Journal of Polish Safety and Reliability Association
Summer Safety and Reliability Seminars, Volume 7, Number 1, 2016

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General approach to Baltic electric cable critical infrastructure network operation process modelling

Keywords

critical infrastructure, network, operation process, energy distribution, high-voltage direct current link

Abstract

In the paper, the critical infrastructure operation process is defined and its main parameters are fixed. Next, a general model of operation process of critical infrastructure network is defined and its parameters are described. A special case of the general when its component critical infrastructures are independent model is considered and applied to the Baltic Electric Cable Critical Infrastructure Network.

1. Introduction

We study the general model of operation process of critical infrastructure network and particularly investigate the case in which its critical infrastructures are independent, with the goal to apply our analysis to power grid.
The power grid provides crucial basis for the functioning of various other sectors, thus making them dependent on it. As a result an interruption in the power grid service will predicatively have a significantly destructive influence on the health, safety, security, economics, and social conditions of large human communities and territories.
As we define it in Section 2, an electric grid can be considered to be a critical infrastructure and the interconnected and interdependent network of electric cables can constitute the critical infrastructure network.
Moreover, in following Sections of the article we: define the joint operation process of a network of critical infrastructures, Section 3, considers a case of the joint operation process when the critical infrastructure's operating processes are independent, Section 5, describes the Baltic Electric Cable Critical Infrastructure Network (BECCIN). In Section 5, we apply the proposed model to the BECCIN operation process.

2. Modelling of critical infrastructure operation process

We assume, that there are k, $k \in N$, of interconnected and interdependent critical infrastructures interacting directly and indirectly at various levels of their complexity and operating activity within this area. Moreover, we suppose that the operation processes of these critical infrastructures have an influence on both the infrastructures' safety and the safety of environments, in which they operate.
To describe their influence, we start by constructing a general model of the Critical Infrastructures Network (CIN) and its general operation process. To do this, we assume that the CIN is composed of k critical infrastructures

$$CI^{(i)}, \ i = 1,2,...,k,$$

that during their operation process can take respectively $\nu^{(i)}$, $\nu^{(i)} \in N$, different operation states

$$z_1^{(i)}, z_2^{(i)}, ..., z_{\nu^{(i)}}^{(i)}, \quad i = 1,2,...,k.$$

Further, we define respectively those critical infrastructures' operation processes

$$Z^{(i)}(t),\ t \in (0,+\infty),\ i = 1,2,...,k, \tag{1}$$

with discrete operation states from the sets

$$Z^{(i)} = \{z_1^{(i)}, z_2^{(i)},..., z_{\nu^{(i)}}^{(i)}\},\ i = 1,2,...,k. \tag{2}$$

To model this processes, we can assume that the critical infrastructure operation processes $Z^{(i)}(t)$, $t \in (0,+\infty)$, $i = 1,2,...,k,$ are semi-Markov processes [4]-[6] with the conditional sojourn times $\theta_{bl}^{(i)}$ at the operation states $z_b^{(i)}$ when its next operation state is $z_l^{(i)}$, $b,l = 1,2,...,\nu^{(i)}$, $b \neq l.$
Under these assumptions, the particular critical infrastructure $CI^{(i)}$, $i = 1,2,...,k,$ with the operation process $Z^{(i)}(t)$, $t \in (0,+\infty)$, $i = 1,2,...,k,$ may be described by the following parameters:
- the vector

$$[p_b^{(i)}(0)]_{1x\nu^{(i)}} = [p_1^{(i)}(0), p_2^{(i)}(0),..., p_{\nu^{(i)}}^{(i)}(0)] \tag{3}$$

of the initial probabilities

$$p_b^{(i)}(0) = P(Z^{(i)}(0) = z_b^{(i)}), \quad b = 1,2,...,\nu^{(i)},\ i = 1,2,...,k, \tag{4}$$

of the critical infrastructure $CI^{(i)}$, $i = 1,2,...,k,$ operation process $Z^{(i)}(t)$ staying at particular operation states at the moment $t = 0$;
- the matrix

$$[p_{bl}^{(i)}]_{\nu^{(i)} \times \nu^{(i)}} = \begin{bmatrix} p_{11}^{(i)} & p_{12}^{(i)} & \cdots & p_{1\nu^{(i)}}^{(i)} \\ p_{21}^{(i)} & p_{22}^{(i)} & \cdots & p_{2\nu^{(i)}}^{(i)} \\ \cdots & & & \\ p_{\nu^{(i)}1}^{(i)} & p_{\nu^{(i)}2}^{(i)} & \cdots & p_{\nu^{(i)}\nu^{(i)}}^{(i)} \end{bmatrix} \tag{5}$$

of the probabilities

$$p_{bl}^{(i)},\ b,l = 1,2,...,\nu^{(i)},\ b \neq l, \tag{6}$$

That the critical infrastructure operation process $Z^{(i)}(t)$ transitions between the operation states $z_b^{(i)}$ and $z_l^{(i)}$, where by formal agreement

$$p_{bb}^{(i)} = 0 \text{ for } b = 1,2,...,\nu^{(i)},\ i = 1,2,...,k, \tag{7}$$

- the matrix

$$[H_{bl}^{(i)}(t)]_{\nu^{(i)} \times \nu^{(i)}} = \begin{bmatrix} H_{11}^{(i)}(t) & H_{12}^{(i)}(t) & \cdots & H_{1\nu^{(i)}}^{(i)}(t) \\ H_{21}^{(i)}(t) & H_{22}^{(i)}(t) & \cdots & H_{2\nu^{(i)}}^{(i)}(t) \\ \cdots & & & \\ H_{\nu^{(i)}1}^{(i)}(t) & H_{\nu^{(i)}2}^{(i)}(t) & \cdots & H_{\nu^{(i)}\nu^{(i)}}^{(i)}(t) \end{bmatrix} \tag{8}$$

of the conditional distribution functions

$$H_{bl}^{(i)}(t) = P(\theta_{bl}^{(i)} < t),\ ,\ b,l = 1,2,...,\nu^{(i)}, \quad b \neq l,\ i = 1,2,...,k, \tag{9}$$

of the critical infrastructure operation process $Z^{(i)}(t)$ conditional sojourn times $\theta_{bl}^{(i)}$ at the operation states $z_b^{(i)}$ when the next operation state is $z_l^{(i)}$, and where by formal agreement

$$H_{bb}^{(i)}(t) = 0 \text{ for } b = 1,2,...,\nu^{(i)},\ i = 1,2,...,k.$$

Under these assumptions, the mean values of the critical infrastructure operation process conditional sojourn times $\theta_{bl}^{(i)}$ are given by

$$M_{bl}^{(i)} = E[\theta_{bl}^{(i)}] = \int_0^\infty t dH_{bl}^{(i)}(t), \quad b,l = 1,2,...,\nu^{(i)},\ b \neq l,\ i = 1,2,...,k. \tag{10}$$

By the formula for total probability the unconditional distribution functions of the sojourn times $\theta_b^{(i)}$ of the critical infrastructure operation process $Z^{(i)}(t)$ at the operation states z_b, $b = 1,2,...,\nu^{(i)}$, are given by

$$H_b^{(i)}(t) = \sum_{l=1}^{\nu^{(i)}} p_{bl}^{(i)} H_{bl}^{(i)}(t), \quad b = 1,2,...,\nu^{(i)},\ i = 1,2,..k. \tag{11}$$

Hence, the mean values $E[\theta_b^{(i)}]$ of the critical infrastructure operation process unconditional sojourn times $\theta_b^{(i)}$ in the particular operation states are given by

$$M_b^{(i)} = E[\theta_b^{(i)}] = \sum_{l=1}^{\nu^{(i)}} p_{bl}^{(i)} M_{bl}^{(i)}, \quad b = 1,2,...,\nu^{(i)},\ i = 1,2,...,k, \tag{12}$$

where $M_{bl}^{(i)}$ are defined by (10).

Moreover, it is well known that the limit values of the critical infrastructure operation process transient probabilities at the particular operation states

$$p_b^{(i)}(t) = P(Z^{(i)}(t) = z_b),\ t \in <0,+\infty),$$
$$b = 1,2,\ldots,v^{(i)},\ i = 1,2,\ldots,k,$$

are given by

$$p_b^{(i)} = \lim_{t\to\infty} p_b^{(i)}(t), \quad (13)$$
$$b = 1,2,\ldots,v^{(i)},\ \ i = 1,2,\ldots,k.$$

Other interesting characteristics of the operation process $Z^{(i)}(t)$ possible to obtain are its total sojourn times $\hat{\theta}_b^{(i)}$ in the particular operation states z_b, $b = 1,2,\ldots,v^{(i)}$, $i = 1,2,\ldots,k.$ It is well known [6] that the critical infrastructure operation process total sojourn times $\hat{\theta}_b^{(i)}$ in the particular operation states z_b, for sufficiently large operation time θ, have approximately normal distribution with the expected value given by

$$\hat{M}_b^{(i)} = E[\hat{\theta}_b^{(i)}] = p_b^{(i)}\theta, \quad (14)$$
$$b = 1,2,\ldots,v^{(i)},\ \ i = 1,2,\ldots,k,$$

where $p_b^{(i)}$ are given by (13).

3. General joint model of CIN operation process

Next steps in modelling the CIN operation process, taking into account just defined processes $Z^{(i)}(t)$, $t \in <0,+\infty)$, $i = 1,2,\ldots,k,$ interactions and interdependences, the joint operation process of this network of critical infrastructures can be defined in the form of the vector

$$Z(t) = [Z^{(1)}(t),\ Z^{(2)}(t),\ \ldots,\ Z^{(k)}(t)],\ t \in <0,+\infty),$$

with $v = v^{(1)} \cdot v^{(2)} \cdots v^{(k)}$ indicating discrete operation states from the set

$$Z = \{\breve{z}_1, \breve{z}_2, \ldots, \breve{z}_v\}, \quad (15)$$

where

$$\breve{z}_1 = z_{\underbrace{11\ldots1}_{k}},\ \breve{z}_2 = z_{\underbrace{21\ldots1}_{k}},\ \breve{z}_3 = z_{31\ldots1}, \ldots,\ \breve{z}_{v^{(1)}} = z_{v^{(1)}1\ldots1},$$
$$\ldots,\ \breve{z}_v = z_{v^{(1)}v^{(2)}\ldots v^{(k)}},$$

and

$$Z(t) = \breve{z}_b\ ,\ \breve{z}_b = z_{b_1 b_2 \ldots b_k}\ , b_i = 1,2,\ldots,v^{(i)},$$
$$i = 1,2,\ldots,k,$$

if and only if

$$Z^{(1)}(t) = z_{b_1}^{(1)} \wedge Z^{(2)}(t) = z_{b_2}^{(2)} \ldots \wedge Z^{(k)}(t) = z_{b_k}^{(k)},$$
$$t \in <0,+\infty).$$

To model this processes, we can assume that the joint operation processes of the network $Z(t)$, $t \in (0,+\infty)$, is a semi-Markov processes [4], [6] with the conditional sojourn times $\breve{\theta}_{bl}$ at the operation states $\breve{z}_b$ where the next operation state is $\breve{z}_l$, $b,l = 1,2,\ldots,v,\ b \neq l.$
Under these assumptions, the joint operation process $Z(t)$, $t \in <0,+\infty)$, may be described by the following parameters:

- the vector of the initial probabilities

$$[\breve{p}_b(0)]_{1xv} = [\breve{p}_1(0), \breve{p}_2(0), \ldots, \breve{p}_v(0)] \quad (16)$$

where

$$\breve{p}_b(0) = \breve{p}_{b_1 b_2 \ldots b_k}(0),\ b_i = 1,2,\ldots,v^{(i)},\ i = 1,2,\ldots,k,$$

and

$$\breve{p}_{b_1 b_2 \ldots b_k}(0)$$
$$= P(Z^{(1)}(0) = z_{b_1}^{(1)}, Z^{(2)}(0) = z_{b_2}^{(2)}, \ldots, Z^{(k)}(0) = z_{b_k}^{(k)}),$$
$$b_i = 1,2,\ldots,v^{(i)},\ i = 1,2,\ldots,k, \quad (17)$$

of the joint operation process $Z(t)$, $t \in <0,+\infty)$, staying at particular operation states $\breve{z}_b$, $b = 1,2,\ldots,v$, at the moment $t = 0$;

- the matrix of the transition probabilities $\breve{p}_{bl}$, $b,l = 1,2,\ldots,v,\ b \neq l,$ of the joint operation process $Z(t)$ from the operation state $\breve{z}_b$ into the operation state $\breve{z}_l$,

$$[\breve{p}_{bl}]_{vxv} = \begin{bmatrix} \breve{p}_{11} & \breve{p}_{12} & \cdots & \breve{p}_{1v} \\ \breve{p}_{21} & \breve{p}_{22} & \cdots & \breve{p}_{2v} \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ \breve{p}_{v1} & \cdots & \cdots & \breve{p}_{vv} \end{bmatrix}, \quad (18)$$

where by formal agreement

$$\breve{p}_{bb} = 0 \text{ for } b = 1,2,...,\nu;$$

- the matrix of joint conditional distribution functions $\breve{H}_{bl}(t_1, t_2,..., t_k)$, $b, l = 1,2,...,\nu,$ $b \neq l,$ of the critical infrastructure network operation process $Z(t)$ conditional sojourn times $\breve{\theta}_{bl}$ at the operation states $\breve{z}_b$ when the next operation state is $\breve{z}_l$,

$$[\breve{H}_{bl}(t)]_{\nu \times \nu} = \begin{bmatrix} \breve{H}_{11}(t) & \breve{H}_{12}(t) & \cdots & \breve{H}_{1\nu}(t) \\ \breve{H}_{21}(t) & \breve{H}_{22}(t) & \cdots & \breve{H}_{2\nu}(t) \\ \cdots & \cdots & \cdots & \cdots \\ \breve{H}_{\nu 1}(t) & \breve{H}_{\nu 2}(t) & \cdots & \breve{H}_{\nu\nu}(t) \end{bmatrix} \quad (19)$$

where

$$\breve{H}_{bl}(t) = P(\breve{\theta}_{bl} < t), \quad (20)$$
$$b_i, l_i = 1,2,..., \nu^{(i)}, \; b_i \neq l_i, \; i = 1,2,...,k,$$

and by formal agreement

$$\breve{H}_{bb}(t) = \breve{H}_{b_1 b_2 ... b_k \;\; b_1 b_2 ... b_k}(t) = 0 \quad (21)$$
$$b_i = 1,2,..., \nu^{(i)}, \; i = 1,2,...,k.$$

The mean value of the critical infrastructure network joint operation process conditional sojourn times $\breve{\theta}_{bl}$ is given by

$$\breve{M}_{bl} = E[\breve{\theta}_{bl}] = \int_0^\infty t d\breve{H}_{bl}(t), \quad (22)$$
$$b, l = 1,2,...,\nu, \;\; b \neq l, \;\; i = 1,2,...,k.$$

By the formula for total probability the unconditional joint distribution functions of the sojourn times $\breve{\theta}_b$ of the critical infrastructure joint operation process $Z(t)$ at the operation states $\breve{z}_b$, $b = 1,2,...,\nu,$ are given by

$$\breve{H}_b(t) = \sum_{l=1}^{\nu} \breve{p}_{bl} \breve{H}_{bl}(t), \;\; b = 1,2,...,\nu. \quad (23)$$

Hence, the mean values $E[\breve{\theta}_b]$ of the critical infrastructure network joint operation process unconditional sojourn times $\breve{\theta}_b$ in the particular operation states $\breve{z}_b$, $b = 1,2,...,\nu,$ are given by

$$\breve{M}_b = E[\breve{\theta}_b] = \sum_{l=1}^{\nu} \breve{p}_{bl} \breve{M}_{bl}, \; b = 1,2,...,\nu. \quad (24)$$

where $\breve{M}_{bl}$ are defined by (22).

Moreover, we introduce the joint operation process of critical infrastructure network transient probabilities at the particular operation states $\breve{z}_b$, $b = 1,2,...,\nu,$ as follows

$$\breve{p}_b(t) = P(Z(t) = \breve{z}_b)$$
$$= P(Z^{(1)}(t) = z^{(1)}_{b_1}, Z^{(1)}(t) = z^{(2)}_{b_2},..., Z^{(k)}(t) = z^{(k)}_{b_k}),$$
$$t \in < 0, +\infty), \;\; b = 1,2,...,\nu,$$

and their limit values

$$\breve{p}_b = \lim_{t \to \infty} \breve{p}_b(t), \;\; b = 1,2,...,\nu. \quad (25)$$

Other possible to obtain interesting characteristics of the joint operation process of network $Z(t)$ are its total sojourn times $\hat{\theta}_b$ in the particular operation states $\breve{z}_b$, $b = 1,2,...,\nu.$ It is well known [6] that the critical infrastructure network operation process total sojourn times $\hat{\theta}_b$ in the particular operation states $\breve{z}_b$, for sufficiently large operation time θ, have approximately normal distribution with the expected value given by

$$\hat{M}_b = E[\hat{\theta}_b] = \breve{p}_b \theta, \; b = 1,2,...,\nu, \quad (26)$$

4. Joint model of CIN operation process with independent components

In the case when the critical infrastructure's operating processes $Z^{(i)}(t)$, $i = 1,2,...,k,$ are independent, the joint operation process $Z(t),$ $t \in (0, +\infty),$ may be described as follows

- the vector of the initial probabilities

$$[\breve{p}_b(0)]_{1x\nu}$$

where by (17)

$$\breve{p}_b(0) = \prod_{i=1}^{k} p^{(i)}_{b_i}(0), \quad (27)$$
$$b = 1,2,...,\nu, \; b_i = 1,2,..., \nu^{(i)}, \; i = 1,2,...,k,$$

where $p_{b_i}^{(i)}(0) = P(Z^{(i)}(0) = z_{b_i}^{(i)})$, are initial probabilities of the critical infrastructure operation process $Z^{(i)}(t)$ staying at particular operation states $z_{b_i}^{(i)}$ at the moment $t = 0$;

- the matrix of the transition probabilities $\breve{p}_{bl}$, $b,l = 1,2,...,\nu$, $b \neq l$, of the joint operation process $Z(t)$ from the operation state $\breve{z}_b$ into the operation state $\breve{z}_l$,

$$[\breve{p}_{bl}]_{\nu x \nu},$$

where by (18)

$$\breve{p}_{bl}(t) = \prod_{i=1}^{k} p_{b_i l_i}^{(i)}(t), \quad (28)$$

$$b_i, l_i = 1,2,...,\nu^{(i)},\ b_i \neq l_i,\ i = 1,2,...,k,$$

and where by formal agreement

$$\breve{p}_{bb} = 0 \ \text{ for } \ b = 1,2,...,\nu;$$

- the matrix of joint conditional distribution functions $\breve{H}_{bl}(t_1, t_2,..., t_k)$, $b,l = 1,2,...,\nu$, $b \neq l$, of the critical infrastructure network operation process $Z(t)$ conditional sojourn times $\breve{\theta}_{bl}$ at the operation states $\breve{z}_b$ when the next operation state is $\breve{z}_l$,

$$[\breve{H}_{bl}(t)]_{\nu \times \nu}$$

where by (19)-(21)

$$\breve{H}_{bl}(t) = \prod_{i=1}^{k} H_{b_i l_i}^{(i)}(t),$$

(29)

$$b_i, l_i = 1,2,...,\nu^{(i)},\ b_i \neq l_i,\ i = 1,2,...,k,$$

and where by formal agreement

$$\breve{H}_{bb}(t) = 0 \ \text{ for } \ b = 1,2,...,\nu;$$

$H_{b_i l_i}^{(i)}(t) = P(\theta_{b_i l_i}^{(i)} < t)$ are the conditional distribution functions of the critical infrastructure operation process $Z^{(i)}(t)$ conditional sojourn times $\theta_{bl}^{(i)}$ at the operation states $z_{b_i}^{(i)}$ when the next operation state is $z_{l_i}^{(i)}$, i.e. the marginal conditional distribution of the critical infrastructure network joint operation process $Z(t)$ are the conditional distribution functions of the critical infrastructure operation process $Z^{(i)}(t)$, $i = 1,2,...,k.$

In this case the mean value of the critical infrastructure network joint operation process conditional sojourn times $\breve{\theta}_{bl}$ is given by

$$\breve{M}_{bl} = E[\breve{\theta}_{bl}] = \int_0^{\infty} t d\breve{H}_{bl}(t), \quad (30)$$

where $\breve{H}_{bl}(t)$ are defined by (29).

By the formula for total probability the unconditional joint distribution functions of the sojourn times $\breve{\theta}_b$ of the critical infrastructure network joint operation process $Z(t)$ at the operation states $\breve{z}_b$, $b = 1,2,...,\nu,$ are given by

$$\breve{H}_b(t) = \sum_{l=1}^{\nu} \breve{p}_{bl} \breve{H}_{bl}(t), \ b = 1,2,...,\nu. \quad (31)$$

Hence, the mean values $E[\breve{\theta}_b]$ of the critical infrastructure network joint operation process unconditional sojourn times $\breve{\theta}_b$ in the particular operation states $\breve{z}_b$, $b = 1,2,...,\nu,$ are given by

$$\breve{M}_b = E[\breve{\theta}_b] = \sum_{l=1}^{\nu} \breve{p}_{bl} \breve{M}_{bl}, \ b = 1,2,...,\nu.$$

where $\breve{M}_{bl}$ are defined by (30).

Moreover, the limit values of the network joint operation process of independent critical infrastructures transient probabilities at the particular operation states $\breve{z}_b$, $b = 1,2,...,\nu,$

$$\breve{p}_b(t)$$
$$= P(Z^{(1)}(t) = z_{b_1}, Z^{(1)}(t) = z_{b_2}),..., Z^{(k)}(t) = z_{b_k})$$
$$= \prod_{i=1}^{k} p_{b_i}^{(i)}(t) \quad (32)$$

where $p_{b_i}^{(i)}(t) = P(Z^{(i)}(t) = z_{b_i})$, $t \in <0,+\infty)$, $b_i = 1,2,...,\nu^{(i)}$, $i = 1,2,...,k,$

and by (25) , are given by

$$\breve{p}_b = \prod_{i=1}^{k} \lim_{t \to \infty} p_{b_i}^{(i)}(t), \ b = 1,2,...,\nu. \quad (33)$$

It is well known [6] that the critical infrastructure network operation process total sojourn times $\hat{\theta}_b$ in the particular operation states $\breve{z}_b$, for sufficiently large operation time θ, have approximately normal distribution with the expected value given by

$$\hat{M}_b = E[\hat{\theta}_b] = \breve{p}_b \theta, \; b = 1,2,...,\nu, \tag{34}$$

where $\breve{p}_b$ are given by (33).

5. Baltic Electric Cable Critical Infrastructure Network

Power grid is a sector upon which the other sectors dependent and any interruption of its service can have significant destructive influence on the health, safety and security, economics and social conditions of large human communities and territories. Short term outages and more serious and extended disruptions may have cascading effects on the whole society through different sectors. Thus, according to the definition given in Section 2, electric grid can be considered as a critical infrastructure and the interconnected and interdependent network of electric cables can constitute the critical infrastructure network. Further, the electric gird operating in the Baltic Sea Region is called BECCIN. The considered BECCIN placed at the Baltic seaside is composed of 11 electric critical infrastructures:

- the Electric Cable Critical Infrastructure ($ECCI^{(1)}$) EstLink 1 with Espoo in Finland and Harku in Estonia converter stations,
- the Electric Cable Critical Infrastructure ($ECCI^{(2)}$) EstLink 2 with Anttila in Finland and Püssi in Estonia converter stations,
- the Electric Cable Critical Infrastructure ($ECCI^{(3)}$) NordBalt with Nybro in Sweden and Klaipeda in Lithuania converter stations,
- the Electric Cable Critical Infrastructure ($ECCI^{(4)}$) LitPol Link with Ełk in Poland and Alytus in Lithuania converter stations,
- the Electric Cable Critical Infrastructure ($ECCI^{(5)}$) SwePol Link with Stärnö in Sweden and Slupsk in Poland converter stations,
- the Electric Cable Critical Infrastructure ($ECCI^{(6_1)}$) Fenno-Skan 1 with Dannebo in Sweden and Rauma in Finland converter stations, ($ECCI^{(6_2)}$) Fenno-Skan 2 with Finnböle in Sweden and Dannebo in Sweden converter stations,
- the Electric Cable Critical Infrastructure ($ECCI^{(7_1)}$) Konti-Skan from Gothenburg in Sweden to Aalborg in Denmark, ($ECCI^{(7_2)}$) Konti-Skan with Lindome in Sweden and Vester Hassing in Denmark converter stations,
- the Electric Cable Critical Infrastructure ($ECCI^{(8)}$) Great Belt Power Link (Storebælt HVDC) with Fraugde, Funen and Herslev, Zealand in Denmark converter stations,
- the Electric Cable Critical Infrastructure ($ECCI^{(9)}$) Kontek with Bjaeverskov in Denmark and Bentwisch in Germany converter stations,
- the Electric Cable Critical Infrastructure ($ECCI^{(10)}$) Baltic Cable with Kruseberg in Sweden and Herrenwyk in Germany converter stations,
- the Electric Cable Critical Infrastructure ($ECCI^{(11)}$) Vyborg Link with Yllikkälä, Kymi in Finland and Vyborg in Russia converter stations.

The maximal transmission capacity of Electric Cable Critical Infrastructures defined above are presented in *Table 1*:

Table 1. Technical details of Electrical Cable Critical Infrastructures based on ENTSO-E report [2]

Name of the HVDC link	Commi-sioning year	Market connection (Y/N)	Rated power, monopolar (MW)	Parallel monopolar capacity (MW)	Bipolar Capacity (MW)	Defined export direction (N-S, E-W)
Estlink 1	2006	Y	350			N-S
Estlink 2	2014	Y	650	1000		N-S
NordBalt	2015/ 2016	Y	700			E-W
LitPol	2015	Y	1000			E-W
Swepol	2000	Y	600			N-S
Feno-Skan 1	1989	Y	550			E-W
Feno-Skan 2	2011	Y	800	1350	1350	E-W
Konti-Skan 1	2008	Y	370			E-W
Konti-Skan 2	1988	Y	370	740		E-W
Storebaelt	2010	Y	600			E-W
Kontek	1986	Y	600			N-S
Baltic Cable	1994	Y	600			N-S
Vyborg Link	1981 - 2000	Y	1400			E-W

As it is defined in the ENTSO-E report [2] the technical capacity of the High-Voltage Direct Current (HVDC) link is the maximum energy ($E_{\max}$), presented in *Table 1*, that can be transmitted from the Alternating Current (AC) grid to the converter station, including all HVDC link losses. The technical capacity of the link is defined in [2] as a theoretical value and can be divided into technical capacity used for transmission (E_T) and technical capacity not used E_{TCNU}, and unavailable technical capacity (E_U). The unavailable technical capacity E_U is due to outages or limitations.

Figure 1 presents the overview of the availability and unavailability of HVDC link's statistics. In this statistical summary of 2014 it has not been included NordBalt and LitPol Link because these interconnections have just been completed at the end of 2015.

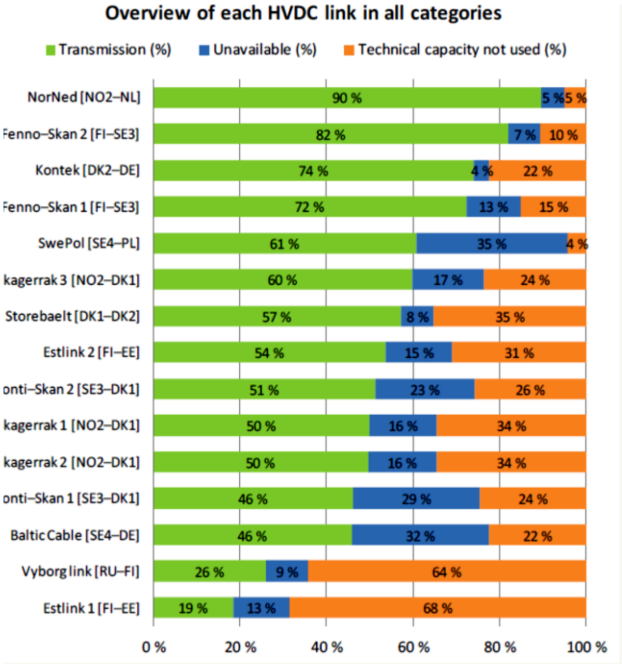

Figure 1. Annual overview of each HVDC link sorted by highest transmission in 2014 based on ENTSO-E report [2]

6. Baltic Electric Cable Critical Infrastructure Network operation process

We assume that when the critical infrastructure's operation processes $Z^{(i)}(t)$, $i=1,2,...,11$, are independent the joint operation process $Z(t)$, $t \in (0,+\infty)$, may be described as follows

- the vector of the initial probabilities

$$[\breve{p}_b(0)]_{1x\nu},\ \nu = \nu^{(1)} \cdot \nu^{(2)} \cdots \nu^{(11)},$$

where by (17)

$$\breve{p}_b(0) = \prod_{i=1}^{11} p_{b_i}^{(i)}(0), \tag{35}$$

$$b = 1,2,...,\nu,\ b_i = 1,2,...,\nu^{(i)},\ i = 1,2,...,11,$$

where $p_{b_i}^{(i)}(0) = P(Z^{(i)}(0) = z_{b_i}^{(i)})$, are initial probabilities of the critical infrastructure operation process $Z^{(i)}(t)$ staying at particular operation states $z_{b_i}^{(i)}$ at the moment $t = 0$;

- the matrix of the transition probabilities $\breve{p}_{bl}$, $b,l = 1,2,...,\nu$, $b \neq l$, of the joint operation process $Z(t)$ from the operation state $\breve{z}_b$ into the operation state $\breve{z}_l$,

$$[\breve{p}_{bl}]_{\nu x \nu},$$

where by (19)

$$\breve{p}_{bl} = \prod_{i=1}^{11} p_{b_i l_i}^{(i)}, \tag{36}$$

$$b_i, l_i = 1,2,...,\nu^{(i)},\ b_i \neq l_i,\ i = 1,2,...,11,$$

and where by formal agreement

$$\breve{p}_{bb} = 0 \text{ for } b = 1,2,...,\nu;$$

- the matrix of joint conditional distribution functions $\breve{H}_{bl}(t)$, $b,l = 1,2,...,\nu$, $b \neq l$, of the critical infrastructure network operation process $Z(t)$ conditional sojourn times $\breve{\theta}_{bl}$ at the operation states $\breve{z}_b$ when the next operation state is $\breve{z}_l$,

$$[\breve{H}_{bl}(t)]_{\nu \times \nu}$$

where by (21)

$$\breve{H}_{bl}(t) = \prod_{i=1}^{11} H_{b_i l_i}^{(i)}(t), \tag{37}$$

$$b_i, l_i = 1,2,...,\nu^{(i)},\ b_i \neq l_i,\ i = 1,2,...,11,$$

and where by formal agreement

$$\breve{H}_{bb}(t) = 0 \text{ for } b = 1,2,...,\nu;$$

$H_{b_i l_i}^{(i)}(t) = P(\theta_{b_i l_i}^{(i)} < t)$ are the conditional distribution functions of the critical infrastructure operation process $Z^{(i)}(t)$ conditional sojourn times $\theta_{bl}^{(i)}$ at the operation states $z_{b_i}^{(i)}$ when the next operation state is $z_{l_i}^{(i)}$, i.e. the marginal conditional distribution of the critical infrastructure network joint operation process $Z(t)$ are the conditional distribution functions of the critical infrastructure operation process $Z^{(i)}(t)$, $i = 1,2,...,11$.

After identification the unknown parameters of the BECCN defined by (35)-(37), its basic operation characteristics can be found by application of the formulae (30)-(34).

Conclusion

In the paper, we have established the importance of the development of the general model of operation process of critical infrastructure network and its application to power grid sector. What makes the topic of our study critical to be analysed is the fact that proper functioning of power grid is vital to the operation of various other sectors, many of which are the basis for allowing people to inhabit different territories.

In the article, we have considered a case in which the critical infrastructures are independent, and showed how it can be applied to the BECCIN operation process.

Looking forward and considering the importance of the applications of our model, our next steps will include the investigation of the multistate safety function and to create the joint operation and safety model for BECCIN.

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