Vibrations in Physical Systems Vol.26 (2014)

Damped Vibration of a Non-prismatic Beam with a Rotational Spring

Wojciech SOCHACKI
Institute of Mechanics and Fundamentals of Machinery Design
University of Technology, Czestochowa, Poland
sochacki@imipkm.pcz.pl

Marta BOLD
Institute of Mechanics and Fundamentals of Machinery Design
University of Technology, Czestochowa, Poland
bold@imipkm.pcz.pl

Abstract

In this paper a problem pertaining to the damped lateral vibrations of a beam with different boundary conditions and with a rotational spring is formulated and solved. In the adopted model the vibration energy dissipation derives from the internal damping of the viscoelastic material (Kelvin–Voigt rheological model) of the beam and from the resistance motion in the supports. The rotational spring can be mounted at any chosen position along the beam length. The influence of step changes in the cross-section of the beam on its damped lateral vibrations is also investigated in the paper. The damped vibration frequency and the vibration amplitude decay level are calculated. Changes in the eigenvalues of the beam vibrations along with the changes in the damping ratio and the change in the model geometry observed on it are also presented. The considered beam was treated as Euler- Bernoulli beam.

Keywords: Vibration damping, non-prismatic beam, rotational spring.

1. Introduction

The transverse vibration of prismatic and non-prismatic beams with additional discrete elements has been investigated in a number of studies. Study [1] presents the transverse vibration of a beam with a stepped cross-section together with the phenomenon of damped vibration in the body where the system is present. The problem of the vibration and dynamic stability of beams with different boundary conditions with additional discrete elements was presented in study [2]. Study [3] concerned the modal analysis of a semi-infinite Euler-Bernoulli beam with discrete elements in the form of a rotational and a translational spring. Investigations concerning damped vibration were discussed in [4-7]. Study [4] discussed the effect of small internal and external damping on the stability of non-conservative beam systems. The authors of study [5] demonstrated the effect of internal damping on the vibrations of a supported beam with a mass attached to the free end of the beam. Study [6] examined the vibration of an axially-loaded Timoshenko beam with local internal damping. The effect of constructional damping of the fixations on free vibration of the Bernoulli-Euler beam was presented in study [7].

This study formulates and solves the problems of transverse damped vibration in a C-P (clamped-pinned) beam with a stepped cross-section and with a rotational spring. Dissipation of the vibration energy occurs as a result of the simultaneous internal damping of the viscoelastic material of the beam and structural damping in the support. The constructional damping was modelled using a rotational viscous damper. The study analyses the simultaneous effect of the structural damping and internal damping, the spring rigidity and its location and the effect of the location of the stepped cross-section of the beam on the properties of the considered system. The results obtained in the study are presented as 2D figures and spatial presentations.

2. Mathematical model

A scheme of the considered C-P beam is presented in Fig. 1.

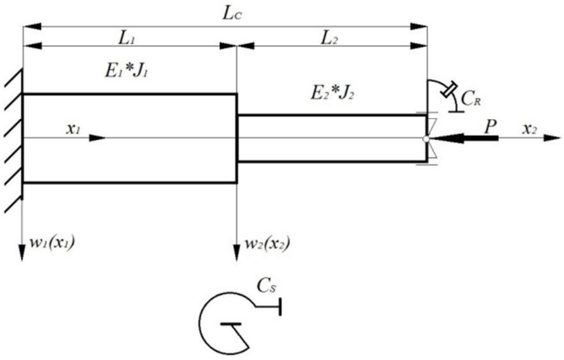

Figure 1. Model of the C-P beam with step changes in the cross-section with a rotational spring C_S and rotational viscous damper C_R

Viscoelastic material was characterized by the Young's modulus E_n and the viscosity coefficient E_n* of the beam material. The coefficient of constructional damping in the rotational viscous damper was denoted as C_R.

The vibration equation for the two parts of a beam is known and has the following form:

$$E_n J_n \frac{\partial^4 W_n(x,t)}{\partial x^4} + E_n^* J_n \frac{\partial^5 W_n(x,t)}{\partial x^4 \partial t} + \\ + P_n \frac{\partial^2 W_n(x,t)}{\partial x^2} + \rho_n A_n \frac{\partial^2 W_n(x,t)}{\partial t^2} = 0 \tag{1}$$

where:

$W_n(x,t)$ – the lateral displacement of beam,
A_n – the cross-section area of the beam,
J_n – the moment of inertia for the beam section,
ρ_n – the density of the beam material,
P_n – longitudinal forces in beam,
n = 1,2
x – space coordinate,
t – time,

Solutions to equations (1) take the form:

$$W_n(x,t) = w_n(x)e^{i\omega^* t} \tag{2}$$

where: ω^* – the complex eigenvalue of the system, $i = \sqrt{-1}$

Substitution of (2) into (1) leads to:

$$w_n^{IV}(x) + \beta_n^2 w_n^{II}(x) - \gamma_n w_n(x) = 0 \tag{3}$$

where:

$$\gamma_n = \frac{\rho_n A_n \omega^{*2}}{(E_n + iE_n^* \omega^*)J_n}, \quad \beta_n = \sqrt{\frac{P_n}{(E_n + iE_n^* \omega^*)J_n}} \tag{4}$$

Boundary conditions:

$$\begin{aligned} &w_1(0) = 0, \quad w_1(l_1) = w_2(0), \quad w_1^I(0) = 0, \quad w_2(l_2) = 0, \\ &w_1^I(l_1) = w_2^I(0), \quad (E_2 + iE_2^*\omega^*)J_2\, w_2^{II}(l_2) = -C_R\, i\, \omega^* w_2^I(l_2), \\ &(E_1 + iE_1^*\omega^*)J_1\, w_1^{III}(l_1) + (E_2 + iE_2^*\omega^*)J_2\, w_2^{III}(0) = 0, \\ &(E_1 + iE_1^*\omega^*)J_1\, w_1^{II}(0) + C_S\, w_1^I(l_1) = (E_2 + iE_2^*\omega^*)J_2\, w_2^{II}(0) \end{aligned} \tag{5}$$

The solution to equations (3) is expressed in the form of functions:

$$w_n(x) = D_{1n}e^{\lambda_n x} + D_{2n}e^{-\lambda_n x} + D_{3n}e^{i\bar{\lambda}_n x} + D_{4n}e^{-i\bar{\lambda}_n x} \tag{6}$$

where:

$$\lambda_n = \sqrt{-\frac{\beta_n^2}{2} + \sqrt{\frac{\beta_n^4}{4} + \gamma_n}}, \quad \bar{\lambda}_n = \sqrt{\frac{\beta_n^2}{2} + \sqrt{\frac{\beta_n^4}{4} + \gamma_n}} \tag{7}$$

By substituting (6) into (5) a homogeneous system of equations was obtained with respect to unknown constants D_{kn} , and can be written in the matrix form as:

$$[A](\omega^*)\, D = 0 \tag{8}$$

where:

$$A(\omega^*) = [a_{pq}], \quad (p,q = 1,2..8), \quad D = [D_{kn}]^{\mathrm{T}}, \; k = 1,2-4 \tag{9}$$

The system has a nontrivial solution when the matrix determinant of coefficients is equal to zero with constants D_{kn}.

$$\det A(\omega^*) = 0 \tag{10}$$

Finding the complex eigenvalues of matrix $A(\omega^*)$ leads to the determination of damped vibration frequency $Re(\omega^*)$ and the vibration amplitude decay level $Im(\omega^*)$ of the considered system.

3. Numerical calculation results

Computations were carried out assuming the following dimensionless quantities:

$$\eta = \frac{E_n^*}{aE_n}, \quad a^2 = L_C^{\,4}\frac{\sum_n \rho_n A_n}{\sum_n E_n J_n}, \quad \mu = \frac{C_R}{L_C\sqrt{\rho_2 A_2 (E_2 + iE_2^* \omega^*) J_2}}, \tag{11}$$

$$c = \frac{C_S\, L_C}{(E_1 + iE_1^* \omega^*) J_1}, \quad J = \frac{J_2}{J_1}, \quad l = \frac{L_1}{L_C}, \quad p = \frac{P}{P_C} \tag{12}$$

where: P_C – the critical load of the tested beam with a constant cross-section.

The results of the calculations are presented in Figs. 2 to 6. Investigations were carried out for different ratios of the moments of inertia for the two parts of the beam (J=0.5, J=5) and for a beam with a constant cross-section (J=1). The system was loaded with longitudinal force P (p=0.05). The dependency of the eigenvalues (real parts $Re(\omega^*)$ and imaginary parts $Im(\omega^*)$) on the coefficients of constructional damping μ, spring rigidity c and location of the change in the beam cross-section l was also determined.

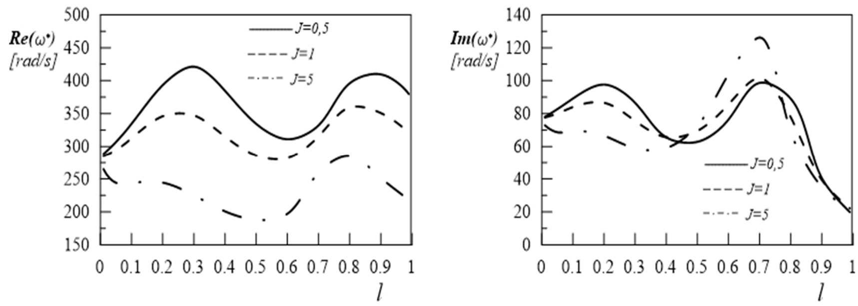

Figure 2. The dependency of real parts ($Re(\omega^*)$) and imaginary parts ($Im(\omega^*)$) of the first beam eigenvalue on the coefficient l at η=0.002, μ=0.3, c=10

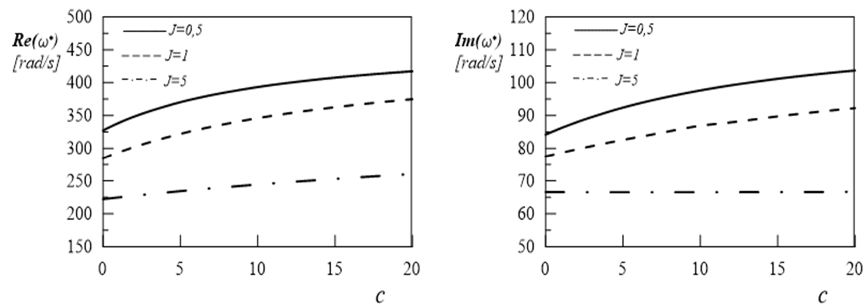

Figure 3. The dependency of real parts ($Re(\omega^*)$) and imaginary parts ($Im(\omega^*)$) of the first beam eigenvalue on the spring rigidity coefficient c at η=0.002, μ=0.3 and l=0.2

Figures 5 and 6 present collective diagrams of the dependency of eigenvalues (*Re(ω*)* and *Im(ω*)*) in the studied system on the change in the rigidity of elastic support *c* and constructional damping *μ*. The calculations were carried out for selected values of internal damping and for a central location of the rotational spring and two values of the relation of the moments of inertia (*J*=5 and *J*=0.5). The results are presented as spatial diagrams.

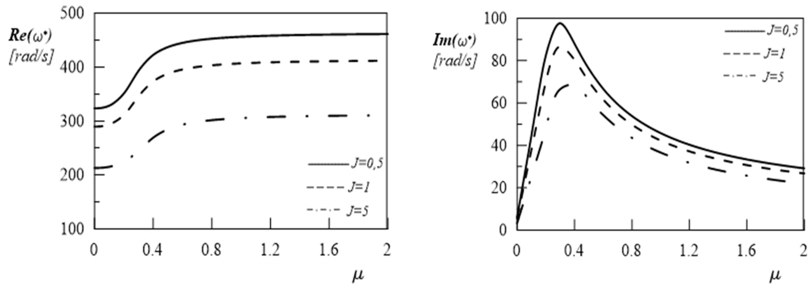

Figure 4. The dependency of real parts (*Re(ω*)*) and imaginary parts (*Im(ω*)*) of the first beam on structural damping *μ* at *η*=0.002, *c*=10 and *l*=0.2

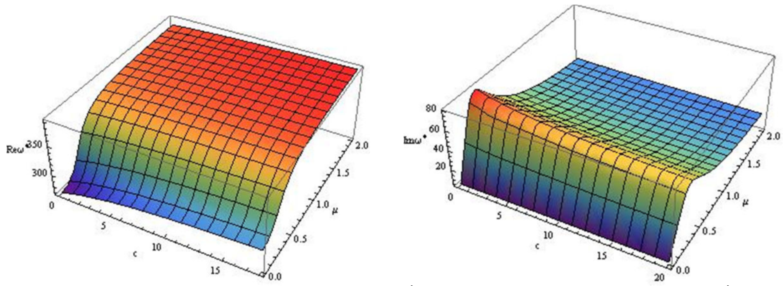

Figure 5. The dependency of real parts (*Re(ω*)*) and imaginary parts (*Im(ω*)*) of the first eigenvalue of the beam on the coefficient of structural damping *μ* and spring rigidity coefficient *c* for *l*=0.5 and *J*=0.5, *η*=0.002

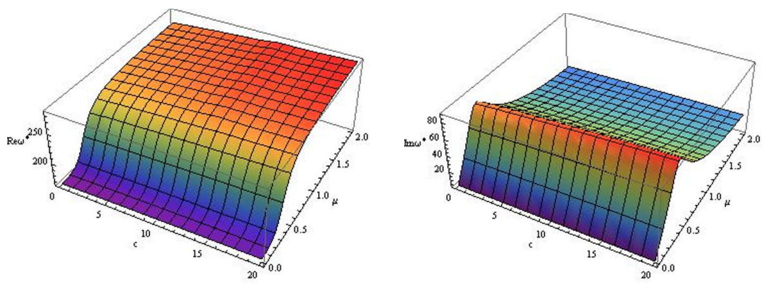

Figure 6. The dependency of real parts (*Re(ω*)*) and imaginary parts (*Im(ω*)*) of the first eigenvalue of the beam on the coefficient of structural damping *μ* and spring rigidity coefficient *c* for *l*=0.5 and *J*=5, *η*=0.002

4. Conclusions

The damped frequencies of system *Re*(ω^*) and the degree of amplitude decay *Im*(ω^*) in the system depend on the location of the rotational spring along the beam. No uniform tendency for changes was observed in the case studied (Fig. 2). Improved spring rigidity causes a constant increase in the damped frequencies of the first eigenvalue of the system (for selected values of coefficients η, μ and l). The degree of amplitude decay in this case depends on the ratio of rigidity J for the two beam parts. For the central location of the change in the cross-section (l=0.5), an increase in c causes a decrease in the coefficient of the amplitude decay for J=0.5, and an increase for J=5 (Fig. 5 and 6). The constructional damping of the fixation points with selected values of spring rigidity causes much more substantial changes in the eigenvalues of the system than in the reverse case (the change in coefficient c for selected value μ). The results presented in the study help determine the geometric parameters and values of the coefficients that characterize the damping and elasticity of the system for which the maximum degree of amplitude decay is maintained.

Acknowledgments

This research was supported in 2014 by the Ministry of Science and Higher Education, Warsaw, Poland.

References

1. M. I. Friswell, A. W. Lees, *The modes of non-homogeneous damped beams*, Journal of Sound and Vibration **242**(2) (2001) 355-361.
2. W. Sochacki, *Dynamic stability of discrete-continuous mechanical systems as working machine models*, Monographic series **147**, Czestochowa University of Technology, Czestochowa 2008.
3. A. Pau, F. Vestroni, *Modal Analysis of a Beam with Radiation Damping: Numerical and Experimental Results*, Journal of Vibration and Control **13**(8) (2007), 1109-1125.
4. O. N. Kirrllov, A. O. Seyranin, *The effect of small internal and external damping on the stability of distributed non-conservative systems*, Journal of Applied Mathematics and Mechanics **69** (2005), 529-552.
5. M. Gürgöze, A. N. Doğruoğlu, S. Aeren, *On the eigencharacteristics of a cantilevered visco-elastic beam carrying a tip mass and its representation by a spring-damper-mass system*, Journal of Sound and Vibration **301** (2007), 420-426.
6. Wei-Ren Chen, *Bending vibration of axially loaded Timoshenko beams with locally distributed Kelvin–Voigt damping*, Journal of Sound and Vibration **330** (2011), 3040–3056.
7. G. Oliveto, A. Santini, E. Tripodi, *Complex modal analysis of flexural vibrating beam with viscous end conditions*, Journal of Sound and Vibration **200** (1997), 327-345.