u)}{\overline{F}_2(w(u-s))} f_1(u)du.$$

and the mean life is given by

$$E(T_s)=\int_0^s \overline{F}_1(t)dt + \int_s^\infty \{\overline{F}_1(t)+\int_s^t \overline{F}_{m2}(u-s)\frac{\overline{F}_2(w(u-s)+t-u)}{\overline{F}_2(w(u-s))} \times f_1(u)du\}dt \quad (6)$$

We consider special cases and numerical examples. First, we consider a special case in which the failure distributions of units in active state are exponential distributions with $f_i(t)=\lambda_i e^{-\lambda_i t}$, $i=1,2,$, respectively. In this case, if we assume that $\gamma(t)=(\lambda_0/\lambda_2)t$, where $\lambda_0<\lambda_2$, then we have $\overline{F}_{m2}(t)=e^{-\lambda_0 t}$ and $f_{m2}(t)=\lambda_0 e^{-\lambda_0 t}$. Hence, in this case, the distributions of units in both active and warm standby states are exponential distributions. Note that, in this case, the distribution of the standby unit under active state does not depend on the function $w(u)$ since its distribution after activation follows an exponential distribution. So it is unnecessary to define the function $w(u)$ in this case.

In this case, the system reliability is given by

$$R_s(t)=e^{-\lambda_1 t}+\int_s^t e^{-\lambda_0(u-s)}e^{-\lambda_2(t-u)}\lambda_1 e^{-\lambda_1 u}du = e^{-\lambda_1 t}+\frac{\lambda_1}{\lambda_0+\lambda_1-\lambda_2}\{e^{-\lambda_2 t-(\lambda_1-\lambda_2)s}-e^{-(\lambda_0+\lambda_1)t+\lambda_0 s}\}. \quad (7)$$

The mean life of the system is given by

$$ET_s=\int_0^s e^{-\lambda_1 t}dt+\int_s^\infty\{e^{-\lambda_1 t}+\frac{\lambda_1}{\lambda_0+\lambda_1-\lambda_2}(e^{-\lambda_2 t-(\lambda_1-\lambda_2)s}-e^{-(\lambda_0+\lambda_1)t+\lambda_0 s})\}dt = \frac{1}{\lambda_1}+\frac{\lambda_1 e^{-\lambda_1 s}}{(\lambda_0+\lambda_1)\lambda_2} \quad (8)$$

From the equation (8), we can easily find that the optimal switching time which maximizes the mean time to failure of system is given by 0.

In the case of s=0, the system reliability is given by

$$R_s(t)=e^{-\lambda_1 t}+\frac{\lambda_1}{\lambda_0+\lambda_1-\lambda_2}\{e^{-\lambda_2 t}-e^{-(\lambda_0+\lambda_1)t}\}$$

The mean life of the system is given by

$$ET_s=\frac{1}{\lambda_1}+\frac{\lambda_1}{(\lambda_0+\lambda_1)\lambda_2}$$

The above two results are identical to the results give n in [8] and [4]. As another particular case, we consi der Weibull distributions with IFR, $F_i(t)=1-e^{-(\lambda_i t)^2}, \lambda_1=\lambda_2=1$, $\gamma(t)=\alpha t$ and $w(u)=\beta t$. The system reliability is given by

$$R_s(t)=\begin{cases} e^{-t^2}, if & t<s \\ e^{-t^2}+\int_s^t e^{-\alpha^2(u-s)^2-(\beta(u-s)+t-u)^2+(\beta(u-s))^2-u^2}2udu, if & t>s \end{cases}$$

And the expected life is given by

$$ET_s=\int_0^\infty e^{-u^2}du+\int_s^\infty\int_s^t e^{-\alpha^2(u-s)^2-(\beta(u-s)+t-u)^2+(\beta(u-s))^2-u^2}2ududt.$$

As a special case, we consider the case with $\alpha=\beta=0.5$. *Figure 2* shows the reliability function in case that s=0.5. *Figure 3* shows the expected life as a function of s and we can find the fact that the delayed starting can lengthen the expected system life. From *Figure 3*, when the distribution of the components follow IFR Weibull, there exists a unique optimal switching time which maximizes the expected system life time.

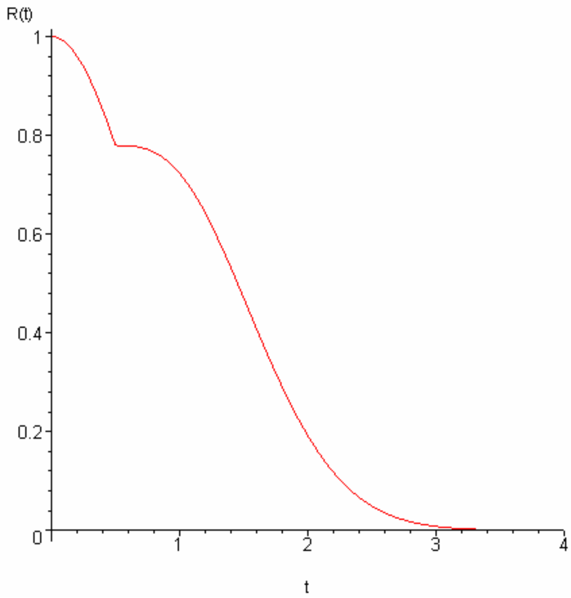

Figure 2. Reliability function (s=0.5)

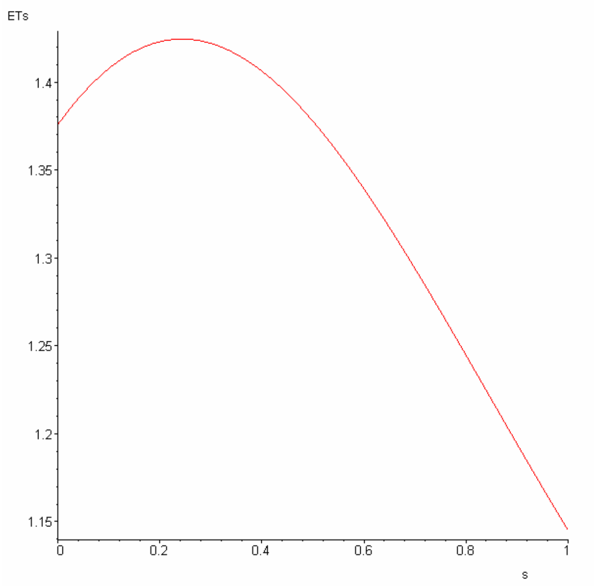

Figure 3. Expected system life

4. Conclusion

A standby structure is adopted to improve system performance. In almost all of the studies on standby systems, only exponential distribution has been considered for the distributions of the units composing standby systems and the Markov methods are used to obtain performance. In this paper, the distributions of lifetimes of units can have arbitrary continuous distributions. Considering the situation when the state change from cold standby to active state is not smooth and continuous, a switching time is adopted, which makes a continuous operation of the system after the failure of the main component. The reliability function and mean time to failure of the standby system has been derived. Furthermore, the problem of determining the optimal switching time has been investigated. The case of IFR Weibull was considered and it has been illustrated that there exists a unique optimal switching time which maximizes the mean time to failure of the standby system.

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