Paper

An Extended Version of the Proportional Adaptive Algorithm Based on Kernel Methods for Channel Identification with Binary Measurements

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Abstract — In recent years, kernel methods have provided an important alternative solution, as they offer a simple way of expanding linear algorithms to cover the non-linear mode as well. In this paper, we propose a novel recursive kernel approach allowing to identify the finite impulse response (FIR) in non-linear systems, with binary value output observations. This approach employs a kernel function to perform implicit data mapping. The transformation is performed by changing the basis of the data in a high-dimensional feature space in which the relations between the different variables become linearized. To assess the performance of the proposed approach, we have compared it with two other algorithms, such as proportionate normalized least-mean-square (PNLMS) and improved PNLMS (IPNLMS). For this purpose, we used three measurable frequency-selective fading radio channels, known as the broadband radio access network (BRAN C, BRAN D, and BRAN E), which are standardized by the European Telecommunications Standards Institute (ETSI), and one theoretical frequency selective channel, known as the Macchi's channel. Simulation results show that the proposed algorithm offers better results, even in high noise environments, and generates a lower mean square error (MSE) compared with PNLMS and IPNLMS.

Keywords — binary measurement, BRAN channel identification, kernel methods, PNLMS, phase estimation

1. Introduction

Numerous measurement-related challenges faced in digital communication systems have been effectively resolved with the help of adaptive filtering algorithms dealing with signal enhancement, acoustic noise cancellation, echo cancellation, channel estimation, blind channel equalization, and system identification [1]–[4]. System identification is of crucial interest in the field of automatic control [5], used to determine the most adequate mathematical model based on the inputs, outputs, and perturbations of a simulated real system. The least mean squares (LMS) algorithm [6] and its variants – normalized LMS (NLMS) [7] and recursive least squares (RLS) – [8] are the most popular methods employed for identification of linear systems due to the statistical conceptual clarity of the mean square error cost function, simple mathematical operations required, stability, and easy implementation [1]. Unfortunately, this algorithm is not valid for sparse system identification.

An attempt was made to overcome this limitation by proposing a novel adaptive technique for model system identification with sparse impulse response using an adaptive delay filter [9]. After that, Duttweiler introduced the concept of updating the proportional NLMS (PNLMS) [10] algorithm for network echo cancellation applications. Unfortunately, its convergence rate begins to slow down considerably after the fast initial period, finally becoming even slower than in the case of NLMS. To resolve this drawback, several versions of the PNLMS algorithm were developed. The examples include the well-known improved PNLMS (IPNLMS) algorithm [11], which uses a controlled mixture of PNLMS and NLMS algorithms, the μ-law PNLMS (MPNLMS) algorithm [12], an improved IPNLMS algorithm [13], an improved μ-law proportionate NLMS algorithm [14], l_0-LMS [15] , and the evaluation of block-sparse systems with an improved μ-law PNLMS algorithm (BS-MPNLMS) [16].

In addition, to exploit the sparsity characteristics of the estimated systems, certain subtypes of these techniques operating based on core idea presented above have also been introduced [17]–[19] with zero-attractors. In the field of system identification, the requirement for a high degree of precision in large complex systems has driven the need for good models that are capable of representing the non-linear structure of numerous real systems [20], [21]. For example, the Hammerstein model has been employed in diverse non-linear system applications and many related research works exist – see, for instance, [22]–[30].

Non-linear system identification continues to be a hot topic in the scientific community [31]. Volterra filters [32]–[34] and neural networks [35] are two of the most well-known techniques gaining a lot of attention. Each technique has its own set of benefits and drawbacks. For example, in the case of Volterra filters, the number of parameters to be estimated is determined by the filter order and its complexity. This allows to incorporate a high degree of complexity as far as the range of parameters is concerned. The weak point of neural networks lies in the choice of the parametric form, as this can often only be performed in a more or less arbitrary way. A false decision may degrade performance.

The development of Kernel methods [36]–[38] has been speeding up in recent years, as they serve as an important tool for the advancement of new technologies, especially in terms of the reduction of computational time required to handle difficult tasks [39]. These techniques greatly increase the accuracy of processing thanks to their ability to detect any existing commonalities in the treated information. They depend on a key principle known as the kernel trick, which was first applied to the support vector machine (SVM) [40], [41], and was soon after used to recast many classical linear methods in high dimensional the reproducing kernel Hilbert space (RKHS) [42]–[44], and was then reformulated as an inner product to yield more powerful non-linear extensions [38]. The kernel trick makes it possible to attribute the non-linear nature to many previously classical linear techniques and, with no restraint, it can only be represented in the scalar products form of data measurement.

Up to date, a number of kernel adaptive-filtering (KAF) algorithms have been suggested in the research literature. The examples include, inter alia, kernel least mean square (KLMS) [45], kernel recursive least square (KRLS) [46] and kernel affine projection algorithm (KAPA) [47] used in the field of non-linear signal processing [48]. In order to achieve the highest level of performance of fundamental kernel algorithm varieties, different subtypes of these categories were identified, including quantized kernel least mean square (QKLMS) [49], quantized kernel recursive least square (QKRLS) [50], regularized kernel least mean square based on multiple time delay feedback (RKLMS-MDF) [51], kernel least mean square with adaptive kernel size (KLMS-AKS) [52], extended kernel recursive least square (Ex-KRLS) [53] and reduced kernel recursive least square (RKRLS) [54] that are used for channel identification [24]–[29], [55] and equalization in non-linear systems.

In this paper, we propose a novel recursive algorithm that is based on the positive definite kernel function, with our primary focus being on the improved proportionate normalized least mean square (IPNLMS) variety developed in [11]. As far as we are aware, the application of kernel-based adaptive nonlinear system identification methods with binary-valued output observations of the IPNLMS has not been studied yet. For validity and test purposes, the proposed algorithm is compared with the proportional normalized least mean square (PNLMS) algorithm and with its improved version (IPNLMS), where the goal is to identify impulse response parameters of Macchi and ETSI BRAN channels. The relation of the proposed algorithm with other algorithms described in the literature will be demonstrated based on two examples. First, an example using the Hammerstein system, in which the input sequence is randomly generated with a uniform distribution, will highlight how the proposed algorithm is capable of estimating, with good accuracy, the impulse response parameters of the practical frequency selective fading channel (BRAN) and the theoretical frequency selective channel (Macchi). Second, it will be shown how the proposed algorithm is capable of converging faster and yielding a smaller estimation error than both PNLMS and IPNLMS.

This paper is arranged as follows. In Section 2 we introduce the architecture of a non-linear system (Hammerstein model) identification problem with binary-valued output observations and noise. In Section 3, we give an overview of some fundamental notations of the kernel methods, with that overview followed by a description of PNLMS, IPNLMS, and kernel extended IPNLMS algorithms. The effectiveness of the proposed recursive kernel algorithm is discussed based on some simulation results in Section 4. Finally, Section 5 concludes this paper.

2. Problem Statement and Assumptions

Let us consider the single-input single-output (SISO) Hammerstein model presented in Fig. 1. It is made up of a non-linear static function followed by a known-order finite impulse response (FIR).

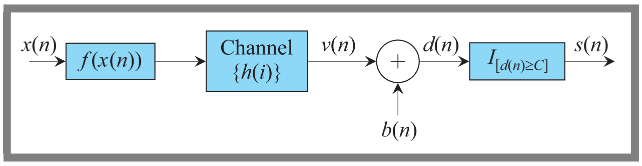

Fig. 1. Block diagram of a Hammerstein model with binary outputs and noises.

From Fig. 1, the output of the desired system is given by:

$$\begin{cases} v(n) = \displaystyle\sum_{i=0}^{L-1} h(i) f(x(n-i)), \\ d(n) = v(n) + b(n), \quad n = 0, 1, 2, \ldots, N \end{cases}, \tag{1}$$

where $x(n)$ represents the input signal, $d(n)$ the output, $h(i)_{(i=0,1,\ldots,L-1)}$, L the coefficients of the finite impulse response filter, $f(.)$ is the nonlinearity, and $b(n)$ is the measurement noise.

A binary-valued sensor $I_{[.]}$ with a fixed threshold $C \in \mathbb{R}$ can be used to measure the system's output $d(n)$. The output with a binary value $s(n)$ can be represented by the following formula:

$$s(k) = I_{[d(n) \geqslant C]} = \begin{cases} 1 & \text{if } d(n) \geqslant C \\ -1 & \text{otherwise} \end{cases}. \tag{2}$$

The following are the main assumptions that were made for the system model:

- input sequence $\{x(n)\}$, is i.i.d. (independent and identically distributed) bounded random process with zero mean,
- additive noise $\{b(n)\}$ is suggested, Gaussian and independent of $\{x(n)\}$ (bounded) and $\{d(n)\}$ (bounded),
- let $f(.)$ be invertible and continuous for any finite x,
- the system model does not include any delays, i.e. $h(0) \neq 0$,
- C value is available (known).

The above-mentioned assumptions are made in order to facilitate the analysis of the system and to obtain the best results in terms of the mean square error and the channel identification framework considered. The main purpose of this paper is to construct a recursive identification algorithm for finite impulse response (FIR) systems based on positive definite kernels and binary-valued observations $s(n)$, in order to recursively estimate the channel's parameters.

3. Proposed Adaptive Filtering Algorithm

In this section, we start by presenting the general idea behind kernel methods. We will define what a kernel is, specifying its properties and those of the kernel spaces. Next, we describe the adaptive algorithms used to identify channel impulse responses, i.e. the proportional normalized LMS algorithm (PNLMS) and the improved PNLMS algorithm (IPNLMS). This derivation order is equally the historical arrangement of the algorithms that were previously extended according to [10], [11]. Then, the kernel methods are incorporated into the IPNLMS algorithm strategy in order to produce a kernelized version of the improved proportionate normalized LMS algorithm based on binary measurements.

Kernel methods can be used to solve non-linear adaptive filtering problems in high dimensional spaces. The problem of dimensionality, referring to the number of parameters to be estimated, is then reduced to the amount of learning data available. A fundamental characteristic of kernel methods is that the resulting model is a linear combination of kernel functions whose order is identical to the size of the learning data, where all sources of input information $\{x(i)\}_{i=1}^{N} \in \mathcal{X}$ were mapped (implicit) into a high dimensional space $\mathcal{H}$ (an inner product space) by taking advantage of the idea that the Mercer kernel function could be used to express an inner product in Hilbert spaces. Based on Mercer's theorem, the mapping $\Psi(.)$ that was introduced by means of $\kappa(x(i), x(j))$ will be expressed by the following relationship [36], [63]:

$$\kappa(x(i), x(j)) = \langle \Psi(x(i)), \Psi(x(j)) \rangle_{\mathcal{H}}, \quad \forall x(i), x(j) \in \mathcal{X}, \quad (3)$$

where $\kappa(.,.)$ is a kernel function and $\Psi(.)$ mapped $\mathcal{X}$ to a space $\mathcal{H}$ with an inner product $\langle .,. \rangle$. Commonly, dimension of $(\mathcal{X})$ is much smaller than the dimension of $(\mathcal{H})$.

The block diagram shown in Fig. 2 illustrates an adaptive kernel-based channel estimation using the proposed algorithm, where $e(n)$ represents the estimation error and $y(n)$ is the estimated desired response. The learning procedure is conducted in two distinct phases at each time n (Fig. 2):

1) Initially, using the Hammerstein system (HS) with binary--valued output observations and noise, we obtain the binary output $s(n)$.
2) During the next phase, based on the transformation of the measured data into non-linear spaces (RKHSs) employing a Mercer kernel κ, channel coefficients $\theta(n)$ are adjusted according to the functional cost minimization principle.

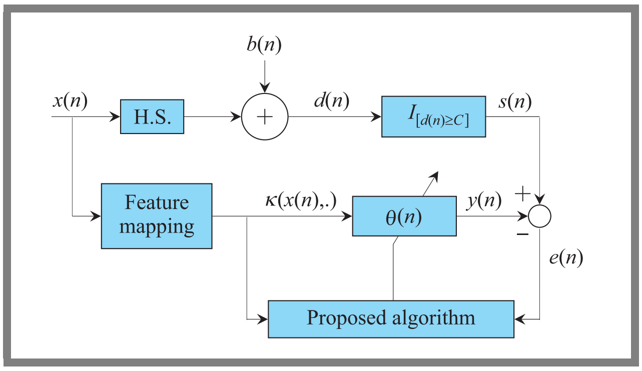

Fig. 2. Block schematic of kernel adaptive filter.

Let us start with some necessary preliminaries that need to be employed in the proposed algorithm in order to successfully determine the existing functional space $\mathcal{H}$.

Definition 1 positive definite kernel. A kernel is said to be positive definite, if it satisfies the following condition for each input data point $\{x(i)\}_{i=1}^{N} \in \mathcal{X}$:

$$\sum_{i,j=1}^{N} \alpha_i \alpha_j \kappa(x(i), x(j)) \geqslant 0, \quad (4)$$

for all $N \in \mathbb{N}$, $\{x(1), \ldots, x(N)\} \subseteq \mathcal{X}$ and $\{\alpha_1, \ldots, \alpha_N\} \subseteq \mathbb{R}$.

In particular, if $\kappa : \mathcal{X} \times \mathcal{X} \longmapsto \mathbb{R}$ is positive definite, then it can be expressed as an inner product in the feature space $\mathcal{H}$, where the data are projected. On the other hand, if we define a correspondence between the input data and a vector space, then the inner product in this vector space will be a positive definite kernel.

According to the Mercer theorem [36], [63], any kernel $\kappa(x(i), x(j))$ can be redefined as follows:

$$\kappa(x(i), x(j)) = \sum_{i=1}^{\infty} \zeta_i \Psi_i(x(i)) \Psi_i(x(j)), \quad (5)$$

where ζ_i and Ψ_i, $i = 1, 2 \ldots$, denote the non-negative eigenvalues and the eigenfunctions, respectively.
Mapping Ψ_i in the reproducing kernel Hilbert space can be created as:

$$\Psi(x) = \left[\sqrt{\zeta_1} \Psi_1(x), \sqrt{\zeta_2} \Psi_2(x), \ldots \right]^{\top}. \quad (6)$$

Definition 2 reproducing kernel Hilbert spaces. Let $\mathcal{H}$ denote a Hilbert space of real functions defined on an indexed set $\mathcal{X}$:

$$\mathcal{H} = \left\{ \sum_{j=1}^{n} \alpha_j \kappa(x(j), .) : \; n \in \mathbb{N}, \; x(j) \in \mathcal{X}, \right. \quad (7)$$
$$\left. \alpha_j \in \mathbb{R}, \; j = 1, \ldots, n \right\}.$$

$\mathcal{H}$ is considered to be a reproducing kernel Hilbert space with an inner product noted $\langle .,. \rangle_{\mathcal{H}}$ and the norm $\|f\|_{\mathcal{H}} = \sqrt{\langle f, f \rangle_{\mathcal{H}}}$ if there exists a function $\kappa : \mathcal{X} \times \mathcal{X} \longrightarrow \mathbb{R}$ that has the following two properties:

1) for any element $x \in \mathcal{X}$, $\kappa(x, .)$ belongs to $\mathcal{H}$,

2) function κ is a reproducing kernel function, i.e. for any function $f \in \mathcal{H}$, we have: $\langle f, \kappa(x, .)\rangle_{\mathcal{H}} = \sum_{j=1}^{n} \alpha_j \kappa(x(j), x) = f(x)$.

3.1. Derivation of the PNLMS Algorithm

The PNLMS algorithm was first proposed by assigning a step parameter to each coefficient using a diagonal step control matrix $G(n) \in \mathbb{R}^{(L)\times(L)}$ [10]. This algorithm is capable of exploiting low impulse response density to achieve a better adaptation than that observed in the case of the classical NLMS algorithm. The PNLMS algorithm needs more operations than the NLMS algorithm but has the benefit of converging faster than the latter. The PNLMS algorithm's practical update equations are given by:

$$e(n) = s(n) - \theta^{\top}(n-1)\mathbf{x}(n), \tag{8}$$

$$\mathbf{D}(n-1) = \mathrm{diag}(d_0(n-1), d_1(n-1), \ldots, d_{L-1}(n-1)), \tag{9}$$

$$\theta(n) = \theta(n-1) + \frac{\mu\mathbf{D}(n-1)\mathbf{x}(n)e(n)}{\delta_{\text{PNLMS}} + \mathbf{x}^{\top}(n)\mathbf{D}(n-1)\mathbf{x}(n)}, \tag{10}$$

where $\mathbf{x}(n) = [x(n), x(n-1), \ldots, x(n-L+1)]^{\top}$ represents the input signal, where the superscript $(.)^{\top}$ is the transpose operator, $e(n)$ is the estimation error, $\mu \in \mathbb{R}_{+}^{*}$ is the fixed step-size, $d_l(n) \in \mathbb{R}_{+}^{*}$, and δ_{PNLMS} is a regularization parameter:

$$\delta_{\text{PNLMS}} = \frac{\delta_{\text{NLMS}}}{L}.$$

The original definition of the diagonal matrix element $\mathbf{D}(n)$ is:

$$d_l(n) = \frac{k_l(n)}{\frac{1}{L}\sum_{i=0}^{L-1} k_i(n)}, \quad l = 0, 1, \ldots, L-1, \tag{11}$$

with

$$k_l(n) = \max\{\{|\theta_l(n)|\}, \rho\max\{\delta_p, |\theta_0(n)|, \ldots, |\theta_{L-1}(n)|\}\}. \tag{12}$$

Parameters δ_p and ρ are used to protect $\theta_l(n)$ from stalling during the initialization step. The typical value of δ_p is equal to 0.01 and ρ ranges from $\frac{1}{L}$ to $\frac{5}{L}$.

3.2. Derivation of the IPNLMS Algorithm

Convergence speed of the PNLMS algorithm degrades significantly when dealing with non-sparse impulse responses. Improved PNLMS (IPNLMS) is proposed to avoid degradation in a scenario in which the impulse underlying the response is non-sparse. In the improved PNLMS algorithm, the diagonal element of $\mathbf{D}(n)$ is:

$$d_l(n) = \frac{1-\alpha}{2L} + \frac{|\theta_l(n)|(1+\alpha)}{2\sum_{i=0}^{L-1}|\theta_i(n)| + \delta_{\text{IPNLMS}}}, \tag{13}$$

where $\alpha \in [-1, 1]$ and $\delta_{\text{IPNLMS}} = \dfrac{1-\alpha}{2L}\delta_{\text{NLMS}}$ is a one of the small positive numbers in order to prevent dividing by zero. We will utilize the kernel-based method to extend the improved PNLMS algorithm in the manner described in the next subsection.

3.3. Projection Over Kernel Methods

The proposed algorithm is presented in this section. The main idea is to operate the improved proportionate NLMS algorithm in the Gaussian kernel feature space that is linked to a reproducing kernel κ (continuous, normalized and symmetric), using the feature map $\Psi(.)$ that enables us to transform the sample sequence as:

$$\Psi \;:\; \mathcal{X} \longrightarrow \mathcal{H}$$
$$x(i) \longrightarrow \kappa(x(i), .),\; 0 \leqslant i \leqslant N. \tag{14}$$

In order to generate the infinite-dimensional space model of the reproducing kernel, there exist various types of kernels such as sigmoid and radial Gaussian kernels defined, respectively, by:

$$\kappa_{a,c}^{sig}(x(i), x(j)) = \text{than}\left(a\langle x(i), x(j)\rangle + c\right), \quad \forall a, c \in \mathbb{R}, \tag{15}$$

$$\kappa_{\sigma}^{G}(x(i), x(j)) = \mathrm{e}^{-\dfrac{\|x(i)-x(j)\|^2}{2\sigma^2}}, \quad \forall (x(i), x(j)) \in \mathcal{X}^2, \tag{16}$$

where $\sigma > 0$ is the bandwidth of the kernel used to specify the form of the kernel function, a is the sigmoid scale, and c is the bias.

Throughout the remainder of the paper, we will use the Gaussian radial basis function (RBF) kernel, which the favorite option mostly thanks to its perfect approximation character and its numerical stability. In [57], one can find a more comprehensive list of Mercer kernels. $\Psi(.)$ mapping generated by this type of kernel is a bit special. In fact, a data item will be mapped onto a Gaussian function that represents the data's similarity to all data in $\mathcal{X}$. Fig. 3 represents the application of the Gaussian radial-basis function kernel to the two pieces of data $x(i)$ and $x(j)$.

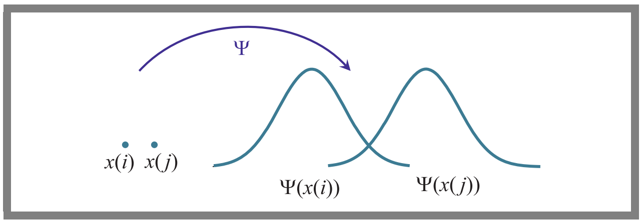

Fig. 3. Definition of characteristic map.

The four steps that make up the proposed identification algorithm are:

1) In the initial step, a transformation of the measured data inputs from the space $\mathcal{X}$ into a high dimensional space (feature space $\mathcal{H}$) is realized to produce the following input data:

$$\{(\Psi(x(1)), s(1)), (\Psi(x(2)), s(2)), \ldots, (\Psi(x(n)), s(n)), \ldots\}. \tag{17}$$

2) In the second step, by applying the methodology of the improved proportionate NLMS algorithm to the input data sequence described in Eq. (17), we can minimize the cost function by:

$$E[|s(k) - \langle(\Psi(x(k))), \theta\rangle_{\mathcal{H}}|^2],$$

where θ denotes the weight vector in the feature space $\mathcal{H}$.

3) Next, we are proceeding directly in the feature space $\mathcal{H}$, under the assumption that our data has already been successfully modeled in the RKHS by means of the Ψ mapping function, i.e.:

$$\mathcal{X} \ni x \longrightarrow \Psi(x(n)) := \kappa(x,.) \in \mathcal{H}. \quad (18)$$

4) The estimate of $\theta(n)$ is produced and is noted $\hat{\theta}(n)$:

$$\hat{\theta}(n) = \hat{\theta}(n-1) + \frac{\mu \mathbf{D}(n-1)\kappa(x(n),.)e(n)}{\delta_{\text{KE-IPNLMS}} + \kappa(x(n),.)^\top \mathbf{D}(n-1)\kappa(x(n),.)}, \quad (19)$$

$$\mathbf{D}(n-1) = \text{diag}(d_0(n-1), d_1(n-1), \ldots, d_{L-1}(n-1)), \quad (20)$$

where:

$$d_l(n) = \frac{1-\alpha}{2L} + \frac{|\hat{\theta}_l(n)|(1+\alpha)}{2\sum_{i=0}^{L-1}|\hat{\theta}_i(n)| + \delta_{\text{KEIPNLMS}}}, \quad (21)$$

where α is the adjusting parameter, and $\delta_{\text{KE-IPNLMS}}$ is a small value used to avoid a denominator equaling zero.

The proposed identification algorithm update equations in kernel Hilbert space is summarized as Algorithm 1.

Algorithm 1. Kernel extended IPNLMS algorithm for channel identification.

Input: samples $\{x(n), s(n)\}$, $n = 1, 2, \ldots, N$
Initialization: channel parameter $\theta(0)$ with zeros, adjusting parameter α, kernel bandwidth σ, and threshold C
Computation:
while $\{x(n), s(n)\}_{n=1}^{N}$ available **do**
1. compute $y(n)$ as: $y(n) = \kappa(x(n),.)^\top \hat{\theta}(n-1)$
2. compute the prediction error as: $e(n) = s(n) - y(n)$
3. update weight vector using Eq. (19)
4. compute the diagonal matrix $\mathbf{D}$ using Eq. (20)
end while

4. Results of Numerical Simulations

The main aim of this section was to investigate the effectiveness of the proposed kernel extended IPNLMS algorithm in terms of channel impulse response identification and to compare it with that of PNLMS and IPNLMS algorithms. Performance was measured using mean squares error (MSE) in decibels, expressed as:

$$\text{MSE} = 10 \log \left[\frac{1}{N} \sum_{n=1}^{N} (s(n) - y(n))^2 \right], \quad (22)$$

where N represents the data length, $s(n)$ is the binary output and $y(n)$ is the estimated desired response. The procedure involved 50 runs of Monte Carlo experiments to reduce measurement uncertainty. We used two models for the linear part: the Macchi channel and the ETSI BRAN channel to simulate the Hammerstein model. A typical non-linear part function is the hyperbolic function $\tanh(x)$, defined by:

$$f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}. \quad (23)$$

The parameter settings selected for the simulations are: threshold is $C = 0.5$, step-size parameter for all the algorithms is $\mu = 0.05$, regularization parameter $\delta_{\text{NLMS}} = 0.01$, adjusting parameter $\alpha = -0.75$, kernel width $\sigma = 0.2$, data length $N = 2^{10}$, and $SNR = 16$ dB. Note that when we modify one of these simulation parameters, the others remain constant. The simulations are performed using Matlab software and are conducted for various *SNR*s defined as follows:

$$SNR = 10 \log \left[\frac{E(v^2(n))}{E(b^2(n))} \right], \quad (24)$$

where $E[.]$ is the mathematical expectation.

4.1. Macchi Channel

To investigate the theoretical performance of the presented approach, we have relied on the Macchi channel. The impulse response of this channel is defined by vector $\mathbf{H} = [h(0), \ldots, h(L-1)]^\top$ of coefficients $h(i)$ [64]:

$$\mathbf{H} = [0.8264, -0.1653, 0.8512, 0.1636, 0.81]^\top.$$

Figure 4 shows the characteristics of this channel. It has four zeros, two of them are outside of the unit circle, which implies that the channel is of the non-minimum phase. This channel's amplitude response is quite deep fading, and its phase response is far from linear.

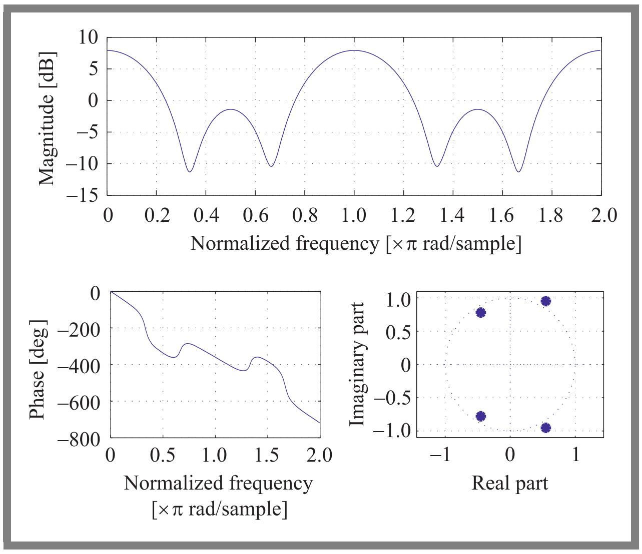

Fig. 4. Macchi channel.

Figure 5 shows the estimation of the magnitude and phase of the Macchi channel impulse response parameters, using the three algorithms for a data length of $N = 2^{10}$ and $SNR = 16$ dB.

We observe that the magnitude and phase estimated with the use of the proposed KE-IPNLMS algorithm follow the true model in perfect agreement with the measured data. But for both other algorithms (PNLMS and IPNLMS), we can see a significant difference between the measured and estimated parameters.

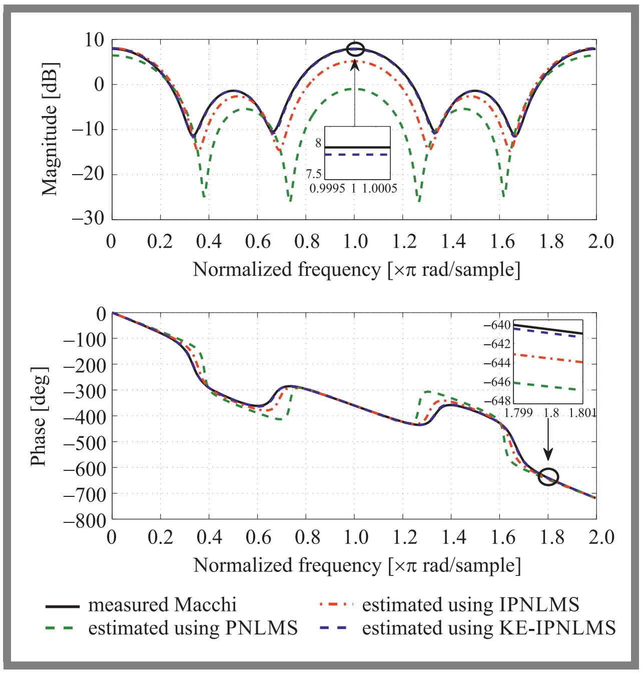

Fig. 5. Macchi channel magnitude and phase estimation for $N = 2^{10}$ and $SNR = 16$ dB.

4.2. ETSI BRAN Channel

The robustness of identification algorithms cannot be fully evaluated by simulating them on theoretical channels. As a result, we looked into mobile radio channel models. We focused on three models (ETSI BRAN C, BRAN D, and BRAN E) that represent fading radio channels. The associated model data are measured for 4G systems [65], [66]. The impulse response parameters of the ETSI BRAN radio channel are described by:

$$h(n) = \sum_{i=0}^{L-1} M_i \delta(n - \tau_i), \; p = 18, \tag{25}$$

where L is the number of paths present, $M_i \in N(0, 1)$ is the magnitude of path i, τ_i is its delay time and $\delta(n)$ is the Dirac function. The magnitudes and time delays of 18 targets of the BRAN C, D and E channels are represented in Tables 1, 2 and 3, respectively.

Tab. 1. Delay and magnitudes of 18 targets of BRAN C radio channel.

Delay τ_i [ns]	**Magnitude** M_i [dB]	**Delay** τ_i [ns]	**Magnitude** M_i [dB]
0	−3.3	230	−3.0
10	−3.6	280	−4.4
20	−3.9	330	−5.9
30	−4.2	400	−5.3
50	0	490	−7.9
80	−0.9	600	−9.7
110	−1.7	730	−13.2
140	−2.6	880	−16.3
180	−1.5	1050	−21.2

Tab. 2. Delay and magnitudes of 18 targets of BRAN D radio channel.

Delay τ_i [ns]	**Magnitude** M_i [dB]	**Delay** τ_i [ns]	**Magnitude** M_i [dB]
0	0	230	−9.4
10	−10	280	−10.8
20	−10.3	330	−12.3
30	−10.6	400	−11.7
50	−6.4	490	−14.3
80	−7.2	600	−15.8
110	−8.1	730	−19.6
140	−9.0	880	−22.7
180	−7.9	1050	−27.6

Tab. 3. Delay and magnitudes of 18 targets of BRAN E radio channel.

Delay τ_i [ns]	**Magnitude** M_i [dB]	**Delay** τ_i [ns]	**Magnitude** M_i [dB]
0	−4.9	320	0
10	−5.1	430	−1.9
20	−5.2	560	−2.8
40	−0.8	710	−5.4
70	−1.3	880	−7.3
100	−1.9	1070	−10.6
140	−0.3	1280	−13.4
190	−1.2	1510	−17.4
240	−2.1	1760	−20.9

Figures 6, 7 and 8 illustrate BRAN C, D and E channel zeros, respectively.

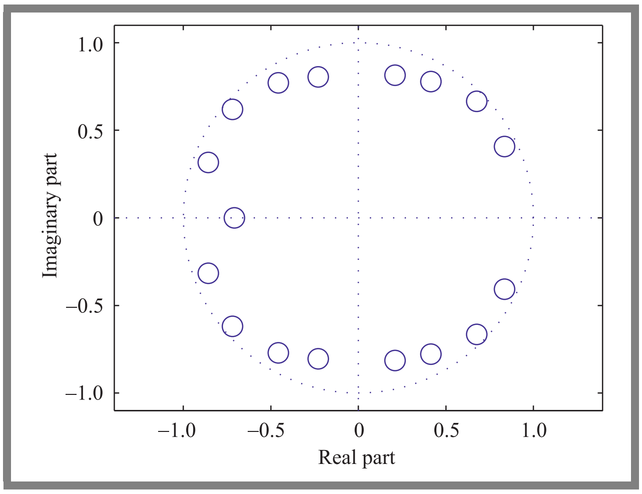

Fig. 6. Zeros of the BRAN C model.

To assess the accuracy of the proposed KE-IPNLMS algorithm, we looked at four different BRAN models with defined properties (i.e. known parameters), then we tried to recuperate these parameters under an additive noise.

Gaussian for an $SNR = 16$ dB and data length $N = 2^{10}$, and we compared them with the two other algorithms proposed in the literature during 50 Monte Carlo runs. Figure 9 illus-

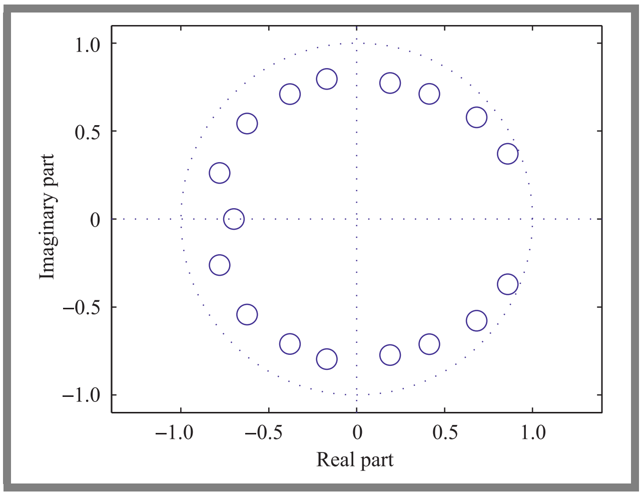

Fig. 7. Zeros of the BRAN D model.

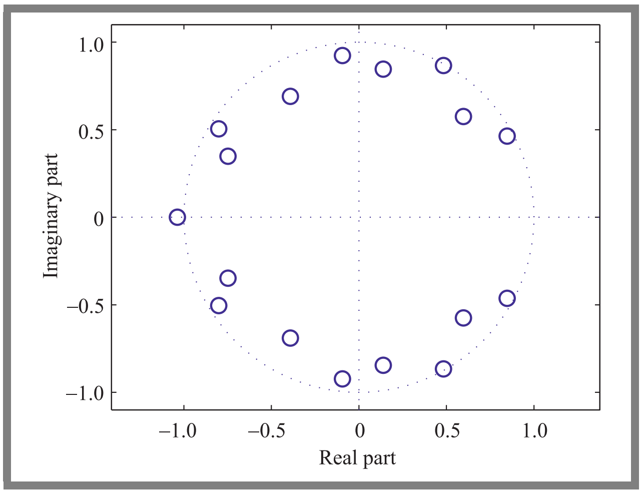

Fig. 8. Zeros of the BRAN E model.

trates the estimated magnitude and phase parameters of the BRAN C radio channel impulse response, using the algorithms presented previously, for a data length of $N = 2^{10}$ and an $SNR = 16$ dB. When the proposed KE-IPNLMS algorithm is used, the magnitude and phase response is estimated with reasonable precision, but several fluctuations are observed when PNLMS and IPNLMS algorithms are used.

Figure 10 demonstrates the magnitude and phase estimation of the BRAN D radio channel impulse response obtained using the proposed KE-IPNLMS algorithm, compared with PNLMS and IPNLMS algorithms for a data length of $N = 2^{10}$ and for $SNR = 16$ dB. The estimated magnitude and phase curves, obtained using the proposed algorithm (KE-IPNLMS), follow the real model with only a slight deviation. When BRAN D radio channel impulse response Parameters are estimated using the IPNLMS algorithm, some minor differences exist between the estimated magnitude and the real model (measured values), and an obvious difference is evident if the PNLMS algorithm is employed. In practical channels when multipath fading is severe for the learning sequence duration, the estimates could yield poor quality results.

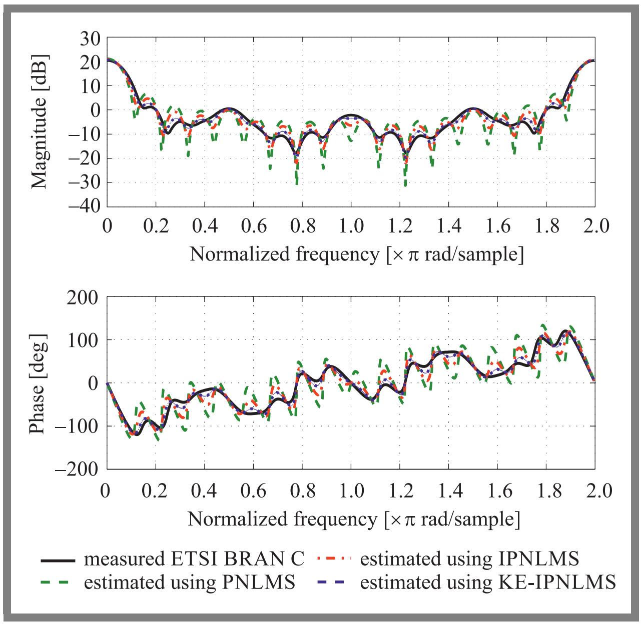

Fig. 9. BRAN C channel magnitude and phase estimation for $N = 2^{10}$ and $SNR = 16$ dB.

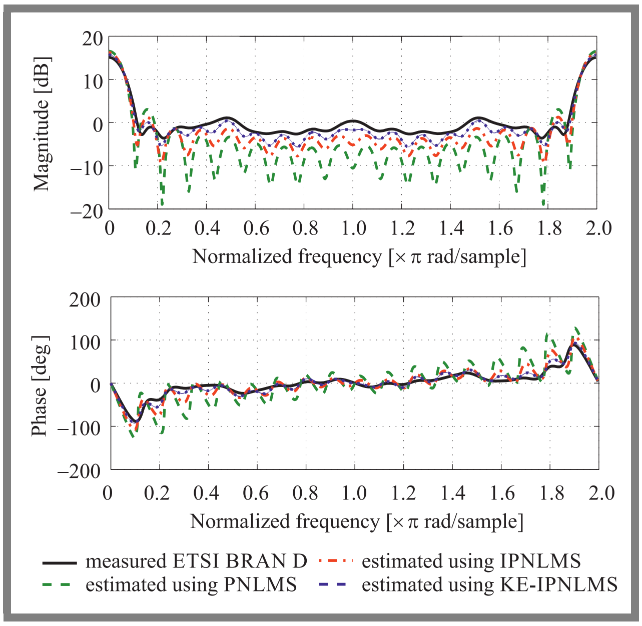

Fig. 10. BRAN D channel magnitude and phase estimation for $N = 2^{10}$ and $SNR = 16$ dB.

Figure 11 shows the estimated magnitude and phase of the BRAN E radio channel impulse response parameters, for a data length of $N = 2^{10}$ and for $SNR = 16$ dB. It should be observed that with the proposed KE-IPNLMS algorithm, the estimated magnitude and phase have the same forms as those measured. When compared with the PNLMS and IPNLMS algorithm, we note that the estimated magnitude follows the variations of the real model's parameters. Performance of the PNLMS algorithm degrades during phase estimation and a large difference between the estimated BRAN E radio channel impulse response and the measured phase is observed. To summarize, Gaussian noise exerts a significant impact on the phase, but only a minor impact on the amplitude estimates.

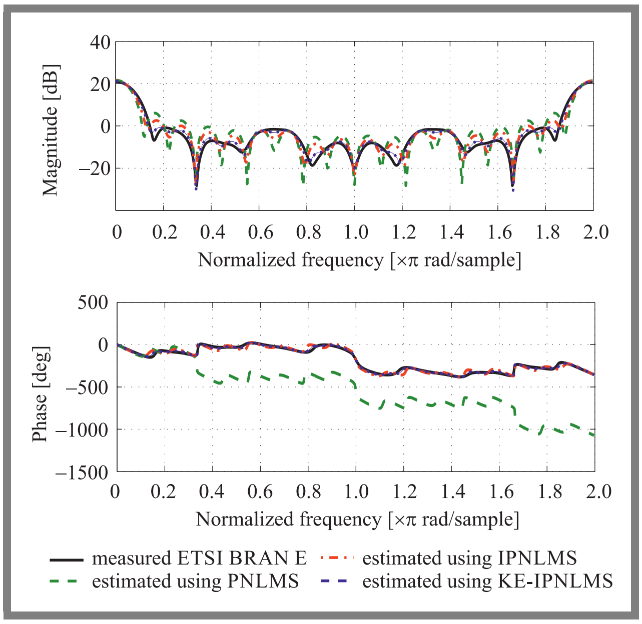

Fig. 11. BRAN E channel magnitude and phase estimation for $N = 2^{10}$ and $SNR = 16$ dB.

4.3. Performance in Noisy Environment

Here, we test the performance of the algorithms in a Gaussian noise environment, where *SNR* varies from 0 to 30 dB and for a fixed data length of $N = 2^{10}$. The results are summarized in Tables 4–7 for 50 Monte Carlo runs. With all these different results taken into consideration, we have several more important points to make.

The proposed KE-IPNLMS algorithm has offers excellent convergence performance in comparison to its PNLMS and IPNLMS counterparts, for all signal-to-noise ratio values, even in a high noise environment ($SNR = 0$ dB), since the MSE values of the proposed KE-IPNLMS algorithm are very low, contrary to those obtained by means of PNLMS and IPNLMS algorithms.

When *SNR* is adjusted from 0 to 30, even if the MSE criterion decreases for the three algorithms, the influence of the Gaussian noise disappears and the proposed KE-IPNLMS algorithm demonstrates its superiority over the remaining varieties.

As shown in Tables 4–7, performance of the proposed KE-IPNLMS solution is substantially better than that of other algorithms. For example, in the case of Macchi channel, with $SNR = 30$ dB, MSE values obtained using the proposed KE-IPNLMS algorithm are seven and four times lower than MSE values obtained by means of PNLMS and IPNLMS algorithms, respectively.

Based on Tables 6 and 7, we have observed that when $SNR = 10$ dB, the MSE value achieved by the proposed KE-IPNLMS algorithm equals only 21% and 37% of the MSE value obtained using PNLMS and IPNLMS algorithms, respectively, in the case of the BRAN D impulse response channel, as well as 38% and 56% of the MSE value using PNLMS and IPNLMS algorithms, respectively, in the cases of the BRAN E impulse response channel. These results give a clear indication regarding the high accuracy of the proposed KE-IPNLMS algorithm.

Tab. 4. MSE values of all algorithms for different *SNR* and a data length $N = 2^{10}$ in the case of the Macchi channel.

***SNR* [dB]**	**Algorithm**	**MSE [dB]**
0	PNLMS	−01.39
	IPNLMS	−04.25
	Proposed	**−09.88**
10	PNLMS	−02.53
	IPNLMS	−04.75
	Proposed	**−18.41**
20	PNLMS	−02.59
	IPNLMS	−04.87
	Proposed	**−22.12**
30	PNLMS	−02.81
	IPNLMS	−05.16
	Proposed	**−22.47**

Tab. 5. MSE values of all algorithms for different *SNR* and a data length $N = 2^{10}$ in the case of the BRAN C channel.

***SNR* [dB]**	**Algorithm**	**MSE [dB]**
0	PNLMS	−03.80
	IPNLMS	−04.90
	Proposed	**−05.17**
10	PNLMS	−07.64
	IPNLMS	−09.44
	Proposed	**−12.06**
20	PNLMS	−07.79
	IPNLMS	−10.15
	Proposed	**−13.87**
30	PNLMS	−08.19
	IPNLMS	−10.26
	Proposed	**−14.42**

Tab. 6. MSE values of all algorithms for different *SNR* and a data length $N = 2^{10}$ in the case of the BRAN D channel.

***SNR* [dB]**	**Algorithm**	**MSE [dB]**
0	PNLMS	−03.49
	IPNLMS	−05.26
	Proposed	**−07.54**
10	PNLMS	−04.32
	IPNLMS	−06.73
	Proposed	**−11.04**
20	PNLMS	−04.25
	IPNLMS	−06.95
	Proposed	**−11.18**
30	PNLMS	−04.64
	IPNLMS	−07.09
	Proposed	**−11.49**

Tab. 7. MSE values of all algorithms for different *SNR* and a data length $N = 2^{10}$ in the case of the BRAN E channel.

SNR [dB]	Algorithm	MSE [dB]
0	PNLMS	−02.00
	IPNLMS	−04.14
	Proposed	**−04.79**
10	PNLMS	−07.52
	IPNLMS	−09.16
	Proposed	**−11.61**
20	PNLMS	−8.00
	IPNLMS	−10.51
	Proposed	**−14.28**
30	PNLMS	−08.21
	IPNLMS	−10.68
	Proposed	**−14.79**

4.4. Performance Using Different Data Lengths

Now, we shall focus on the impact that parameter factor N exerts on the performance of the proposed KE-IPNLMS algorithm. Note that N is a data length that impacts the estimated channel parameters and the level of the mean square error. The results are averaged by means of 50 Monte Carlo trials.

The MSE evolution curves of these three algorithms are plotted in Figs. 12–15 for different channel impulse responses. From these results, we may observe that the impact of N is evident, which is linked to the regularity of the evaluated mean square error. It is clearly seen that the proposed algorithm offers the best performance and is also statistically important. For example, in Fig. 12, if N is 8000, MSE is lower than -30 dB in the case of the proposed KE-IPNLMS algorithm. However, we get an MSE that is close to -15 dB and just below -10 dB when we use IPNLMS and PNLMS algorithms, respectively.

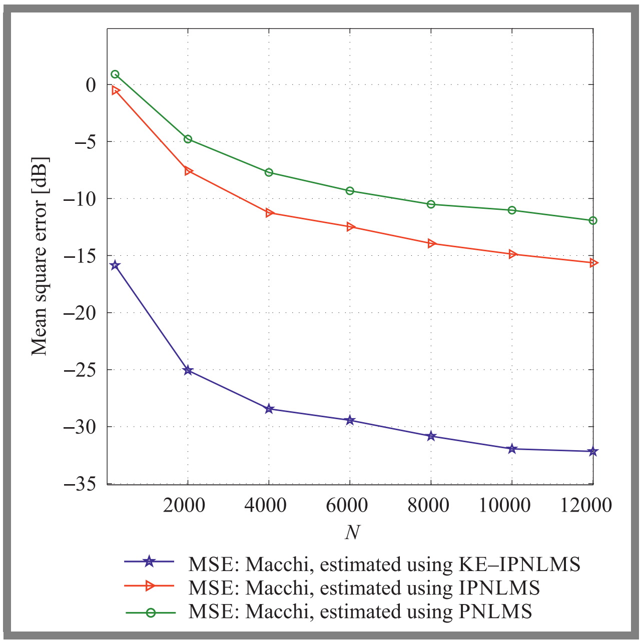

Fig. 12. Comparison of algorithms in terms of MSE for various data lengths N and for a fixed *SNR* = 16 dB, Macchi channel.

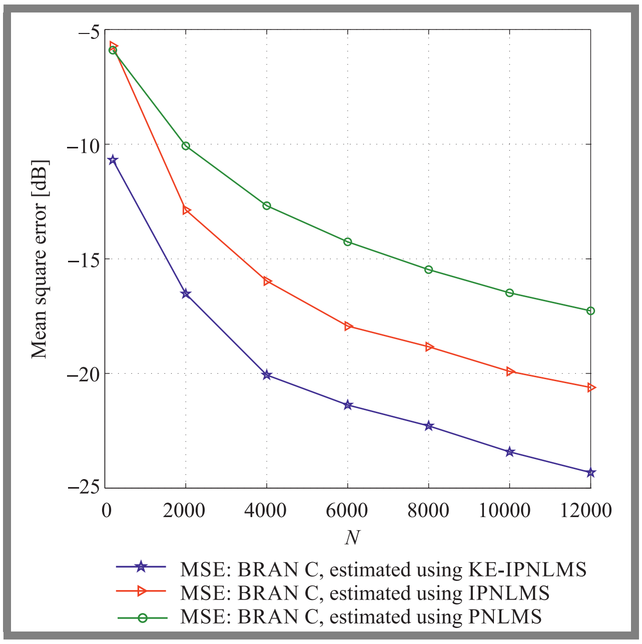

Fig. 13. Comparison of algorithms in terms of MSE for various data lengths N and for a fixed *SNR* = 16 dB, BRAN C channel.

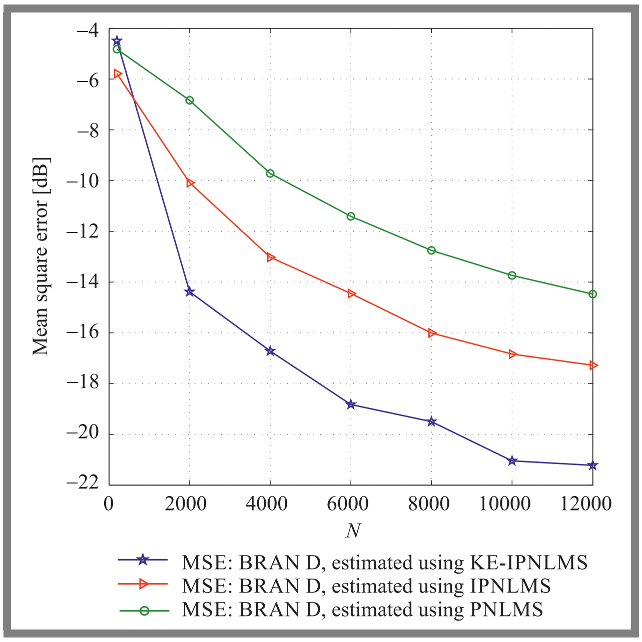

Fig. 14. Comparison of algorithms in terms of MSE for various data lengths N and for a fixed *SNR* = 16 dB, BRAN D channel.

From Figs. 13–15 it is evident that the data length is low ($N \leqslant 2000$), a very slow convergence is observed. Each time we increase the data length, we notice an improvement in the convergence speed. This shows that the speed of convergence of the three algorithms is proportional to data length. We can evidently see that the proposed KE-IPNLMS algorithm converges most quickly and has the lowest mean square error. During this time, the mean square error values of the IPNLMS algorithm are inferior to those of the PNLMS algorithm, but

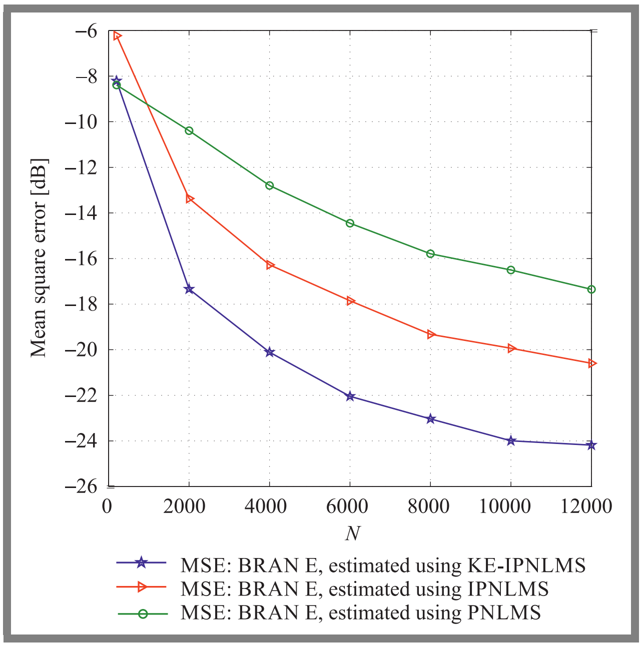

Fig. 15. Comparison of algorithms in terms of MSE for various data lengths N and for a fixed *SNR* = 16 dB, BRAN E channel.

it converges at a slow rate, which implies that the parameters estimated using the proposed KE-IPNLMS algorithm are very close to the exact values when compared to those given by the PNLMS and IPNLMS algorithms. It is very important to select an appropriate value of N and *SNR* in order to achieve a successful result.

Based on our study of the performance and convergence speed of these algorithms, we remarked that this proposed algorithmic version yields good experimental results in terms of channel identification from output binary measurements.

5. Conclusion and Future Scope

Numerical simulations for the Hammerstein system identification problem with binary measurements on the output have confirmed that the proposed KE-IPNLMS algorithm outperforms PNLMS and IPNLMS in terms of identification of magnitude and phase of channel impulse response parameters (BRAN (C, D and E) and Macchi channels), while only requiring linear computational complexity. In all simulations, we obtained good results in terms of channel identification even more in highest noise power (i.e low *SNR*) by using the proposed KE-IPNLMS algorithm.

The future work will focus on the development of an extension of this algorithm to MIMO systems and on comparing it with the existing methods, including quantized kernel recursive least squares (QKRLS), quantized kernel Lleast lncosh (QKLL) and cumulant-based methods.

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