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# USE OF DIMENSIONAL ANALYSIS FOR DETERMINING THE DYNAMIC COEFFICIENT OF THE CRANE SUPPORTING STRUCTURE CAUSED BY DRIVING OVER UNEVEN RAILS

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#### Abstract

The paper presents a way of describing the vertical dynamic load of the crane supporting structure caused by driving over uneven surfaces with the use of a dimensional analysis and similarity theory model. The applied method allows to conduct an initial, theoretical and qualitative analysis and to designate dimensionless parameters affecting the studied process. The obtained values were compared with the dynamic coefficients calculated on the basis of current standards.

#### Keywords:

cranes, dynamic load, dimensional analysis

#### INTRODUCTION

Cranes are an important group of engineering machines. They are used for horizontal and vertical transfer of loads, including lifting, lowering and moving them. The cyclical operation of cranes leads to the development of variable dynamic loads caused not only by the working motion connected with the relocation of loads, but also by the movements related to driving.

Some imperfections in the form of thresholds or crevices, which are the cause of the development of vertical dynamic loads in the load bearing structure, may occur on crane rail joints. The knowledge of these dynamic loads is essential for the correct dimensioning of cranes.

The primary sources and standards [1,2,4-6] present dynamic models for the calculation of dynamic factors or they give the correct values of the factors increasing computational static loads depending on unladen mass. The value of these dynamic loads is influenced by numerous factors related to, e.g. the construction of the load bearing structure and the crane wheels and steering system (the complexity of a wheel set and its distribution on the railway subgrade) [1].

The tool which is frequently used in engineering to search for new solutions to investigated issues is the dimensional analysis. Its use may help to adopt the correct model [3,7] and appropriately plan an experiment by selecting the factors which significantly influence the examined process. The results of the research conducted in this way can be divided into the classes of objects and processes meeting the adopted criteria of model probability.

The dimensional analysis and probability theory allow to conduct a preliminary qualitytheoretical analysis and to select dimensionless parameters influencing the examined processes and phenomena.

Obtained results provide limited, and sometimes even trivial, solutions [3]. Despite its simplicity, the discussed method, when used in the investigation of new issues, requires certain experience and a deeper understanding of the essence of the examined process.

# 1. MODEL FOR THE DETERMINATION OF VERTICAL DYNAMIC LOADS OF THE CRANE SUPPORTING STRUCTURE CAUSED BY DRIVING OVER UNEVEN RAILS

Dynamic loads  $F_d$  are determined by building discrete models of various numbers of degrees of freedom. Both flat and spatial models are used in calculations. The development of the models always has to be preceded by a detailed analysis and selection of the factors which have a direct influence on the needed dynamic loads values occurring at some selected points, bonds and elements of the crane supporting structures. It is important to use a simple dynamic model [4,5] in the cranes for which it is appropriate.

A single mass dynamic model (Fig. 1.), which is a counterpart of the model presented in the standard [5], was used in the examination of the dynamic loads of supporting structures resulting from driving on uneven surfaces, conducted with the use of the dimensional analysis.

The following assumptions were adopted in the model presented in Fig. 1:

- the crane drives over an uneven place with constant velocity v the change of velocity at the uneven place is negligibly small;
- cz is the vertical substitute rigidity of the crane supporting structure;
- reduced mass mz is the sum of lifted mass, winch mass and reduced to the gravity centre range of crane mass;
- dynamic impact of lines was neglected it was assumed that the lifted mass is at its highest position;
- h roughness function describing a threshold or a crevice.



**Fig. 1.** Single mass dynamic model of the crane:  $m_z$  – reduced crane mass and lifted load mass, v – velocity,  $c_z$  – vertical substitute rigidity of the supporting structure, h – roughness function

Source: [5]

# 2. DESCRIPTION OF THE DYNAMIC LOADS OF THE CRANE SUPPORTING STRUCTURE CAUSED BY DRIVING ON UNEVEN SURFACES WITH THE USE OF THE DIMENSIONAL ANALYSIS

Taking into consideration the above assumptions adopted in the model construction, the vertical dynamic loads of crane supporting structure  $F_d$  can be expressed by function  $\Phi$ :

$$F_d = \Phi[m_z, g, c_z, v, h] \tag{1}$$

where:

F<sub>d</sub> – vertical dynamic load of the crane supporting structure [N];

m<sub>z</sub> - reduced mass [kg];

g – acceleration due to gravity [ms<sup>-2</sup>];

c<sub>z</sub> – vertical substitute rigidity of the crane supporting structure [Nm<sup>-1</sup>];

v – crane velocity [ms<sup>-1</sup>];

h – roughness function [m].

are dimensional arguments of function  $\Phi$  describing the examined process.

The issue is reduced to finding a relation between the arguments of the following dimensions:

$$[m_z] = [m^0 s^0 kg^1];$$
  

$$[g] = [m^1 s^{-2} kg^0];$$
  

$$[c_z] = [m^0 s^{-2} kg^1];$$
  

$$[v] = [m^1 s^{-1} kg^0];$$
  

$$[h] = [m^1 s^0 kg^0].$$

The very dimensional analysis does not provide any particular indications as to the choice of the dimensional basis (dimensionally independent values) describing the issue under discussion. For the purpose of determining the optimum dimensional basis, operations were performed on a few bases. The operations were based on experience and the knowledge of the examined phenomenon.

On this basis three values  $m_z$ , g,  $c_z$  were selected, they allow to properly describe the phenomena taking place in the analysed issue. The selected arguments are dimensionally independent because the determinant of the matrix of power exponents is different from zero 0:

$$\begin{bmatrix} 0 & 0 & 1 \\ 1 & -2 & 0 \\ 0 & -2 & 1 \end{bmatrix} = -2 \neq 0$$
(2)

The other dimensional values v, h should be expressed by  $m_z$ , g,  $c_z$ . In the case of v the following dependence is obtained:

$$v = \varphi_1[m_z, g, c_z] \tag{3}$$

$$v = \varphi_1 m_z^{a_1} g^{a_2} c_z^{a_3}$$
(4)

The comparison of the left and right side of equation (4):

$$m^{1}s^{-1} = kg^{a_{1}}(ms^{-2})^{a_{2}}(kg^{1}s^{-2})^{a_{3}}$$

the following system of equations is obtained:

$$\begin{cases} a_1 + a_3 = 0\\ a_2 = 1\\ -2a_2 - 2a_3 = -1 \end{cases}$$
(5)

its solution is as follows:

$$a_1 = \frac{1}{2}$$
  $a_2 = 1$   $a_3 = -\frac{1}{2}$ 

after the substitution of  $a_1$ ,  $a_2$ ,  $a_3$  in equation (4)

$$v = \varphi_1 m_z^{\frac{1}{2}} g^1 c_z^{-\frac{1}{2}}$$
(6)

and using theorem  $\pi$  [7] the following formula is obtained:

$$\varphi_1 = \sqrt{\frac{c_z}{m_z}} \frac{v}{g}$$
(7)

where  $\sqrt{\frac{c_z}{m_z}} = \omega$  corresponds with the first frequency of the angular frequency of the supporting structure, then:

$$\varphi_1 = \frac{\mathbf{v}\omega}{g} \tag{8}$$

The procedure with the other dimensionally dependent value *h* is analogical:

$$h = \varphi_2[m_z, g, c_z] \tag{9}$$

$$h = \varphi_2 m_z^{a_1} g^{a_2} c_z^{a_3}$$
(10)

$$m^{1} = kg^{a_{1}} (ms^{-2})^{a_{2}} (kg^{1}s^{-2})^{a_{3}}$$

$$\begin{cases} a_{1} + a_{3} = 0 \\ a_{2} = 1 \\ -2a_{2} - 2a_{3} = 0 \end{cases}$$
(11)

The solution of the system of equations is as follows:

 $a_1 = 1$   $a_2 = 1$   $a_3 = -1$ 

After the substitution of  $a_1$ ,  $a_2$ ,  $a_3$  in equation (10) and using theorem  $\pi$  the following formula is obtained:

$$\varphi_2 = \frac{c_z h}{m_z g} \tag{12}$$

after transformations:

$$\varphi_2 = \frac{\omega^2 \mathbf{h}}{g} \tag{13}$$

Taking into account the above dimensionless coefficients  $\varphi_1$  and  $\varphi_2$  and using equation (1), it can be stated:

$$F_{d} = f_{d}(\varphi_{1}, \varphi_{2})m_{z}^{a_{1}}g^{a_{2}}c_{z}^{a_{3}}$$
(14)

where:

$$f_d$$
 – numerical function of dimensionless arguments  $\varphi_1 = \frac{\nabla \omega}{g}$ ;  $\varphi_2 = \frac{\omega^2 h}{g}$ .

As a result of the determination of power exponents  $a_1$ ,  $a_2$ ,  $a_3$  in equation (14), a dimensionally invariant homogenous form of the function describing the dynamic loads of the crane supporting structure caused by driving on uneven rails was obtained:

$$F_{d} = \left[\frac{\mathbf{v}\boldsymbol{\omega}}{g}, \frac{\boldsymbol{\omega}^{2}\mathbf{h}}{g}\right] m_{z}g$$
(15)

The analysis of dependence (15) allows to observe that in the case of the first numerical function, the value of vertical dynamic loads of the crane supporting structure caused by driving on uneven rails is proportional to the velocity of the crane and the free vibration frequency of the supporting structure, while in the second case it is proportional to the square of the free vibration frequency and the shape and size of the uneven place.

It should be noted that the obtained dependencies (8) and (13) can represent the formulae allowing to determine the value of the dynamic load coefficient of the supporting structure caused by driving on uneven rails.

### 3. RESULTS AND DISCUSSION

#### 3.1. Standard approach to the determination of the analysed dynamic loads

According to the binding standards [4,5], the value of dynamic loads caused by driving on uneven rails should be determined by multiplying the gravity force of the crane by coefficient  $\Phi_4$ , whose values are stated in relevant European standards. Actually its values will be defined for particular cranes with account for the detailed tolerances of rails and foundation conditions. In the case of a lack of coefficient  $\Phi_4$  value, it is possible to determine this value using the simplified single mass dynamic model of the crane supporting structure (Fig. 1) and calculate it in accordance with the relevant formula (16), in which component  $\delta_4$  defines the dynamic coefficient (dynamic surplus) of the crane supporting structure caused by driving over obstacles, such as thresholds or crevices, which is equal to the ratio of dynamic force to static load [2].

$$\phi_4 = 1 + \delta_4 \tag{16}$$

After the transformation of standard dependencies [4,5], the value of coefficient  $\delta_4$  can be expressed in the following way:

- in the case of a threshold:

$$\delta_4 = \left(\frac{\pi}{2}\right)^2 \frac{v^2}{gR} \xi_s \tag{17}$$

– in the case of a crevice:

$$\delta_4 = \left(\frac{\pi}{2}\right)^2 \frac{v^2}{gR} \xi_G \tag{18}$$

where:

v – constant velocity of the crane [m/s],

R – road wheel radius [m],

 $g - \text{gravity} [\text{m/s}^2],$ 

 $\xi_s(\alpha_s)$ ,  $\xi_G(\alpha_G)$  – coefficients of the roughness function defining the maximum excitation after a wheel goes over an uneven rail.

The values of coefficients  $\xi_S(\alpha_S)$ ,  $\xi_G(\alpha_G)$  can be determined analytically using the expression given in standard [5] or on the basis of the chart (Fig. 5 in standard [4]), additionally:

$$\xi_s = \frac{\alpha_s^2}{1 - \alpha_s^2} \sqrt{2 - 2\cos(2\pi\alpha_s)}$$
(19)

$$\xi_G = \frac{\alpha_G^2}{1 - \alpha_G^2} \sqrt{2 - 2\cos(2\pi\alpha_G)}$$
(20)

$$\alpha_G = \frac{f_q e_G}{v} \tag{21}$$

$$\alpha_s = \frac{2f_q h}{v} \sqrt{\frac{2R}{h}}$$
(22)

$$f_q = \frac{1}{2\pi} \sqrt{\frac{c_u}{m_z}}$$
(23)

where:

h – height of the threshold [m],

 $e_G$  – crevice width [m],

 $f_q$  – vibration frequency of the single mass crane model [Hz],

 $c_z$  – substitute rigidity of the crane supporting structure [N/m],

 $m_z$  – substitute mass of the crane and the lifted load [kg].

## 3.2. Validation of results and discussion

The validation of the dynamic loading with the values determined on the basis of the binding standards was conducted for the purpose of the determination of the usefulness and correctness of the dependence for the calculation of the dynamic loads of the crane supporting structure, caused by driving over uneven rails, obtained on the basis of the dimensional analysis.

Table 1 presents sample values of dynamic coefficients  $\delta_4$  calculated according to formulae (8), (13) and (17). The values were determined for the data relevant for overhead travelling cranes. It was assumed that all wheels go over a threshold which is h=0.001 m high at the same time, road wheel radius is R=0.1 m. Because the analysed formulae do not take into account the influence of the number of wheels simultaneously driving over the uneven place, in the calculations the most unfavourable, and at the same time the least probable case when all wheels drive over the same obstacle at the same time, was assumed.

The values of dynamic coefficients calculated on the basis of the formulae used in standards and the conducted dimensional analysis, presented in Table 1, allow to observe that the free vibration of the crane supporting structure has a significant influ-

ence on them. The velocity of driving results also in increased dynamic loads in the case of formulae (8) and (17), however, the values obtained from dependence (8) significantly deviate from those determined on the basis of the standards. Hence, it can be stated that the values of the dynamic coefficient determined on the basis of dependence (8) are not very useful.

	Free vibration	Free vibration	Calculated values of dynamic		
Crane velocity			coefficients $\delta_4$		
	frequency of	frequency of			Using
	the crane	the crane	Using	Using	standards
	supporting	supporting	formula	formula	[4,5] and
	structure	structure	(8)	(13)	formula
			(-)	()	(17)
m/s	Hz	rad/s	_	-	-
0.3	2	12.6	0.38	0.02	0.02
	5	31.4	0.96	0.10	0.10
	10	62.8	1.92	0.40	0.33
0.5	2	12.6	0.64	0.02	0.02
	5	31.4	1.60	0.10	0.10
	10	62.8	3.20	0.40	0.37
1.0	2	12.6	1.28	0.02	0.02
	5	31.4	3.20	0.10	0.10
	10	62.8	6.40	0.40	0.39

**Table 1.** Values of the calculated dynamic coefficients  $\delta_4$ , R=0.1 m, h=0.001 m

# Source: Own work

In the case of the low frequency of the free vibration of the crane supporting structure for velocities of 0.3 and 0.5 [m/s], the values of dynamic surpluses determined from formula (13) are similar to those calculated using formula (17) (Fig. 2). However, the increase of the vibration frequency of dynamic coefficient  $\delta_4$  obtained from dependence (13) results in the higher values of surpluses.

With the increase in crane velocity, the dynamic loads calculated using dependence (13), obtained from the dimensional analysis, are comparable with the standard ones.

On the basis of Fig. 2, it can be observed that in the adopted calculation variant, crane velocity has a small impact on the value of dynamic loads. The dynamic loads of the crane supporting structure caused by driving on uneven rails grow with the increase in velocity, however, the growth is limited by a threshold value. When this value is exceeded, they reach their maximum and remain at a fixed level, they are also analogous to the function described by formula (13).



Source: Own study

# CONCLUSIONS

The dimensional analysis allows to conduct the quality-theoretical analysis of the dynamic loads of the crane supporting structure caused by driving on uneven rails.

The dependencies determined using the presented method allow to adequately define the values of vertical dynamic loads.

The use of the dimensional analysis, despite its simplicity and elementary nature, requires certain experience and in-depth understanding of the essence of the investigated phenomena, and in some cases the obtained results may be wrong or they may not be applicable.

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# **BIOGRAPHICAL NOTE**

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