

# EVALUATION OF THE USEFULNESS OF THE „MILLIMETER GPS” SYSTEM FOR DETERMINATION OF ELEVATIONS OF POINTS IN TOPOGRAPHIC SURVEYS

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## 1. INTRODUCTION

Commencement of the ASG-EUPOS national network in 2008 resulted in rapid development of utilisation of satellite technologies for topographic surveys. Surveying experiments, performed in 2008 and 2009 at the chair of Engineering Geodesy and Topographic Surveys pointed to high usefulness of the RTK technology for topographic surveys. Research works, carried out at the surveying test site close to Grybów at the Nowy Sącz District, performed with the use of NAWGEO services of the ASG-EUPOS network proved that the expected maximum mean errors of horizontal locations and the ellipsoidal elevation of points, may be of the order of approximately 0.04m and 0.06 m, respectively (Malarski et al., 2010). Considering additional errors of the quasigeoid model, the issue is to obtain the centimetre accuracy in the system of normal elevations, which is required for topographic surveys. It seems that the proposal of Topcon company, concerning the mmGPS technology (Millimeter GPS) may meet those requirements.

## 2. DESCRIPTION OF THE „MILLIMETER GPS” SYSTEM

The mmGPS System has been created with the objective to apply it for controlling machinery. It serves for determination of horizontal locations with the accuracy of centimetre order and the point elevation with the accuracy of the millimetre order, i.e. with better accuracy than it may be obtained with the use of RTK measurements.



Fig. 1. The PZL-1 laser transmitter and the PZS-1 detector of a laser beam.

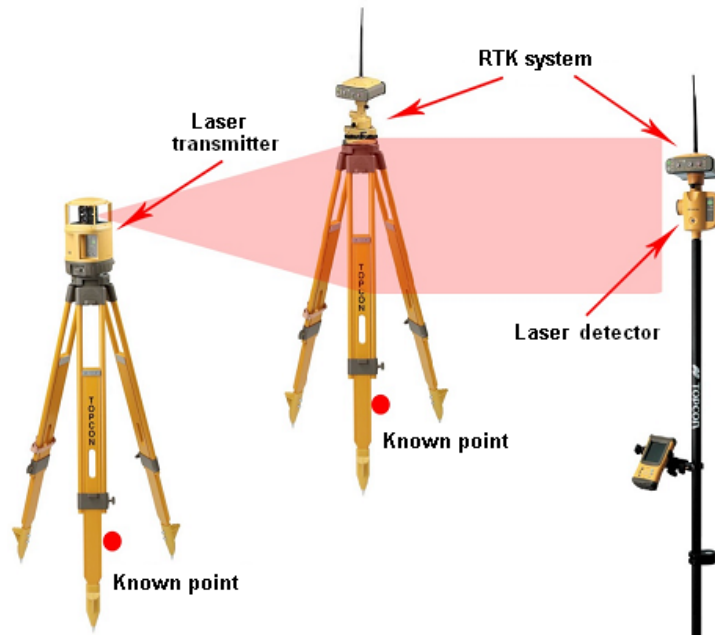


Fig. 2. Location of the mmGPS System elements.

The system combines RTK measurements and laser technology. Elevations are calculated in that system basing on an inclined distance and a vertical angle, similarly to the case of tacheometric surveys. The difference concerns calculations of distances with the use of data from RTK measurements and determination of a vertical angle with the use of a transmitted laser beam, which is received by a detector (Fig. 1) (Topcon TPI, 2011). Location of the mmGPS System elements is presented in Fig. 2.

### 3. RESEARCH WORKS

An attempt to evaluate the real accuracy of the mmGPS System in field operations, has been made for the control base, consisting of 27 points located within the Pola Mokotowskie Park in Warszawa (Dąbkowski and Gaładyk, 2010). Elevations of points of the control network were determined by means of precise direct levelling with the accuracy which was higher by one order than the expected errors of measurements performed with the use of the mmGPS System.

Field testing of the mmGPS System included:

- testing the influence of the distance on the accuracy of determination of elevations of points,
- testing the drift of the device,
- testing the influence of instrumental errors on the accuracy of determination of elevations of points,
- testing the influence of the size of the vertical angle on the accuracy of determination of elevations of points,
- testing the influence of terrain barriers.

Testing the influence of the distance on the accuracy of determination of elevation using the mmGPS System has been performed by multiple determination (in different days and at different time) of elevations of points from the first group (points 2-16), located every 25 m, along a straight line, with the transmitter located in point 1 (Fig. 3). An independent calibration of the transmitter has been performed before each measuring cycle. Results of the experiment are presented in Fig. 4.

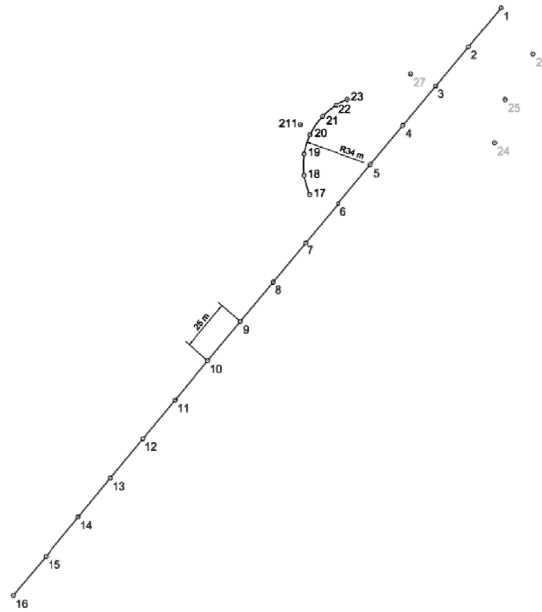


Fig. 3. Sketch of location of the points of the control network.

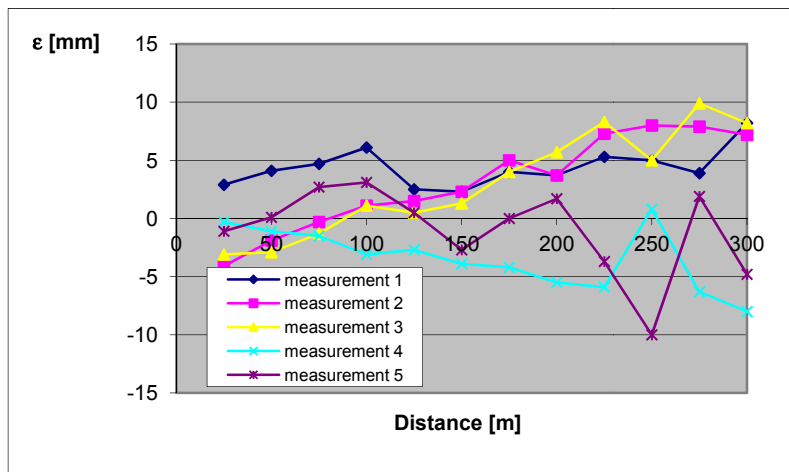


Fig. 4. Diagram of true errors of elevations for various distances of the transmitter.

Trends of changes of true errors, which are close to linear trends, proportional to the increase of the distance, visible in Fig. 4, suggest that systematic reasons of such changes exist. TPI Ltd. Company, being the main distributor of Topcon equipment, confirmed that the correction of the Earth curvature has not been considered. As it turns out from Fig. 4, this has not been the only reason of the existing true errors. Identical decreasing trends, which are typical for the lack of that correction, have not been observed for all cycles. It was expected that the linear variability of true errors might have been caused by the constant *drift of the device*, i.e. the variations of determined elevations in time. On the other hand, diversified inclination of diagrams of true errors might have been the result of variability of directions of that drift in various days and at different time. The significant influence of that drift on the linearity and inclination of diagrams of true errors has excluded the experiment which was performed later for the same base. Within 2 hours in three cycles of measurement, double measurements of elevations of points were performed, in forward and backward directions. The transmitter was independently calibrated before each cycle. True errors of relative elevations, determined in that experiment, are presented in Fig. 5.

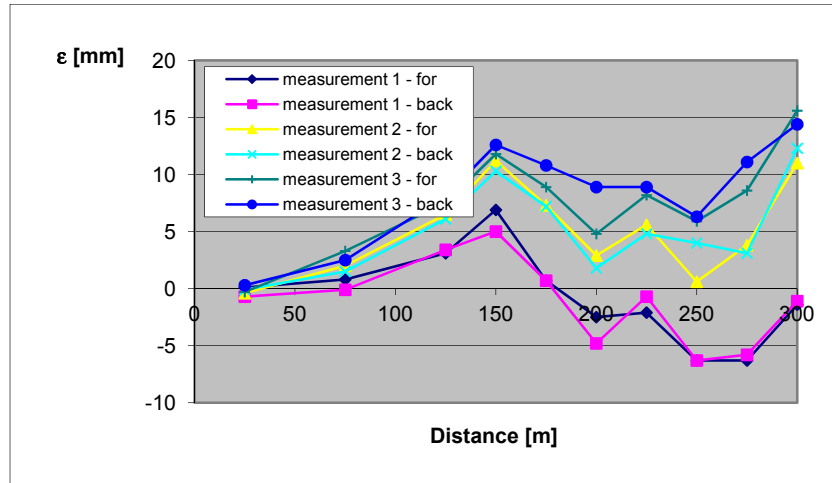


Fig. 5. Diagram of true errors of elevations for various distances of the transmitter (the repeated experiment).

Similarly to the first experiment, variable inclination of diagrams of true errors in various cycles may be observed, but it is compliant at the level of several millimetres, for both, forward and backward directions. The inclination of diagrams is so big, that – in the case when the drift of the specified sign and value occurs, reverse trends for the forward and backward directions should occur. Coincidence of the inclination of diagrams for both directions negates the existence of the drift. Therefore, the inclination of diagrams of true errors of elevations was caused by another, systematic reason. The repeated experiment, described above, was to check the existence of the drift. In further investigations, which tested the relations between the elevation accuracy and variations of distanced, the first experiment was considered as more reliable, due to higher independence of measurements (various days and times of day).

Considering the exclusion of significant influence of the instrumental drift, further detailed analysis was to prove the possible *influence of systematic instrumental errors*: deflection of the axis of rotation of the optical transmitting system and collimation of the laser beam<sup>1</sup>. Vertical deflection of the axis of rotation is a residual error which has not been removed in the process of calibration of the laser transmitter. Influence of that error is proportional to the increase of the distance and the value and sign of that influence depend on the accidental angle of deflection of the transmitter towards the receiver. That error reversely influences the results of surveys in reverse directions, i.e. in one direction it raises the laser beam, what results in increase of the point elevation, and in the reverse direction, it lowers the laser beam – and decreases the elevation. Therefore, in order to detect such type of error, the survey „from the centre” was performed (the transmitter located in point 8), towards both edges of the control base (Fig. 6). The second error results from conic type deflection of the beam and it is caused by incorrect location of rotating elements of the optical transmitting system of the laser beam with respect to the axis of rotation. The influence of that error is also proportional to the distance, but it is identical in all directions.

The influence of deflection of the axis of rotation is confirmed by the results of analysis of differences in elevation between points which are symmetrically distributed around the transmitter – the measurements “from the centre”. Fig. 7 presents obtained true errors of differences of elevations. Differences of elevations between symmetric points are not influenced by systematic influences of symmetric type errors, i.e. the influence of the Earth curvature, influence of collimation of the beam and the influence of

<sup>1</sup> The name introduced similarly to the collimation error of a theodolite aiming line

refraction, which importance is low for such distances. In the majority of cycles, close to linear and “not horizontal” diagrams of true errors of differences of elevations, confirm the influence of vertical deflection of the axis of rotation.

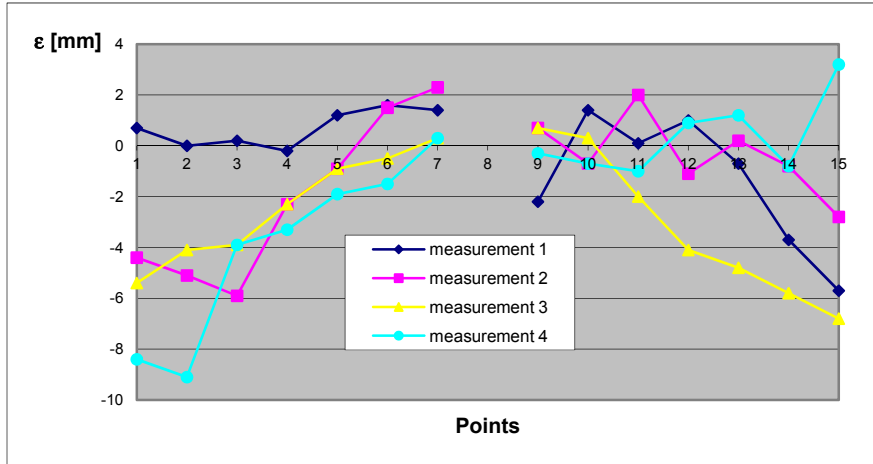


Fig. 6. Diagram of true errors of elevation of points, symmetrically located around the transmitter, for the measurements „from the centre”.

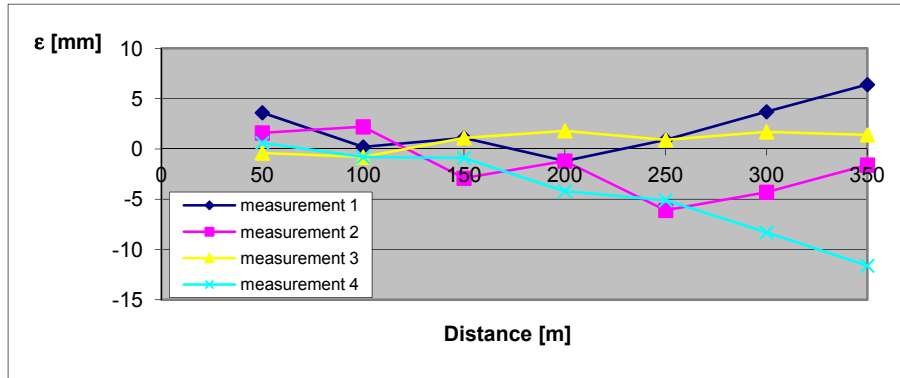


Fig. 7. Diagram of true errors of differences of elevations of points symmetrically located around the transmitter.

In order to confirm the influence of collimation of the laser beam, an attempt was made to model the true errors, obtained from the measurements „from the centre” by means of simultaneous influences of the Earth curvature, vertical deflection of the axis of rotation and collimation of the laser beam. The equation applied for modelling the true errors in one part of the base had the form:

$$\frac{D_i}{\rho''} a + \frac{D_i}{\rho''} c - \frac{D_i^2}{2R} = \varepsilon_i \quad (1)$$

where:  $\varepsilon$  – true error,

$D$  – distance from the transmitter,

$a$  – vertical deflection of the axis of rotation in seconds,

$c$  – collimation of the laser beam in seconds,

$R$  – the Earth radius.

For the “backward” and “forward” observations the first element was applied with opposite signs. For the created model, an accidental, i.e. different value of the angle of deflection of the axis of rotation ( $a$ ) was assumed for each cycle of measurements; the constant value of the collimation angle of the laser beam ( $c$ ) was assumed for all measurements, which resulted from the fixed placement of the transmitting system with

respect to the axis of rotation. The following model parameters were obtained:  $a_1 = 2,2''$ ,  $a_2 = -2,2''$ ,  $a_3 = 1,0''$ ,  $a_4 = -5,2''$ ,  $c = -1,6''$ .

The non-zero value of the collimation angle of the laser beam confirms the existence of that systematic error in above measurements. Values of true errors for the measurements “from the centre”, after introduction of corrections for the Earth curvature and after elimination of influences of systematic instrumental errors, are presented in Fig. 8.

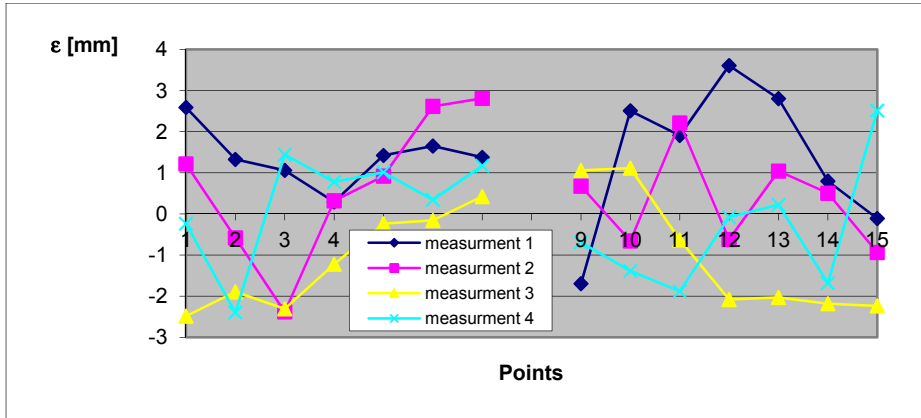


Fig. 8. Diagram of true errors for points symmetrically located around the transmitter, after elimination of systematic errors.

Values of true errors, obtained from „forward” measurements are also influenced by systematic errors. Since measurements are performed in one direction only, it is not possible to determine those errors and eliminate their influence. Therefore it would not be correctly to perform the accuracy analysis of determination of point elevation as depending on the distance from the transmitter, basing on calculation of mean errors. That is why the analysis based on the maximum values of modules of obtained true errors was applied (Fig. 9).

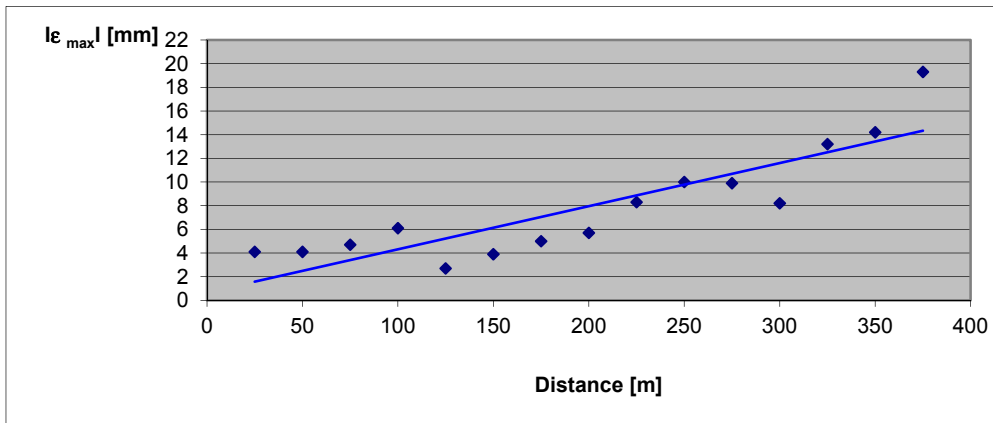


Fig. 9. Diagram of the maximum values of modules of true errors of point elevations, for various distances from the transmitter.

The next aspect of the measuring experiment concerned the investigation of the influence of the vertical angle of incidence of the laser beam on the accuracy of elevation measurements. Similarly to the previous case, multiple measurements of elevations were performed; in this case they were performed for points from the second group (point numbers between 17 and 23). In order to eliminate the variable influence of the distance on the measurement results, those points were located on the circle of the radius equal to approx. 34 m, with the transmitter located in point 5 (Fig. 3). Evaluation of the true

errors, resulting from the increase of the vertical angle, does not point to existence of systematic influences. The diagram of changes of the maximum values of modules of the true errors, together with changes of vertical angles, are presented in Fig. 10.

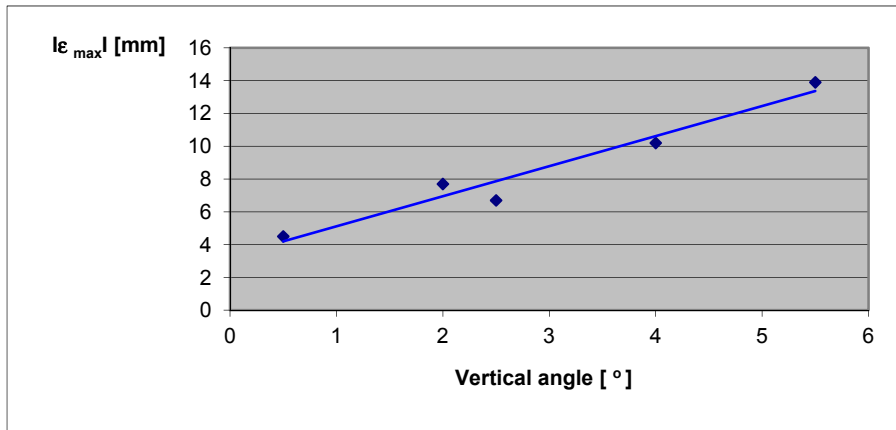


Fig. 10. Diagram of the maximum values of modules of the true errors for various vertical angles.

The last investigated issue concerned *the influence of terrain barriers on the accuracy of determination of elevations using the mmGPS System*. In order to evaluate that aspect, measurements were performed for points from the third group (point numbers between 24 and 27), with the transmitter placed in point 3 (Fig. 3). Among others points were located close to a tree, a street lamp and behind a bush. The points were selected in such a way that they were characterised by the small distance and small vertical angle with respect to the transmitter, in order to minimise influences of those factors, which were investigated earlier. When analysing results of measurements, it may be stated that terrain barriers did not visibly influence the deterioration of the accuracy of determined elevations. The obtained maximum value of the module of the true error, equal to 4.5 mm within the radius of 40 metres from the transmitter, where investigated points were located, corresponds to the influence of the distance on the elevation accuracy, which was already analysed earlier.

Knowing the results of presented analysis of the influence of the distance and vertical angle of the laser beam on the true error of elevations, the foreseen accuracy of the mmGPS System was determined, for the extremely disadvantageous conditions, however, within the range of the System operations, i.e. 300m, as specified by the System manufacturer. Results of that analysis are presented in Fig. 11.

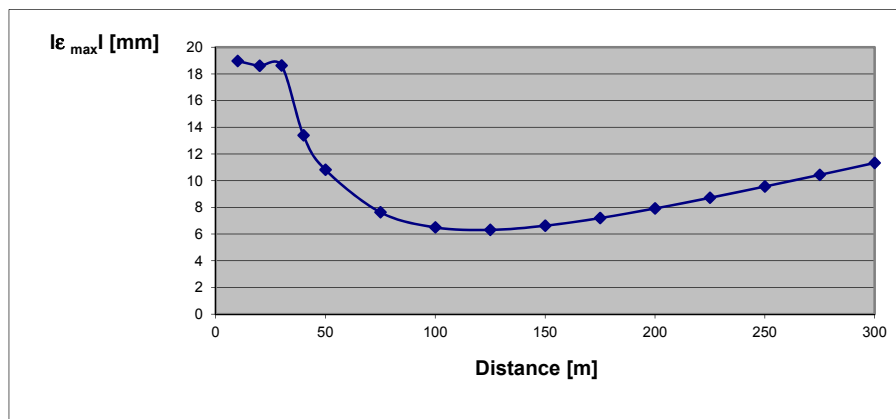


Fig. 11. Diagram of the foreseen total influence of the distance and vertical angle on the value of the maximum true error of elevation, for the extreme conditions.

The analysis was performed assuming the maximum beam incidence angle equal to  $10^\circ$  for the distance of 30 m; for longer distances – the value of the angle corresponding to the limit value of the elevation differences ( $\pm 5$  m) with respect to the transmitter, was assumed. The disadvantageous conditions of summing up the influences of the distance and vertical angle on the determined elevation, was also assumed.

#### 4. FINAL REMARKS

The following recommendations concerning measurements, may be concluded basing on the results of the above experiments:

1) In order to achieve the accuracy of determination of relative elevations of points, not worse than 1 cm, the radius of measurements should be limited to 250 m, avoiding measurements in areas of high elevation differences between the levels of the transmitter and the detector;

2) Due to similar reasons, in the process of determination of the transmitter elevation – basing on measurements between two points of known elevations – in order to achieve the millimetre accuracy – points should be selected within the distance not longer than 50 m in such a way that the vertical angle does not exceed  $1^\circ$  (the elevation difference of about 1 m for 50 m of the distance);

3) Although the mmGPS system is a technological solution for precise elevation surveys, in the case of surveys with big values of vertical angles, much attention should be paid to the verticality of the pole, due to the high influence of the distance error on elevation differences, which occurs in such conditions;

4) Due to instability of surveys (even within short periods), in particular for long distances, multiple surveys are recommended.

Due to achieved accuracy, the mmGPS technology may be used for establishment measurement networks, for surveys of investments, performed within small areas, as well as for creation building and road machinery controlling systems. What refers to application of the discussed technology, which was the reason for performing the above investigations, i.e. surveys of elevations of topographic details, the system allows for obtaining the accuracy not worse than 1 cm, with some limitations of the vertical angle. Despite the above disadvantages of the mmGPS System, those results may be considered as satisfactory comparing to the accuracy of elevation measurements using the RTK technology.

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