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On safety of critical infrastructures modelling with application to port oil transportation system

Keywords

safety, multistate system, ageing, operation process, dependability, critical infrastructure, piping transport

Abstract

A new approach to safety investigations of multistate complex systems with dependent components at variable operation conditions called critical infrastructures is proposed. The safety function of the critical infrastructure system is defined and determined for an exemplary "*m* out of *l*" critical infrastructure. In the developed model, it is assumed that the system components have the multistate exponential safety functions with interdependent departures rates from the subsets of the safety states. The approach is adapted to safety prediction of oil piping transportation system operating at a maritime port.

1. Introduction

Currently, the newest trends in the safety investigations of complex technical systems analysis are directed to the critical infrastructures. In general, a critical infrastructure is a single complex system of large scale or a network of complex large systems (set of hard or soft structures) that function collaboratively and synergistically in order to ensure to a continuous production flow of essentials goods and services. These are complex systems that significant features are inside-system dependencies and outside-system dependencies that in the case of damage have significantly destructive influence on the health, safety and security, economics and social conditions of large human communities and territory areas. These systems are made of large number of interacting components and even small perturbations can trigger large scale consequences in critical infrastructures that may cause multiple threats in human life and activity. For the above reason, as an extended failure within one of these infrastructures may result in the critical incapacity or destruction and can significantly damage many aspects of human life and further cascading across the critical infrastructure boundaries, they have the potential for multi-infrastructural collapse with unprecedented and transnational dangerous consequences.

Many technical systems belong to the class of complex critical infrastructure systems as a result of the large number of interacting components and subsystems they are built of and their complicated operating processes having significant influence on their safety. This complexity and the inside-infrastructure and outside-infrastructure dependencies and hazards cause that there is a need to develop new comprehensive approaches and general methods of analysis, identification, prediction, improvement and optimization for these complex system safety. We meet complex critical infrastructure systems, for instance, in piping transportation of water, gas, oil and various chemical substances, in port and maritime transportation. Optimization of the structures, operation processes and maintenance strategies of critical infrastructures with respect to their safety and costs is very important and very often also complicated and often not possible to perform by practitioners because of the mathematical complexity of the applied methods. In addition, analyzing the critical infrastructures in their variable operation conditions and considering their changing in time safety structures and their among components and subsystems dependability and resulting in changes of their safety characteristics becomes much more complicated. Adding to this analysis, the outside of the critical infrastructures hazards coming from other systems, from natural cataclysm and from other dangerous events makes the problem essentially more difficult to become solved in order to improve and to ensure high level of these systems safety.

From the point of view of more precise analysis of the safety and effectiveness of critical infrastructures, the developed methods should be based on a multistate approach [4]-[5], [11]-[14] to these complex systems safety analysis instead of normally used two-state approach. This will enable different critical infrastructure inside and outside safety states to be distinguished, such that they ensure a demanded level of the system operation effectiveness with accepted consequences of the dangerous accidents for the environment, population, etc.

In most safety analyses, it is assumed that components of a system are independent. But in reality, especially in the case of critical infrastructures, this assumption is not true, so that the dependencies among the critical infrastructure systems components and subsystems should be assumed and considered. It is a natural assumption, as after decreasing the safety state by one of components in a subsystem, the inside interactions among the remaining components may cause further components safety states decrease [9]-[10]. In reality, in the critical infrastructures, it may even cause the whole system safety state dangerous degradation.

To tie the results of investigations of the critical infrastructures inside-dependences together with the results coming from the assumed in the critical infrastructures outside-dependencies, the semi-Markov models [1]-[3], [7]-[8] can be used to describe the complex systems operation processes. This linking of the inside and outside the critical infrastructures dependencies and including other outside dangerous events and hazards coming from the environment and from other dangerous processes, under the assumed their structures multi-state models, is the main idea of the critical infrastructures safety analysis methodology [5]-[6].

2. Multistate approach to safety analysis

In the multistate safety analysis to define a system composed of n, $n \in N$, ageing components we assume that:

- E_i, $i = 1,2,\ldots,n$, are components of a system,
- all components and a system under consideration have the set of safety states $\{0,1,...,z\}$, $z \geq 1$,
- the safety states are ordered, the state 0 is the worst and the state z is the best,
- the component and the system safety states degrade with time t,
- $T_i(u)$, $i = 1,2,\ldots,n$, $n \in N$, are random variables representing the lifetimes of components E_i in the safety state subset $\{u, u+1,..., z\}$, while they were in the safety state z at the moment $t = 0$,
- $T(u)$ is a random variable representing the lifetime of a system in the safety state subset $\{u, u+1,..., z\}$, while it was in the safety state z at the moment $t = 0$,
- $s_i(t)$ is a component E_i safety state at the moment t, $t \in <0, \infty)$, given that it was in the safety state z at the moment $t = 0$,
- $s(t)$ is the system safety state at the moment t, $t \in <0, \infty)$, given that it was in the safety state z at the moment $t = 0$.

The above assumptions mean that the safety states of the ageing system and components may be changed in time only from better to worse.

Definition 1. A vector

$$S_i(t,\cdot) = [S_i(t,0), S_i(t,1), \ldots, S_i(t,z)] \quad (1)$$

for $t \in <0, \infty)$, $i = 1,2,\ldots,n$, where

$$S_i(t,u) = P(s_i(t) \geq u \mid s_i(0) = z) = P(T_i(u) > t) \quad (2)$$

for $t \in <0, \infty)$, $u = 0,1,\ldots,z$, is the probability that the component E_i is in the safety state subset $\{u, u+1,..., z\}$ at the moment t, $t \in <0, \infty)$, while it was in the safety state z at the moment $t = 0$, is called the multistate safety function of a component E_i.

Definition 2. A vector

$$\boldsymbol{S}(t,\cdot) = [\boldsymbol{S}(t,0), \boldsymbol{S}(t,1), \ldots, \boldsymbol{S}(t,z)], \ t \in <0, \infty), \quad (3)$$

where

$$\boldsymbol{S}(t,u) = P(s(t) \geq u \mid s(0) = z) = P(T(u) > t) \quad (4)$$

for $t \in <0, \infty)$, $u = 0,1,...,z$, is the probability that the system is in the safety state subset $\{u, u+1,..., z\}$ at the moment t, $t \in <0, \infty)$, while it was in the safety state z at the moment $t = 0$, is called the multi-state safety function of a system.

The safety functions $S_i(t,u)$ and $\boldsymbol{S}(t,u)$, $t \in <0, \infty)$, $u = 0,1,...,z$, defined by (2) and (4) are called the coordinates of the components and the system multistate safety functions $S_i(t,\cdot)$ and $\boldsymbol{S}(t,\cdot)$ given by respectively (1) and (3). It is clear that from *Definition 1* and *Definition 2*, for $u = 0$, we have $S_i(t,0) = 1$ and $\boldsymbol{S}(t,0) = 1$.

Moreover, is the mean lifetime of the system in the safety state subset $\{u, u+1, ..., z\}$ is defined by

$$\mu(u) = \int_0^\infty \boldsymbol{S}(t,u)dt, \; u = 1,2,...,z, \quad (5)$$

and is the standard deviation of the system lifetime in the safety state subset $\{u, u+1, ..., z\}$ is given by

$$\sigma(u) = \sqrt{n(u) - [\mu(u)]^2}, \; u = 1,2,...,z, \quad (6)$$

where

$$n(u) = 2\int_0^\infty t\, \boldsymbol{S}(t,u)dt, \; u = 1,2,...,z. \quad (7)$$

Moreover, the mean lifetimes of the system in the safety state u, $u = 1,2,...,z$,

$$\bar{\mu}(u) = \int_0^\infty p(t,u)dt, \; u = 1,2,...,z, \quad (8)$$

where

$$p(t,u) = P(s(t) = u \mid s(0) = z) = \boldsymbol{S}(t,u) - \boldsymbol{S}(t,u+1),$$

for $u = 0,1,...,z-1$, $t \in <0,\infty)$, can be found from the following relationships [5]

$$\bar{\mu}(u) = \mu(u) - \mu(u+1), \; u = 0,1,...,z-1,$$

$$\bar{\mu}(z) = \mu(z). \quad (9)$$

Definition 3. A probability

$$\boldsymbol{r}(t) = P(s(t) < r \mid s(0) = z) = P(T(r) \le t), \quad (10)$$
$$t \in <0,\infty),$$

that the system is in the subset of safety states worse than the critical safety state r, $r \in \{1,...,z\}$ while it was in the safety state z at the moment $t = 0$ is called a risk function of the multi-state system [5].
Under this definition, from (4), we have

$$\boldsymbol{r}(t) = 1 - P(s(t) \ge r \mid s(0) = z) = 1 - \boldsymbol{S}(t,r), \quad (11)$$
$$t \in <0,\infty),$$

and if τ is the moment when the system risk exceeds a permitted level δ, then

$$\tau = \boldsymbol{r}^{-1}(\delta), \quad (12)$$

where $\boldsymbol{r}^{-1}(t)$ is the inverse function of the system risk function $\boldsymbol{r}(t)$.

3. Safety of "*m* out of *l*" system with dependent components

One of the basic multistate safety structures with components ageing in time are "*m* out of *l*" systems.

Definition 4. A multi-state system is called "*m* out of *l*" system if its lifetime $T(u)$ in the safety state subset $\{u, u+1, ..., z\}$ is given by

$$T(u) = T_{(l-m+1)}(u), \; m = 1,2,...,l, \; u = 1,\dots,z,$$

where $T_{(l-m+1)}(u)$ is the l-m+1-th order statistic in the sequence of the component lifetimes $T_1(u)$, $T_2(u)$, . . ., $T_l(u)$.
The above definition means that the multistate "*m* out of *l*" system is in the safety state subset $\{u, u+1, ..., z\}$ if and only if at least *m* out of its *l* components are in this safety state subset.

Definition 5. A multi-state "*m* out of *l*" system is called homogeneous if its components E_i have the same safety function

$$S_i(t,\cdot) = [1, S_i(t,1), \dots, S_i(t,z)]$$

for $t \in <0,\infty)$, $i = 1,2,\dots,l$, with the coordinates

$$S_i(t,u) = S(t,u) \text{ for } t \in <0,\infty), \; u = 1,\dots,z,$$
$$i = 1,2,...,l.$$

In a multi-state "*m* out of *l*" system with dependent components we may consider the dependency of the changes of their ageing safety states and assume that after changing the safety state subset by one of the system components to the worse safety state subset, the lifetimes of the remaining system components in this safety state subsets decrease. More exactly, we assume that if $\upsilon, \upsilon = 0,1,2,\dots,l-1$, components of the system are out of the safety state subset $\{u, u+1, ..., z\}$, the mean values of the lifetimes $T_i'(u)$ in this safety state subset of the system remaining components are given by

$$E[T_i'(u)] = E[T_i(u)] - \frac{\upsilon}{l} E[T_i(u)] = \frac{l-\upsilon}{l} E[T_i(u)]$$

for $i = 1,2,\dots,l$, $u = 1,2,\dots,z$.
Hence, for the case when components have exponential safety functions given by

$$S_i(t,\cdot) = [1, S_i(t,1), \ldots, S_i(t,z)], \; t \in <0,\infty), \quad (13)$$

for $i = 1,2,\ldots,l,$ where

$$S_i(t,u) = \begin{cases} 1, & t < 0 \\ \exp[-\lambda(u)t], \, t \geq 0, \lambda(u) \geq 0, \\ i = 1,2,\ldots,l \end{cases} \quad (14)$$

with the intensity of departure $\lambda(u)$ from the safety state subset $\{u, u+1,\ldots,z\}$, we get the following formula for the intensities of departure from this safety state subset of the remaining components

$$\lambda^{(\upsilon)}(u) = \frac{l}{l-\upsilon}\lambda(u) \text{ for } v = 0,1,2,\ldots,l-1, \quad (15)$$
$$u = 1,2,\ldots,z.$$

Proposition 1 [6]. If in a homogeneous multi-state "*m* out of *l*" system

(i) the components have exponential safety function given by (13)-(14),
(ii) the components are dependent,
(iii) the intensities of departure from the safety state subsets of the system components are given by (15),

then the multistate system safety function is given by the formula

$$\boldsymbol{S}(t,\cdot) = [1, \boldsymbol{S}(t,1), \ldots, \boldsymbol{S}(t,z)],$$

where

$$\boldsymbol{S}(t,u) = \sum_{\upsilon=0}^{l-m} \frac{[l\lambda(u)t]^{\upsilon}}{\upsilon!} \exp[-l\lambda(u)t], \; t \geq 0, \quad (16)$$
$$u = 1,\ldots,z.$$

4. System operation at variable conditions

We assume that the system during its operation process is taking $v, v \in N,$ different operation states $z_1, z_2, \ldots, z_v$. Further, we define the system operation process $Z(t)$, $t \in <0,+\infty)$, with discrete operation states from the set $\{z_1, z_2, \ldots, z_v\}$. Moreover, we assume that the system operation process $Z(t)$ is a semi-Markov process [5] with the conditional sojourn times θ_{bl} at the operation states z_b when its next operation state is z_l, $b, l = 1,2,\ldots,v,$ $b \neq l.$

Under these assumptions, the system operation process may be described by:

– the vector $[p_b(0)]_{1\times v}$ of the initial probabilities $p_b(0) = P(Z(0) = z_b),$ $b = 1,2,\ldots,v,$ of the system operation process $Z(t)$ staying at the operation states at the moment $t = 0$;
– the matrix $[p_{bl}]_{v\times v}$ of probabilities p_{bl}, $b, l = 1,2,\ldots,v,$ $b \neq l,$ of the system operation process $Z(t)$ transitions between the operation states z_b and z_l;
– the matrix $[H_{bl}(t)]_{v\times v}$ of conditional distribution functions $H_{bl}(t) = P(\theta_{bl} < t)$, $b, l = 1,2,\ldots,v,$ $b \neq l,$ of the system operation process $Z(t)$ conditional sojourn times θ_{bl} at the operation states.

The mean values of the conditional sojourn times θ_{bl} of the system operation process $Z(t)$ are given by

$$M_{bl} = E[\theta_{bl}] = \int_0^{\infty} t dH_{bl}(t), \; b,l = 1,2,\ldots,v, \; b \neq l. \quad (17)$$

From the formula for total probability, it follows that the unconditional distribution functions of the sojourn times θ_b, $b = 1,2,\ldots,v,$ of the system operation process $Z(t)$ at the operation states z_b, $b = 1,2,\ldots,v,$ are given by [5]

$$H_b(t) = \sum_{l=1}^{v} p_{bl} H_{bl}(t), \; b = 1,2,\ldots,v. \quad (18)$$

Hence, the mean values $E[\theta_b]$ of the system operation process $Z(t)$ unconditional sojourn times θ_b, $b = 1,2,\ldots,v,$ at the operation states are given by

$$M_b = E[\theta_b] = \sum_{l=1}^{v} p_{bl} M_{bl}, \quad (19)$$

where M_{bl} are defined by the formula (17).

The limit values of the system operation process $Z(t)$ transient probabilities at the particular operation states

$$p_b(t) = P(Z(t) = z_b), \; t \in <0,+\infty), \; b = 1,2,\ldots,v,$$

are given by [5]

$$p_b = \lim_{t\to\infty} p_b(t) = \frac{\pi_b M_b}{\sum_{l=1}^{v} \pi_l M_l}, \; b = 1,2,\ldots,v, \quad (20)$$

where M_b, $b = 1,2,\ldots,v,$ are given by (19), while the steady probabilities π_b of the vector $[\pi_b]_{1\times v}$ satisfy the system of equations

$$\begin{cases}[\pi_b] = [\pi_b][p_{bl}] \\ \sum_{l=1}^{\nu} \pi_l = 1.\end{cases} \quad (21)$$

5. Safety of multistate system at variable operation conditions

We assume that the changes of the system operation process $Z(t)$ states have an influence on the system multistate components E_i, $i = 1,2,...,n$, safety and the system safety structure as well. We mark by $T_1^{(b)}(u)$, $T_2^{(b)}(u)$, $...,T_n^{(b)}(u)$ the system components $E_1, E_2, ..., E_n$ conditional lifetimes in the safety states subset $\{u, u+1, z\}$, $u = 1,2,...,z$, and by $T^{(b)}(u)$ the system conditional lifetimes in the safety states subset $\{u, u+1,..., z\}$, $u = 1,2,...,z$, while the system is at the operation state z_b, $b = 1,2,...,\nu$. Further, we define the conditional safety function of the system multi-state component E_i, $i = 1,2,...,n$, while the system is at the operation state z_b, $b = 1,2,...,\nu$, by the vector [5]

$$[S_i(t, \cdot)]^{(b)} = [1, [S_i(t,1)]^{(b)}, ..., [S_i(t,z)]^{(b)}], \quad (22)$$

where

$$[S_i(t,u)]^{(b)} = P(T_i^{(b)}(u) > t | Z(t) = z_b) \quad (23)$$

for $t \in < 0,\infty)$, $u = 1,2,...,z$, E_{ij}, and the conditional safety function of the multistate system while the system is at the operation state z_b, $b = 1,2,...,\nu$, by the vector [5]

$$[\boldsymbol{S}(t,\cdot)]^{(b)} = [1, [\boldsymbol{S}(t,1)]^{(b)}, ..., [\boldsymbol{S}(t,z)]^{(b)}], \quad (24)$$

where

$$[\boldsymbol{S}(t,u)]^{(b)} = P(T^{(b)}(u) > t | Z(t) = z_b) \quad (25)$$

for $t \in < 0,\infty)$, $u = 1,2,\dots,z$, $b = 1,2,...,\nu$.
The safety function $[S_i(t,1)]^{(b)}$ is the conditional probability that the component E_i lifetime $T_i^{(b)}(u)$ in the safety state subset $\{u, u+1,..., z\}$ is greater than *t*, while the process *Z*(*t*) is at the operation state z_b. Similarly, the safety function $[\boldsymbol{s}(t,u)]^{(b)}$ is the conditional probability that the system lifetime $T^{(b)}(u)$ in the safety state subset $\{u, u+1,..., z\}$ is greater than *t*, while the process *Z*(*t*) is at the operation state z_b. Consequently, we mark by $T(u)$ the system unconditional lifetime in the safety states subset $\{u, u+1,..., z\}$, $u = 1,2,...,z$, and we define the system unconditional safety function by the vector

$$\boldsymbol{S}(t,\cdot) = [1, \boldsymbol{S}(t,1), ..., \boldsymbol{S}(t,z)], \quad (26)$$

where

$$\boldsymbol{S}(t,u) = P(T(u) > t) \text{ for } t \in < 0,\infty), \quad (27)$$
$$u = 1,2,...,z,$$

In the case when the system operation time θ is large enough, the system unconditional safety function coordinates are given by

$$\boldsymbol{S}(t,u) \cong \sum_{b=1}^{\nu} p_b [\boldsymbol{S}(t,u)]^{(b)} \text{ for } t \geq 0, \quad (28)$$
$$u = 1,2,...,z,$$

where $[\boldsymbol{S}(t,u)]^{(b)}$, $u = 1,2,...,z$, $b = 1,2,...,\nu$, are the coordinates of the system conditional safety functions defined by (24)-(25) and p_b, $b = 1,2,...,\nu$, are the system operation process limit transient probabilities given by (20).

6. Safety of multistate "*m* out of *l*" system with dependent components at variable conditions

Proposition 1, may be generalized in the following way [6].
Proposition 2. If in a homogeneous multi-state "*m* out of *l*" system with the shape parameters $m^{(b)}$, $l^{(b)}$ at the operation state z_b, $b = 1,2,...,\nu$,
(i) the components have at the operation state z_b, $b = 1,2,...,\nu$, the exponential safety function given by

$$[S_i(t, \cdot)]^{(b)} = [1, [S_i(t,1)]^{(b)}, ..., [S_i(t,z)]^{(b)}] \quad (29)$$

for $t \in < 0,\infty)$, $i = 1,2,\dots,l^{(b)}$, where

$$[S_i(t,u)]^{(b)} = \begin{cases} 1, & t < 0 \\ \exp[-[\lambda(u)]^{(b)} t], & t \geq 0, \\ [\lambda(u)]^{(b)} \geq 0, i = 1,2,...,l^{(b)} \end{cases} \quad (30)$$

with the intensity of departure $[\lambda(u)]^{(b)}$ from the safety state subset $\{u, u+1,..., z\}$,
(ii) the components are dependent in such a way that after the departure from the safety state subset $\{u, u+1,..., z\}$ by υ components of the "*m* out of *l*"

system the intensities $[\lambda(u)]^{(b)}$ of departures from this safety states subset of this system remaining components at the operation stare z_b increase according to the formula

$$[\lambda^{(\upsilon)}(u)]^{(b)} = \frac{l^{(b)}}{l^{(b)} - \upsilon}[\lambda(u)]^{(b)}, \tag{31}$$
$$\upsilon = 0,1,2,\ldots,l^{(b)}-1,\ \ u = 1,2,\ldots,z,$$

then the multistate system safety function is given by the formula

$$\boldsymbol{S}(t,\cdot) = [1, \boldsymbol{S}(t,1),\ldots,\boldsymbol{S}(t,z)], \tag{32}$$

where

$$\boldsymbol{S}(t,u) \cong \sum_{b=1}^{\nu} p_b [\sum_{\upsilon=0}^{l^{(b)}-m^{(b)}} \frac{[l^{(b)}\lambda(u)t]^{\upsilon}}{\upsilon!}\exp[-l^{(b)}\lambda(u)t]]$$
$$t \geq 0,\ \ u = 1,\ldots,z. \tag{33}$$

7. Safety of port oil piping transportation system

7.1. Piping system description

The considered oil piping transportation system is operating at one of the Baltic Oil Terminals that is designated for the reception from ships, the storage and sending by carriages or cars the oil products. It is also designated for receiving from carriages or cars, the storage and loading the tankers with oil products such like petrol and oil. The considered terminal is composed of three parts *A*, *B* and *C*, linked by the piping transportation system with the pier. The scheme of this terminal is presented in *Figure 1.*

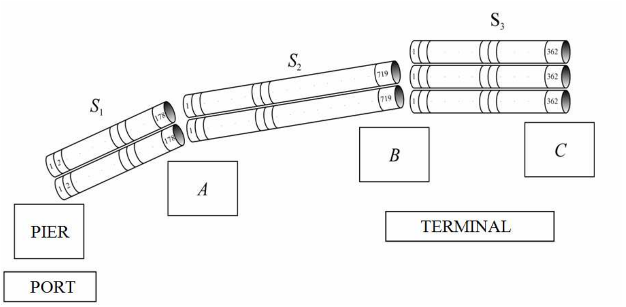

Figure 1. The scheme of the port oil transportation system

The unloading of tankers is performed at the pier placed in the port. The pier is connected with terminal part *A* through the transportation subsystem S_1 built of two piping lines composed of steel pipe segments with diameter of 600 mm. In the part *A* there is a supporting station fortifying tankers pumps and making possible further transport of oil by the subsystem S_2 to the terminal part *B*. The subsystem S_2 is built of two piping lines composed of steel pipe segments of the diameter 600 mm. The terminal part *B* is connected with the terminal part *C* by the subsystem S_3. The subsystem S_3 is built of one piping line composed of steel pipe segments of the diameter 500 mm and two piping lines composed of steel pipe segments of diameter 350 mm. The terminal part *C* is designated for the loading the rail cisterns with oil products and for the wagon sending to the railway station of the port and further to the interior of the country.

Thus, the port oil pipeline transportation system consists of three subsystems:

- the subsystem S_1 composed of two pipelines, each composed of 178 pipe segments and 2 valves,
- the subsystem S_2 composed of two pipelines, each composed of 717 pipe segments and 2 valves,
- the subsystem S_3 composed of three pipelines, each composed of 360 pipe segments and 2 valves.

The subsystems S_1, S_2, S_3, indicated in Figure 1 are forming a general series port oil pipeline system safety structure presented in *Figure 2.*

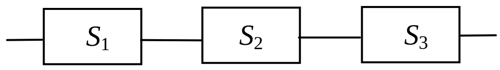

Figure 2. General scheme of the port oil pipeline system safety structure

The system is a series system composed of two series-parallel subsystems S_1, S_2, each containing two pipelines and one series-"2 out of 3" subsystem S_3.

7.2. Piping system operation process

The subsystems S_1, S_2 and S_3 are forming a general series port oil pipeline system safety structure presented in Figure 2. However, the pipeline system safety structure and its subsystems and components safety depend on its changing in time operation states [5].

Taking into account expert opinions on the varying in time operation process of the considered piping system, we distinguish the following as its eight operation states [2]:

- an operation state z_1 – transport of one kind of medium from the terminal part B to part C using two out of three pipelines of the subsystem S_3,
- an operation state z_2 – transport of one kind of medium from the terminal part C to part B using one out of three pipelines of the subsystem S_3,
- an operation state z_3 – transport of one kind of medium from the terminal part B through part A

to pier using one out of two pipelines of the subsystem S_1 and one out of two pipelines of the subsystem S_2,

- an operation state z_4 – transport of one kind of medium from the pier through parts A and B to part C using one out of two pipelines of the subsystem S_1, one out of two pipelines in subsystem S_2 and two out of three pipelines of the subsystem S_3,
- an operation state z_5 – transport of one kind of medium from the pier through part A to B using one out of two pipelines of the subsystem S_1 and one out of two pipelines of the subsystem S_2,
- an operation state z_6 – transport of one kind of medium from the terminal part B to C using two out of three pipelines of the subsystem S_3, and simultaneously transport one kind of medium from the pier through part A to B using one out of two pipelines of the subsystem S_1 and one out of two pipelines of the subsystem S_2,
- an operation state z_7 – transport of one kind of medium from the terminal part B to C using one out of three pipelines of the subsystem S_3, and simultaneously transport second kind of medium from the terminal part C to B using one out of three pipelines of the subsystem S_3.

The influence of the above system operation states changing on the changes of the pipeline system safety structure is as follows.

At the system operation states z_1 and z_7, the system is composed of the subsystem S_3, that is a series-"2 out of 3" system containing three series subsystems with the scheme showed in *Figure 3.*

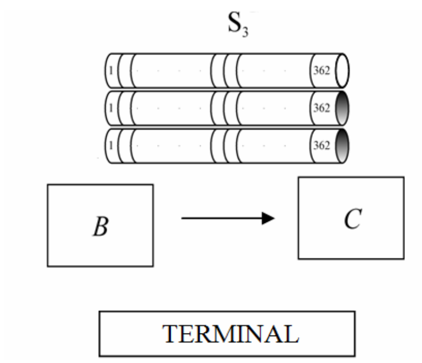

Figure 3. The scheme of the port oil piping transportation system at the operation states z_1 and z_7

At the system operation state z_2, the system is composed of a series-parallel subsystem S_3, which contains three pipelines with the scheme showed in *Figure 4.*

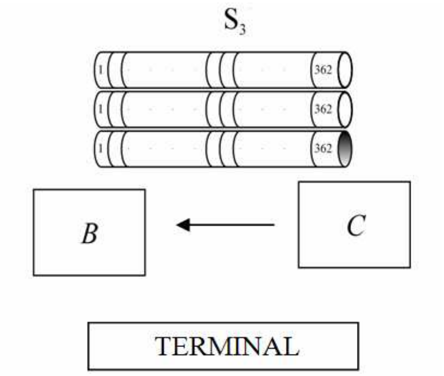

Figure 4. The scheme of the port oil piping transportation system at the operation state z_2

At the system operation states z_3 and z_5, the system is series and composed of two series-parallel subsystems S_1, S_2, each containing two pipelines with the scheme showed in *Figure 5.*

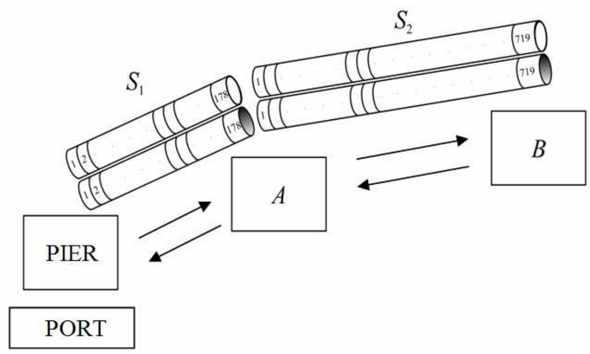

Figure 5. The scheme of port oil piping transportation system at the operation states z_3 and z_5

At the system operation states z_4 and z_6, the system is series and composed of two series-parallel subsystems S_1, S_2, each containing two pipelines and one series-"2 out of 3" subsystem S_3 with the scheme showed in *Figure 6.*

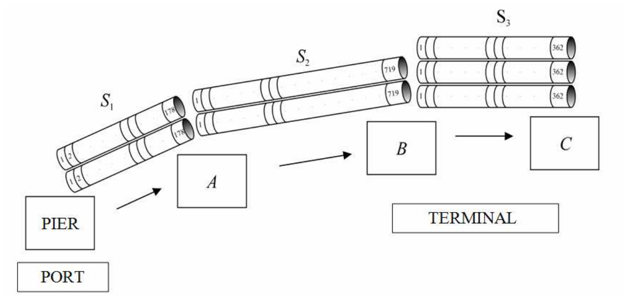

Figure 6. The scheme of the port oil piping transportation system at the operation states z_4 and z_6

To identify the unknown parameters of the port oil piping transportation system operation process the suitable statistical data coming from its real realizations should be collected. The lack of sufficient statistical data about the port oil piping transportation system operation process causes that it is not possible to estimate exactly its operation parameters. However, even on the basis of the fragmentary statistical data coming from experts, the port oil piping transportation system operation process probabilities p_{bl} of transitions from the operation state z_b into the operation state z_l, $b, l = 1,2,...,7,$ $b \neq l,$ can be evaluated approximately. Their approximate evaluation are given in the matrix below

$$[p_{bl}] = \begin{bmatrix} 0 & 0.022 & 0.022 & 0 & 0.534 & 0.111 & 0.311 \\ 0.2 & 0 & 0 & 0 & 0 & 0 & 0.8 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0.488 & 0.023 & 0 & 0.023 & 0 & 0.233 & 0.233 \\ 0.095 & 0 & 0 & 0 & 0.667 & 0 & 0.238 \\ 0.531 & 0.062 & 0 & 0 & 0.219 & 0.188 & 0 \end{bmatrix}. \quad (34)$$

Unfortunately, it is not possible to identify completely the matrix of the conditional distribution functions $[H_{bl}(t)]_{7x7}$ of the sojourn times θ_{bl} for $b, l = 1,2,...,7,$ $b \neq l,$ and consequently, it is also not possible to determine the vector $[H_b(t)]_{1x7}$ of the unconditional distribution functions of the sojourn times θ_b of this system operation process at the operation states z_b, $b = 1,2,...,7,$ defined by (18). However, on the basis of data coming from practice and collected by experts operating this piping system, some hypotheses on the forms of the distributions describing the system operation process conditional sojourn times θ_{bl}, $b, l = 1,2,...,7,$ $b \neq l,$ at the particular operation states can be formulated and accepted. In this case, having these distributions identified, it is possible to evaluate the mean values $M_{bl} = E[\theta_{bl}]$ of the conditional sojourn times θ_{bl} at the particular operation states, using the general formula (17). Otherwise, if the collected statistical data is not sufficient to test and to accept the forms of the distributions of the piping system operation process conditional sojourn times θ_{bl}, their mean values $M_{bl} = E[\theta_{bl}]$ may be estimated by applying the formula for the empirical man values of the conditional sojourn times at the particular operation states. As the results of using the last of these two possibilities, the approximate evaluations of these mean values are as follows:

$$\begin{array}{lll} M_{12} = 1920, & M_{13} = 480, & M_{15} = 1999.4, \\ M_{16} = 1250, & M_{17} = 1129.6, & M_{21} = 9960, \\ M_{27} = 810, & M_{31} = 575, & M_{47} = 380, \\ M_{51} = 874.7, & M_{52} = 480, & M_{54} = 300, \\ M_{56} = 436.3, & M_{57} = 1042.5, & M_{61} = 325, \\ M_{65} = 510.7, & M_{67} = 438, & M_{71} = 850.9, \\ M_{72} = 510, & M_{75} = 2585.7, & M_{76} = 2380. \end{array} \quad (35)$$

This way, the port oil piping transportation system operation process is approximately defined and we may predict its main characteristics. Namely, applying (19), (34) and (35), the unconditional mean sojourn times of the piping system operation process at the particular operation states are:

$$M_1 = E[\theta_1]$$

$$= p_{12}M_{12} + p_{13}M_{13} + p_{15}M_{15} + p_{16}M_{16} + p_{17}M_{17}$$

$$= 0.022 \cdot 1920 + 0.022 \cdot 480 + 0.534 \cdot 1999.4$$

$$+ .111 \cdot 1250 + 0.311 \cdot 1129.6 \cong 1610.52,$$

$$M_2 = E[\theta_2] = p_{21}M_{21} + p_{27}M_{27}$$

$$= 0.2 \cdot 9960 + 0.8 \cdot 810 \cong 2640,$$

$$M_3 = E[\theta_3] = p_{31}M_{31} = 1 \cdot 575 = 575,$$

$$M_4 = E[\theta_4] = p_{47}M_{47} = 1 \cdot 380 = 380,$$

$$M_5 = E[\theta_5]$$

$$= p_{51}M_{51} + p_{52}M_{52} + p_{54}M_{54} + p_{56}M_{56} + p_{57}M_{57}$$

$$= 0.488 \cdot 874.7 + 0.023 \cdot 480 + 0.023 \cdot 300$$

$$+ 0.233 \cdot 436.3 + 0.233 \cdot 1042.5 \cong 789.35,$$

$$M_6 = E[\theta_6] = p_{61}M_{61} + p_{65}M_{65} + p_{67}M_{67}$$

$$= 0.095 \cdot 325 + 0.667 \cdot 510.7 + 0.238 \cdot 438$$

$$\cong 475.76,$$

$$M_7 = E[\theta_7] = p_{71}M_{71} + p_{72}M_{72} + p_{75}M_{75} + p_{76}M_{76}$$

$$= 0.531 \cdot 850.9 + 0.062 \cdot 510 + 0.219 \cdot 2585.7$$

$$+0.188 \cdot 2380 \cong 1497.16. \quad (36)$$

Considering (34) in the system of equations (21) that takes the form

$$\begin{cases} [\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6, \pi_7] \\ = [\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6, \pi_7][p_{bl}]_{7x7} \\ \pi_1 + \pi_2 + \pi_3 + \pi_4 + \pi_5 + \pi_6 + \pi_7 = 1, \end{cases}$$

we get its following solution

$$\pi_1 \cong 0.291, \ \pi_2 \cong 0.027, \ \pi_3 \cong 0.006, \ \pi_4 \cong 0.007,$$

$$\pi_5 \cong 0.301, \ \pi_6 \cong 0.144, \ \pi_7 \cong 0.224. \quad (37)$$

Hence and from (36), after applying (20), it follows that the limit values of the piping system operation process transient probabilities $p_b(t)$ at the operation states z_b, $b = 1,2,...,7$, are given by

$$p_1 = 0.395, \ p_2 = 0.060, \ p_3 = 0.003, \ p_4 = 0.002,$$

$$p_5 = 0.200, \ p_6 = 0.058, \ p_7 = 0.282. \quad (38)$$

7.3. Piping system safety

After considering the comments and opinions coming from experts, taking into account the effectiveness and safety aspects of the operation of the oil pipeline transportation system, we distinguish the following three safety states $(z = 2)$ of the system and its components:

- a safety state 2 – piping operation is fully safe,
- a safety state 1 – piping operation is less safe and more dangerous because of the possibility of environment pollution,
- a safety state 0 – piping is destroyed.

Moreover, by the expert opinions, we assume that there are possible the transitions between the components safety states only from better to worse ones and we assume that the system and its components critical safety state is $r = 1$.

The port oil piping transportation system safety structure and its subsystems and components safety depend on its changing in time operation states. The influence of the system operation states changing on the changes of the system safety structure and its components safety functions is as follows.

At the system operation state z_1, the system is composed of the subsystem S_3 illustrated in Figure 3, which contains three series subsystems $(l^{(1)} = 3)$, each composed of 362 components with the exponential safety functions given below and is a "2 out of 3" system $(m^{(1)} = 2)$ of these subsystems. The subsystem S_3 consists of 3 pipelines and in each pipeline there are:

- 360 pipe segments with conditional three-state safety functions co-ordinates

$$[S^{(3)}(t,1)]^{(1)} = \exp[-0.0059t],$$

$$[S^{(3)}(t,2)]^{(1)} = \exp[-0.0074t],$$

- 2 valves with conditional three-state safety functions co-ordinates

$$[S^{(3)}(t,1)]^{(1)} = \exp[-0.0166t],$$

$$[S^{(3)}(t,2)]^{(1)} = \exp[-0.0181t].$$

Consequently, we determine the three-state safety functions of the system series subsystems/components E_i, $i = 1,2,3$, at the operation state z_1 in the form of the vector

$$[S_i(t,\cdot)]^{(1)} = [1, \ [S_i(t,1)]^{(1)}, [S_i(t,2)]^{(1)}], t \geq 0, \quad (39)$$

for $i = 1,2,3$, with the exponential coordinates

$$[S_i(t,1)]^{(1)} = \exp[-(360 \cdot 0.0059 + 2 \cdot 0.0166)t]$$

$$= \exp[-2.1572t], \ i = 1,2,3, \quad (40)$$

$$[S_i(t,2)]^{(1)} = \exp[-(360 \cdot 0.0074 + 2 \cdot 0.0181)t]$$

$$= \exp[-2.7002t], \ i = 1,2,3. \quad (41)$$

Considering (39)-(41) the subsystems dependence of the form (31) and applying the formulae either (16) or (33), we get the piping system conditional safety function at the operation state z_2 of the form

$$[\boldsymbol{S}(t,\cdot)]^{(1)} = [1, [\boldsymbol{S}(t,1)]^{(1)}, \ [\boldsymbol{S}(t,2)]^{(1)}], \ t \geq 0, \quad (42)$$

where

$$[\boldsymbol{S}(t,1)]^{(1)} = [\boldsymbol{S}^{(3)}(t,1)]^{(1)}$$

$$= \sum_{\upsilon=0}^{1} \frac{[3 \cdot 2.1572t]^{\upsilon}}{\upsilon!} \exp[-3 \cdot 2.1572t]$$

$$= \exp[-6.4716t] + 6.4716t \exp[-6.4716t], \quad (43)$$

$$[\boldsymbol{S}(t,2)]^{(1)} = [\boldsymbol{S}^{(3)}(t,2)]^{(1)}$$

$$= \sum_{\upsilon=0}^{1} \frac{[3 \cdot 2.7002t]^{\upsilon}}{\upsilon!} \exp[-3 \cdot 2.7002t]$$

$$= \exp[-8.1006t] + 8.1006t \exp[-8.1006t]. \quad (44)$$

The expected values of the pipeline system conditional lifetimes in the safety state subsets $\{1,2\}$, $\{2\}$ at the operation state z_1, calculated from the results given by (42)-(44), according to (5), respectively are:

$$\mu_1(1) \cong 0.309, \ \mu_1(2) \cong 0.247 \text{ year}, \quad (45)$$

and further, using (9) and (45), the mean values of the conditional lifetimes in the particular safety states 1, 2 at the operation state z_1, respectively are:

$$\bar{\mu}_1(1) \cong 0.062, \ \bar{\mu}_1(2) \cong 0.247 \text{ year}. \quad (46)$$

At the system operation state z_2, the system is composed of the subsystem S_3 illustrated in Figure 4, which contains three series subsystems $(l^{(2)} = 3)$, each composed of 362 components with the exponential safety functions the same as at the operation state z_1 and is a parallel system $(m^{(2)} = 1)$ of these subsystems.

Considering (39)-(41) the subsystems dependence of the form (31) and applying the formulae either (16) or (33), we get the piping system conditional safety function at the operation state z_2 of the form

$$[\boldsymbol{S}(t,\cdot)]^{(2)} = [1, [\boldsymbol{S}(t,1)]^{(2)}, \ [\boldsymbol{S}(t,2)]^{(2)}], \ t \ge 0, \quad (47)$$

where

$$[\boldsymbol{S}(t,1)]^{(2)} = [\boldsymbol{S}^{(3)}(t,1)]^{(2)}$$

$$= \sum_{\upsilon=0}^{2} \frac{[3 \cdot 2.1572t]^{\upsilon}}{\upsilon!} \exp[-3 \cdot 2.1572t]$$

$$= \exp[-6.4716t] + 6.4716t \exp[-6.4716t]$$

$$+ \frac{[6.4716t]^2}{2} \exp[-6.4716t], \quad (48)$$

$$[\boldsymbol{S}(t,2)]^{(2)} = [\boldsymbol{S}^{(3)}(t,2)]^{(2)}$$

$$= \sum_{\upsilon=0}^{1} \frac{[3 \cdot 2.7002t]^{\upsilon}}{\upsilon!} \exp[-3 \cdot 2.7002t]$$

$$= \exp[-8.1006t] + 8.1006t \exp[-8.1006t]$$

$$+ \frac{[8.1006t]^2}{2} \exp[-8.1006t], \quad (49)$$

The expected values of the pipeline system conditional lifetimes in the safety state subsets $\{1,2\}$, $\{2\}$ at the operation state z_1, calculated from the results given by (47)-(49), according to (5), respectively are:

$$\mu_2(1) \cong 0.464, \ \mu_2(2) \cong 0.370 \text{ year}, \quad (50)$$

and further, using (9) and (50), the mean values of the conditional lifetimes in the particular safety states 1, 2 at the operation state z_2, respectively are:

$$\bar{\mu}_1(1) \cong 0.094, \ \bar{\mu}_1(2) \cong 0.370 \text{ year}. \quad (51)$$

At the system operation state z_3, the piping is a series system composed of two series-parallel subsystems S_1 and S_2 illustrated in *Figure 5*. The subsystem S_1 contains two series subsystems $(l^{(3)} = 2)$, each composed of 178 components with the exponential safety functions given below and is a parallel system $(m^{(3)} = 1)$ of these subsystems. The subsystem S_1 consists of 2 pipelines and in each pipeline there are:

- 176 pipe segments with conditional three-state safety functions co-ordinates

$$[S^{(1)}(t,1)]^{(3)} = \exp[-0.0062t],$$

$$[S^{(1)}(t,2)]^{(3)} = \exp[-0.0088t],$$

- 2 valves with conditional three-state safety functions co-ordinates

$$[S^{(1)}(t,1)]^{(3)} = \exp[-0.0167t],$$

$$[S^{(1)}(t,2)]^{(3)} = \exp[-0.0182t].$$

Consequently, we determine the three-state safety functions of the subsystem S_1 series subsystems/components E_i, $i = 1,2,$ at the operation state z_3 in the form of the vector

$$[S_i(t,\cdot)]^{(3)} = [1, [S_i(t,1)]^{(3)}, [S_i(t,2)]^{(3)}], \ t \ge 0, \quad (52)$$

for $i = 1,2,$ with the exponential coordinates

$$[S_i(t,1)]^{(3)} = \exp[-(176 \cdot 0.0062 + 2 \cdot 0.0167)t]$$

$$= \exp[-1.1246t], \quad i = 1,2, \tag{53}$$

$$[S_i(t,2)]^{(3)} = \exp[-(176 \cdot 0.0088 + 2 \cdot 0.0182)t]$$

$$= \exp[-1.5852t], \quad i = 1,2. \tag{54}$$

Considering (52)-(54) the subsystems dependence of the form (31) and applying the formulae either (16) or (33), we get the piping subsystem S_1 conditional safety function at the operation state z_3 of the form

$$[\boldsymbol{S}^{(1)}(t,\cdot)]^{(3)} = [1, [\boldsymbol{S}^{(1)}(t,1)]^{(3)}, \ [\boldsymbol{S}^{(1)}(t,2)]^{(3)}], \quad t \geq 0, \tag{55}$$

where

$$[\boldsymbol{S}^{(1)}(t,1)]^{(3)} = \sum_{\upsilon=0}^{1} \frac{[2 \cdot 1.1246t]^{\upsilon}}{\upsilon!} \exp[-2 \cdot 1.1246t]$$

$$= \exp[-2.2492t] + 2.2492t \exp[-2.2492t], \tag{56}$$

$$[\boldsymbol{S}^{(1)}(t,2)]^{(3)} = \sum_{\upsilon=0}^{1} \frac{[2 \cdot 1.5852t]^{\upsilon}}{\upsilon!} \exp[-2 \cdot 1.5852t]$$

$$= \exp[-3.1704t] + 3.1704t \exp[-3.1704t]. \tag{57}$$

The subsystem S_2 contains two series subsystems $(l^{(3)} = 2)$, each composed of 719 components with the exponential safety functions given below and is a parallel system $(m^{(3)} = 1)$ of these subsystems. The subsystem S_2 consists of 2 pipelines and in each pipeline there are:

- 717 pipe segments with conditional three-state safety functions co-ordinates

$$[S^{(2)}(t,1)]^{(3)} = \exp[-0.0062t],$$

$$[S^{(2)}(t,2)]^{(3)} = \exp[-0.0088t],$$

- 2 valves with conditional three-state safety functions co-ordinates

$$[S^{(2)}(t,1)]^{(3)} = \exp[-0.0166t],$$

$$[S^{(2)}(t,2)]^{(3)} = \exp[-0.0181t].$$

Consequently, we determine the three-state safety functions of the subsystem S_2 series subsystems/components E_i, $i = 1,2,$ at the operation state z_3 in the form of the vector

$$[S_i(t,\cdot)]^{(3)} = [1, \ [S_i(t,1)]^{(3)}, [S_i(t,2)]^{(3)}], \ t \geq 0, \tag{58}$$

for $i = 1,2,$ with the exponential coordinates

$$[S_i(t,1)]^{(3)} = \exp[-(717 \cdot 0.0062 + 2 \cdot 0.0166)t]$$

$$= \exp[-4.4786t], \quad i = 1,2, \tag{59}$$

$$[S_i(t,2)]^{(3)} = \exp[-(717 \cdot 0.0088 + 2 \cdot 0.0181)t]$$

$$= \exp[-6.3458t], \quad i = 1,2. \tag{60}$$

Considering (58)-(60) the subsystems dependence of the form (31) and applying the formulae either (16) or (33), we get the piping subsystem S_1 conditional safety function at the operation state z_3 of the form

$$[\boldsymbol{S}^{(2)}(t,\cdot)]^{(3)} = [1, [\boldsymbol{S}^{(2)}(t,1)]^{(3)}, [\boldsymbol{S}^{(2)}(t,2)]^{(3)}], \quad t \geq 0, \tag{61}$$

where

$$[\boldsymbol{S}^{(2)}(t,1)]^{(3)} = \sum_{\upsilon=0}^{1} \frac{[2 \cdot 4.4786t]^{\upsilon}}{\upsilon!} \exp[-2 \cdot 4.4786t]$$

$$= \exp[-8.9572t] + 8.9572t \exp[-8.9572t], \tag{62}$$

$$[\boldsymbol{S}^{(2)}(t,2)]^{(1)} = \sum_{\upsilon=0}^{1} \frac{[2 \cdot 6.3458t]^{\upsilon}}{\upsilon!} \exp[-2 \cdot 6.3458t]$$

$$= \exp[-12.6916t] + 12.6916t \exp[-12.6916t]. \tag{63}$$

Since at the system operation state z_3, the piping is a series system composed of two series-parallel subsystems S_1 and S_2 with the conditional safety functions respectively given by (55)-(57) and (61)-(63), then the piping system conditional safety function at the operation state z_3 is of the form

$$[\boldsymbol{S}(t,\cdot)]^{(3)} = [1, [\boldsymbol{S}(t,1)]^{(3)}, \ [\boldsymbol{S}(t,2)]^{(3)}], \ t \geq 0, \tag{64}$$

where

$$[\boldsymbol{S}(t,1)]^{(3)} = [\boldsymbol{S}^{(1)}(t,1)]^{(3)} \ [\boldsymbol{S}^{(2)}(t,1)]^{(3)}$$

$$= [\exp[-2.2492t] + 2.2492t \exp[-2.2492t]]$$

$$[\exp[-8.9572t] + 8.9572t \exp[-8.9572t]]$$

$$= \exp[-11.2064t] + 11.2064t\exp[-11.2064t]$$

$$+ 20.1465t^2\exp[-11.2064t] \quad (65)$$

$$[\boldsymbol{S}(t,2)]^{(3)} = [\boldsymbol{S}^{(1)}(t,2)]^{(3)} \; [\boldsymbol{S}^{(2)}(t,2)]^{(3)}$$

$$= [\exp[-3.1704t] + 3.1704t\exp[-3.1704t]]$$

$$[\exp[-12.6916t] + 12.6916t\exp[-12.8616t]]$$

$$= \exp[-15.8620t] + 15.8620t\exp[-15.8620t]$$

$$+ 40.2374t^2\exp[-15.8620t] \quad (66)$$

The expected values of the pipeline system conditional lifetimes in the safety state subsets $\{1,2\}$, $\{2\}$ at the operation state z_3, calculated from the results given by (64)-(66), according to (5), respectively are:

$$\mu_3(1) \cong 0.207, \; \mu_3(2) \cong 0.146 \text{ year}, \quad (67)$$

and further, using (9) and (67), the mean values of the conditional lifetimes in the particular safety states 1, 2 at the operation state z_1, respectively are:

$$\bar{\mu}_3(1) \cong 0.061, \; \bar{\mu}_3(2) \cong 0.146 \text{ year}. \quad (68)$$

At the system operation state z_4, the piping is a series system composed of two series-parallel subsystems S_1 and S_2, and one series-"2 out of 3" subsystem S_3 illustrated in Figure 6. The subsystems S_1 and S_2 components have the same safety functions as they have at the operation state z_3 and the subsystem S_3 components have the same safety functions as they have at the operation state z_1. Thus, considering the results (64)-(66) and (42)-(44), we conclude that the conditional safety function of the piping system at the operation state z_4 is of the form

$$[\boldsymbol{S}(t,\cdot)]^{(4)} = [1, [\boldsymbol{S}(t,1)]^{(4)}, \; [\boldsymbol{S}(t,2)]^{(4)}], \; t \geq 0, \quad (69)$$

where

$$[\boldsymbol{S}(t,1)]^{(4)} = [\boldsymbol{S}^{(1)}(t,1)]^{(3)} \; [\boldsymbol{S}^{(2)}(t,1)]^{(3)} \; [\boldsymbol{S}^{(3)}(t,1)]^{(1)}$$

$$= [\exp[-2.2492t] + 2.2492t\exp[-2.2492t]]$$

$$[\exp[-8.9572t] + 8.9572t\exp[-8.9572t]]$$

$$[\exp[-6.4716t] + 6.4716t\exp[-6.4716t]]$$

$$= \exp[-17.6780t] + 17.6780t\exp[-17.6780t]$$

$$+ 92.6692t^2\exp[-17.6780t]$$

$$+ 130.3801t^3\exp[-17.6780t] \quad (70)$$

$$[\boldsymbol{S}(t,2)]^{(4)} = [\boldsymbol{S}^{(1)}(t,2)]^{(3)} \; [\boldsymbol{S}^{(2)}(t,2)]^{(3)} \; [\boldsymbol{S}^{(3)}(t,2)]^{(1)}$$

$$= [\exp[-3.1704t] + 3.1704t\exp[-3.1704t]]$$

$$[\exp[-12.6916t] + 12.6916t\exp[-12.8616t]]$$

$$[\exp[-8.1006t] + 8.1006t\exp[-8.1006t]]$$

$$= \exp[-23.9626t] + 23.9626t\exp[-23.9626t]$$

$$+ 168.7291t^2\exp[-23.9626t]$$

$$+ 326.4331t^3\exp[-23.9626t]. \quad (71)$$

The expected values of the pipeline system conditional lifetimes in the safety state subsets $\{1,2\}$, $\{2\}$ at the operation state z_4, calculated from the results given by (69)-(71), according to (5), respectively are:

$$\mu_4(1) \cong 0.156, \; \mu_4(2) \cong 0.114 \text{ year}, \quad (72)$$

and further, using (9) and (72), the mean values of the conditional lifetimes in the particular safety states 1, 2 at the operation state z_4, respectively are:

$$\bar{\mu}_4(1) \cong 0.042, \; \bar{\mu}_4(2) \cong 0.114 \text{ year}. \quad (73)$$

At the operation state z_5, the piping is a series system composed of two series-parallel subsystems S_1 and S_2 illustrated in Figure 5. The piping system safety structure and its components safety functions are the same as at the operation state z_3. Thus, according to (64)-(66), the piping system conditional safety function at the operation state z_5 is given by

$$[\boldsymbol{S}(t,\cdot)]^{(5)} = [1, [\boldsymbol{S}(t,1)]^{(5)}, \; [\boldsymbol{S}(t,2)]^{(5)}], \; t \geq 0, \quad (74)$$

where

$$[\boldsymbol{S}(t,1)]^{(5)} = \exp[-11.2064t]$$

$$+ 11.2064t\exp[-11.2064t]$$

$$+ 20.1465t^2 \exp[-11.2064t] \quad (75)$$

$$[\boldsymbol{S}(t,2)]^{(5)} = \exp[-15.8620t]$$

$$+ 15.8620t \exp[-15.8620t]$$

$$+ 40.2374t^2 \exp[-15.8620t]. \quad (76)$$

The expected values of the pipeline system conditional lifetimes in the safety state subsets $\{1,2\}$, $\{2\}$ at the operation state z_3, calculated from the results given by (74)-(76), according to (5), respectively are:

$$\mu_5(1) \cong 0.207, \ \mu_5(2) \cong 0.146 \text{ year}, \quad (77)$$

and further, using (9) and (71), the mean values of the conditional lifetimes in the particular safety states 1, 2 at the operation state z_1, respectively are:

$$\bar{\mu}_5(1) \cong 0.061, \ \bar{\mu}_5(2) \cong 0.146 \text{ year}. \quad (78)$$

At the system operation state z_6, the piping is a series system composed of two series-parallel subsystems S_1 and S_2, and one series-"2 out of 3" subsystem S_3 illustrated in Figure 6. The subsystems' structures and their components safety functions are the same as at the operation state z_4. Thus, considering the results (69)-(71), we conclude that the conditional safety function of the piping system at the operation state z_6 is of the form

$$[\boldsymbol{S}(t,\cdot)]^{(6)} = [1, [\boldsymbol{S}(t,1)]^{(6)}, \ [\boldsymbol{S}(t,2)]^{(6)}], \ t \geq 0, \quad (79)$$

where

$$[\boldsymbol{S}(t,1)]^{(6)} = \exp[-17.6780t]$$

$$+ 17.6780t \exp[-17.6780t]$$

$$+ 92.6692t^2 \exp[-17.6780t]$$

$$+ 130.3801t^3 \exp[-17.6780t] \quad (80)$$

$$[\boldsymbol{S}(t,2)]^{(6)} = \exp[-23.9626t]$$

$$+ 23.9626t \exp[-23.9626t]$$

$$+ 168.7291t^2 \exp[-23.9626t]$$

$$+ 326.4331t^3 \exp[-23.9626t]. \quad (81)$$

The expected values of the pipeline system conditional lifetimes in the safety state subsets $\{1,2\}$, $\{2\}$ at the operation state z_1, calculated from the results given by (79)-(81), according to (5), respectively are:

$$\mu_6(1) \cong 0.156, \ \mu_6(2) \cong 0.114 \text{ year},$$

and further, using (9) and (82), the mean values of the conditional lifetimes in the particular safety states 1, 2 at the operation state z_6, respectively are:

$$\bar{\mu}_6(1) \cong 0.042, \ \bar{\mu}_6(2) \cong 0.370 \text{ year}.$$

At the system operation state z_7, the piping is a series system composed of the subsystem S_3 illustrated in Figure 3. The subsystem structure and its components' safety functions are the same as at the operation state z_1. Thus, considering the results (41)-(43), we conclude that the conditional safety function of the piping system at the operation state z_7 is of the form

$$[\boldsymbol{S}(t,\cdot)]^{(7)} = [1, [\boldsymbol{S}(t,1)]^{(7)}, \ [\boldsymbol{S}(t,2)]^{(7)}], \ t \geq 0, \quad (84)$$

where

$$[\boldsymbol{S}(t,1)]^{(7)} = \exp[-6.4716t]$$

$$+ 6.4716t \exp[-6.4716t], \quad (85)$$

$$[\boldsymbol{S}(t,2)]^{(7)} = \exp[-8.1006t]$$

$$+ 8.1006t \exp[-8.1006t]. \quad (86)$$

The expected values of the pipeline system conditional lifetimes in the reliability state subsets $\{1,2\}$, $\{2\}$ at the operation state z_7, calculated from the results given by (84)-(86), according to (5), respectively are:

$$\mu_7(1) \cong 0.309, \ \mu 7(2) \cong 0.247 \text{ year},$$

and further, using (9) and (87), the mean values of the conditional lifetimes in the particular reliability states 1, 2 at the operation state z_7, respectively are:

$$\bar{\mu}_7(1) \cong 0.062, \ \bar{\mu}_7(2) \cong 0.247 \text{ year}.$$

Finally, considering the results (38), (42)-(44), (47)-(49), (64)-(66), (69)-(71), (74)-(76), (79)-(81), (89)-(91) and applying the formula (28), we get the piping system unconditional safety function

$$\boldsymbol{S}(t,\cdot) = [1, \boldsymbol{S}(t,1), \boldsymbol{S}(t,2)],\ t \geq 0, \tag{89}$$

where

$$\boldsymbol{S}(t,1) = 0.395[\exp[-6.4716t]$$

$$+ 6.4716t\exp[-6.4716t]] + 0.060[\exp[-6.4716t]$$

$$+ 6.4716t\exp[-6.4716t]$$

$$+ \frac{[6.4716t]^2}{2}\exp[-6.4716t]]$$

$$+0.003[\exp[-11.2064t] + 11.2064t\exp[-11.2064t]$$

$$+ 20.1465t^2\exp[-11.2064t]]$$

$$+0.002[\exp[-17.6780t] + 17.6780t\exp[-17.6780t]$$

$$+ 92.6692t^2\exp[-17.6780t]$$

$$+ 130.3801t^3\exp[-17.6780t]]$$

$$+0.200[\exp[-11.2064t] + 11.2064t\exp[-11.2064t]$$

$$+ 20.1465t^2\exp[-11.2064t]]$$

$$+ 0.058[\exp[-17.6780t] + 17.6780t\exp[-17.6780t]$$

$$+ 92.6692t^2\exp[-17.6780t]$$

$$+ 130.3801t^3\exp[-17.6780t]]$$

$$+0.282[\exp[-6.4716t]$$

$$+ 6.4716t\exp[-6.4716t]] \text{ for } t \geq 0, \tag{90}$$

$$\boldsymbol{S}(t,2) = 0.395[\exp[-8.1006t]$$

$$+ 8.1006t\exp[-8.1006t]] + 0.060[\exp[-8.1006t]$$

$$+ 8.1006t\exp[-8.1006t]$$

$$+ \frac{[8.1006t]^2}{2}\exp[-8.1006t]]$$

$$+0.003[\exp[-15.8620t] + 15.8620t\exp[-15.8620t]$$

$$+ 40.2374t^2\exp[-15.8620t]]$$

$$+0.002[\exp[-23.9626t] + 23.9626t\exp[-23.9626t]$$

$$+ 168.7291t^2\exp[-23.9626t]$$

$$+ 326.4331t^3\exp[-23.9626t]]$$

$$+0.200[\exp[-15.8620t] + 15.8620t\exp[-15.8620t]$$

$$+ 40.2374t^2\exp[-15.8620t]]$$

$$+0.058[\exp[-23.9626t] + 23.9626t\exp[-23.9626t]$$

$$+ 168.7291t^2\exp[-23.9626t]$$

$$+ 326.4331t^3\exp[-23.9626t]]$$

$$+0.282[\exp[-8.1006t]$$

$$+ 8.1006t\exp[-8.1006t]] \text{ for } t \geq 0. \tag{91}$$

The coordinates of the piping system unconditional safety function are presented in *Figure 7.*

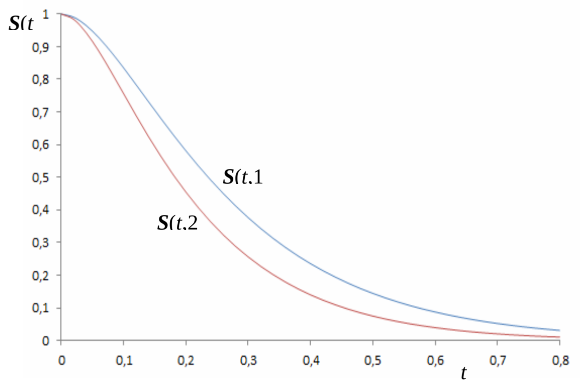

Figure 7. The graph of the piping system unconditional safety function

The expected values of the pipeline system unconditional lifetimes in the safety state subsets $\{1,2\}$, $\{2\}$, calculated from the results given by (89)-(91) according to (5) and using the results (45), (50), (67), (72), (77), (82), (92), respectively are:

$$\mu(1) \cong 0.395\ 0.309 + 0.060\ 0.464 + 0.003\ 0.207$$

$$+ 0.002\ 0.156 + 0.200\ 0.207 + 0.058\ 0.156$$

$$+ 0.282\ 0.309 = 0.288 \text{ year,}$$

$$\mu(2) \cong 0.395\ 0.247 + 0.060\ 0.370 + 0.003\ 0.146$$

$$+\ 0.002\ 0.114 + 0.200\ 0.146 + 0.058\ 0.114$$

$$+\ 0.282\ 0.247 = 0.226 \text{ year}, \qquad (92)$$

and further, using (9) and (92), the mean values of the unconditional lifetimes in the particular safety states 1, 2, respectively are:

$$\bar{\mu}(1) \cong 0.062,\ \bar{\mu}(2) \cong\ 0.226 \text{ year}. \qquad (93)$$

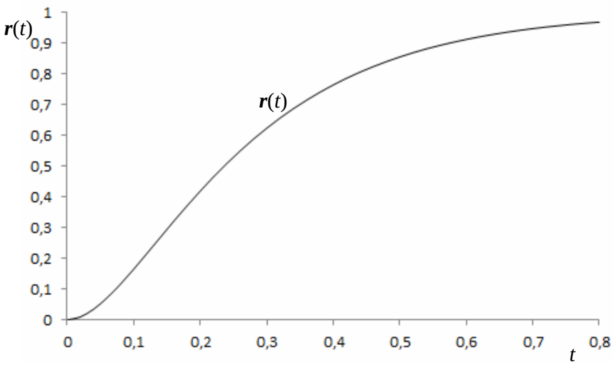

Figure 8. The graph of the piping system risk function

As the critical safety state is r =1, then the system risk function, according to (11), is given by

$$\boldsymbol{r}(t) = 1 - \boldsymbol{S}(t,1) \qquad (94)$$

where $\boldsymbol{S}(t,1)$ is given by (90) and by (12) the moment when the system risk exceeds a permitted level δ = 0.05 is

$$\tau = \boldsymbol{r}^{-1}(0.05) = 0.049.$$

8. Conclusions

Presented in this paper results are partly coming from the general analytical models of complex technical multi-state systems safety [5] and their applications to safety analysis of critical infrastructures [6]. The material given in this paper delivers the procedures and algorithms that allow to find the main an practically important safety characteristics of the complex technical systems with dependent components at the variable operation condition. The safety characteristics of the port oil transportation system with dependent components predicted in this paper are different from those determined in [5] for this system with independent components. This fact justifies the sensibility of considering the complex technical systems with dependent components at the variable operation conditions that is appearing out in a natural way from practice. This approach, upon the good accuracy of the systems' operation processes and their components safety parameters identification, makes their safety characteristics prediction more precise.

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