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MATRIX CALCULUS AND SPREADSHEET IN PLANNING AND CONTROLLING OF OPERATING COSTS

10.1 INTRODUCTION

Planning and controlling are among the main functions of management [2, 7]. They may be related to a vast array of diverse operating aspects of an organisation. One of the most significant spheres in which these functions should by no means be neglected is undoubtedly the overall body of financial problems. Being fully aware of present and projected revenues, costs or cash flows is prerequisite for both the correct assessment of the enterprise's current standing as well as for making appropriate decisions affecting its future position. In this sphere, some of the most complex problems one may encounter is the planning of operating costs related to the projected level of the organisation's operations along with their subsequent controlling on an ongoing basis. This problem proves to be particularly complex when it comes to enterprises which have deployed broad-scale production processes or whose selection of products or services is considerably diversified, for which creating a reliable plan and being able to verify it in the follow-up involve the necessity of taking a large number of different factors and their interrelations into consideration. In similar situations, when creating the cost plan, one may decide to apply a solution proposed in the literature of the subject [9] which makes use of the matrix calculus and extends the concepts previously developed and applied in this respect [4, 5, 6]. Application of the matrix calculus also allows for subsequent ongoing cost controlling which makes it possible to identify specific types of deviations enabling accurate verification of the assumptions adopted at the planning stage. The types of such identifiable deviations have been discussed in the following paragraphs of the article along with the method proposed for their calculation as well as the range of data required for this purpose. Both the creation and the follow-up verification of the cost plan involve the necessity of data processing in large volumes. In order to successfully pursue the foregoing goals by following the aforementioned approach, one must use a sufficient IT tool. Spreadsheets, for instance, are well known tools commonly used in nearly every organisation. They may be applied to support very diverse aspects of operations [1, 3, 8, 10]. A sample solution enabling practical application of the cost planning and controlling concepts discussed in the paper with the support of a spreadsheet has been provided in the final section of the article.

10.2 APPLICATION OF THE MATRIX CALCULUS FOR COST PLANNING AND CONTROLLING

In accordance with the approach explained in the literature [9], when creating a plan of costs, one can perceive an enterprise as a set of interlinked quantitative components and cost components. The quantitative components may correspond to the work performed, the energy consumed, the operations completed as well as materials, products, services etc. The main characteristic feature of this group of components is that their flow can be represented in natural units other than a currency. The cost components reflect the costs incurred by the enterprise, and their flow can only be expressed by means of a currency. All components may be connected through consumption standards which define the flow of a secondary component assuming a unit flow of a primary component. In the process of establishing the standards, the quantitative components may be either the primary or the secondary ones. The cost components can only be secondary components. With the component structure thus defined, in order to create a cost plan, i.e. in fact a plan of flows for the cost components, it is required that external flows be defined for the quantitative components. The external flows are defined by taking different groups of targets into consideration, and these may include [9] sales, stock level changes for products, semi-finished products etc. as well as fixed consumption of individual components. In each group, as needed, different targets may be included.

Assuming that k corresponds to the number of targets for external flows, m is the number of cost components and n is the number of quantitative components, before the cost plan can be developed, matrices $A_{n\times n}$:, $B_{m\times n}$, $D_{k\times 3}$ are created. Matrix A represents the consumption standards defined for quantitative components. Every element “a_{ij}” of this matrix expresses the magnitude of flow for component “i” in the case of a unit flow of component “j”. Matrix B corresponds to the standards for cost components. Every element “b_{ij}” of this matrix expresses the flow of cost component “i” in the case of a unit flow of quantitative component “j”. Matrix D is used to assign all the identified external targets to three main groups of targets. Element „d_{ij}” assumes the value of “1” in the event that target “i” belongs to group “j”. Each target may only belong to one group, therefore the sum of elements in a row must equal 1. With matrices A, B and D in disposal, in order to create a cost plan, one should plan the values for elements of matrix $C_{n\times k}$. This matrix constitutes a plan of all external flows. The value of element “c_{ij}” corresponds to the quantitative flow which has been planned for element “i” in the target of “j”. Having the above matrices, the cost plan comprising the flows planned for cost components may be represented using matrix G calculated based on formula 10.1.

$$G = B \cdot (I - A)^{-1} \cdot C \cdot D \tag{10.1}$$

This matrix contains 3 columns, each corresponding to one of three groups of external targets. Elements in one column are the planned flows of cost components resulting from the plan of external flows towards targets which belong to one group of targets. An overall cost plan may be developed based on formula 10.2.

$$h = G \cdot x \tag{10.2}$$

However, vector x is a 3-element column vector filled with the values of 1.

Once the period for which the plan has been prepared is over, real-life cost values may be represented as m-element vector hr, where hr_i is the real-life cost, i.e. the real-life flow of cost component "i". Vector ho, calculated with reference to formula 10.3, is the vector of total deviations between real-life costs and the planned ones.

$$ho = h - hr \tag{10.3}$$

Total cost deviations may result from a number of reasons, e.g. external flows being different than the planned ones (type 1), non-fulfilment of consumption standards for quantitative components (type 2) or non-fulfilment of cost-related standards (type 3). In order to determine the magnitude of individual types of deviations, one should establish the $Cr_{n\times k}$ matrix of external real-life flows as well as the n-element vector of total real-life flows designated as pr.

Deviations of the first type may be represented in a breakdown into all external flow targets as matrix Qo1 calculated with reference to formula 10.4.

$$Qo1 = B \cdot (I - A)^{-1} \cdot (Cr - C) \tag{10.4}$$

These deviations may be aggregated into three groups of external targets and represented by means of matrix Go1 calculated based on formula 10.5,

$$Go1 = Qo1 \cdot D \tag{10.5}$$

or aggregated into individual values for each cost component and represented as vector ho1 determined with reference to formula 10.6.

$$ho1 = Go1 \cdot x \tag{10.6}$$

Deviations of the second type may be represented as vector ho2 calculated with reference to formula 10.7.

$$ho2 = B \cdot \left(pr - (I - A)^{-1} \cdot Cr \cdot y\right) \tag{10.7}$$

In this case, vector y is a k-element column vector filled with the values of 1. Values of elements of vector ho2 provide an answer to the question what part of deviation for each of the costs generally results from the failure to fulfil consumption standards. However, it delivers no information about the extent to which this deviation results from non-fulfilment of consumption standards for individual quantitative components. Such information may only be obtained by following formula 10.8 to build the V matrix of unit costs with elements v_{ij}, and by creating the r vector of quantitative deviations with elements r_i in accordance with formula 10.9,

$$V = \left(I - A^T\right)^{-1} \cdot B^T \tag{10.8}$$

$$r = pr - Cr \cdot y - A \cdot pr \qquad (10.9)$$

and then matrix S with elements s_{ij} which meet the following condition:

$$s_{ij} = v_{ij} \cdot r_i \qquad (10.10)$$

Every element s_{ij} carries information what part of the second type deviation of cost component "j" results from the failure to fulfil the consumption standards set for quantitative component "i".

Deviations of the third type may be represented as vector ho3 calculated with reference to formula 10.11.

$$ho3 = hr - B \cdot pr \qquad (10.11)$$

10.2 SPREADSHEET IN COST PLANNING AND CONTROLLING

The method proposed to be applied for planning and controlling of costs may involve using a spreadsheet. A sample system of matrices and vectors whose values must be user defined has been provided in Figure 10.1.

Worksheet "A"

quant component \ quant component	1	2	3	4	5	6	7
1	0	1,2	0	0	0	0	0,9
2	0	0	0	0	0	1	0
3	0	0	0	0	0	12	0
4	0	0,5	0	0	0	0	0
5	0	0	0	0	0	0	0,2
6	0	0	0	0	0	0	3
7	0	0	0	0	0	0	0

Worksheet "B"

cost component \ quant component	1	2	3	4	5	6	7
1	0,7	0	0	0	0	0	0
2	0	0	1,4	0	0	0	0
3	0	5	0	0	0	1	0
4	0	0	0	39	21	0	0
5	0	0	0	26	0	0	0

Worksheet "C"

quant component \ ext flow targets	1	2	3	4	5
1	0	0	0	0	18
2	0	0	0	0	0
3	0	0	-99	0	0
4	0	0	0	0	17
5	0	0	0	0	0
6	45	0	0	23	0
7	85	25	0	0	0

Worksheet "D"

ext flow targets \ targets group	1	2	3
1	1	0	0
2	1	0	0
3	0	1	0
4	0	1	0
5	0	0	1

Worksheet "Cr"

quant component \ ext flow targets	1	2	3	4	5
1	0	0	0	0	20
2	0	0	0	0	0
3	0	0	50	0	0
4	0	0	0	0	16
5	0	0	0	0	0
6	42	0	0	24	0
7	87	19	0	0	0

Worksheet "pr"

quant component	
1	580,20
2	394,00
3	4718,00
4	217,00
5	21,20
6	384,00
7	106,00

Worksheet "hr"

cost component	
1	410,00
2	6557,00
3	2366,00
4	8923,20
5	5662,00

Fig. 10.1 System of matrices and vectors with user-defined values

Source: author's own materials

The system of matrices and vectors being calculated, linked with the system shown in Figure 10.1, has been provided in Figures 10.2, 10.3 and 10.4 along with the formulas applied.

Worksheet "h" (FA)

	cost component	C
1	1	416,22
2	2	6547,80
3	3	2388,00
4	4	8886,00
5	5	5616,00

Worksheet "ho" (FB)

	cost component	C
1	1	-6,22
2	2	9,20
3	3	-22,00
4	4	37,20
5	5	46,00

Worksheet "ho1" (FC)

	cost component	C
1	1	-12,88
2	2	-26,60
3	3	-84,00
4	4	-328,80
5	5	-208,00

Worksheet "ho2" (FD)

	cost component	C
1	1	2,80
2	2	84,00
3	3	50,00
4	4	351,00
5	5	234,00

Worksheet "ho3" (FE)

	cost component	C
1	1	3,86
2	2	-48,20
3	3	12,00
4	4	15,00
5	5	20,00

FA	{=MMULT(G!C3:E7;{1;1;1})}
FB	=hr!C1-h!C1
FC	{= MMULT('Go1'!C3:E7;{1;1;1})}
FD	{= MMULT (B!C3:I7;pr!C1:C7-MMULT(MMULT(MINVERSE(IF(A!C2:I2=A!B3:B9;1;0)-A!C3:I9);Cr!C3:G9);JEŻELI(B1:B5<>"";1;1)))}
FE	{=hr!C1:C5- MMULT(B!C3:I7;pr!C1:C7)}

Fig. 10.2 System of vectors h, ho, ho1, ho2 and ho3 along with the formulas applied

Source: author's own materials

Worksheet "G" (FG)

cost component	targets group 1	targets group 2	targets group 3
1	384,30	19,32	12,60
2	6300,00	247,80	0,00
3	2250,00	138,00	0,00
4	7774,50	448,50	663,00
5	4875,00	299,00	442,00

Worksheet "Qo1" (FH)

cost component	ext flow targets 1	2	3	4	5
1	3,78	-18,9	0	0,84	1,4
2	50,4	-302,4	208,6	16,8	0
3	18	-108	0	6	0
4	66,9	-376,2	0	19,5	-39
5	39	-234	0	13	-26

Worksheet "Go1" (FI)

cost component	targets group 1	targets group 2	targets group 3
1	-15,1	0,84	1,4
2	-252	225,4	0
3	-90	6	0
4	-309	19,5	-39
5	-195	13	-26

FG	{= MMULT(MMULT(MMULT(B!C3:I7; MINVERSE(IF(A!C2:I2=A!B3:B9;1;0)-A!C3:I9));'C'!C3:G9);D!C3:E7)}
FH	{= MMULT(MMULT(B!C3:I7;MINVERSE(IF(A!C2:I2=A!B3:B9;1;0)-A!C3:I9));Cr!C3:G9-'C'!C3:G9)}
FI	{= MMULT('Qo1'!C3:G7;D!C3:E7)}

Fig. 10.3 System of matrices G, Qo1 and Go1 along with the formulas applied

Source: author's own materials

Worksheet "V" (FJ)

quant component	cost component 1	2	3	4	5
1	0,70	0,00	0,00	0,00	0,00
2	0,84	0,00	5,00	19,50	13,00
3	0,00	1,40	0,00	0,00	0,00
4	0,00	0,00	0,00	39,00	26,00
5	0,00	0,00	0,00	21,00	0,00
6	0,84	16,80	6,00	19,50	13,00
7	3,15	50,40	18,00	62,70	39,00

Worksheet "r" (FK)

quant component	C
1	-8,00
2	10,00
3	60,00
4	4,00
5	0,00
6	0,00
7	0,00

Worksheet "S" (FL)

quant component	cost component 1	2	3	4	5
1	-5,60	0,00	0,00	0,00	0,00
2	8,40	0,00	50,00	195,00	130,00
3	0,00	84,00	0,00	0,00	0,00
4	0,00	0,00	0,00	156,00	104,00
5	0,00	0,00	0,00	0,00	0,00
6	0,00	0,00	0,00	0,00	0,00
7	0,00	0,00	0,00	0,00	0,00

FJ	{= MMULT(MINVERSE (IF(A!B3:B9=A!C2:I2;1;0)-TRANSPOSE(A!C3:I9));TRANSPOSE(B!C3:I7))}
FK	{=pr!C1:C7- MMULT(Cr!C3:G9; IF(Cr!B3:B7<>"";1;1))- MMULT(A!C3:I9;pr!C1:C7)}
FL	='r'!$C1*V!C3

Fig. 10.4 System of matrices V and S and of vector r along with the formulas applied

Source: author's own materials

10.3 CONCLUSIONS

As evidenced above, the matrix calculus may be applied while the cost plan is being created as well as at the stage of its subsequent controlling. According to the approach described in the paper, the total deviation between real-life costs and planned costs may be broken down into the deviation being a consequence of the fact that the level of external flows is different than the planned one, a failure to fulfil consumption standards set for quantitative components as well as non-fulfilment of consumption standards defined for cost components. The solution proposed requires IT support, which may be ensured by application of a tool created in a spreadsheet. In such a tool, one can use array formulas and functions like MMULT(), MINVERSE(), IF() and TRANSPOSE(). This approach may be applied at any production or service company where the production process can be represented as a series of interlinked components for which one can define consumption standards and identify quantitative flows on an ongoing basis.

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Abstract: *The article provides a discussion on a concept of using the matrix calculus for purposes of planning and controlling of operating costs. The opening paragraphs of the paper describe creating a plan of costs for an enterprise perceived as a series of interlinked components whose mutual relations are represented in corresponding matrices. Further sections of the article are focused on an analysis of the deviations emerging while planned costs are being compared with the reported actual costs. Different types of deviations comprising the total deviation have been identified. It is possible to determine these deviations based on the data contained in matrices used at the planning stage as well as with reference to appropriate real-life data acquired. For individual groups of deviations, the manner in which they can be determined has been explained. The final paragraphs of the article address potential applications of spreadsheets, both in creating the plan of costs and for purposes of its subsequent controlling.*

Keywords: *cost planning, cost controlling, IT tools*

RACHUNEK MACIERZOWY I ARKUSZ KALKULACYJNY W PLANOWANIU I KONTROLI KOSZTÓW OPERACYJNYCH

Streszczenie: *W artykule przedstawiono koncepcję wykorzystania rachunku macierzowego w zakresie planowania i kontroli kosztów operacyjnych. W pierwszej kolejności w opracowaniu omówione zostało tworzenie planu kosztów dla przedsiębiorstwa traktowanego jako ciąg powiązanych komponentów, których wzajemne powiązania są ujmowane w postaci odpowiednich macierzy. W dalszej części artykułu skupiono się na analizie odchyleń występujących podczas porównania kosztów planowanych z zarejestrowanymi kosztami rzeczywistymi. Zidentyfikowano różne rodzaje odchyleń składających się na odchylenie całkowite. Odchylenia te są możliwe do ustalenia na podstawie danych zawartych w macierzach wykorzystywanych na etapie planowania oraz na podstawie odpowiednich zgromadzonych danych rzeczywistych. Dla poszczególnych grup odchyleń przedstawiono sposób ustalenia ich wartości. W końcowej części artykułu omówiono możliwość wykorzystania arkuszy kalkulacyjnych zarówno w zakresie tworzenia planu kosztów, jak i jego późniejszej kontroli. .*

Słowa kluczowe: *planowanie kosztów, kontrola kosztów, narzędzia informatyczne*