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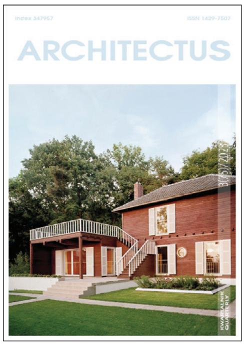

Wojciech Januszewski*

An outline of the geometric proportion systems in architecture

Introduction

Reflections on the intellectual foundations that inform the creation of architectural form occupy an important place among various trends of the theory of architecture. Numerous principles, which were formulated in the language of practical geometry or simple arithmetic over the centuries, served logical mastery of compositional problems. Over the last 50 years, the formerly common knowledge and understanding of these principles have undergone erosion[1]. According to some scholars, the separation of the art of design and mathematical methods leads to the lack of balance in the creative process and consequently to a general reduction in design skills [2].

The subject of the study covers proportion systems, i.e. principles of organizing the architectural form regarding mutual relations of measures of the part and the whole (Fig. 1). These principles may be geometric or arithmetic in nature and – in connection with the proportions of the human body – anthropometric as well.

The purpose of this research is to present a synthetic outline of the issues of geometric proportion systems in architecture, including the contemporary concept of these systems. The work aims at presenting important theoretical aspects taking into account their practical applications. The article may not claim to be a comprehensive study, however, it is an attempt at showing the essence of the subject. A huge variety of proportion systems, their variants and interpretations required selection of the source material. The significance of individual proportion systems in the theory of architecture as well as their representa-

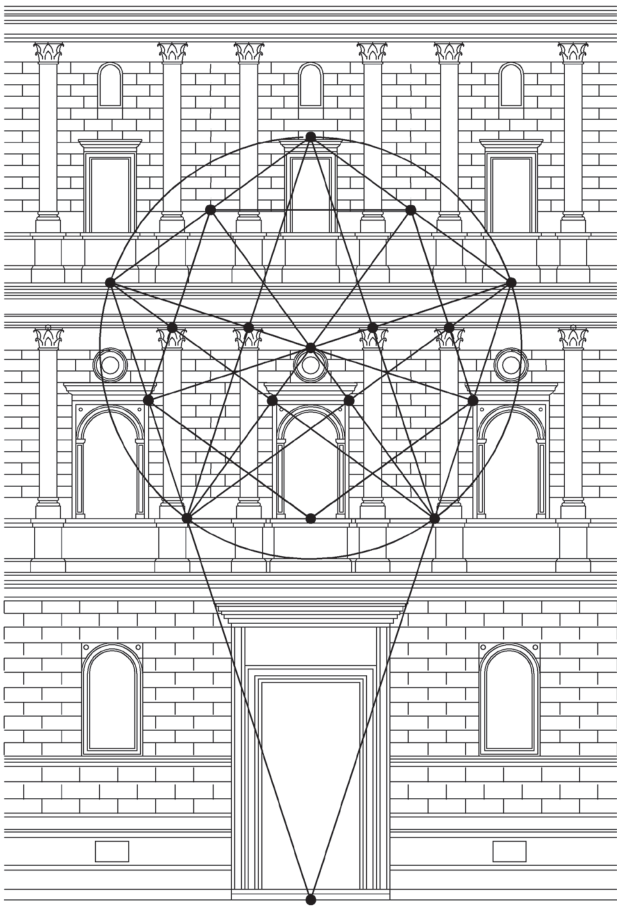

Fig. 1. Analysis of the façade of the Chancellery Palace in Rome by Piotr Biegański using regular pentagon geometry and the golden ratio (elaborated by W. Januszewski based on [3])

Il. 1. Analiza fasady Pałacu Kancelaryjnego w Rzymie autorstwa Piotra Biegańskiego wykorzystująca geometrię pięciokąta foremnego i złoty podział (oprac. W. Januszewski na podstawie [3])

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[1] As Peter Märkli emphasizes, a reference to proportion systems in the architect's practice was "self-evident – across every epoch" [1, p. 117].

tiveness for the proposed types of geometric principles (regulating lines, similar shapes, regular figures and dynamic symmetry) were adopted as the selection criterion.

The study is based on an interdisciplinary approach, combining the history and theory of architecture with elements of practical geometry. The method includes the analysis of literature and the research on the application of proportion systems in solving compositional problems. The article proposes an original approach to geometric proportion systems, interpreting them as a coherent theory which is based on interdependent elements. Unlike the current understanding of these elements as competitive principles, the author claims that they are integral components of the theory based on the paradigm of rationally organized architectural form.

The state of research

The theory of proportion systems is a vast and multi-trend field. The subject of proportions has been a central concept of the theory of architecture. It appeared in numerous works from the early Renaissance till the end of the 18th century. The classic work by Rudolf Wittkower entitled *Architectural principles in the age of humanism* is devoted to these theories [4]. The 19th century brought a number of geometric speculations regarding proportion systems in ancient and medieval architecture. The review of these theories is included in the English Gwilt's *Encyclopedia* [5]. New concepts were also formulated in the 1st decade of the 20th century, e.g. Macody Lund's, Jay Hambidge's and Ernst Moessel's systems [6]–[8] and after World War II as well. Due to the volume of the material, compilation and cross-sectional studies are invaluable [9]–[11]. Contemporary studies on proportion systems are published in "Nexus Network Journal" which is devoted to architecture and mathematics [12]. Among the Polish authors, the idea of proportions was discussed to a limited extent by, inter alia, Piotr Biegański [3] and Kazimierz Ciechanowski [13]. The work by Rafał Mazur, which describes the problem from the perspective of architectural philosophy, is also noteworthy [14]. Publications by Juliusz Żórawski, a pioneer of the theory of architecture perception, constitute a significant – although indirect – Polish contribution to this issue [15], [16]. Valuable resources that offer insights into proportion systems from the perspective of contemporary architectural practice include a collection of conversations with architects titled *Proportions and Cognition in Architecture and Urban Design*, published in 2019, with an interview with Swiss architect Peter Märkli [1]. Among recognized contemporary authors, Rob Krier is also a popularizer of proportion systems [17].

An outline of the theory of geometric systems

Theory versus history

The dynamic development of the modern theory of geometric proportion systems began in around the 2nd half of the 19th century and reached its culmination in the 1st decade after World War II along with the activities of Le Corbusier, Rudolf Witkower, and others [18, pp. 1–14]. This theory should be treated as a separate field from the history of architecture because its main purpose is not to determine historical facts, but to find formulas useful in design. The history of architecture is treated instrumentally – as an experimental field which makes it possible to find formulas for the present. This may explain the certain freedom with which numerous authors approached historical matter and how they justified their concepts using it. Reconstructions of old systems in which literature abounds are most often not supported by sources and are based almost exclusively on measurements of preserved buildings. Their value as a certain historical knowledge is debatable[2].

However, we do have decisive evidence that the proportion systems were in use in ancient times and in the Middle Ages. Apart from the famous record in Vitruvius [19, pp. 13, 14], they include, inter alia, the inscription regarding the construction of the arsenal in Piraeus which is assigned to Philo [20, pp. 388–391] (Fig. 2a), documents of the construction of the Milan Cathedral containing the construction of *ad triangulum* from the 14th century (Fig. 2b), and the brochure of Master Mathes Roritczer from 1486 which described the method of drawing the pinnacle cross-sections by transforming the square (Fig. 2c). Details of these methods and their importance are the subject of thorough historical research. According to some researchers, they first of all had a symbolic meaning connected with old cosmological ideas. Others emphasize their practical side. Due to imperfections of measuring methods the point was to facilitate the process of staking out a building by means of simple tools, e.g. a rope, as well as easy communication between the architect and builders [21, pp. 46–59]. The theorists of the Renaissance postulated proportions based on simple numerical ratios which derived from Pythagorean music intervals (1:2 – octave, 2:3 – fourth interval, 3:4 – fifth interval, etc.) [4, pp. 113–142]. The numerical Renaissance theory was later undermined as based on a false analogy between hearing and seeing [9, pp. 72–75].

The reborn proportion theory which developed from the mid-19th century until the 1950s abandoned numerical systems and limited itself to visual aspects of the form, i.e. to the geometric structure [9, pp. 82–125]. This study is focused on this trend of the modern theory of proportions[3]. In the further part of the article, elements of the theory of geometric systems will be presented. Then – on the examples of practical applications – the relationships between these elements will be discussed.

[2] It is difficult to ascertain whether the matching of a given system to the existing building really indicates the use of it by builders, or whether it results from the random convergence of dimensions. Moreover, the search for "hidden" systems often led to different conclusions in relation to the same buildings. Famous buildings such as the Parthenon or the Cathedral in Chartres received a few or several different geometric interpretations [11, pp. 75–95].

[3] Therefore, two important contemporary systems were omitted: the Le Corbusier's *Modulor* which combines geometric aspects with anthropometry as well as Hans van der Laan's *plastic number* system as arithmetic and at the same time based on a relatively complicated theory [22], [23].

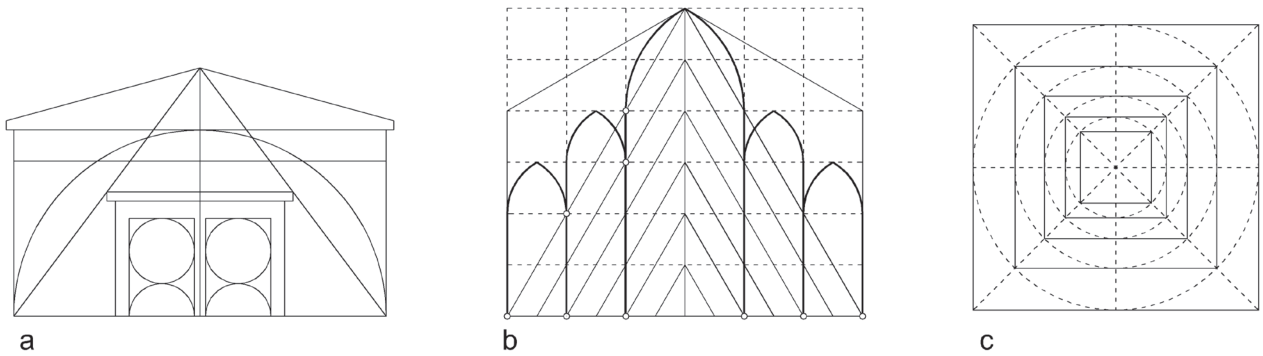

Fig. 2. Historical geometric constructions in architecture:
a) reconstruction of the Arsenal Façade in Piraeus according to Auguste Choisy,
b) geometric scheme of the section of the Cathedral in Milan according to the drawing by mathematician Gabrielle Stornalocco from 1391,
c) quadrature – geometric construction of a square with an area equal to half of the original square
(elaborated by W. Januszewski)

Il. 2. Historyczne konstrukcje geometryczne w architekturze:
a) rekonstrukcja fasady arsenału w Pireusie według Augusta Choisy,
b) schemat geometryczny przekroju katedry w Mediolanie według rysunku matematyka Gabrielle Stornalocco z 1391 r.,
c) tzw. kwadratura – geometryczna konstrukcja kwadratu o polu równym połowie kwadratu wyjściowego (oprac. W. Januszewski)

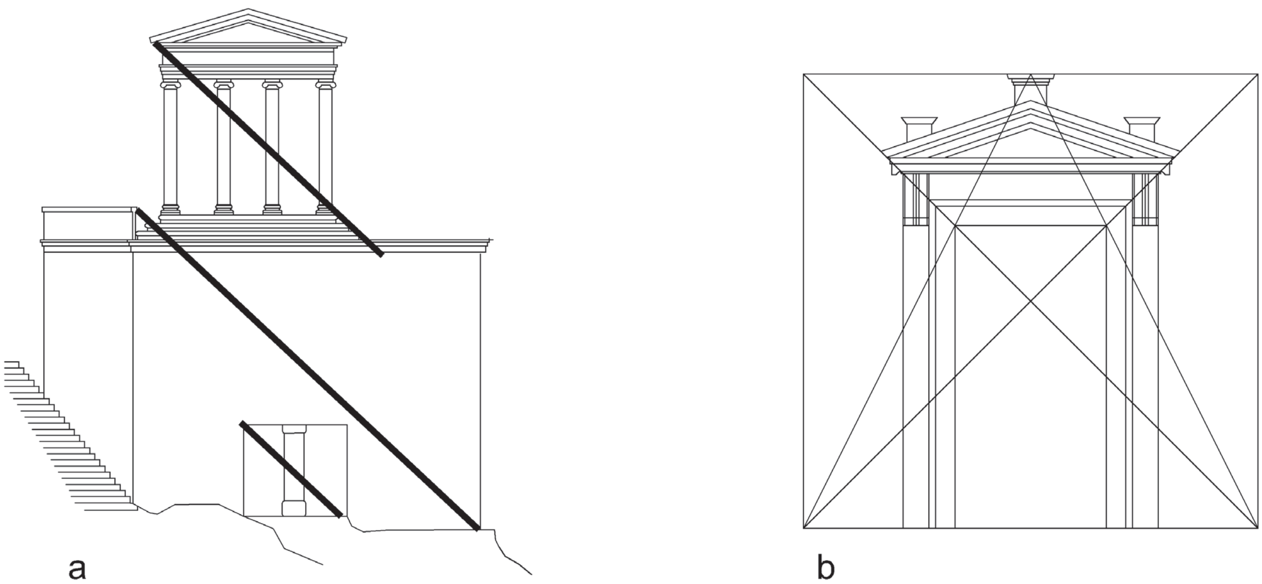

Fig. 3. Shapes and lines in geometric proportion systems:
a) similar rectangles in the composition of the Temple of Nike on the Acropolis according to [24],
b) regulatory lines of the portal in the drawing by Sebastiano Serlio according to [25]
(elaborated by W. Januszewski)

Il. 3. Kształty i linie w geometrycznych systemach proporcji:
a) prostokąty podobne w kompozycji fasady świątyni Nike na Akropolu według [24],
b) linie regulacyjne portalu w rysunku Sebastiana Serlia według [25] (oprac. W. Januszewski)

Similar shapes

German theoretician August Thiersch was the first to present the concept of similar figures. He noticed that the composition which consisted of rectangles with the same ratios of sides gave the impression of being harmonious. He illustrated his theses with drawings in which he signaled the similarity of rectangles by means of parallel or perpendicular diagonals (Fig. 3a) [24, pp. 37–81].

The originality of Thiersch's thought was expressed in the belief that the source of harmony was not a specific mathematical ratio but the equality of several ratios[4]. Harmony was supposed to result not from the character of individual shapes but from the relationship between shapes. From the point of view of the perceptive theory of form which was developed by, inter alia, Żórawski, we can talk about an attempt at creating a *limited complexity* of the work of architecture. This is aimed at regulating the amount of visual information and the balance

[4] That is, proportion in the strict sense.

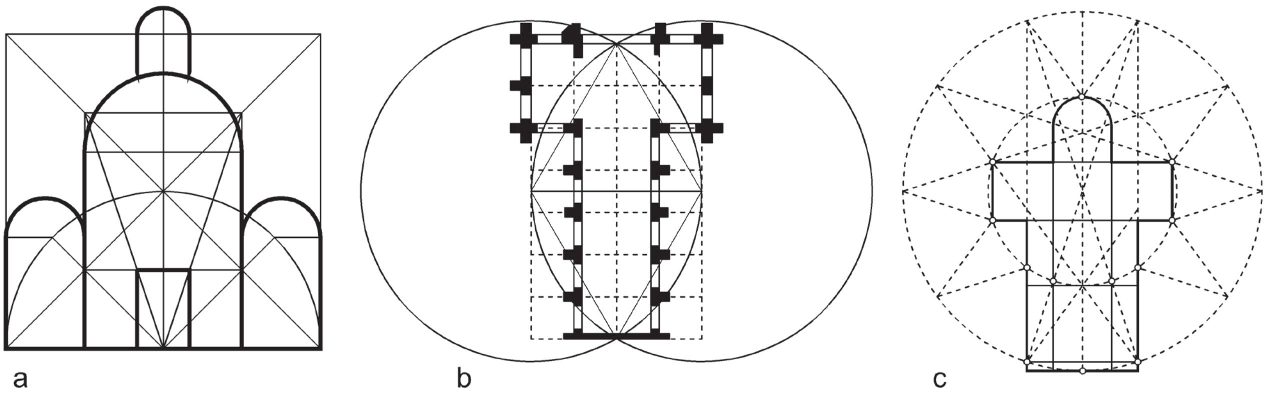

Fig. 4. Regular figures:
a) scheme of the church façade construction from Francesco di Giorgio's treatise using the square's form, its diagonals, and arches,
b) equilateral triangles and *vesica piscis* in the 19th-century analysis of New College Chapel in Oxford (2nd half of the 14th century) [5, p. 1014],
c) one of the "Gothic" plan types based on dividing the circle into 10 parts according to Moessell [8, p. 18]
(elaborated by W. Januszewski)

Il. 4. Figury regularne:
a) schemat konstrukcji elewacji kościoła z traktatu Francesco di Giorgio z wykorzystaniem formy kwadratu, jego przekątnych i łuków,
b) trójkąty równoboczne i *vesica piscis* w XIX-wiecznej analizie proporcji kaplicy New College w Oksfordzie (2. połowa XIV w.) [5, p. 1014],
c) jeden z typów planu „gotyckiego" oparty na podziale koła na 10 części według Moessela [8, p. 18]
(oprac. W. Januszewski)

between a deficiency of stimuli and their abundance [16, pp. 81–85].

The principle proposed by Thiersch is transparent, easy to be applied and flexible, therefore, it was very popular. Heinrich Wolflin expressed his great recognition for this principle [9, p. 104] and Le Corbusier applied it in the Schwob Villa project from 1916, which was published in *Towards a new architecture* [26, p. 80].

Regulating lines

The term *regulating lines* was popularized by Le Corbusier in *Towards a New Architecture* [26, pp. 63–83]. The idea itself has a former origin, i.e. regulating lines appear in the writings by Sebastiano Serlio, Francesco di Giorgio, Philibert de L'Orme [25, pp. 95–128], François Blondel and others. The regulating line is a graphic tool which is used to organize elements of a composition in the drawing plane. The line arrangement is often subject to a more general principle that is proper for a given system, e.g. the geometry of the regulating figure – a triangle, square, etc. Regulating lines usually connect important composition points, e.g. wall corners, tops of window openings, important architectural elements, etc. (Fig. 3b).

The regulating lines *cannot be seen* in the implemented building, that is, they are not the content of the visual stimulus projected onto the retina. However, these lines can help an architect in the anticipation of the perception process. On the basis of Żórawski's theory, they can be identified as an element of the so-called *network of straight lines* stretched between *key points*, on which observer's sight moves in the perception process [16, pp. 21–25]. Regulating lines can also form the so-called *structural frame* of the perceived shape [27, pp. 92–95]. Therefore, they serve to model perception and their layout conditions individual perception. According to Żórawski, […] *proportions, which perform the role of visual communication in architecture and give rise to aesthetic feelings in a viewer, always take place between points that are in correspondence thanks to their location on one straight line* [16, p. 31]. Also, […] *straight lines make it easier for us to quickly orient ourselves in architectural wholes and strengthen the combination of parts* [15, p. 92].

Regular figures

One of the methods of controlling the form is to inscribe it in a simple geometric figure, i.e. a circle, an equilateral triangle, a square, a regular pentagon, etc. This method in *ad triangulum* and *ad quadratum* variants was used in medieval architecture. The forms of regular polygons, as well as of stars constructed by means of them, are repeated in great abundance in medieval decorative art, the excellent example of which is presented by Gothic rosettes. The square and its divisions also appear in the Renaissance tractates (Fig. 4a). Starting from the middle of the 19th century, a number of theories about geometric rules of old architecture have been presented. Various figures appear in them – apart from those already mentioned, there is also "fish bladder" (*vesica piscis*) which was created by means of overlapping of two circles (Fig. 4b) and – as proposed by Violett-le-Duc – the so-called Egyptian triangle [28, pp. 402–471]. The reconstruction of the *ad quadratum* method was made by Norwegian architect Macody Lund [6]. Then, German researcher Ernst Moessel presented the method of a circle division – *Kreuzteilung*[5]. Moessel believed that he not only discovered the

[5] By dividing a circle, a number of rectangles with specific proportions can be constructed. For example, a division into six parts generates a rectangle with proportions of sides $\sqrt{3}$, into eight parts – rectangle $\sqrt{2}$, and into five – rectangle φ ("golden"). More complicated charts generate

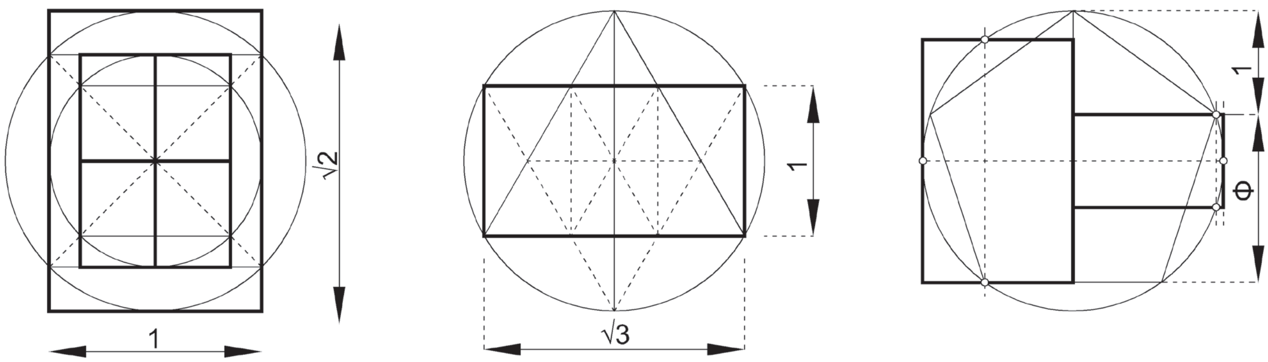

Fig. 5. Some proportional relationships in regular shapes (square, equilateral triangle, and pentagon) and the construction of characteristic rectangles ($\sqrt{2}$, $\sqrt{3}$, and φ, respectively) (drawing by W. Januszewski)

Il. 5. Niektóre relacje proporcjonalne w figurach foremnych (kwadratu, trójkąta równobocznego i pięciokąta) oraz konstrukcja charakterystycznych prostokątów (odpowiednio $\sqrt{2}$, $\sqrt{3}$ i φ) (oprac. W. Januszewski)

geometric system, but the archaic *ritual* of erecting buildings. The construction of a temple or a house was supposed to be preceded by designating a regulating circle in the terrain, which determined the shape of the whole and parts (Fig. 4c). Moessel's theories, like many other reconstructions of old systems, constitute speculations which are difficult to be verified due to the lack of sources.

According to Hambidge, systems based on simple figures are usually radial, i.e. […] *this is concentric symmetry, ordered distribution of shapes or compositional units around the center. These units are almost always parts or logical divisions of regular figures, namely an equilateral triangle, a square, and a regular pentagon* [7, p. 138]. The application of regular figures requires gaining knowledge of these elementary *logical divisions* (Fig. 5).

Dynamic symmetry

The system of *dynamic symmetry* is an important achievement of the modern theory of proportions. Its inventor, American artist Jay Hambidge, like many other researchers – tried to reconstruct the geometric principle of ancient art. By studying the profiles of ancient vases and Greek architecture, he created a flexible system which made it possible to construct complicated geometric layouts with a limited number of elements. Hambidge continued Thiersch's trend – he employed similar rectangles and diagonals while simultaneously incorporating a rigorous mathematical discipline into his system [7, pp. 11–29]. The system is based on the properties of selected rectangles. For example, $\sqrt{2}$ *rectangle*[6] is divided into two smaller $\sqrt{2}$ rectangles, $\sqrt{3}$ rectangle into three $\sqrt{3}$ rectangles, and φ rectangle[7] into a square and a smaller φ rectangle. These divisions can be multiplied each time obtaining elements belonging to the system but in different scales. Diagonal grids specific for the "motif" applied help in determining divisions – different for φ, different for $\sqrt{2}$ or $\sqrt{3}$, etc. (Fig. 6a). On the basis of this principle, it is possible to create more complicated compositions. For example, if we draw φ rectangle in a square with a longer side common with the side of the square, the remaining part will consist of a smaller square and a smaller φ rectangle. Similar layouts can be created on motifs $\sqrt{2}$ and $\sqrt{3}$ (Fig. 6b).

Related principles

Although the rules described above are often theoretically considered as separate elements, it would be a mistake to treat them as independent or mutually competitive. There is a clear and significant mutual relationship between them in geometric systems. This relation naturally results from objective geometric relationships, however, it is beneficial to consciously combine various principles, which is conducive to the creation of more coherent architectural compositions. This will be shown on the example of practical applications of these principles in solving simple formal problems in architecture.

And thus, the principle of similar figures is inextricably linked with regulating lines which constitute a tool for the graphical determination of the similarity relationship of rectangles on a vertical or horizontal projection. In this method, however, the *regulating line* can be used in accordance with its basic compositional function, i.e. it can combine three or more key points of the system. This was presented in the example in which the principle of *similar figures* was applied, and which was illustrated by parallelism and perpendicularity of *regulating lines*. At the same time, regulating lines connect significant corners of volumes and windows, giving the façade a harmonious appearance (Fig. 7).

Greater complexity may occur along with the application of the *regular figure* principle, as in the example in Figure 8. In this case, to design the gable wall of the house, an equilateral triangle was used which determined the points of the wall and roof ridge base. The remaining

smaller divisions of the same ratios. Moessel's method is clearly explained by R. Krier [17, pp. 181–190].

[6] In literature, it was accepted to describe rectangles by means of a ratio of a larger side to a smaller one, e.g. in $\sqrt{2}$ rectangle this ratio is $\sqrt{2}$: 1

[7] φ – the usual term determining the golden ratio, i.e. approximately 1.618…

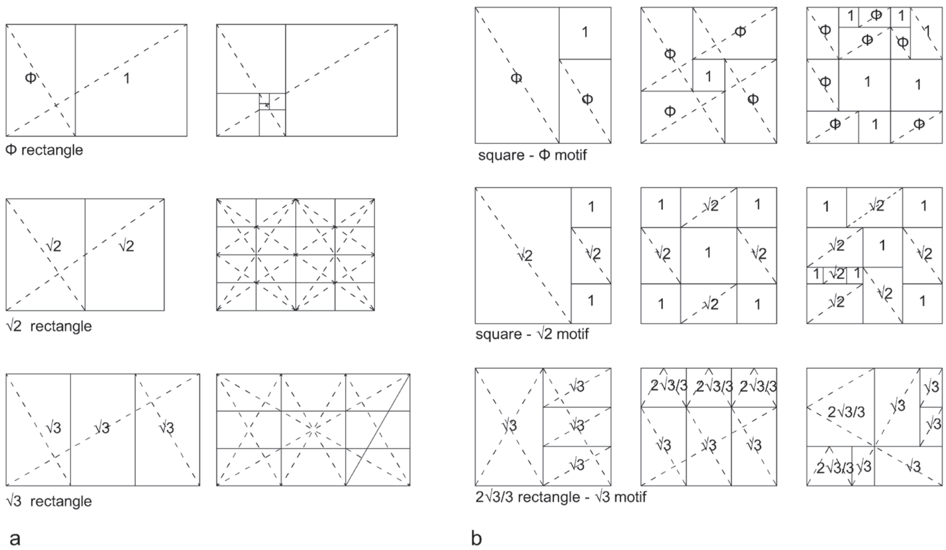

Fig. 6. Dynamic symmetry: a) basic divisions of the "golden" rectangle φ and rectangles $\sqrt{2}$ and $\sqrt{3}$, b) variants of square decomposition based on φ and $\sqrt{2}$, and the decomposition of the rectangle $2\sqrt{3}/3$ based on $\sqrt{3}$ (drawing by W. Januszewski)

Il. 6. Symetria dynamiczna: a) podstawowe podziały „złotego" prostokąta φ oraz prostokątów $\sqrt{2}$ i $\sqrt{3}$, b) warianty dekompozycji kwadratu na motywie φ i $\sqrt{2}$, oraz dekompozycje prostokąta $2\sqrt{3/3}$ na motywie $\sqrt{3}$ (rys. W. Januszewski)

points were determined as a result of secondary divisions of the triangle and hexagram. The effect is a harmonious composition consisting of similar shapes (rectangle $2\sqrt{3}/3$ described on an equilateral triangle), which is shown by the arrangement of parallel *regulating lines* in Figure 2b.

Although Jay Hambidge emphasized the separateness of the *dynamic symmetry* he invented from the previous *static* systems [7], in practice these rules may also be strongly connected, which is shown in the example in Figure 9. The dimensions of a simple church projection are determined by three squares drawn in accordance with the principle of *ad quadratum*, i.e. total length is the side of the largest square, the length of the nave body – the side of the medium square, and its width – the side of the smaller square. As a result, the nave body has the proportions of dynamic rectangle $\sqrt{2}$, which makes it possible to decompose it into smaller rectangles $\sqrt{2}$ and thus obtaining divisions of bays of the nave and aisles. However, the "eastern" part is divided into two squares, i.e. "towers" and a vestibule with proportions $\sqrt{2}$. The principle of similar figures is met as well.

In the case of asymmetrical compositions, *dynamic symmetry* is a more appropriate output principle than *regular figures*. This is illustrated by the geometric system organizing the horizontal projection of the house in Figure 10. *Dynamic symmetry* on the φ motif was applied here. Individual spaces are constructed by means of the diagonals of rectangle φ oriented horizontally or vertically. However, here, too, a *regular figure*, i.e. a square closely connected with the geometry of the golden rectangle, plays an important role in creating a sense of harmony.

The principles of *similar figures*, *regulating lines*, *regular figures* and *dynamic symmetry* constitute together an integral basis of geometric proportion systems. They are used to achieve one goal, i.e. creating a geometric structure in which components of the composition are included and which defines their relations.

The value of proportion systems

Proportion systems are one of the most controversial topics of the theory of architecture. Disputes on this subject took place already in the 17th century, for example, the debate between Claude Perrault and François Blondel [29, pp. 6–9]. The 18th century brought aesthetic revolution and the view that beauty is a subjective category and independent of objective mathematical principles.

Interest in proportion systems revived in the mid-19th century and reached its peak in the 1st decade of the 20th century. They constituted the subject of studies which were conducted by pioneers of modernism such as Hendrik Petrus Berlage [30, pp. 185–215] or Peter Behrens [31, p. 448]. Le Corbusier's *Modulor* gained great publicity, however, it failed to reverse general skepticism towards the idea of mathematical order in architecture [18, pp. 1–14]. Nowadays, the debate about proportion systems has become less popular, but even today we can encounter diametrically different views on this issue. Contemporary authors

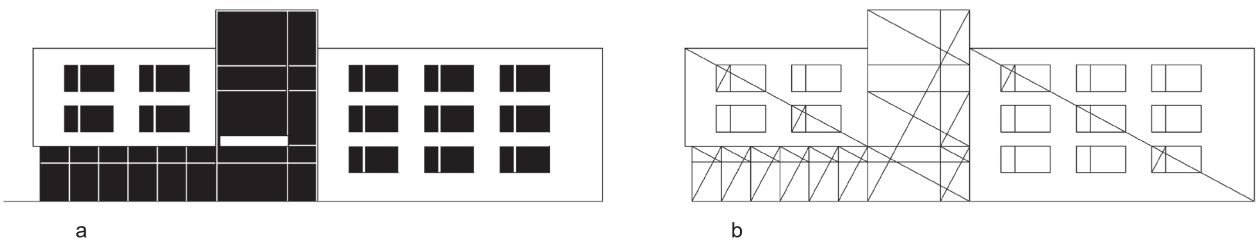

Fig. 7. Application of the principle of similar figures: a) façade; b) regulating lines (drawing by W. Januszewski)

Il. 7. Zastosowanie zasady figur podobnych: a) rysunek fasady, b) linie regulacyjne (rys. W. Januszewski)

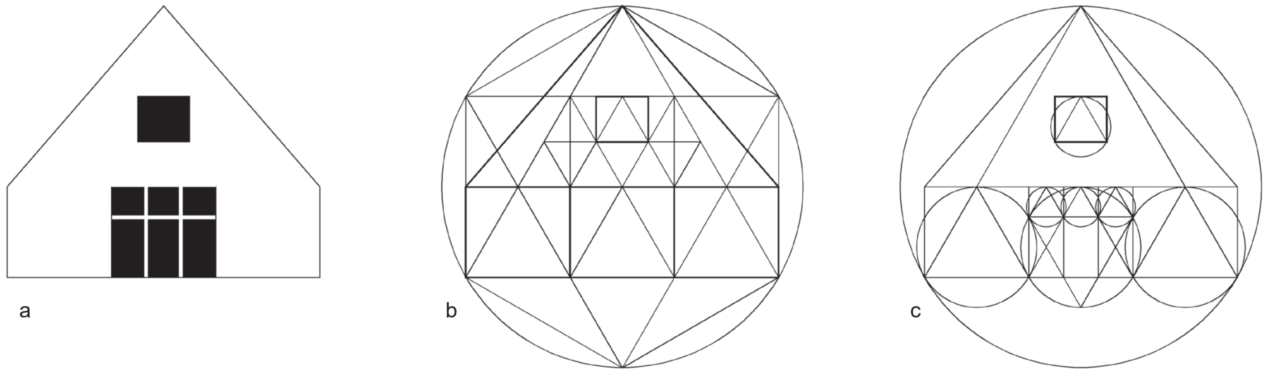

Fig. 8. Application of the principle of the equilateral triangle:
a) façade, b) division construction, c) regulating lines (drawing by W. Januszewski)

Il. 8. Zastosowanie zasady trójkąta równobocznego:
a) rysunek fasady, b) konstrukcja podziałów, c) linie regulacyjne (rys. W. Januszewski)

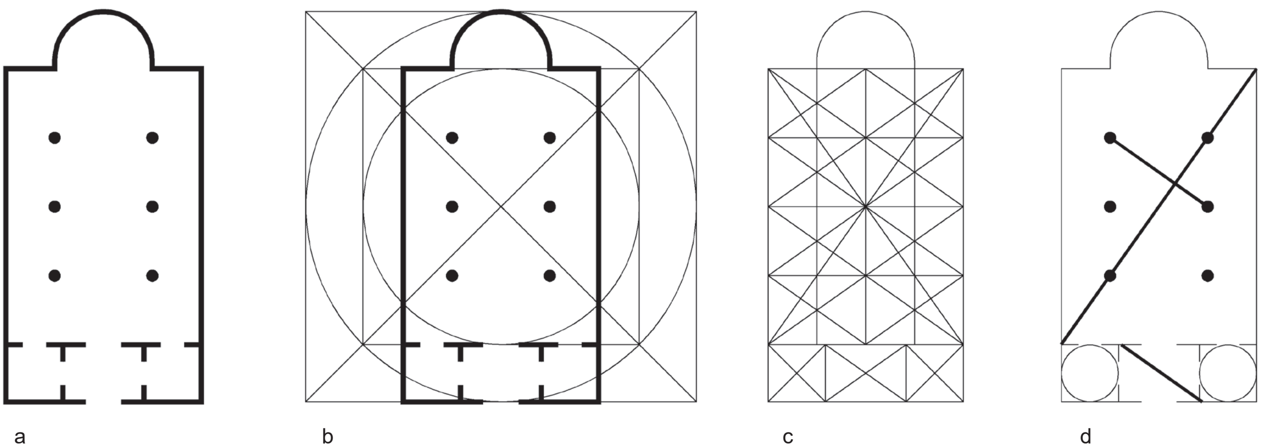

Fig. 9. Application of the square principle and dynamic symmetry based on $\sqrt{2}$:
a) plan, b) *ad quadratum* construction, c) $\sqrt{2}$ decomposition grid, d) regulating lines (drawing by W. Januszewski)

Il. 9. Zastosowanie zasady kwadratu i symetrii dynamicznej na motywie $\sqrt{2}$:
a) rysunek planu, b) konstrukcja *ad quadratum*, c) siatka dekompozycyjna $\sqrt{2}$, d) linie regulacyjne (rys. W. Januszewski)

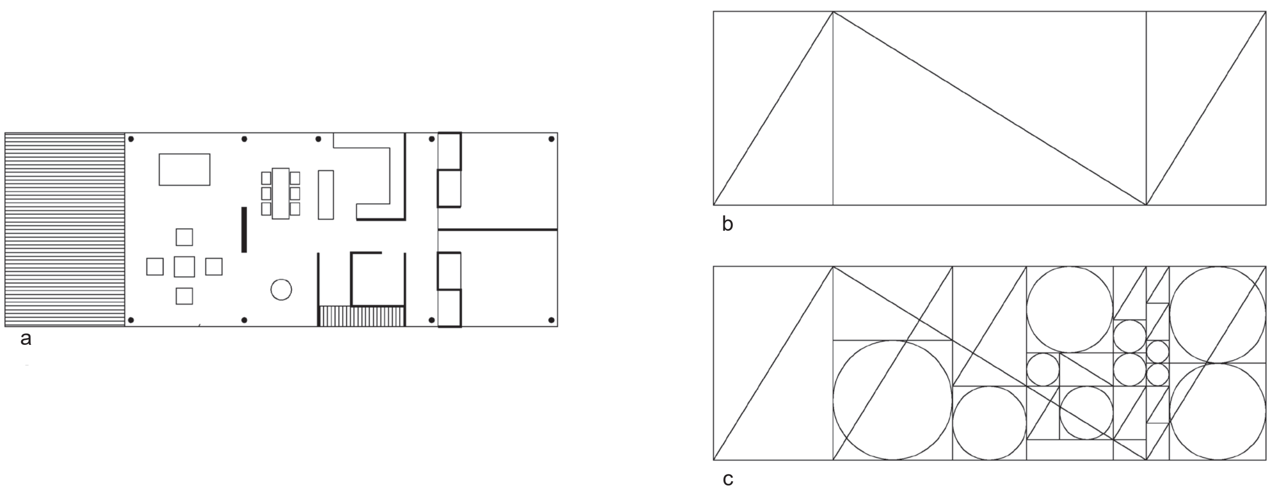

Fig. 10. Application of the dynamic symmetry principle based on *φ*:
a) plan; b) construction of basic divisions; c) construction of detailed divisions (drawing by W. Januszewski)

Il. 10. Zastosowanie zasady symetrii dynamicznej na motywie *φ*:
a) rysunek planu, b) konstrukcja podstawowych podziałów, c) konstrukcja podziałów szczegółowych (rys. W. Januszewski)

and theoreticians who emphasize the significance of proportion systems are represented by, inter alia, Richard Padovan, Rob Krier, and Peter Märkli [1, pp. 117–130].

The issues of proportion systems are inseparably connected with the field of aesthetics and the search for beauty. This relationship has its roots in the classical definition of beauty which is formulated as the right proportion between components[8]. Vitruvius' connection of *eurythmia* – a beautiful appearance and *symmetria* – an objective mathematical structure determining proportional relationships between parts and the whole was fundamental in the context of the theory of architecture [19, pp. 13, 14]. Nevertheless, a popular image of proportion systems as a "formula for beauty" seems to be far from sufficient to explain a deeper role of these systems. There are at least several theories about proportion systems and their connections with the concept of beauty.

As part of the naturalistic approach, aesthetic experiences caused by specific proportions were supposed to result from the fact that these proportions were appropriate for forms occurring in nature [10, pp. 87–110]. This approach, which is part of the ancient idea of *mimesis*, gained popularity in the 19th century due to the works by German psychologist Adolf Zeising – a propagator of the idea of golden division as a "principle of Nature" [33, pp. 141–147]. Although this myth is often repeated in popular culture, it has no strong scientific foundations. In these considerations it is important that this approach puts emphasis on one specific mathematical relationship (1.613…) in place of a more flexible Vitruvian principle of *symmetry* combining elements and the whole of a structure. One of the variants of the naturalistic approach is the anthropomorphic theory which is derived from Vitruvian comparison of a beautiful building to a harmoniously shaped human body. The history of this metaphor and its importance for the theory of architecture were thoroughly described by Joseph Rykwert [34]. Proportion systems which derived from this tradition, from Francesco di Giorgio's theory to *Modulor*, adopted a specific canon of the human body as a model. The arbitrariness of this choice met with criticism in the context of post-structuralism [33, pp. 164–167]. Different justifications for proportion systems are presented by a symbolic trend which was based on elements of Pythagorean philosophy and Platonism [35]. As part of this theory, proportion systems constitute an element of a broader concept of architecture as a mystical practice by means of which a creator can learn the fundamental principles of reality. Beauty is treated as a metaphysical being to which an artist has access by active contemplation. Architectural creation becomes an analogy of the act of creation of the universe, which is symbolically described, for example, in Plato's dialogue *Timaeus*. Numbers and geometry acquire symbolic significance here[9]. Steve Bass presented a full reconstruction of this trend, which was based on various esoteric traditions [36].

In the opinion of skeptics, proportion systems are the phenomenon specific for the non-scientific era. At that time they had a significant practical and symbolic meaning, but in the early modern period they lost their *raison d'etat*. Attempts at reactivating them would be rather sentimental anachronism. Their actual action is unverifiable. According to Matthew A. Cohen, they are *a mental and not visual constructs* [37, p. 3] and their values are invisible or hardly recognizable. Anyway, as Cohen emphasizes, […] *assessments of beauty are not universal across time and geography, as beauty-in-proportion believers assume, but always subjective* [37, p. 3]. Therefore, as Cohen postulates, the paradigm of *beauty-in-proportion* should be abandoned and proportion systems should be studied in a historical perspective exclusively.

Undoubtedly, the uncritical combination of historical research and the search for the modern system was a mistake of the tradition of the 19th century[10]. However, the negation of the universal value of proportion systems is rather untimely. The argument that the proportion systems *cannot be seen* seems to be too simplified, as is the uncritical adoption of the subjective theory of beauty. And thus, for example, the perceptive theory of art which was developed by Rudolf Arnheim or Żórawski is based on "invisible" concepts such as the *hidden structure* or *network of straight lines*. These are components of individual perceptions but resulting from objective features of things, i.e. from the geometric structure. Żórawski describes these elements as *supraindividual*, i.e. specific to each perception of a given shape. The source of a subjective impression of order is an objectively ordered structure. This does not necessarily mean experiencing beauty because, according to Żórawski, […] *beauty is neither synonymous with the strength of form, nor with its coherence, readability, or unambiguity. It is something more and something else. Complying with the inclination – which is given to us by nature – to straight lines or to any other tendencies causes readability and facilitates orientation* [15, p. 92]. Thus, a rational geometric structure can be a component of experiencing beauty, however, only when cohesion and harmony are desirable features. One of the functions of architecture is to establish visual relations in the space of human life – understandable and clear relationships[11]. Architecture is therefore looking for methods to agree on a visual order which is perceived sensually and a rational order which is described in the abstract language of mathematics. Both in practice and architectural theory, many attempts at combining these orders were made, which resulted in proportion systems. They operate at the interface

[8] For example, in the formulation of St. Thomas Aquinas: *For beauty consists of three components: first, the totality and perfection of things; what is defective and lacking is considered ugly; secondly, maintaining proper proportion, i.e. a harmonious arrangement of things; third, eye-catching splendor or brightness* – Summa Theologica. Vol. 3, 1, qu. 39, a. 8 [32].

[9] And thus, 1 defines the unity of the universe, 2 – its diversity, 3 – a relationship between elements (structure), 4 – phenomenal reality (phenomena). The corresponding geometric structures consist in successive divisions of basic forms, i.e. a circle and a square [36, p. 34].

[10] This tradition, also continued in the 20th century, led to risky speculations, e.g. to the search for dimensions of *Modulor* in ancient buildings by Le Corbusier or measures of the plastic number in the ruins of Stonehenge by Van der Laan [23, pp. 185–204].

[11] According to Richard Padovan, architecture […] *simply gives a measure to our spatial environment* [11, p. 372].

between universal principles of mathematics and supraindividual components of the perception process, whereas their practical or symbolic meaning is secondary.

The concept of proportion systems as auxiliary tools for organizing the perception of architectural form implies their specific place in the design process:

1. Proportion systems should be subordinated to the wholeness of the process. They do not constitute an end in themselves, but they are of a service nature. Their application can broaden the architect's instrumentation in the field of solving form problems.

2. Individual proportion systems should be treated interchangeably and a choice of the system should be dictated by a specific design situation. In the past, a lot of effort was made to look for the "only" system but various results of these searches discredited the very idea of proportion systems [16, p. 34].

3. Proportion systems do not constitute a generator of the architectural form. The form results from the type of building, utility program, structure, general idea and many other factors. The application of the proportion system does not guarantee the creation of a work with a high objective value or positive emotional reaction. It will not correct functional errors or replace the lack of a clear idea. The proportion system, as a tool for coordinating the building measures, can help to compose the output elements as a coherent whole[12].

4. Rational and intuitive methods are equal elements of the design process. It is important for a designer to be able to use both of these methods in a flexible way and adapt them to specific design requirements. Moreover, the proportion system can only be effective if it is consistent with the aesthetic intuition of a designer. The use of mathematical methods does not exclude trial and error methods but rather requires a designer to submit the applied method to a critical assessment.

There are a number of detailed problems which go beyond the framework of this work and require further studies. These include the issue of proportion systems in the light of digital tools supporting the design process. A traditional methodology of proportion systems is adapted to the work with a flat drawing (horizontal projection, cross-section, façade drawing, etc.). The modern fully three-dimensional work environment of the architect makes us ask a question about a method how these traditional principles can be adapted to the dynamic virtual space. Another issue is the development of the so-called parametric architecture which is based on mathematical form modeling. It is worth noticing that the relationship between proportion systems and this trend seems rather superficial. Proportion systems strive for the clarity of the form, whereas in parametric architecture a designer takes control over the generative process and the form becomes the result of this process. The rational process does not always lead to a clear and intuitive form, which could be seen in the architecture of the first decade of the parametric trend. For the pioneers of parametric architecture such as Greg Lynn or Peter Eisenman, who were connected with deconstructionism and *blob* trend, digital tools and non-Euclidian geometry resulted rather from the radical negation of the classical concept of architecture, including visual order resulting from proportions [33, pp. 65–67]. Reconciling parametric methods with the traditional theory of proportions is a matter of the future.

Conclusions

This study expands the state of knowledge about geometric proportions systems presenting an approach which orders the theoretical foundations of these systems. The presented findings provide arguments which confirm the thesis that geometric proportion systems constitute a coherent group of methods based on interdependent and interchangeable principles (*similar figures principle*, *regulating lines*, *regular figures*, *and dynamic symmetry*). The way these systems function consists in reducing the number of shapes and placing key points on straight lines. The purpose of these operations is to introduce *limited complexity* in the architectural structure.

The work abandoned a traditional presentation of individual systems as separate or even competitive and searching for the perfect system. In the presented view, these systems constitute a set of tools for any use rather than the idealized "beauty formula". Their goal is to organize the architectural form understood in perceptual terms, which can be achieved by various means. This distinguishes the presented approach from the above-mentioned symbolic, naturalistic or anthropomorphic explanations. These approaches, although interesting in historic or general terms, place the theory of proportion systems in the sphere of highly abstract considerations or even pseudoscience and esotericism. The presented outline better responds to the need to spread knowledge about proportion systems and their wider application. It is also an alternative to a skeptical approach which accuses these systems of having no universal value and emphasizes their historical condition only.

The analysis of proportion systems in architecture and the presentation of their applications shows the wealth of this concept and its potential in the context of a creative design process. The discussed methods are based on the elementary geometric knowledge and are relatively easy to share and acquire. Combination of various methods and their interchangeability makes it possible to apply systems in a flexible way. Although it is not possible to clearly state that proportion systems are indispensable in designing, they seem to be a valuable extension of the architect's instruments. At the same time, we must remember that the application of proportion systems should be coordinated with other design factors as one of the elements of rich knowledge and architectural skills.

Translated by
Bogusław Setkowicz

[12] According to Kazimierz Ciechanowski, proportion systems […] *sometimes help to organize plans and projections of buildings, however, they cannot be treated as a substitute for design art* [13, p. 83]. Le Corbusier expresses the same thought much more pointedly, i.e. *Wisdom is for the wise* [18, p. 3].

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Abstract

An outline of the geometric proportion systems in architecture

Proportional systems are arithmetic or geometric methods of organizing architectural form that determine the mutual proportional relationships of the parts and the whole of an architectural work. The theory of these systems, developed over the centuries, constitutes an essential component of architectural theory, deserving of rediscovery and reinterpretation in our era.

The purpose of this study is to present a synthetic overview of the issues related to geometric proportional systems in architecture. The article adopts an interdisciplinary approach, integrating the history and theory of architecture with elements of practical geometry. The research method encompasses a literature analysis and an examination of selected geometric methods applied to specific design problems. The article offers an original perspective on geometric proportional systems, interpreting them as a coherent theory based on interconnected elements, such as similarity of figures, regulating lines, regular shapes, and dynamic symmetry.

The study portrays proportional systems as a set of flexible design methods rooted in elementary geometric principles, empowering architects to better control the visual relationships of their designed objects of architecture. Simultaneously, the application of such methods necessitates their harmonious integration with other factors in the design process and subjecting them to critical aesthetic evaluation.

Key words: architecture, geometry, proportion systems, regulating lines, dynamic symmetry

Streszczenie

Zarys problematyki geometrycznych systemów proporcji w architekturze

Systemy proporcji są arytmetycznymi lub geometrycznymi metodami organizacji formy architektonicznej, które określają wzajemne relacje miarowe części i całości dzieła architektury. Rozwijana przez wieki teoria tych systemów stanowi istotny komponent teorii architektury, zasługujący na ponowne odkrywanie i reinterpretację w naszej epoce.

Autor artykułu miał na celu przedstawienie syntetycznego zarysu problematyki geometrycznych systemów proporcji w architekturze. Wykorzystano podejście interdyscyplinarne, łącząc historię i teorię architektury z elementami praktycznej geometrii i psychologią percepcji. Metoda badawcza objęła analizę literatury przedmiotu oraz badanie wybranych metod geometrycznych zastosowanych do konkretnych problemów projektowych. W artykule zaprezentowano oryginalne spojrzenie na geometryczne systemy proporcji, interpretując je jako spójną teorię opartą na wzajemnie powiązanych elementach, takich jak podobieństwo figur, linie regulacyjne, figury regularne i symetria dynamiczna. Ukazano systemy proporcji jako zestaw elastycznych metod projektowych oparty na elementarnych zasadach geometrycznych, dzięki któremu architekt może lepiej kontrolować relacje wizualne projektowanego obiektu. Jednocześnie stosowanie tego rodzaju metod wymaga ich harmonijnego połączenia z innymi czynnikami procesu projektowego i poddania ich krytycznej ocenie estetycznej.

Słowa kluczowe: architektura, geometria, systemy proporcji, linie regulacyjne, symetria dynamiczna