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THE INFLUENCE OF STATIC FRICTION ON THE SPEED AND INCLINATION ANGLE OF A BELT CONVEYOR

WPŁYW TARCIA STATYCZNEGO NA PRĘDKOŚĆ I KĄT NACHYLENIA PRZENOŚNIKA TAŚMOWEGO

Key words: friction test, coefficient of wall friction, static friction, inclined conveyors, elastomeric belt.

Abstract: The article describes experimental studies of effective belt tension between the mined hard coal and a rubber conveyor belt. Effective belt tension has a critical impact on the safety and operational reliability of belt conveyors transporting excavated material after ascending or descending. Three samples of flame-retardant conveyor belts from three manufacturers and raw power coal material were used in the tests. In order to determine the significance of the influence of variable parameters on the values of static friction coefficients, the hypothesis on the equality of mean values of the studied variable in three populations was verified by means of the analysis of variance. On the basis of the obtained results, the maximum belt speed was determined as a function of the permissible inclination angle of the belt conveyor.

Słowa kluczowe: test tarcia, współczynnik tarcia zewnętrznego, tarcie statyczne, przenośnik nachylony, taśma elastomerowa.

Streszczenie: W artykule opisano badania doświadczalne sprzężenia ciernego między materiałem sypkim i gumową taśmą przenośnikową. Sprzężenie cierne między materiałem sypkim i taśmą przenośnikową jest jednym z podstawowych czynników istotnych dla zapewnienia bezpiecznej i niezawodnej eksploatacji przenośnika taśmowego transportującego materiał sypki pod określonym kątem nachylenia. Badaniom poddano trzy płaskie fragmenty taśm przenośnikowych pochodzących od różnych producentów. Do obliczeń statystycznych zastosowano analizę wariancji. Na podstawie wyników określono zależność maksymalnej prędkości taśmy w funkcji dopuszczalnego kąta nachylenia przenośnika taśmowego.

INTRODUCTION

Effective belt tension between bulk material and a conveyor belt is one of the basic factors affecting the safety and reliability of work during start-up, braking, and the steady-state movement of conveyor belts transporting bulk materials after ascending or descending **[L. 1]**. Due to the currently applied constructional solutions (higher efficiency of conveyors, increased speed and inclination angle of the belt), and primarily the lack of theoretical and experimental studies on effective belt tension between bulk material and a conveyor belt, the identification of effective belt tension is an important research problem for the proper design of industrial belt conveyor systems **[L. 2–3]**. In order to solve this problem the following were prepared:

- An experimental method of measuring the coefficients of static and kinetic friction between a specific bulk material and conveyor belts of different constructions and surface characteristics was developed; and,
- A series of experimental studies was carried out to determine the relationship between the value of the coefficient of static and kinetic external friction and the influence factors, such as the type of coal, the weight of the sample, the grain size of the sample, the total moisture of the material sample, the type of new belt, the surface condition of the belt (surface roughness), and the inclination of the belt.

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The subject of the paper is to determine the significance of the influence between the value of the coefficient of external static friction and the type of a new conveyor belt of the same type from different manufacturers and to determine the maximum belt speed as a function of the permissible longitudinal angle of the inclination of the conveyor belt on the basis of the obtained results of experimental studies. Selected results of experimental studies presented in the article have been excerpted from a doctoral dissertation **[L. 4]**.

DESCRIPTION OF THE RESEARCH METHODOLOGY

The test stand consists of a frame (for bulk material) placed in a rigid guide on a tilting plate (inclined plane) with a fixed sample of a conveyor belt **[L. 5–6]**. The values measured on the test stand are the angle of the inclination of the inclined plane – φ, and gravitational-motor force – F_{gm}.

On the example of a single measurement (measurement No. 8 from **Table 4**), the method of experimental determination of the static friction coefficient between the mined coal and conveyor belt is presented.

While tilting the inclined plane, the dynamometer measures the gravitational-motor force as a function of the inclination angle (**Fig. 1**), ensuring a constant position of the frame with the excavated material in relation to the belt. From the moment the static effective belt tension is broken at angle φ_s, the gravitational-motor force starts to increase until the maximum angle of inclination φ_{max} is reached.

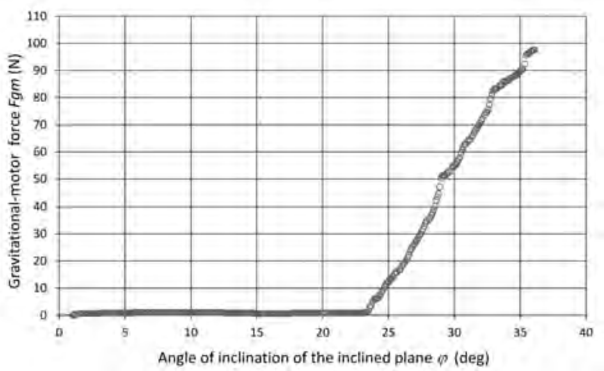

Fig. 1. Graph of the gravitational-motor force as a function of the inclination angle of the inclined plane, $F_{gm} = f(\varphi)$ [L. 4]

Rys. 1. Wykres siły grawito-motorycznej w funkcji kąta nachylenia równi pochyłej, $F_{gm} = f(\varphi)$ **[L. 4]**

In order to determine the apparent angle of static friction – φ_s, the linear regression method was applied (rejecting the initial phase of the graph for $\varphi \leq \varphi_s$, when $F_{gm} = 0$). The results of statistic regression analysis are presented in **Table 1** and graphically in **Figure 2**.

Table 1. Results of statistical regression analysis between gravitational-motor force F_{gm} and the inclination of the inclined plane φ [L. 4]

Tabela 1. Wyniki statystycznej analizy regresji między siłą grawito-motoryczną F_{gm} a nachyleniem równi pochyłej φ **[L. 4]**

Linear model: $y = ax + b + \zeta$	Parameter evaluation	Standard error
Parameter a:	8.120	0.061
Parameter b:	- 190.344	1.833
Regression equation: $F_{gm} = 8.120\ \varphi - 190.344$		
Coefficient of determination: $R^2 = 0.993$		
Evaluation of the standard deviation of a random component: 2.599		

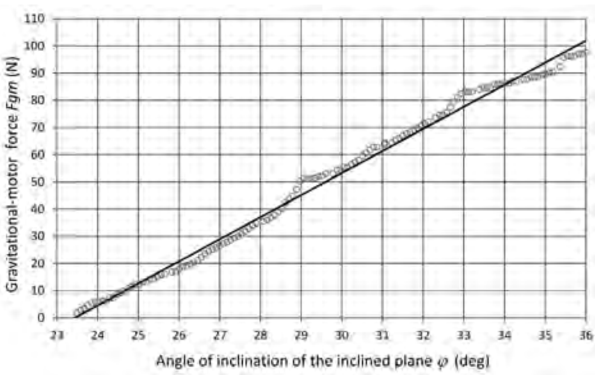

Fig. 2. The values of the apparent angle of static friction φ_s = 23.44° calculated from the regression equation are shown in the graph $F_{gm} = f(\varphi)$ (Table 1) [L. 4]

Rys. 2. Przedstawienie na wykresie $F_{gm} = f(\varphi)$ wartości pozornego kąta tarcia statycznego φ_s = 23,44° obliczonego z równania regresji (tab. 1) **[L. 4]**

The value of the static friction coefficient was calculated on the basis of the apparent angle of friction **[L. 7]**:

$$\mu_s = \frac{m_n + m_r}{m_n} \cdot tg\varphi_s - \frac{F_{gm} + F_{or}}{m_n \cdot g \cdot \cos\varphi_s}, \qquad (1)$$

where

m_s – coefficient of static friction (-)

m_n – mass of bulk solid sample (kg)

m_r – mass of box (kg)

F_{gm} – gravitational-motor force, (for $\varphi \leq \varphi_s$, $F_{gm} = 0$ and for $\varphi_s < \varphi \leq \varphi_{max}$, $F_{gm} > 0$) (N)

F_{or} – force of resistance to motion of the empty box (N)

φ_s – apparent angle of static friction (deg)

DESCRIPTION OF THE EXAMINED SAMPLES

Raw hard coal mining material (type 31.1., tailing type) of apparent density of 1.30–1.37 (g/cm[3]), uniaxial compression strength of 9.1–34.2 (MPa), shear strength $\tau = 0.817\sigma_n$, and internal friction angle of 39.4±5.2 (deg), ash content of 4.93 (%), total sulphur content of 0.87 (%) and total moisture content of 11.7 (%) was used for experimental studies **[L. 8]**.

Three samples of new smooth rubber conveyor belts of the same type produced by three manufacturers were tested. In order to characterise the surface features of the applied conveyor belt samples, the Shore A hardness was determined and the surface structure was analysed using a non-contact laser profilograph. The parameters characterising the spatial distribution of the roughness of the belt are presented in **Table 2**.

Table 2. Selected properties of conveyor belts [L. 4]
Tabela 2. Wybrane właściwości badanych taśm przenośnikowych **[L. 4]**

No.	Properties	Unit	Conveyor belt sample		
			Sample A (Manufacturer A)	Sample B (Manufacturer B)	Sample C (Manufacturer C)
1	Type	(-)	Non- inflammable rubber fabric conveyor belt with increased fire resistance		
2	Shore A hardness	(°Sh A)	72	66	65
3	S_a	(µm)	1.36	2.16	2.03
4	S_z	(µm)	56.7	42.5	30.3
5	S_t	(µm)	66.4	72.6	39.5
6	S_{dq}	(µm/µm)	1.11	0.684	1.32

where:

S_a – arithmetic mean deviation of the height of surface irregularities from the reference plane (µm),

S_z – height of surface irregularities for 10 points (five highest elevations and five lowest depressions) (µm),

S_t – vertical distance between the top of the highest vertex and the lowest surface depression (µm),

S_{dq} – mean square inclination of surface irregularities (measured in the direction of the maximum slope of each individual surface element) (µm/µm).

Figure 3 graphically presents the roughness of a selected cross-section of the conveyor belt samples.

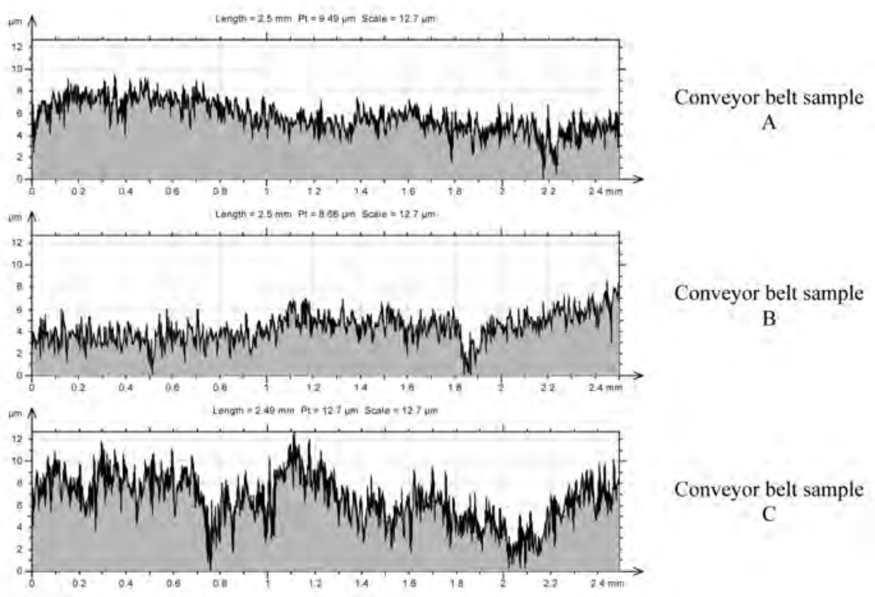

Fig. 3. Surface roughness of the selected cross-section of the conveyor belt samples [L. 4]
Rys. 3. Chropowatość powierzchni wybranych przekrojów próbek taśm przenośnikowych **[L. 4]**

RESULTS OF STATIC FRICTION TESTS

Experimental studies of the static friction coefficient μ_s were carried out for the following friction pair:

- Conveyor belt sample: Sample A (Manufacturer A), Sample B (Manufacturer B), Sample C (Manufacturer C),
- Sample of the handled material: Raw hard coal mining material.

Independent variable: Conveyor belt,
Dependent variable: Coefficient of static friction μ_s.

Summary results of the measurement series are presented in **Tables 3, 4,** and **5**.

Table 3. Experimental results: coal versus belt sample A (Manufacturer A) [L. 4]
Tabela 3. Wyniki badań doświadczalnych: węgiel versus próbka taśmy A (Producent A) **[L. 4]**

No.	φ_s (deg) ($F_{gm}=0$)	$\overline{\varphi_s}$ (deg)	$s_{\overline{\varphi_s}}$ (deg)	μ_s	$\overline{\mu_s}$	$s_{\overline{\mu_s}}$	F_{gm} (N) ($\varphi=\varphi_{max}$)	$\overline{F_{gm}}$ (N)	$s_{\overline{F_{gm}}}$ (N)
1	26.77	25.61	0.38	0.570	0.542	0.009	68.0	85.5	2.4
2	23.74			0.497			92.6		
3	27.38			0.585			82.8		
4	24.57			0.516			91.8		
5	24.68			0.519			89.6		
6	24.68			0.519			85.2		
7	27.04			0.577			91.0		
8	25.86			0.547			88.0		
9	25.85			0.547			78.0		
10	25.58			0.541			87.6		

Table 4. Experimental results: coal versus belt sample B (Manufacturer B) [L. 4]
Tabela 4. Wyniki badań doświadczalnych: węgiel versus próbka taśmy B (Producent B) **[L. 4]**

No.	φ_s (deg) ($F_{gm}=0$)	$\overline{\varphi_s}$ (deg)	$s_{\overline{\varphi_s}}$ (deg)	μ_s	$\overline{\mu_s}$	$s_{\overline{\mu_s}}$	F_{gm} (N) ($\varphi_s=\varphi_{s\,max}$)	$\overline{F_{gm}}$ (N)	$s_{\overline{F_{gm}}}$ (N)
1	23.00	23.04	0.46	0.479	0.481	0.011	92.2	100.4	2.6
2	20.91			0.431			108.6		
3	22.20			0.461			106.0		
4	25.57			0.540			81.6		
5	20.98			0.433			107.8		
6	23.09			0.481			101.4		
7	23.92			0.501			102.2		
8	23.44			0.490			102.6		
9	24.38			0.512			105.0		
10	22.96			0.478			97.0		

where:
φ_s – apparent angle of static friction (deg)
$\overline{\varphi_s}$ – arithmetic mean of the apparent angle of static friction (deg)
$s_{\overline{\varphi_s}}$ – standard deviation of the mean apparent angle of static friction (deg)
μ_s – coefficient of static friction (-)
$\overline{\mu_s}$ – arithmetic mean of the coefficient of static friction (-)
$s_{\overline{\mu_s}}$ – standard deviation of the mean coefficient of static friction (-)
F_{gm} – gravitational-motor force, (for $\varphi \le \varphi_s$, $F_{gm} = 0$ and for $\varphi_s < \varphi \le \varphi_{max}$, $F_{gm} > 0$) (N)
φ – angle of tilt (deg)
$\overline{F_{gm}}$ – arithmetic mean of the gravitational-motor force (N)
$s_{\overline{F_{gm}}}$ – standard deviation of the mean gravitational-motor force (N)

Table 5. Experimental results: coal versus belt sample C (Manufacturer C) [L. 4]
Tabela 5. Wyniki badań doświadczalnych: węgiel versus próbka taśmy C (Producent C) **[L. 4]**

No.	φ_s (deg) ($F_{gm} = 0$)	$\overline{\varphi_s}$ (deg)	$s_{\overline{\varphi_s}}$ (deg)	μ_s	$\overline{\mu_s}$	$s_{\overline{\mu_s}}$	F_{gm} (N) ($\varphi_s = \varphi_{s\,max}$)	$\overline{F_{gm}}$ (N)	$s_{\overline{F_{gm}}}$ (N)
1	22.59	22.05	0.25	0.470	0.457	0.006	112.6	109.7	1.7
2	21.36			0.441			118.0		
3	20.85			0.430			116.2		
4	22.41			0.466			109.8		
5	21.08			0.435			110.6		
6	21.94			0.455			107.8		
7	23.28			0.486			102.4		
8	22.22			0.461			103.2		
9	21.86			0.453			104.0		

STATISTICAL ANALYSIS

In order to determine the significance of the influence of the type of the new belt on the values of the static friction coefficient, the hypothesis on the equality of mean values of the studied variable in three populations was verified by means of the single-factor analysis of variance for dependent groups. The basic conditions of using the test include the following: measurement on an interval scale, the normality of distribution in each examined population and a dependent model. All the results were determined on an interval scale, so the first condition is met. The second condition is the normality of distribution in each examined population. For this purpose, a Lilliefors normality test was carried out for each population, consisting in the verification of the hypothesis of the insignificance of the difference between the examined variable distribution (empirical distribution) and the normal distribution (theoretical distribution). The Lilliefors test is a corrected Kolmogorov-Smirnov test applicable when the mean value and the standard deviation for the general population from which the sample is derived are not known and these values are estimated from the sample. The third condition for using the test is a dependent model, which means that the measurements of a given characteristic are called dependent (related) when they are taken several times for the same objects. The measurements of the examined variable were repeated ten times (k = 10) each time, so the third condition is also met.

The following hypotheses were used for the analysis of variance:

$H_0: \overline{\mu_1} = \overline{\mu_2} = \overline{\mu_3}$

H_1:not all $\overline{\mu_j}$ are equal to each other (j = 1, 2, …k),

where

$\overline{\mu_1}, \overline{\mu_2}, \overline{\mu_3}$ – mean values of the examined variable in populations.

The test statistics shall be in the following form **[L. 9]**:

$$F = \frac{MS_{BC}}{MS_{res}}, \tag{2}$$

where

$$MS_{BC} = \frac{SS_{BC}}{df_{BC}}, \tag{3}$$

$$MS_{BC} = \frac{SS_{BC}}{df_{BC}}, \tag{4}$$

$$SS_{BC} = \sum_{j=1}^{k}\left(\frac{\left(\sum_{i=1}^{n} x_{ij}\right)^2}{n}\right) - \frac{\left(\sum_{j=1}^{k}\sum_{i=1}^{n} x_{ij}\right)^2}{N}, \tag{5}$$

$$SS_{res} = SS_T - SS_{BS} - SS_{BC}, \tag{6}$$

$$SS_T = \left(\sum_{j=1}^{k}\sum_{i=1}^{n} x_{ij}^2\right) - \frac{\left(\sum_{j=1}^{k}\sum_{i=1}^{n} x_{ij}\right)^2}{N}, \tag{7}$$

$$SS_{BS} = \sum_{i=1}^{n}\left(\frac{\left(\sum_{j=1}^{k} x_{ij}\right)^2}{k}\right) - \frac{\left(\sum_{j=1}^{k}\sum_{i=1}^{n} x_{ij}\right)^2}{N}, \tag{8}$$

$$df_{BC} = k - 1, \tag{9}$$

$$df_{res} = df_T - df_{BC} - df_{BS}, \tag{10}$$

$$df_T = N - 1, \tag{11}$$

$$df_{BS} = n - 1, \tag{12}$$

$$N = nk, \tag{13}$$

where
MS_{BC} – the average of squares between measurements,
MS_{res} – the average of squares for the remainders,
SS_{BC} – the sum of the squares between the measurements,
SS_{res} – the sum of the squares for the remainders,
SS_{T} – the total sum of the squares,
SS_{BS} – the sum of the squares between the objects,
df_{BC} – degrees of freedom (between measurements),
df_{res} – degrees of freedom (for the remainders),
df_{T} – total degrees of freedom,
df_{BS} – degrees of freedom (between objects),
n – number of measurements,
x_{ij} – values of the variable for the i objects ($i = 1, 2, \ldots n$) in the j measurements ($j = 1, 2, \ldots k$).

The statistics are subject to the F-Snedecor distribution with df_{BC} and df_{res} degrees of freedom. The p value determined on the basis of the test statistics is compared with the assumed level of significance α:
If the value $p \leq \alpha \Rightarrow H_0$ is rejected by taking H_1,
If the value $p > \alpha \Rightarrow$ there are no grounds to reject H_0.
Table 6 provides the obtained results of the variance analysis for the static friction coefficient as a function of the conveyor belt type. On the basis of F statistics and the p value of the F-Snedecor distribution, the appropriate statistical hypothesis was selected.

Table 6. Analysis of variance for dependent groups [L. 4, 9]
Tabela 6. Analiza wariancji dla grup zależnych **[L. 4, 9]**

Analysed independent variable	Conveyor belt
Analysed dependent variable	Coefficient of static friction μ_s
Level of significance α	0.05
The total sum of the squares SS_T	0.0588
The sum of the squares between the measurements SS_{BC}	0.0380
The sum of the squares between the objects SS_{BS}	0.0112
The sum of the squares for the remainders SS_{res}	0.0096
Degrees of freedom (between measurements) df_{BC}	2
Degrees of freedom (between objects) df_{BS}	9
Degrees of freedom (for the remainders) df_{res}	18
Degrees of freedom (for the remainders) df_T	29
The average of squares between measurements MS_{BC}	0.0190
The average of squares between the objects MS_{BS}	0.0012
The average of squares for the remainders MS_{res}	0.0005
***F* statistics**	**35.591**
***p* value**	**0.000001**

Result: $(p = 0.000001) \leq (\alpha = 0.05) \Rightarrow$ We reject H_0, and take H_1.

The p value of the conducted analysis of variance indicates that, at the level of significance $\alpha = 0.05$, the zero hypothesis must be rejected, which means that the analysed averages are variable; therefore, there is an effect of the type of the new conveyor belt (independent variable) used on the value of the static friction coefficient μ_s. In the graph (**Fig. 4**) concerning the static friction coefficient, a thick line of the mean value of a given coefficient is marked. The confidence interval for the mean value built for a confidence level of 95% is marked in blue. The graph also shows the range of the mean value plus minus a standard deviation signifying the most probable range of variability of friction coefficients determined in individual tests.

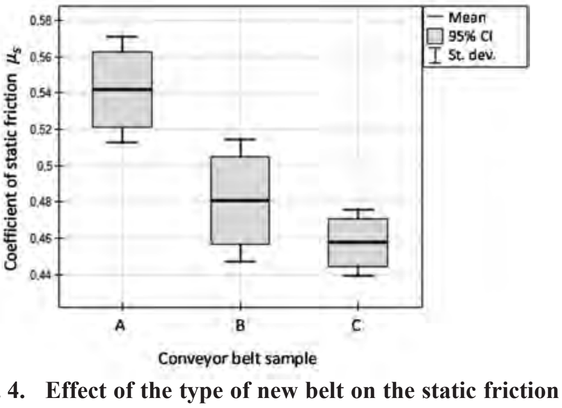

Fig. 4. Effect of the type of new belt on the static friction coefficient between the mined coal material and the conveyor belt [L. 10, 11]
Rys. 4. Wpływ rodzaju nowej taśmy na wartości współczynnika tarcia statycznego między urobkiem węgla i taśmą przenośnikową **[L. 10, 11]**

APPLICATION OF THE RESULTS IN PRACTICE

For the mean values of the static friction coefficient between the mined coal material and three conveyor belts, using the mathematical model of prof. Roberts **[L. 12, 13]**, the maximum belt speed was calculated as a function of the longitudinal angle of inclination of the belt conveyor (**Fig. 5**). The calculations took into account the influence of waving of the excavated material with the belt (the deflection of the belt between the disc sets was assumed at the level of 0.5%) and the acceleration during start-up and braking equal to a_r = 0.5 (m/s^2).

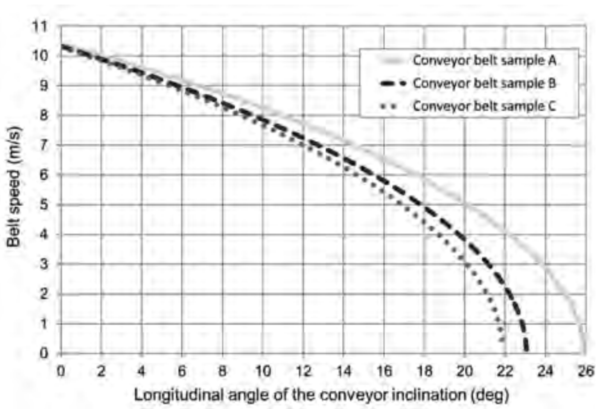

Fig. 5. Maximum belt speed as a function of the longitudinal angle of the conveyor inclination for different values of the friction coefficient (obtained experimentally) between the coal output and the conveyor belt

Rys. 5. Maksymalna prędkość taśmy w funkcji podłużnego nachylenia przenośnika taśmowego dla różnych wartości współczynnika tarcia (wyznaczonych doświadczalnie) pomiędzy urobkiem węgla i taśmą przenośnikową

CONCLUSIONS

It was concluded, based on the conducted analysis of variance that at the level of significance α = 0.05, the zero hypothesis H_0 should be rejected with assuming H_1, which means that the analysed mean values are characterised by variability; thus, there is a significant influence of the type of the new conveyor belt used on the values of the static friction coefficient μ_s.

At the stage of designing an inclined belt conveyor, account should be taken of the fact that conveyor belts of the same type but manufactured by different manufacturers differ significantly in their ability to achieve the maximum permissible longitudinal angle of the conveyor belt inclination. Therefore, a precise determination of the friction coefficient is essential for the proper design of industrial conveyor belt systems. Having the data enabling one to identify the effective belt tension between particular excavated material and a particular belt, inclined belt conveyors can be designed to better utilise their performance capabilities.

For the belt speed of 4 m/s, for example, the maximum permissible conveyor inclination angle is as much as 22° for belt sample A, 19.6° for belt sample B, and only 18.5° for belt sample C (based on **Fig. 5**). The difference in the permissible inclination angle between the belt sample A and belt sample C is as much as 16%.

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