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A mathematical model to optimize the pilot training process

Keywords

optimization of process, pilot training, risk assessment

Abstract

In most NATO countries, pilots are prepared in accordance with a modern system of three-stage training: selection, basic and advanced. For each phase, the purpose and scope of the training shall be defined and the aviation equipment together with training support systems should be appropriate selected. The practical training in the air is performed according to the parallelism of the training, which describes the technological relationships between the individual exercises. The purpose of this contribution is to propose a mathematical model which would enable the selection of exercises for each pilot, the appropriate selection of the aircraft and the moment of beginning of each exercise, so that all required exercises are carried out in the shortest possible time. Additionally, in this paper the concept of pilot's habits and skills improvements is presented.

1. Introduction

The practical training in the air is performed according to the parallelism of the training, which describes the technological relationships between the individual exercises. For each exercise, the exercises are specified, which must be done beforehand. Each of the tasks is performed at most once. The duration of each exercise is fixed and different types of aircraft are at the disposal. For each type, the number of aircraft shall be determined. Exercises are only possible on the appropriate type of aircraft. Exercises can be performed on exactly one aircraft by one pilot. A single aircraft allows one pilot to be trained in a single exercise. The purpose of this contribution is to propose a mathematical model which would enable the selection of exercises for each pilot, the appropriate selection of the aircraft and the moment of beginning of each exercise, so that all required exercises are carried out in the shortest possible time. Additionally, in this paper the concept of pilot's habits and skills improvements is presented.

2. Formulation of the problem

2.1. Pilot training process

The aim of the screening programme is to determine the psychophysical predisposition of candidates for the profession of military pilot. This training, notwithstanding the subsequent specialisation of pilots, is typically conducted on low-cost aircraft, which enables navigation flights and basic aerial aerobatics.
During basic training, future pilots should master the take-off and landing manoeuvre, basic navigation, daytime and at night group flights.
Candidates for pilots of combat planes are also required to perform basic and advanced acrobatics.
During advanced training, students already have a fully mastered piloting technique in all atmospheric conditions. At this stage, the elements of combat ability

are also taught - this includes mainly candidates for pilots of combat aircraft. In countries, where a modern system of training pilots of multi-purpose aircraft is in existence, advanced training takes place on jet-powered, airplane training and combat aircraft witch have a similar to combat aircraft characteristics.

2.2. A military pilot education system at the Polish Air Force Academy (PAF)

The training system in PAF includes the following aspects:

- PAF theoretical training courses in the following specialties: jet pilot, transport airplane pilot, helicopter pilot and navigator, air traffic controller (corrected with simulator training);
- Practical training in 4 School Aviation Wings (4 SLSz) on aircraft (helicopters and aeroplanes).

The curricula of the PAF for theoretical and practical training of pilots as well as the scope and methods of shaping their psyche during their studies must take into account the requirements of operating units. The main goal of training at the Higher School of Air Force Officers in the specialization of "`Aircraft Pilot" is to educate officers of the Polish Army - military pilots on the basis of the personal and professional characteristics of a graduate of the PAF, which, together with a mastered range of knowledge and professional skills, will ensure that they properly perform their tasks on the first professional positions requiring higher education, in accordance with the Act of 11 September 2003 on military service of professional soldiers. In pursuing the above objective, PAF enables their students:

1. Mastering the knowledge and general, tactical and specialist skills necessary to perform the duties of an engineer-military pilot in the first position of service, which allow to understand and interpret properly the phenomena encountered and to make appropriate and rapid decisions while performing tasks in the air.
2. Mastering the knowledge and skills necessary to perform the duties of an officer of the Polish Army on the first official position, including in particular:
   - knowledge of the humanities and basics of military sciences concerning the essence, regularity and functioning problems of an officer in a military unit in peaceful conditions, threats and war;
   - the ability to effectively communicate, negotiate and cooperate, as well as services in a compact team (aircraft staff).
3. Obtaining the necessary knowledge, technical and specialist skills in accordance with the training specialization, i. e.:
   - knowledge in the scope enabling understanding of the operation and correct usage of weapons and technical equipment (aircraft, flight safety systems, air navigation, etc.);
   - construction and technical capabilities of the aircraft, as well as the principles of use and utilization of the exploited aircraft equipment and weapons;
   - general knowledge of the principles of command, preparation, organization and conducting combat at the tactical level;
   - knowledge of procedures, aviation regulations and rules of conducting radio correspondence in air networks (including English) in accordance with the requirements of national and international aviation institutions.
4. Shaping psychophysical aptitude to ensure the effective performance of official duties within the aviation specialty, i. e.:
   - good spatial orientation and imagination;
   - ability to concentrate and divide attention, make quick and accurate decisions;
   - ability to perform tasks in the context of mental stress (e. g. actions in difficult, special conditions).

2.3. Schedule of training in helicopters in the aviation of armed forces

The programme of aviation training in helicopters in the Polish Armed Forces of the Republic of Poland standardizes the process of training of flying personnel in helicopters and constitutes the basis for developing a Long-term plan of air training to achieve class I pilots. The block graphics of the parallelism of pilot training process is presented in *Figure 1*. It is an example of the implementation of the scope of training required to provide a pilot with *Visual Meteorological Condition* (VMC) skills for pilot training. Example of schematic sequence of pilot training process for one module is presented in the following *Figure 2.*

Figure 1. Sketch of the parallelism of pilot training process [1]

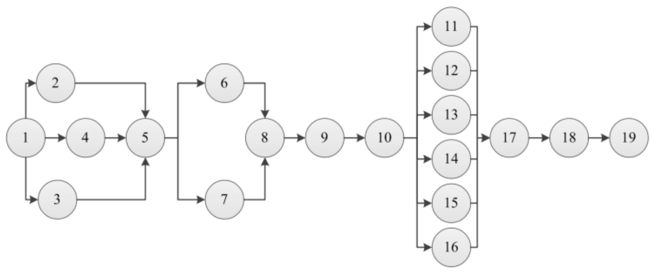

Figure 2. Example of schematic sequence of pilot training process for one module [1]

3. The sketch of the mathematical model

3.1. Mathematical description of the essential attributes

$N \in \mathbb{P}$ - the number of pilots,

$C \in \mathbb{P}$ - the number of exercises,

$$a_{ij} = \begin{cases} 1 & \text{if excercise no. } j \text{ requires excercise no. } i \\ 0 & \text{otherwise} \end{cases}$$

$i, j = \overline{1, C}$,

$$b_c^n = \begin{cases} 1 & \text{if pilot no. } n \text{ has to perform exercise no. } c \\ 0 & \text{otherwise} \end{cases}$$

$c = \overline{1, C}$, $n = \overline{1, N}$,

$H \in \mathbb{P}$ - the number of aircraft types,

$SP_h \in \mathbb{P}$ - the number of aircraft types h, $h = \overline{1, H}$

$d_c \in \mathbb{R}^+$ - the duration of exercise no. c, $c = \overline{1, C}$,

$$e_c^h = \begin{cases} 1 & \text{if the aircraft type } h \text{ may be used for the excercise } c \\ 0 & \text{otherwise} \end{cases},$$

$h = \overline{1, H}$, $c = \overline{1, C}$,

$$f_c^n = \begin{cases} 1 & \text{if pilot no. } n \text{ has performed the excercise no. } c \\ 0 & \text{otherwise} \end{cases},$$

$c = \overline{1, C}$, $n = \overline{1, N}$,

$$x_n^{ch} = \begin{cases} 1 & \text{if is planed to perform excercise no. } c \text{ by the pilot no. } n \text{ using aircraft of type } h \\ 0 & \text{otherwise} \end{cases}$$

$c = \overline{1, C}$, $n = \overline{1, N}$, $h = \overline{1, H}$,

$y_c^n \in$ ˇ $^+$ - the moment when exercise no. c begins by pilot n, $c = \overline{1, C}$, $n = \overline{1, N}$,

$Z \in \mathbb{R}^+$ - the duration of all training.

3.2. Mathematical description of the essential attributes

1. Each pilot may perform the exercise only on the type of aircraft appropriate for the particular exercise:

$$\underset{c=\overline{1,C}}{\forall}\ \underset{n=\overline{1,N}}{\forall}\ \underset{h=\overline{1,H}}{\forall}\ x_n^{ch} \le e_c^h$$

2. The starting moment of each subsequent exercise shall not be less than the sum of the starting moment of the preceding exercise and the duration of the preceding exercise:

$$\underset{i,j=\overline{1,C}}{\forall}\ \underset{n=\overline{1,N}}{\forall}\ \underset{h=\overline{1,H}}{\forall} \left[\left(\sum_{h=1}^{H} x_n^{ih} = 1 \right) \wedge \left(\sum_{h=1}^{H} x_n^{jh} = 1 \right) \wedge \right.$$
$$\left. \wedge \left(a_{ij} = 1 \right) \wedge \left(i \ne j \right) \right] \Rightarrow \left(y_i^n + d_i \le y_j^n \right)$$

3. If the exercise no. j was carried out after exercise no. i and by pilot n, this exercise also had to be planned or performed by pilot n:

$$\underset{i,j=\overline{1,C}}{\forall}\ \underset{n=\overline{1,N}}{\forall} \left[\left(a_{ij} = 1 \right) \wedge \left(\sum_{h=1}^{H} x_n^{jh} = 1 \right) \right]$$
$$\Rightarrow \left[\left(\sum_{h=1}^{H} x_n^{ih} = 1 \right) \vee \left(f_i^n = 1 \right) \right]$$

4. The duration of all trainings should not be less than the moment of the end of each planned exercise:

$$Z = \max\left(y_c^n + d_c\right), \quad n = \overline{1,N}, c = \overline{1,C}$$

5. No more aircraft can be used during the exercise than are available:

$$\underset{h=\overline{1,H}}{\forall} \underset{t\geq 0}{\forall} \sum_{c=1}^{C} \left| \begin{matrix} n = \overline{1,N} : \\ \left(y_c^n \leq t \leq y_c^n + d_c \wedge x_n^{ch} = 1\right) \end{matrix} \right| \leq SP_h$$

6. For each exercise, exactly one single Aircraft is used:

$$\underset{c=\overline{1,C}}{\forall} \underset{n=\overline{1,N}}{\forall} b_c^n = 1 \Rightarrow \sum_{h=1}^{H} x_n^{ch} = 1$$

7. A single Aircraft allows one pilot to be trained on a single flight:

$$\underset{c=\overline{1,C}}{\forall} \underset{n=\overline{1,N}}{\forall} \sum_{h=1}^{H} x_n^{ch} \leq 1$$

8. In accordance with the training schedule, an exercise with an earlier number cannot require an exercise with a later number. Such a requirement may be represented by an acyclical graph:

$$\underset{i>j}{\forall} a_{ij} = 0$$

9. The duration of all training courses should be kept to a minimum:

$$T = Z \xrightarrow[\left\langle \left[x_n^{ch}\right]_{N\times C\times H}, \left[y_c^n\right]_{N\times C} \right\rangle]{} \min$$

3.3. The division of the characteristics into decision variables, criteria and data

List of data:

$$a = \left\langle N, C, \left[a_{ij}\right]_{i=\overline{1,C}, j=\overline{1,C}}, \left[b_c^n\right]_{c=\overline{1,C}, n=\overline{1,N}}, H, \right.$$
$$\left\langle SP_h \right\rangle_{h=\overline{1,H}}, \left\langle d_c \right\rangle_{c=\overline{1,C}}, \left[e_c^h\right]_{c=\overline{1,C}, h=\overline{1,H}},$$
$$\left. \left[f_c^n\right]_{c=\overline{1,C}, n=\overline{1,N}} \right\rangle$$

List of decision variables:

$$q = \left\langle \left[x_n^{ch}\right]_{N\times C\times H}, \left[y_c^n\right]_{N\times C} \right\rangle$$

Criterion:

$$Z$$

3.4. Analysis of the information content

The decision-maker will know the values of all the data. The decision-maker will not know the values of the following attributes:

- Z - the duration of all training, as this is the result of decisions taken,
- $x_n^{ch} = \begin{cases} 1 & \text{if is planned to perform exercise no. } c \text{ by pilot no. } n \text{ using aircraft of type } h \\ 0 & \text{otherwise} \end{cases}$,
  $c = \overline{1,C}, n = \overline{1,N}, h = \overline{1,H}$, since it is a decision that decision-maker wishes to be taken,
- $y_c^n \in ˇ^{+}$ - the moment when exercise no. c begins by pilot no. n, $c = \overline{1,C}, n = \overline{1,N}$, since it is a decision that decision-maker wishes to be taken.

3.5. Definition of sets of correct values, limit values for decision variables and possible criteria values

The set of correct data values may be defined as follows [2]:

$$A = \mathrm{P} \times \mathrm{P} \times \mathrm{B}^{C\times C} \times \mathrm{B}^{C\times N} \times \mathrm{P} \times P^{H} \times \left(ˇ_0^{+}\right)^{C} \times \mathrm{B}^{C\times H} \times \mathrm{B}^{C\times N}$$

A set of acceptable values for decision variables is as follows [3]:

$$\Omega(a) = \left\{ \left\langle \left[x_n^{ch}\right]_{N\times C\times H}, \left[y_c^n\right]_{N\times C}, Z \right\rangle \in \right.$$
$$\in \mathrm{B}^{N\times C\times H} \times \left(R_0^{+}\right)^{N\times C} \times R_0^{+} :$$
$$\left(\underset{c=\overline{1,C}}{\forall} \underset{n=\overline{1,N}}{\forall} \underset{h=\overline{1,H}}{\forall} x_n^{ch} \leq e_h^c \right) \wedge$$
$$\wedge \left\{ \left[\left(\sum_{h=1}^{H} x_n^{ih} = 1 \right) \wedge \left(\sum_{h=1}^{H} x_n^{jh} = 1 \right) \wedge \right. \right.$$
$$\left. \wedge \left(a_{ij} = 1\right) \wedge \left(i \neq j\right) \right] \Rightarrow \left(y_i^n + d_i \leq y_j^n\right) \Big\} \wedge$$

$$\wedge\left\{\left[\left(a_{ij}=1\right)\wedge\left(\sum_{h=1}^{H}x_n^{jh}=1\right)\right]\right.$$

$$\left.\Rightarrow\left[\left(\sum_{h=1}^{H}x_n^{ih}=1\right)\vee\left(f_i^n=1\right)\right]\right\}\wedge$$

$$\wedge\left[Z\le\left(y_c^n+d_c\right),\quad n=\overline{1,N},c=\overline{1,C}\right]\wedge$$

$$\wedge\left(\underset{h=\overline{0,H}}{\forall}\ \underset{t\ge 0}{\forall}\left|U_h(t)\right|\le SP_h\right)\wedge$$

$$\wedge\left(b_c^n=1\Leftrightarrow\sum_{h=1}^{H}x_n^{ch}=1\right)\Bigg\}$$

3.6. A function to assess the achievement of the aim

The task is a one-criteria task of maximizing the achievement of the result, so no function to assess the achievement of the aim is required [4].

3.7. Draft formulation of the optimization task

For the data $a\in A$ determine such

$$q^*=\left\langle\left[x_n^{*ch}\right]_{N\times C\times H},\left[y_c^{*n}\right]_{N\times C}\right\rangle\in\Omega(a)$$

that

$$Z\left(q^*\right)=Z^*=\min_{\left\langle\left[x_n^{ch}\right]_{N\times C\times H},\left[y_c^n\right]_{N\times C}\right\rangle}Z\,.$$

5. Conclusion

In the presented article the model of the training of the pilots in PAF and 4th SLSz is described. The model is based on the current instructions and programmes, which are implemented in Polish Army. Additionally, the algorithm is described, which allows to find the best order of making the exercises.
This contribution stands at the beginning of research thesis on optimization of aviation pilot training process and subsequent development will be the subject of future papers. The content of this paper, introducing the primary mathematical model for pilot training process, is the first paper of the series.
The key feature of the model, is the use of a prediction function to optimize the training of the pilots. Planed simulation studies, as well as comparison with data collected in Polish Air Force Academy, will give the information, if the model can be used in the process of planning the training, or should be optimized.

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