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## SIMULATION OF THE MAGNETOSTRICTIVE ACTUATOR TRANSIENTS

The paper deals with a magnetostrictive fast-acting actuator applied as a driving device for plasma valve. The actuator is characterized by a relatively small displacement (less than 0.1 mm), but with a very short response time – below 0.1 ms. System is designed for so called “high intensity plasma pulses gun” which is applied in the area of plasma physics and material engineering [1]. A structure with an axisymmetrical actuator energised by discharged pulses of a capacitor has been proposed. The field-circuit mathematical model of the dynamic operation of the actuator has been applied. The model includes: the equation of transient electromagnetic field in a non-linear ferromagnetic material and equation of electric circuit. Using the Borland Delphi 9.0 environment, the computer software has been elaborated. Results of simulation are presented.

KEYWORDS: actuators, magnetostriction, electromagnetic field, transient analysis

### 1. INTRODUCTION

The linear motion devices represent a broad class among the electromechanical energy converters. This group includes both: converters performing limited movement called magnetic actuators as well as linear motors. In comparison with the classic drives with a rotating electrical machines the primary advantage resulting from the use of actuators and motors for the direct realization of linear movement is elimination of transmission and coupling mechanisms .

The most common use of electromagnetic actuators (i.e. converters with limited movement) are associated with contactors and hydraulic/pneumatic valve drives [4]. In this case, the basic functional requirements concern the characteristics of the drive, i.e. force-displacement characteristic. In many cases, however, also very fast dynamic operations of the device, it is a short response time is required [6]. In case

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of conventional electromechanical actuators the response time is of order of a millisecond up to hundred milliseconds (the lower values refer to small objects). Obtaining a response time of less than 1 millisecond is very difficult and even impossible without additional supporting springs.

Therefore, there are new structures in which the force is generated by exploitation another phenomena than those resulting from the interaction of electromagnetic fields and moving ferromagnetic elements. For this purpose the piezoelectric, magnetostrictive, ultrasonic piezoceramic elements, as well as electro-rheological actuators are constructed.

The paper deals with a system designed for so called “high intensity plasma pulses gun” which is applied in the area of plasma physics and material engineering [3, 5]. The magnetostrictive actuator is used as a driving device for plasma valve. The actuator is characterized by very short response time – below 0.1 ms, but relatively small displacement – less than 0.1 mm. A mathematical field-circuit model [6, 7] of the dynamic operation of the actuator has been elaborated and results of simulation are presented.

Magnetostriction occurs in all ferromagnetic materials. It consists in a change in the linear dimension of a ferromagnetic element caused by magnetic field. Changes in linear dimensions of the element are different in each direction. In most cases a privileged direction in which the change is greatest can be distinguished. This type is called a linear magnetostriction [3, 5]. This linear length change is expressed in micrometers. Usually, so called coefficient of magnetostriction  $\lambda = \Delta l/l$  is defined, where:  $\Delta l$  - the change in length of the sample  $l$  - total length of the sample [2, 3, 8]. The unit of  $\lambda$  is micrometer per meter ( $\mu\text{m}/\text{m}$ ). In catalogs of manufacturers this unit is denoted by the symbol *ppm*. Modern magnetostrictive devices are made of materials of the so-called giant magnetostriction (GMM) [3]. The values of  $\lambda$  for these materials even exceed 1000 *ppm*. Fig. 1 shows the change in linear dimension as a function of the external magnetic field for the sample made from typical GMM material [3, 5].

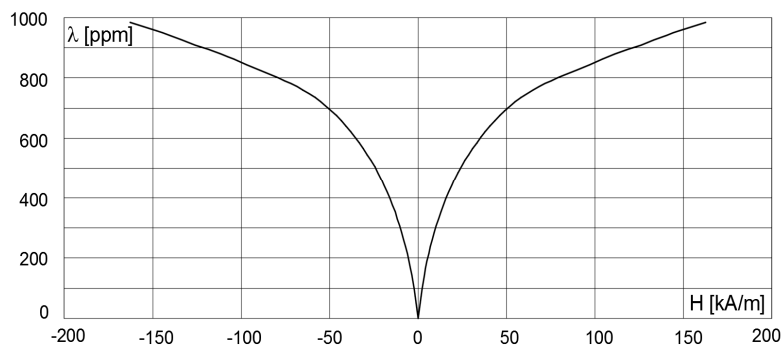


Fig. 1. Relative linear elongation versus magnetic field intensity of the GMM sample

Figure 2 shows a structure of the magnetostrictive actuator. The active element of the actuator (magnetostrictive cylindrical core) made of a GMM material is placed within the solenoid. In order to avoid inducing eddy currents in the dynamic states, that significantly delay the magnetic flux increase and change the length of the core, the other parts of the device should be made of a material with a very high resistivity.

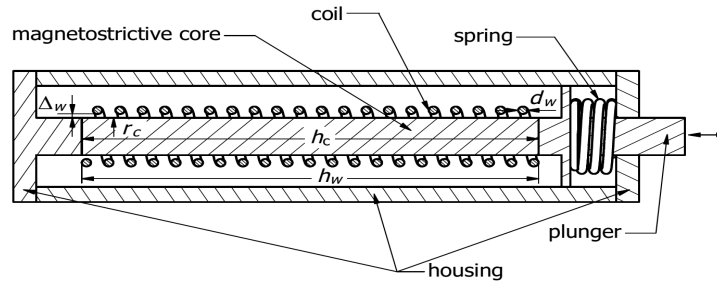


Fig. 2. Structure of axisymmetrical magnetostrictive actuator

## 2. FIELD-CIRCUIT MATHEMATICAL MODEL OF THE CAPACITOR-INDUCTOR SYSTEM

A structure with an axisymmetrical magnetostrictive actuator (Fig.1) has been proposed [4, 7]. The device is supplied with a capacitor battery discharge pulses. During the charging the capacitors in the battery are connected in parallel, while during discharging – in series. This allows for a high initial discharging voltage. Two-dimensional mathematical model of a system in which the field equations are coupled to the equation the electrical circuit consisting of the R, L, C lumped parameters [4, 6, 7].

The magnetic circuit of the actuator contains non-linear ferromagnetic parts. The transient electromagnetic field in the actuator is voltage-excited, and this means that the current  $i(t)$  in the winding is not known in advance, i.e. prior to the electromagnetic field calculation [6]. Mathematical model of the electromagnetic phenomena consist of:

- equation of the transient axisymmetrical electromagnetic field in a non-linear medium

$$\vec{\nabla} \cdot \left( \frac{\nu}{\rho} \vec{\nabla} \Phi \right) = -J + \frac{\gamma}{\rho} \left( \frac{\partial \Phi}{\partial t} + \nu \frac{\partial \Phi}{\partial z} \right) \quad (1)$$

- equation of the electric circuit,

$$\frac{d\Psi}{dt} + Ri = u_c, \quad \frac{du_c}{dt} = -\frac{1}{C}i \quad (2)$$

where:  $\rho = 2\pi r$ ;  $\Phi(r,z,t) = \rho A_{\Phi}(r,z,t)$  is the auxiliary magnetic potential;  $J$  - is the current density;  $\nu, \gamma$  are the reluctivity and conductivity of the considered regions, respectively;  $i, \Psi$ , are the winding current and flux linkage,  $u_c$  is the capacitor voltage;  $R$  is the winding resistance,  $C$  is the capacitance.

It has been assumed that the magnetic circuit of the device is made of nonconductive materials and winding is formed from conductors made of thin wires (litz wire) or very thin tapes, so the second term in the right-hand side in equation (1) representing the eddy currents has been neglected.

There are two kinds of the nonlinearity of the system. The nonlinearity arises from (a) nonlinear  $B-H$  curve and (b) nonlinear function  $\lambda(H)$  where  $\lambda$  is the dimension linear change (elongation) of the magnetostrictive material.

### 3. NUMERICAL IMPLEMENTATION OF FIELD AND CIRCUIT EQUATIONS

The numerical implementation of the algorithm is based on the finite element method and „step by step” Cranck-Nicholson scheme for time derivatives estimation [6].

At the  $n$ -th time-step, system of FEM equations can be written as

$$\mathbf{S}\Phi_n = \mathbf{N}i_n \quad (3)$$

where:  $\mathbf{S}$  is the FEM stiffness matrix;  $\Phi_n$  is the one-column matrix (vector) of nodal potentials;  $\mathbf{N}$  is the vector of "nodal turn numbers" [6].

According to the assumed Cranck-Nicholson scheme the discrete circuit equations at the  $n$ -th time-step take the forms [6]:

$$2 \frac{\Psi_n - \Psi_{n-1}}{\Delta t} \frac{d\Psi}{dt} \Big|_{t_{n-1}} + Ri_n = u_{cn} \quad (4)$$

$$2 \frac{u_{cn} - u_{cn-1}}{\Delta t} \frac{du_c}{dt} \Big|_{t_{n-1n}} = -\frac{1}{C} i_n \quad (5)$$

Substituting  $u_{cn}$  from (5) into (2) finally we get

$$(0.5\Delta t)^{-1} \mathbf{N}^T \Phi_n + Zi_n = V_{n-1} \quad (6)$$

where:  $Z = R + 0.5\Delta t / C$ ,

$$V_{n-1} = u_{cn-1} + (0.5\Delta t)^{-1} \mathbf{N}^T \Phi_{n-1} + 0.5\Delta t \frac{du_c}{dt} \Big|_{t_{n-1}} + \mathbf{N}^T \frac{d\Phi}{dt} \Big|_{t_{n-1}}.$$

Because the field is voltage-excited the current value  $i_n$  in (3) is not know in advance. Therefore, the field equation (3) and circuit equation (6) must be solved at the same time (simultaneously). The global system of the field-circuit equations can be written in the compact form

$$\begin{bmatrix} \mathbf{S} & -\mathbf{N} \\ (0.5\Delta t)^{-1} \mathbf{N}^T & Z \end{bmatrix} \cdot \begin{bmatrix} \Phi_n \\ i_n \end{bmatrix} = \begin{bmatrix} 0 \\ V_{n-1} \end{bmatrix} \quad (7)$$

Because of the nonlinearity of the ferromagnetic core, the stiffness matrix  $\mathbf{S}_n$  in (3) depends on the solution  $\Phi_n$  and therefore the solution (nodal potential vector  $\Phi_n$  and also current  $i_n$ ) must be determined iteratively [6]. In the proposed algorithm, the Newton-Raphson process has been adopted. In the  $k$ -th iteration, the vector  $\Phi_n$  in (3) and current  $i_n$  in (6) are replaced with increases  $\delta\Phi_n^k = \Phi_n^k - \Phi_n^{k-1}$  and  $\delta i_n^k = i_n^k - i_n^{k-1}$ .

The vector  $\delta\Phi_n^k$  satisfies the set of equations

$$\mathbf{H}_n^k \cdot \delta\Phi_n^k = \mathbf{R}_n^k \quad (8)$$

where:  $\mathbf{H}_n^k = \mathbf{H}(\Phi_n^{k-1})$  is the Jacobian matrix of the Newton-Raphson process,  $\mathbf{R}_n^k$  is the residual vector of the equation (3), i.e.:

$$\mathbf{R}_n^k = \mathbf{N}(i_n^{k-1} + \delta i_n^k) - \mathbf{S}_n^k \Phi_n^{k-1} \quad (9)$$

The matrix  $\mathbf{H}_n^k$  represents so called Hessian of the magnetic energy function  $W = 0,5\Phi^T \mathbf{S} \Phi$ . Elements  $h_{ij}$  of this matrix are equal to second order derivatives  $h_{ij} = \partial^2 W / \partial \Phi_i \partial \Phi_j$ , where  $\Phi_i, \Phi_j$  are the potentials of nodes  $Q_i$  and  $Q_j$  [6].

In the  $k$ -th iteration the current increase  $\delta i_n^k$  in (9) is not known in advance and therefore the vector  $\mathbf{R}_n^k$  is not known either.

Thus, also in case of using the increases  $\delta\Phi_n^k, \delta i_n^k$ , the field and circuit equations must be solved simultaneously. The following system of equations is obtained:

$$\begin{bmatrix} \mathbf{H}_n^k & -\mathbf{N} \\ (0.5\Delta t)^{-1} \mathbf{N}^T & Z \end{bmatrix} \cdot \begin{bmatrix} \delta\Phi_n^k \\ \delta i_n^k \end{bmatrix} = \begin{bmatrix} \mathbf{R}_{\Theta n}^{k-1} \\ R_{U n}^{k-1} \end{bmatrix} \quad (10)$$

The right-hand side of this system is explicitly known from the previous time-step and equal to:  $\mathbf{R}_{\Theta n}^{k-1} = \mathbf{N}i_n^{k-1} - \mathbf{S}_n^k \Phi_n^{k-1}$ ,

$$R_{U n}^{k-1} = V_{n-1} - Zi_n^{k-1} - (0.5\Delta t)^{-1} \mathbf{N}^T \Phi_n^{k-1}.$$

Hereunder, the practical, efficient method for solving complex system (10) is presented.

Let  $\delta\tilde{\Phi}_n^k$  be the solution of the equation (9) for the right hand side equal to

$$\tilde{\mathbf{R}}_n^k = \mathbf{N}(i_n^{k-1}) - \mathbf{S}_n^k \Phi_n^{k-1} \quad (11)$$

(which is known from previous iteration), and let  $\delta\Phi_{0n}^k$  be the solution of (9) for the right-hand side equal to  $\mathbf{R}_{0n}^k = \mathbf{N}i_0$ , where  $i_0$  is an arbitrarily chosen value.

Then, denoting  $\delta i_n^k / i_0 = \lambda_n^k$  one can write:

$$i_n^k = i_n^{k-1} + \lambda_n^k i_0 \quad (12)$$

$$\Phi_n^k = \Phi_n^{k-1} + \delta\tilde{\Phi}_n^k + \lambda_n^k \delta\Phi_{0n}^k \quad (13)$$

Putting (12) and (13) into the Kirchhoff's equation (6) the following is obtained:

$$\lambda_n^k = \frac{V_{n-1} - Z i_n^{k-1} - (0.5\Delta t)^{-1} \mathbf{N}^T (\Phi_n^{k-1} + \delta\tilde{\Phi}_n^k)}{Z i_0 + (0.5\Delta t)^{-1} \mathbf{N}^T \delta\Phi_{0n}^k} \quad (14)$$

Finally, current  $i_n^k$  and nodal potential vector  $\Phi_n^k$  can be computed from (12) and (13), and capacitor voltage – from (15):

$$u_{cn} = u_{cn-1} + \frac{\Delta t}{2} \left. \frac{du_c}{dt} \right|_{n-1} - \frac{\Delta t \cdot i_n}{2C} \quad (15)$$

If  $\|\mathbf{R}_n^k\| < \varepsilon_\Theta$ , where  $\varepsilon_\Theta$  is the permissible incorrectness, then the iterative procedure at the  $n$ -th time-step is completed.

In the developed software two repeating „loops” occur: (a) superior recursive loop associated with a step-by-step scheme and (b) interior (secondary) iterative procedure associated with non-linearity of ferromagnetic core.

As a result of field-circuit computation the time-varying vector of nodal potentials  $\Phi(t)$  is obtained. Then the magnetic flux density, and magnetic field intensity are determined. On this basis the waveform  $\lambda(t)$  the core elongation is computed. To this purpose, the magnetostrictive core having the radius  $r_c$  and length  $h_c$  is divided by planes  $z=\text{const}$  into  $l_{T+1}$  layers of thickness  $h_{cl}$   $l = 1, 2, \dots, l_{T+1}$ .

It has been assumed that the number of layers is equal to the number of nodes on grid lines parallel to the actuator axis within the core region. Average magnetic flux density in the  $l$ -th layer assigned to the node  $(r_c, z_l)$  is computed as  $B_{avl} = \Phi(r_c, z_l) / \pi r_c^2$ . According to the B-H curve, the field intensity  $H_{avl}(B_{avl})$  in the  $l$ -th layer is determined, and finally, on this basis of  $\lambda(H)$  characteristics the relative elongation  $\lambda_l = \lambda(H_{avl})$  of  $l$ -th layer and its absolute value  $\Delta h_l = \lambda_l h_l$  are calculated. The total elongation of the magnetostrictive core is:

$$\Delta h = \sum_{l=1}^{l_{T+1}} \Delta h_l \quad (16)$$

#### 4. RESULTS AND CONCLUSIONS

Structure shown in Fig. 1 has been considered. The following parameters of the device have been assumed dimensions:  $h_c=100$  mm,  $h_w=100$  mm,  $r_c=5$  mm,  $\Delta_w=0.1$  mm, number of winding layers  $w=1$ , number of turns in one winding layer  $N_1=100$ , wire diameter  $d_w=0.8$  mm, capacity  $C=60$   $\mu$ F, the initial capacitor voltage  $U_0=460$  V. The magnetostrictive core is made of TERFENOL D with characteristic  $\lambda(H)$  presented in Fig. 1. Other part of the actuator a made of non-conducting and non-ferromagnetic material of type polyamide.

In the designed system the following functional main parameters were required: the response time:  $t_{req} \leq 0.05$  ms and absolute elongation  $\Delta h_{req} \geq 0.05$  mm.

The time response of the actuator, i.e. the waveforms of winding current  $i(t)$  and relative values of the core elongation  $\lambda(t)$  are shown in Fig. 3. The maximum values  $i_{max}=770$  A and  $\lambda_{max}=575$  ppm occur for  $t = t_{max}=0.041$  ms. The absolute maximal core elongation  $\Delta h_{max} = 0.058$  mm. The required elongation  $\Delta h_{req} = 0.05$  mm is reached at time  $t = 0.02$  ms what is less the required value 0.05 ms. This means that obtained parameters meet the established requirements.

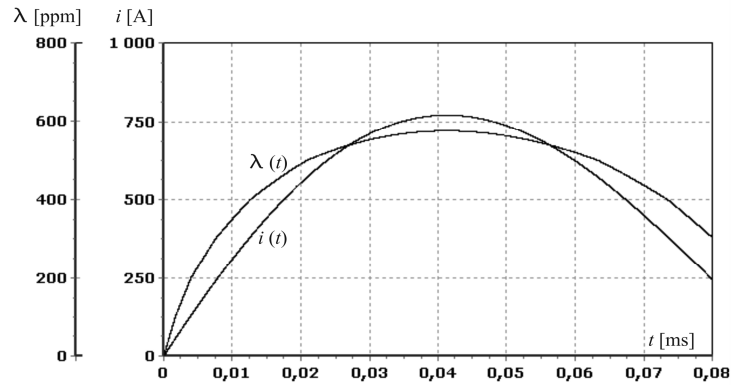


Fig. 3. Time response of the actuator: transient current  $i(t)$  and elongation  $\lambda(t)$

The distribution of the magnetic flux density in the magnetostrictive core at  $t = 0.041$  ms, i.e. for current  $i = 770$  A is show in Fig. 4.

The elaborated mathematical model of a magnetostrictive actuator and implemented computer software can be a good tool for the design and selection of optimal parameters of the actuator. This allows to avoid time-consuming and expensive stage of manufacturing physical models and prototypes of the device.

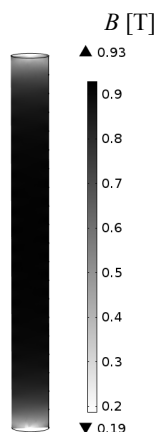


Fig. 4. Distribution of magnetic flux density in the core

In the further research, the other ferromagnetic parts of the actuator can be included. The eddy currents in the magnetostrictive core and other parts, as well as the current displacement on the winding solid conductors will be taken into the consideration. Also the effect of core “movement” on the electromagnetic phenomena will be taken into account.

The improved model will be coupled with an optimization module for optimal synthesis of the actuator.

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