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**GEOMECHANICAL CHARACTERIZATION  
BY ESTIMATING THE  $V_p/V_s$  RATIO OF  
SEDIMENTARY ROCK SAMPLES\*\***

**Abstract:** The velocity ratio  $V_p/V_s$  and elastic parameters are effective quantities for describing various physical parameters and lithological properties of rock samples. Ultrasonic measurement was used to evaluate the  $V_p/V_s$  ratio. Ultrasonic velocity estimation is widely used in the quantitative evaluation of rock materials.

Ultrasonic  $P$  and  $S$ -wave travel times were studied in one direction of cylindrical core samples with a length and diameter of 100 mm. Wave velocity analysis was carried out on 20 crystalline drill cores taken at depths ranging from 10 m to 80 m. The cores were then tested for their ultrasonic velocity. The studied samples were also geomechanically characterized using ultrasonic testing.

In this paper, the method used and a brief description of the measurement geometry are presented. The results of the calculations of  $P$  and  $S$  wave velocities,  $V_p/V_s$  ratio, Young's modulus and Poisson's coefficient are given. Finally, the studied samples were geologically classified and the  $V_p/V_s$  ratio was discussed.

**Keywords:**  $V_p/V_s$  ratio, laboratory ultrasonic measurements, geomechanical parameters, elastic modulus, seismic body waves

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## 1. INTRODUCTION

Laboratory ultrasonic velocity studies are usually performed using the direct transmission technique. This method involves measuring the travel times of the wave between two parallel faces of a cylindrical probe. The method requires careful preparation of the samples and the selection of the proper probe. Meaningful measurement results can be obtained for both longitudinal and transverse waves. However, there is always the possibility of significant errors caused by insufficient coupling of the transducers and their misalignment between the transmitting interfaces. It is reported that the first arrival of transverse wave energy is often difficult to identify, because this event is overlaid by earlier direct or reflected  $P$ -wave events. The mentioned problem can be greatly reduced when shear wave transducers are used. An appropriate ratio of longitudinal to transverse wave velocities ( $V_p/V_s$ ) can be achieved if small rock samples are tested in a loading test [1]. The loading procedure is inappropriate for some groups of rock types and sample sizes.

This paper briefly describes the equipment and procedure for measuring the velocities of compressional and shear waves in rock in the field and estimating the dynamic elastic constants of clayey anisotropic rock. The elastic constants are determined by the exposed method, which may also be referred to as ultrasonics. They are so called because the dominant frequencies of the wave phenomena produced are above the audible range. The ultrasonic elastic constants are calculated from the measured wave velocities and bulk density values.

## 2. MATERIALS AND METHODS

### Significance and previous examples of use

The main advantages of “in situ” ultrasonic testing are that it provides wave velocities and dynamic values for the elastic constants of fully intact rock samples [1, 2]. Elastic constant values may differ from those obtained by static laboratory methods or “in situ” methods – as do seismic velocities, but provide fast, in-field approximations at the laboratory scale. Ultrasonic evaluation of rock properties is useful for the preliminary prediction of static properties and water saturation on pulse velocity. These properties are in turn useful for engineering design. In most cases, ultrasonics involves the application of sound waves with a frequency greater than 20 kHz. The first approach to assess rock quality, such as calculating the dynamic modulus of elasticity, was performed by Long using two transducers attached to the concrete [3]. Since the late 1940s, the ultrasonic method has been developed [2]. Currently, ultrasonic testing belongs to the so-called NDT methods and its usefulness are still being under development.

### Measuring device and method

Estimating the properties of weak or clayey rocks often requires a large core sample, e.g. 100 mm or more in length and up to 100 mm in diameter. Fourteen rock samples were tested. A portable non-destructive digital ultrasonic flaw detector (PUNDIT LE 200), manufactured by Proceq, was coupled to longitudinal (54 kHz) and transverse (40 kHz) wave

transducers (Fig. 1). The transducers were held by hand along the length of the cylindrical samples, with synthetic gel serving as a coupling agent between the transducer and rock surfaces. Travel times for all events were measured. The total length of each recording was 800  $\mu\text{s}$  with a sampling of 0.1  $\mu\text{s}$ . The distance between transducers was equal to the length of the cylindrical samples. The first arrival time was determined using the standard energy threshold method.

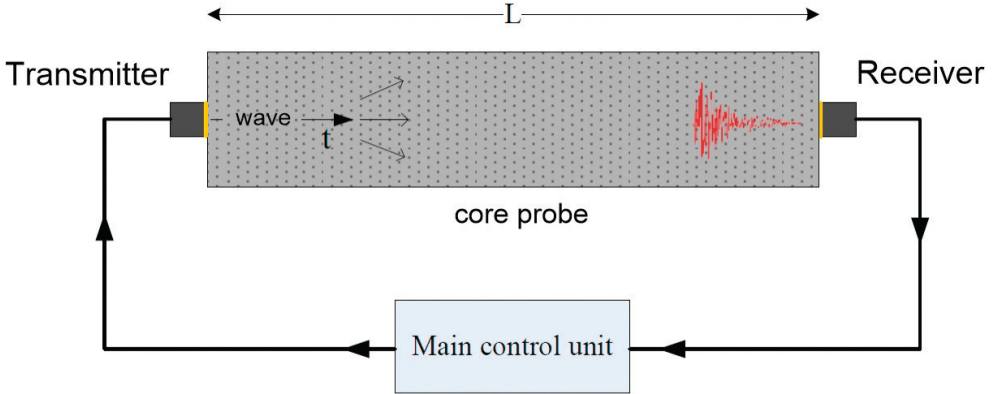


Fig. 1. Geometry and schematic of the ultrasonic system used

### Limitation of the sample dimensions

The path of the ultrasonic signal should not exceed the minimum lateral dimensions. Values greater than this may not allow reliable pulse velocities to be measured. The wavelength corresponding to the dominant frequency of the pulse in the rock is approximately related to the natural resonant frequency of the transducer and the pulse propagation velocity, (longitudinal or transverse) as follows:

$$L = (V_p, V_s) / f \quad (1)$$

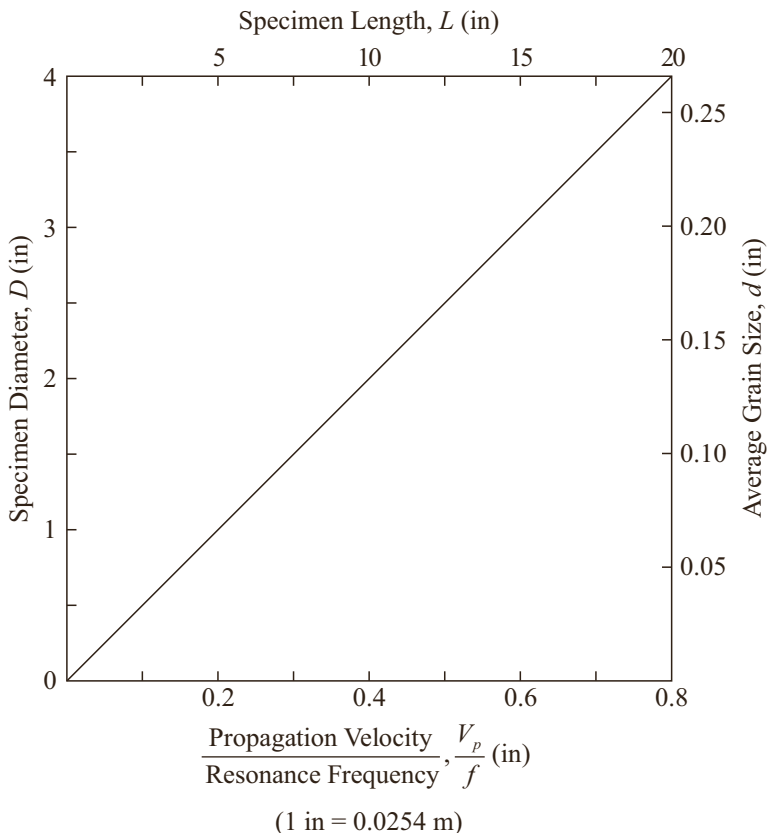
where:

- $L$  – dominant wavelength of the pulse train [in] or [m],
- $V$  – pulse propagation velocity (compression or shear) [in/s] or [m/s],
- $f$  – natural resonant frequency of the transducer [Hz].

For the test, the natural frequency was defined and it was assumed that the maximum velocity would not be greater than 2500 m/s. For this case, the length of the samples should not exceed  $5 \times 0.046$  m for the  $P$ -wave measurement. For the  $S$ -wave measurement, the length of the samples should be  $5 \times 0.037$  m. Due to the high signal attenuation in clayey material, the nominal length of the samples used was reduced by a factor of 2. This means that the probe length varied by 100 mm.

All ultrasonic measurements must be matched to the type of probes. The relationship between specimen length, diameter size and  $V_p/f$  ratio is shown in Figure 2. For a given value

of  $V_p/f$ , the appropriate values of specimen diameter  $D$  lie above the diagonal line, the line of intersection (Fig. 2). For a given diameter, the appropriate values of specimen length  $L$  lie to the left of the diagonal line.



**Fig. 2.** Diagram showing the permissible values of specimen diameter and the grain size as a function of the ratio of propagation velocity to the resonance frequency (from the American Society for Testing and Materials) [3]

### $V_p/V_s$ ratio

The formulas for calculating the velocity of the propagation of longitudinal and transverse waves (based on elastic modulus and rock density) show that the velocity of longitudinal waves is always greater than the velocity of transverse waves because:

$$V_p = V_s \sqrt{\frac{2(1-\nu)}{1-2\nu}} \quad (2)$$

where  $\nu$  – Poisson constant.

Poisson's constant ranges in a small range from 0.0 to 0.5 for solids. For rocks, it is usually in the range of 0.2 to 0.5. Therefore, when analyzing the velocity ratio  $V_p$  to  $V_s$ , it can be concluded that longitudinal waves have a velocity about 1.6 to about 3.5 times higher than transverse waves [5].

The ratio  $V_p/V_s$  represents the ratio of the velocity of the  $P$ -wave to the velocity of the  $S$ -wave. It is an important parameter in the interpretation of seismic and ultrasonic measurements in the field and in the laboratory, and in understanding the mechanism of seismic wave propagation through porous media. The  $V_p/V_s$  ratio is effective in modelling the petrophysical parameters of sedimentary rocks. It is an indicator of lithology because there is a correlation between it and the type of rock [6–8].

For sedimentary rocks it has the following values: the lowest values of  $V_p/V_s$  ratio are characteristic for sandstones (1.59–1.76), for dolomite from 1.78 to 1.84, and the highest values for limestones (1.84–1.99). For shales it has a wide range of values from 1.70 to 3.00 [6]. The  $V_p/V_s$  value for shale tends to be much broader and is higher than the  $V_p/V_s$  value for sandstone, especially in porous clastic interfaces such as Carpathian Flysch. Damage to the rock medium causes a decrease in the velocity of  $V_p$  and  $V_s$  and an increase in the  $V_p/V_s$  ratio. On the other hand, saturation of the pore space with gas can significantly reduce  $V_p/V_s$  (even at low gas volume) [8, 9].

The  $V_p/V_s$  ratio strongly depends on several parameters that affect the propagation of seismic waves or acoustic signals in rock media. These parameters include the length and frequency of the waves and signals, Young's modulus and Poisson's ratio.

### 3. RESULTS AND DISCUSSION

A brief statistical description of the studied samples is given in the tables below. There is also a figure of some correlation parameters of a characteristic sample and a brief description of the same.

#### Ultrasonic $V_p/V_s$ ratio results

Table 1 shows the results of the measurements on the samples cut from the cores. Table 2 presents the basic statistics of the petrophysical investigation results (volume density, velocity of elastic waves: longitudinal  $P$  and transverse  $S$ , the ratio of the above velocities –  $V_p/V_s$ , Poisson's ratio and Young's modulus.

The values of longitudinal wave velocity range from 271.9 to 3786.5 m/s, on average 1251.1 m/s. The values of shear wave velocity from 156.4 to 2009.7 m/s, on average 557.3 m/s. The average velocities of both types of waves for the studied samples are higher, while the medians in both cases are shifted towards lower values, indicating their dominance. The average  $V_p/V_s$  ratio is 2.327 with the min-max values of 1.501–3.596, respectively.

The minimum Poisson's ratio is 0.101, while the maximum is 0.458, with a mean of 0.347. The Young's modulus values range from 0.120 to 21.722 GPa, with a mean of 3.209 GPa. The discussed parameters are within the range of variability for claystones and sandstone occurring at shallow depths [10, 11].

**Table 1**  
Test results on samples from cores

Sample number	Type of rock	$\delta$ [g/cm <sup>3</sup> ]	$V_p$ [m/s]	$V_s$ [m/s]	$V_p/V_s$ [-]	$\nu$ [-]	$E$ [GPa]
1	shale + dusty sand	2.031	1742.504	582.891	2.989	0.437	1.983
2	shale + fine-grained sandstone	2.152	954.496	445.625	2.142	0.361	1.163
3	mud shale + sandstone	2.110	689.680	332.749	2.073	0.348	0.630
4	shale	2.164	939.367	376.633	2.494	0.404	0.862
5	shale + fine-grained sandstone crumbs	2.034	757.506	339.193	2.233	0.375	0.643
6	shale + dusty sand	1.899	1369.760	452.299	3.028	0.439	1.118
7	shale + dusty sand	1.738	390.004	244.103	1.598	0.178	0.244
8	fine-grained sandstone + calcite	2.565	3786.517	1557.781	2.431	0.398	17.405
9	mudstone/claystone	1.963	271.870	156.414	1.738	0.253	0.120
10	claystone	1.915	1344.512	463.479	2.901	0.433	1.179
11	dusty clay + shale	1.863	1573.585	437.566	3.596	0.458	1.040
12	mudstone + sandstone crumbs	2.206	1971.774	627.728	3.141	0.444	2.510
13	shale + sandstone crumbs	2.102	762.082	339.377	2.246	0.376	0.666
14	fine-grained sandstone	2.346	3017.493	2009.709	1.501	0.101	20.872
15	shale	1.846	548.963	255.496	2.149	0.362	0.328
16	fine-grained sandstone + calcite	2.645	3313.725	1778.947	1.863	0.298	21.722
17	shale	1.864	660.047	320.932	2.057	0.345	0.517
18	mud shale and shale	1.987	1456.418	409.902	3.553	0.457	0.973
19	shale + dusty sand	2.069	864.359	329.412	2.624	0.415	0.635
20	shale + dusty sand	2.101	593.156	329.577	1.800	0.277	0.583
21	shale + sandstone	2.108	1079.449	566.741	1.905	0.310	1.774
22	shale + dusty sand	2.007	1109.840	418.284	2.653	0.417	0.995
23	mud shale and shale	1.883	727.745	337.926	2.154	0.363	0.586
24	claystone	2.037	536.713	318.685	1.684	0.228	0.508
25	mudstone + dusty sand	1.950	814.042	501.168	1.624	0.195	1.170

Notations:  $\delta$  – density,  $V_p$  –  $P$ -wave velocity,  $V_s$  –  $S$ -wave velocity,  $V_p/V_s$  – ratio of  $P$  – to  $S$ -wave velocity,  $\nu$  – Poisson’s ratio,  $E$  – Young’s modulus

**Table 2**  
Statistical parameters of the properties of samples

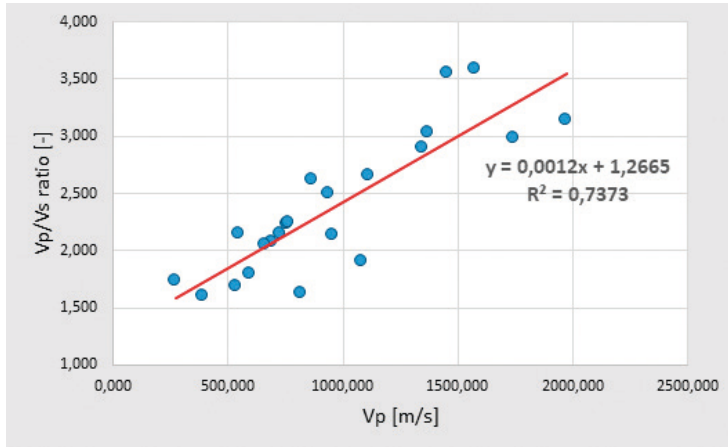
Parameter	Units	Minimum value	Mean	Median	Maximum value	Standard deviation
$\delta$	[g/cm <sup>3</sup> ]	1.738	2.063	2.034	2.645	0.211
$V_p$	[m/s]	271.870	1251.024	939.367	3786.517	909.801
$V_s$	[m/s]	156.414	557.305	409.902	2009.709	478.372
$V_p/V_s$	[-]	1.501	2.327	2.154	3.596	0.601
$\nu$	[-]	0.101	0.347	0.363	0.458	0.096
$E$	[GPa]	0.120	3.209	0.973	21.722	6.385

Notations:  $\delta$  – density,  $V_p$  –  $P$ -wave velocity,  $V_s$  –  $S$ -wave velocity,  $V_p/V_s$  – ratio of wave velocity  $P$  to  $S$ ,  $\nu$  – Poisson’s ratio,  $E$  – Young’s modulus

### Discussion

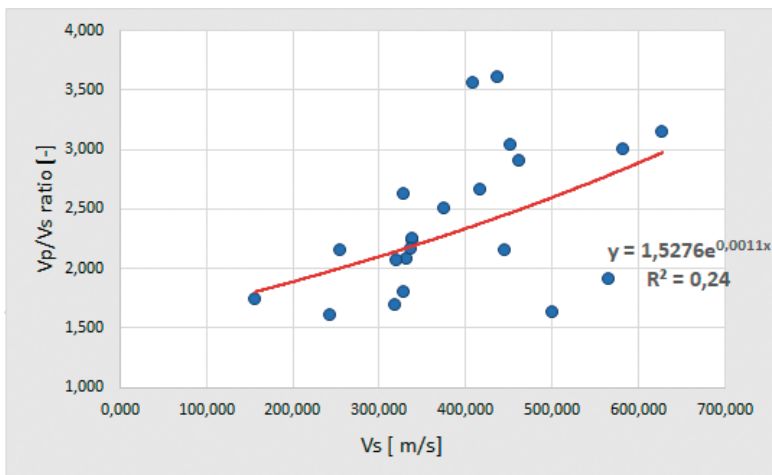
The propagation of the ultrasonic signal depends strongly on the anisotropy. In this case, the measurements must be made with the same geometry, i.e., all between parallel planar surfaces or all across the diameter. The error in  $V_p$  and  $V_s$  caused by both anisotropy and experimental error will then not normally exceed 6%. For common rock types, the respective percentage error according to the  $V_p/V_s$  ratio may be significant. For greater anisotropy, the possible percentage error in the elastic constants would be even greater.

The presented correlations were intended to show the relationship between the discussed parameters. Relationships between velocity ratio ( $V_p/V_s$ ), compressional wave velocity ( $V_p$ ), shear wave velocity ( $V_s$ ), bulk density and elastic modules (Young’s modulus, Poisson’s ratio) were determined (Figs. 3–6). The correlation between the seismic wave propagation velocities is characterized by a sufficient coefficient of determination (Fig. 3).

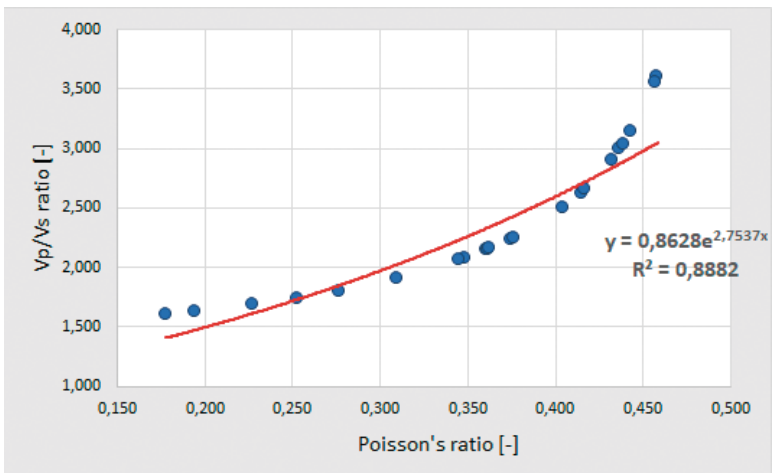


**Fig. 3.**  $P$ -wave velocity ( $V_p$ ) plotted versus  $S$ -wave velocity ( $V_s$ ) for shale from Carpathian Flysch with correlation

A high correlation is observed for the longitudinal wave propagation velocity and the  $V_p/V_s$  ratio (Fig. 4). The correlation between the shear wave propagation velocity and the  $V_p/V_s$  ratio was also verified. The scatter of the points in Figure 4 (right side) is large and the exponential correlation is poorly fitted. The relationship between the Poisson's ratio and the  $V_p/V_s$  ratio is characterized by a high coefficient of determination (Fig. 5), while for the Young's modulus (Fig. 6) a significant scatter of the data (power correlation) is observed and a poor fit is found. Results deviating from the trend line are due to measurement errors or lithological admixtures.

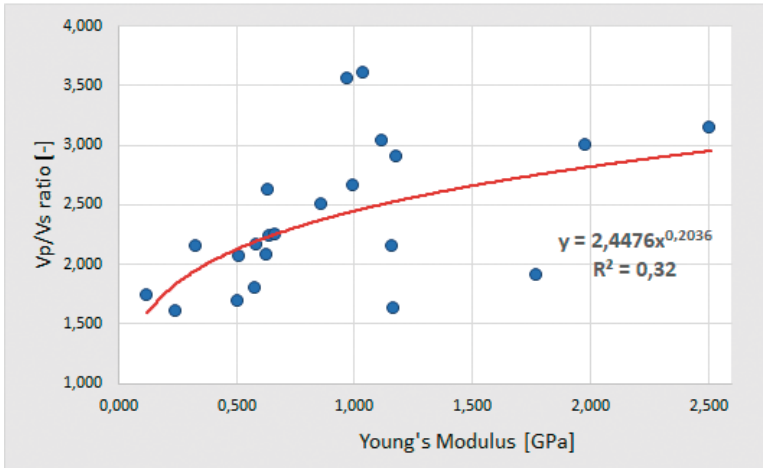


**Fig. 4.**  $P$ -wave velocity ( $V_p$ ) plotted versus  $V_p/V_s$  ratio (left site) and  $S$ -wave velocity ( $V_s$ ) plotted versus  $V_p/V_s$  ratio (right site) for shale from Carpathian Flysch with correlation



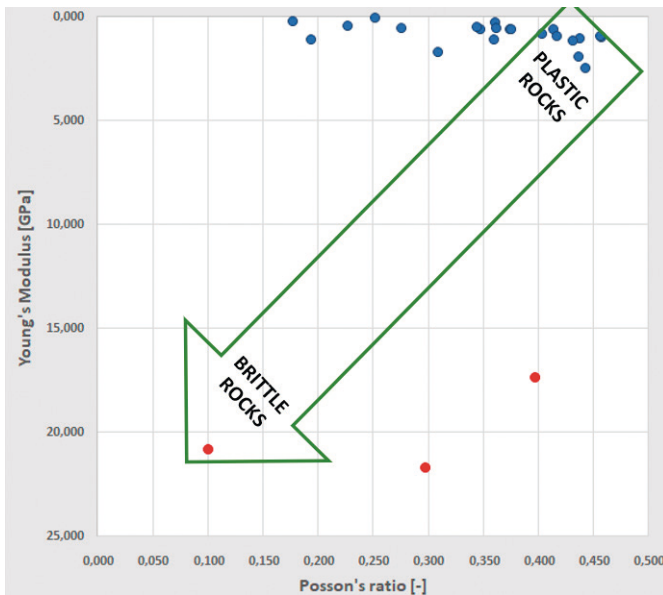
**Fig. 5.** Poisson's ratio plotted versus  $V_p/V_s$  ratio for shale from Carpathian Flysch with correlation





**Fig. 6.** Young's modulus plotted versus  $V_p/V_s$  ratio for shale from Carpathian Flysh with correlation

Figure 7 shows the cross-plot of Young's modulus and Poisson's ratio for the rock samples analysed. The large arrow shows the direction in which the brittleness of the rock increases. The scale of Young's modulus is inverted. Poisson's ratio varies from 0.178 to 0.458 for clay rocks and from 0.101 to 0.398 for sandstone formations, and Young's modulus for clay rocks – from 0.120 GPa to 2.510 GPa, and for sandstone rocks – from 17.405 GPa to 21.722 GPa. The range of changes places the analysed claystone samples in the zone of plastic formations, and the sandstone samples – of medium brittleness.



**Fig. 7.** Cross-plot of Poisson's ratio against Young's modulus for the rock samples analyzed (blue dots represent clay formations, and red dots represent sandstone formations)

#### 4. CONCLUSION

The investigations were carried out in clay and sandstone formations located at shallow depths beneath the earth's surface. The wave velocities  $P$  and  $S$  (271.870–3786.517 m/s and 156.414–2009.709 m/s, respectively) and the resulting dynamic modulus of elasticity (Young's modulus: 0.120–21.722 GPa, Poisson's ratio: 0.101–0.458) and the ratio  $V_p/V_s$  (1.5–3.6) are within the limits reported in the literature. The results show that  $V_p/V_s$  increases significantly with the increase in velocity of longitudinal and transverse waves and increase in Poisson's ratio (with respect to lithology) and increases slightly with the increase of Young's modulus. A comparison of Young's modulus and Poisson's ratio in the form of a cross-plot provides a qualitative assessment of the friability of the studied rocks. The mechanical properties of rocks alter depending on lithological changes. In clay rocks, these properties are worse and the rock is more plastic, while in sandstone and calcite admixtures it is more brittle. The velocities of elastic waves depend on many factors, and therefore their knowledge allows to expand the information about the studied geological medium. Additional information can be obtained by analyzing the relationship between velocities and other petrophysical parameters, such as the correlation between the  $V_p/V_s$  ratio and Poisson's ratio.

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