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THE USE OF AHP METHOD IN THE MULTI-CRITERIA TASK SOLVING PROCESS – CASE STUDY

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Abstract:
In the decision-making process, both single- and multi-criteria tasks are dealt with. In the majority of cases, the selection of a solution comes down to determination of the "best" decision (most often based on the subjective assessment) or to organisation of the set of decisions. The Analytic Hierarchy Process (AHP) is one of the methods used for evaluation of qualitative features in the multi-criteria optimisation processes. This article discusses the possibilities of using the above-mentioned method, illustrated with an example of purchasing technical equipment for one of the municipal landfill sites in the Silesian Province.

Key words: *decision-making process, the method Analytic Hierarchy Process (AHP)*

INTRODUCTION

Taking a decision is an intrinsic element of a conscious activity of any person. It is a complex process and its range includes:

- problem identification,
- delineation of the objective of an activity,
- determination of the variants of decision (the acceptable ones),
- determination of the effects of selection of a variant or variants including the identification of their positive or negative consequences,
- selection of an optimum variant,
- analysis of sensitivity of the adopted solution.

In the decision-making process, the single-criterion model is dealt with when striving after achieving a single objective, while in case of taking into consideration the achievement of two or more objectives – the multi-criteria model. As a general rule, the solution of decision tasks is the set of permissible decisions, which is a sub-set of the decision space, while the set of all obtainable results (considering all possible solutions, which are permissible within the decision space taking into account the preset criterion) is named the set of solutions permissible within the criterion space. Thus, the role of the decision maker is to select the best, in his or her opinion, variant (corresponding to his or her preferences). However, it should to be borne in mind that objectives which the decision maker wants to achieve may, but do not need to, be at variance with each other (conflict objectives), while a solution which is definitely better than the others (the so-called predominating solution) is most often rare. A decision is taken based on the decision criteria (set of criteria) indicated by the decision maker, and in this case the multi-criteria discrete methods can be used as tools.

ANALYTIC HIERARCHY PROCESS – METHOD DESCRIPTION

The Analytic Hierarchy Process is one of the mathematical methods used for solving multi-criteria decision problems. Developed in 1970, this method is classified into the American school of multi-criteria decision-making (MCDM) [6]. The essence of the method devised by Thomas L. Satie [2] is that both the variants of decision and the criteria themselves are compared in pairs to each other – the index of relative significance of criterion *Ki* over *Kj* is expressed by parameter a_{ij} that is the quotient of the above-mentioned indices

$$a_{ij} = \frac{r_i}{r_j} \qquad (1)$$

(r_i – absolute rank (importance) of criterion *Ki*, r_j – absolute rank (importance) of criterion *Kj* for *ij* = 1,2,3.....), whom, in turn, the verbal assessment and numerical value are assigned to (Table 1).

Table 1
Rating scale used in the Analytic Hierarchy Process method [8]

Verbal assessment (variant a as compared to variant b with regard to a defined criterion)	Numerical rating
Extremely	9
Very strongly to extremely	8
Very strongly	7
Strongly to very strongly	6
Strongly	5
Moderately to strongly	4
Moderately	3
Equivalently to moderately	2
Equivalently	1

Thus, the algorithm of proceeding in AHP method consists of four stages [5, 8, 9]:

1. Construction of the hierarchical model. Decomposition of a decision problem and construction of the hierarchy of factors (criteria) affecting problem solution.
2. Determination of predominance (preference) of the main factors.
3. Determination of predominance of variants. Determination of reciprocal priorities (significances) in relation to the criteria and variants of decision.
4. Classification of variants of decision and interpretation of results.

For AHP method it needs to be borne in mind that it is useful especially in the selection of variants of decision where qualitative criteria of evaluations are dealt with, while the risk of achieving results burdened with subjectivity of assessment is significant.

EXAMPLE OF CALCULATIONS

Zakład Gospodarki Odpadami – 1R (the Waste Management Plant) utilises the municipal landfill site of approx. 32 thousand km² located in the north-western part of the Silesian Province. The group of materials deposited on the landfill site includes, but is not limited to:

- waste plastic – code 07 02 13,
- waste printing toner (laser and copier heads) – code 08 03 18,
- ferrous metal filings and turnings – code 12 01 01,
- non-chlorinated emulsions – code 13 01 05,
- mineral-based non-chlorinated hydraulic oils – code 13 01 10,
- lead batteries – code 16 06 01,
- debris – code 17 01 07.

For this plant, as well as any other plant running similar activities, the best method for reduction in the waste volume is compacting (the average density of municipal waste is 350 kg/m³ and after having compacted with a compactor[1] – approx. 1200 kg/m³ to 2500 kg/m³), which eliminates the need to cover rubbish with an intermediate layer every day and limits the generation of biological gases. Due to the surface area of the landfill site and the amount of delivered waste, a decision was made to purchase an extra compactor. Therefore, the features being the subject of multi-criteria assessment were determined:

1. financial outlays,
2. daily tonnage of delivered waste,
3. distance over which the waste is shifted when raking up,
4. type of waste,
5. work safety and comfort for the operator of compactor (reinforced ROPS – FOPS cab, air-conditioning and relevant filters to isolate the operator from harmful and aggressive substances discharged on site),
6. terms and conditions of warranty and service (including the availability of spare parts).

The decision-making process took place in two phases:

Table 2
Comparison of qualitative criteria importance in pairs (non-normalised values)

	ToW[2]	WSCO[3]	TCWS[4]
ToW	1.000	2.000	3.500
WSCO	0.500	1.000	1.000
TCWS	0.286	1.000	1.000

Table 3
Comparison of qualitative criteria importance in pairs (normalised values[5])

	ToW	WSCO	TCWS	Average value
ToW	0.560	0.500	0.636	0.565
WSCO	0.280	0.250	0.182	0.237
TCWS	0.160	0.250	0.182	0.197

- as a part of phase one, the development index was used with regard to the first three features (measurable features) – in this way, 3 variants were selected (Kaelble vg27, Tana G 450 and Cat 826G);
- as a part of phase two, the multi-criteria discrete AHP method was used with regard to the selected variants to evaluate the qualitative features (features no. 4-6) of the machines selected under phase one, comparable with regard to technical parameters, e.g. compactors Kaelble vg27, Tana G 450 and Cat 826G.

The comparison of qualitative criteria importance in pairs is presented in Tables 2 and 3.

The comparison of solution variants with regard to the adopted qualitative criteria is presented in Tables 4 and 5.

Table 4
Comparative matrix for variants of technical solutions with regard to the adopted qualitative criteria (non-standardised values)

ToW				WSCO				TCWS			
	v. I	v. II	v. III		v. I	v. II	v. III		v.I	v. II	v. II
v*. I	1.000	1.000	1.000	v. I	1.000	2.000	0.500	v. I	1.000	1.000	0.250
v. II	1.000	1.000	0.500	v. II	0.500	1.000	0.500	v. II	1.000	1.000	0.333
v. III	1.000	2.000	1.000	v. III	2.000	2.000	1.000	v. III	4.000	3.000	1.000
Total	3.000	4.000	2.500	Total	3.500	5.000	2.000	Total	6.000	5.000	1.583

[*] v - variant

[1] compactor is a machine designed for spreading (raking up) as well as breaking up and compacting the substrate of landfill sites.
[2] ToW – type of waste.
[3] WSCO – work safety and comfort for the operator of compactor.
[4] TCWS – terms and conditions of warranty and service.
[5] in the rating normalisation process, the quotient transformations were used.

Table 5
Comparative matrix for variants of technical solutions with regard to the adopted qualitative criteria (standardised values)

ToW			WSCO						TCWS		
	v. I	v. II		v. I	v. II		v. I	v. II		v. I	v. II
v. I	0.333	0.250	v. I	0.333	0.250	v. I	0.333	0.250	v. I	0.333	0.250
v. II	0.333	0.250	v. II	0.333	0.250	v. II	0.333	0.250	v. II	0.333	0.250
v. III	0.333	0.500	v. III	0.333	0.500	v. III	0.333	0.500	v. III	0.333	0.500

The determined concordance index *c*, which indicates the degree of concordance for the comparisons made, is sufficient ($c \leq 0.1$).

The obtained values allowed the partial rankings and importance ratings (relative weights) to be determined for the adopted criteria (Table 6).

Table 6
Collective summary of partial rankings for qualitative criteria and their importance ratings

	0.565	**0.237**	**0.197**
	ToW	WSCO	TCWS
variant I	0.328	0.312	0.175
variant II	0.261	0.198	0.192
variant III	0.411	0.490	0.633

Thus, the relative weights for the final ranking are as follows:
- for variant I – 0.294,
- for variant II – 0.232,
- for variant III – 0.473.

Based on the obtained results, it can be stated that for qualitative features the predominating (suggested) solution is variant III, i.e. the purchase of Cat 826G compactor.

CONCLUSIONS

Challenges taken up by the decision maker are predominantly probabilistic in nature. Thus, taking a decision, which is a peculiar kind of challenge, is related to a risk (the consequences of a decision are many a time difficult to predict) the reduction in which requires both the access to information which is reliable, up to date and as exhaustive as possible and the use of methods and techniques which allow the possessed data and information resources to be processed.

Therefore, more and more often the qualitative evaluation methods [1] as well as the operational research methods are used in the decision-making processes. The latter group includes, but is not limited to, the multi-criteria discrete methods, including the Analytic Hierarchy Process (AHP) method in which the adopted criteria and possible variants of decision are compared in pairs. In the discussed example, the analysis of qualitative features (financial outlays related to the purchase of compactor, daily tonnage of delivered waste, distance over which the waste is shifted when raking up) was followed by the selection of compactor model taking into consideration the three-element set of qualitative features (type of waste to be compacted, work safety and comfort for the operator of compactor, terms and conditions of warranty and service). Unfortunately, the selection of a variant of decision in AHP method is burdened with subjectivity of assessment of the grading person, which is one of objections to this method. The critical remarks with regard to the above-mentioned method also concern the following [2, 3, 4, 7]:
- lack of theoretical bases used in the hierarchy construction process, which opens up the possibilities for creating various hierarchies, and thus resulting in obtaining various solutions;
- errors in the rating aggregation procedure;
- lack of sufficient confirmation of obtained results in statistical examinations.

Also, adding a new criterion in relation to which all variants are equivalent has an essential impact on both the aggregated weights of variants and the final ranking.

However, having in mind that decisions on the investment works are usually made by persons with relevant formal preparation (education) and most often long job seniority, it can be assumed that the margin of error is acceptable, and the selection made will meet user's expectations.

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