

## UNCERTAINTY OF HYSTERESIS LOOP MEASUREMENTS IN MAGNETIC MATERIALS

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**Abstract:** In this paper an estimation of the uncertainty is presented for the Magnet-Physik REMACOMP<sup>®</sup> C-200 BH-loop tracer. Two configurations were treated for electrical steel in the form of strips: the Epstein frame and the so called MJC yoke. It was revealed that the dominating contribution to the uncertainty budget in both configurations is due to the inaccuracy in measuring of the sample dimensions.

**Keywords:** hysteresis loop, uncertainty of measurement.

## 1. INTRODUCTION

Different kinds of soft magnetic materials (electrical steels, amorphous and nanocrystalline ribbons, etc.) are currently used for magnetic cores of electric and electronic devices. The main magnetic properties of these materials are represented by their hysteresis loops. They provide technical important information on the magnetisation processes in these materials. Besides the so called major loops, symmetric and asymmetric minor loops, initial and anhysteretic curves are examined in practice. Examples of these different kinds of curves are shown in Figure 1. Significant parameters are derived from those experimental curves like the coercivity  $H_c$ , the remanent induction  $B_r$  and the power loss [1-4]. In particular the power loss corresponds to the area of the hysteresis loop.

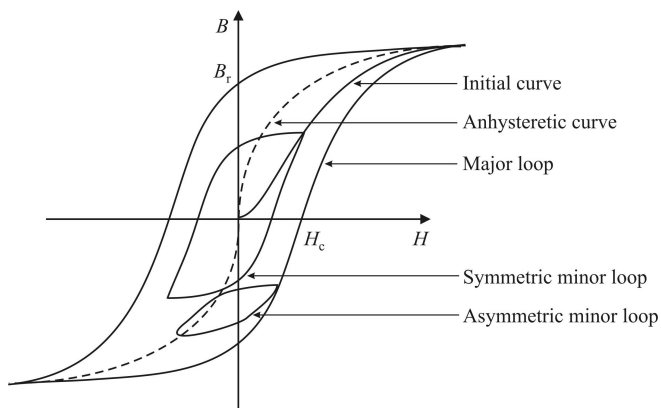


Fig. 1. Hysteresis loops of a magnetic material [4]

The properties of magnetic materials determine their suitability for specific applications and are used in the design

and manufacturing of magnetic cores. Inaccurate information about the material parameters may cause erroneous designs of magnetic cores and, as a consequence, inefficient operation modes of electric devices or excessive material costs. The magnetic properties are usually measured using a computer-aided measuring system. Thereby, the knowledge of the measuring system accuracy is such a crucial issue.

In this paper, the uncertainty of hysteresis loop measurements for a Magnet-Physik REMACOMP<sup>®</sup> C-200 system is estimated.

## 2. DETERMINATION OF THE HYSTERESIS LOOP

The Magnet-Physik REMACOMP<sup>®</sup> C-200 measuring system can be used to determine dynamic hysteresis loops of soft magnetic materials and related magnetic parameters like coercivity, remanence and power loss. The measurements can be carried out for different specimens and measuring fixtures as ring specimens, Epstein frames or Magnet-Physik MJC measuring yokes.

The REMACOMP<sup>®</sup> C-200 system operates based on the oscillographic recording principle. The idea behind this method is presented in Figure 2.

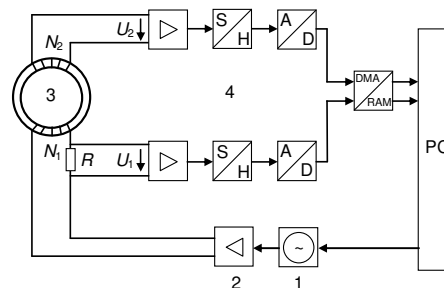


Fig. 2. Operating principle of the REMACOMP<sup>®</sup> C-200 measuring system: 1 – programmable signal generator, 2 – power amplifier, 3 – specimen, 4 – digital sampling system with preamplifiers and analog/digital converters,  $N_{1,2}$  – primary and secondary windings,  $R$  – shunt resistor [5, 6]

The magnetic field strength  $H$  is calculated directly from the measured voltage drop that is caused by a magnetizing current  $I$  flowing through a low-inductance shunt resistor  $R$ :

$$H = \frac{N_1 \cdot I}{l_m} = \frac{N_1 \cdot U_1}{l_m \cdot R} \quad (1)$$

where  $N_1$  – number of turns of the primary winding,  $I$  – current amplitude of the primary windings,  $l_m$  – length of the magnetic path,  $U_1$  – voltage on the shunt resistor  $R$ .

The magnetic flux density  $B$  is determined by integrating the voltage induced into the secondary winding:

$$B(t) = \frac{1}{N_2 \cdot A} \int U_2(t) dt \quad (2)$$

where  $N_2$  – number of turns of the secondary winding,  $A$  – cross-sectional area of the secondary winding,  $U_2$  – voltage induced into the secondary winding,  $t$  – time.

The voltage drop  $U_1$  across the shunt resistor and the voltage  $U_2$  induced in the secondary winding are simultaneously sampled by two analog-to-digital converters. The induced voltage is then numerically integrated to obtain the magnetic flux density  $B$ . More details can be found in [5,6].

### 3. ESTIMATING OF THE UNCERTAINTY OF HYSTERESIS LOOP MEASUREMENTS

The hysteresis loop can be described by a non-linear dependency  $B = f(H)$ . Both quantities  $H$  and  $B$  are measured in the same measurement system indirectly with some uncertainty. The true values are in the ranges  $H_m \pm U_H$  and  $B_m \pm U_B$ , where  $H$  and  $B$  with indexes 'm' are measured values and  $U_H$  and  $U_B$  are the related extended uncertainties. In this paragraph the route how to estimate these uncertainties is presented for the operating rule of the REMACOMP® C-200 loop tracer. This is done for real specimens made of electrical steel in the form of strips, measured by the Epstein frame (case 1 non-oriented strips) and by the Magnet-Physik MJC measuring yoke (case 2 grain-oriented strips), respectively. The results for these two cases are presented in the next two subsections.

#### 3.1. Case 1 (THE EPSTEIN FRAME)

In this case, the measurement of hysteresis loops of non-oriented steel M111-35A, as presented in [6], was used for the estimation of uncertainty.

The value of  $H$  is determined indirectly according to equation (1). The current  $I$  is controlled by the PC to assure that  $H$  does not exceed the range  $\langle -H_{\max}, H_{\max} \rangle$ , which is a priori determined by the operator.

Thereby, according to equation (1), we have four components in the uncertainty budget of  $H$ . In the certificate of calibration [7] the manufacturer declared the relative expanded uncertainties:  $U_{rel-U1} = 0.8\%$  (for a measured value 26.03 mV) and  $U_{rel-R} = 0.4\%$  with a coverage factor  $k=2$ , which for a normal distribution correspond to a coverage probability of approximately 95%. Hence, the relative standard uncertainties of input estimates  $U_1$  and  $R$  are  $u_{rel-U1} = 0.8/2/100 = 0.004$  and  $u_{rel-R} = 0.4/2/100 = 0.002$ . Let's assume that the last turn of the primary winding  $N_1 = 700$  was not fully wended or a part of an extra turn was added to lead the coil wires out. For this reason it is assumed that there is a maximum uncertainty of 1 of a full turn. The

corresponding uncertainty is  $u_{rel-N1} = 1/N_1/\text{sqrt}(3) = 0.0008$  due to an assumption of an uniform distribution. Here "sqrt" denotes a square root function. The magnetic path in [6] is expressed with three digits as 940 mm and the resolution of indication is  $\Delta l_m = 1$  mm. According to the suggestion stated in [8], again an uniform distribution is assumed. Thus the relative standard uncertainty of the magnetic length is  $u_{rel-lm} = \Delta l_m/\text{sqrt}(12)/l_m = 0.0003$ .

The combined standard uncertainty  $u_c(y)$  of  $y = f(x_1, \dots, x_M)$  depending on  $M$  input quantities  $x_i$  is the positive square root of the combined variance  $u_c^2(y)$ , which is given by the equation:

$$u_c^2(y) = \sum_{i=1}^M \left( \frac{\partial f}{\partial x_i} \right)^2 u^2(x_i) \quad (3)$$

This is true for the case that all input quantities are independent. The information about these quantities is taken from independent observations, where the variables  $N_1$ ,  $N_2$  and  $A$  were kept constant. Applying the law of propagation of uncertainty [9] to Eq. (1) and noting that it can be decomposed to principal cases:  $y = y_1/y_2$  and  $y_1 = x_1 \cdot x_2$  and  $y_2 = x_3 \cdot x_4$  we can state that the relative uncertainty of  $H$  can be expressed as:

$$u_{rel-H} = \sqrt{u_{rel-U1}^2 + u_{rel-R}^2 + u_{rel-N1}^2 + u_{rel-lm}^2} = 0.0046 \quad (4)$$

Here we mixed input quantities with normal and uniform distributions. This time, the coverage factor is chosen to  $k=2$  on the basis of the Central Limit Theorem. This theorem states that the output distribution will be approximately normal due to the convolution of even as few as three uniform distributions (being an extreme example of a non-normal distribution) of equal width is approximately normal. A practical consequence of this Central Limit Theorem is that: if the combined standard uncertainty is not dominated by a standard uncertainty component obtained from a Type A evaluation (many observations), or by a standard uncertainty component obtained from a Type B evaluation based on an assumed rectangular distribution, one has to use for the coverage factor a value from the normal distribution [8]. This requirement is satisfied in the analyzed case. Finally, the relative expanded uncertainty of  $H$  is:

$$U_{rel-H} = k \cdot u_{rel-H} = 0.009 < 1\% \quad (5)$$

The magnetic flux density is calculated according to equation (2). This time we have three components in the uncertainty budget of  $B$  as indicated by the equation

$$u_B = \sqrt{u_{relA}^2 + u_{relN2}^2 + u_{rel\int u2}^2} \quad (6)$$

The induced voltage signal measured on the secondary winding is sinusoidal versus time. As a consequence thereof and due to equation (3) this is caused by a cosine type behavior of the induced magnetic flux density versus time.

The range of input voltage of the NI Data Acquisition Board NI PCI-6111 is 5 V and the maximum amplitude of  $U_2$  is 1.952 V [6]. From the certificate of calibration we get a relative expanded uncertainty of 0.2%, with a coverage factor  $k=2$ , referenced to 4.75 V as the closest value to the maximal amplitude of  $U_2$ . Hence, the relative standard

uncertainties of  $U_2$  is  $u_{rel-U_2} = 0.2/2/100 = 0.0010$ . Let us assume that the last turn of  $N_2$  winding is not fully done and  $u_{rel-N_2} = 1/N_2/\sqrt{3} = 0.0085$  and the cross-sectional area  $A = 69.7 \cdot 10^{-6} \text{ m}^2$  is determined with the accuracy of 1%. Thus, a  $u_{rel-A} = 0.01/\sqrt{3} = 0.0058$  is obtained. The assessment of the last component of Eq. (6) is not easy and can not be determined without some simplification. The problem to be solved is how the uncertainty of  $U_2$  propagates through the operation of an integration? At first, let's assume the error of a numerical approximation of the integral can be neglected and we have no information how this operation is realized by the software associated with the measuring equipment. In practice, it depends on the applied numerical solver and a slow rate of the integrated signal in relation to time interval. More discussion on stochastic integrals can be found in [10]. In the following approach we assume that the measuring error of  $U_2$  is additive to its true value.

The uncertainty of the integral of  $U_2$  was evaluated using the Monte Carlo method [11]. We used the Matlab/Simulink environment to simulate a simple circuit as presented in Figure 3. The standard deviation of the output of a block "Integrator" is a measure of the sought uncertainty of the integrator. The number of  $10^6$  samples is enough to satisfy a good approximation of probability of the output random variable [11].

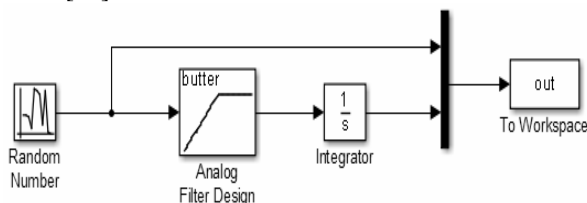


Fig. 3. Generation and propagation of a random signal through a DC removal block and an integrator

The uncertainty of  $U_2$  was modeled by a normally distributed random signal with zero mean and a standard deviation of  $u_{rel-U_2} \cdot 4.75 \text{ V}$ . Additionally, it was assumed that the DC component was removed by a ADC board in the actual measuring system. This functionality was modeled by a high-pass analog filter with an edge frequency (-3 dB) equal to 0.01 Hz. The variance of the integrator output signal is  $u_{rel-intU_2}^2 = 0.000015$  and the distribution of this signal is close to normal. This is in agreement with our expectation which is due to the properties of the "sum of stochastic variables" as expressed by the Central Limit Theorem and the integral-approximation by a sum.

Factor  $k = 2$  was used here for the same reason as above. Finally, the relative expanded uncertainty for  $B$  is obtained:

$$U_{rel\_B} = k \cdot u_{rel\_B} = 0.0205 \approx 2\% \quad (7)$$

### 3.2. Case 2 (THE MJC YOKE)

In this case, the hysteresis loop was measured for the single strip specimen made of grain-oriented steel ET122-30. A picture of the test site is presented in Figure 4.

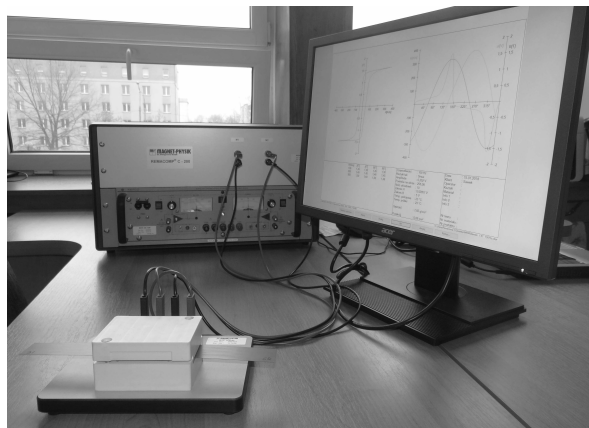


Fig. 4. The REMACOMP C-200 measuring system

The measurements carried out for an arbitrarily chosen frequency of  $f = 100 \text{ Hz}$  are shown in Figure 5. The shape of  $H(t)$  was fully controlled by the system to make sure that both  $U_2(t)$  and finally  $B(t)$  are sinusoidal as required in standard IEC 60404-6. This can be observed in Fig. 5b.

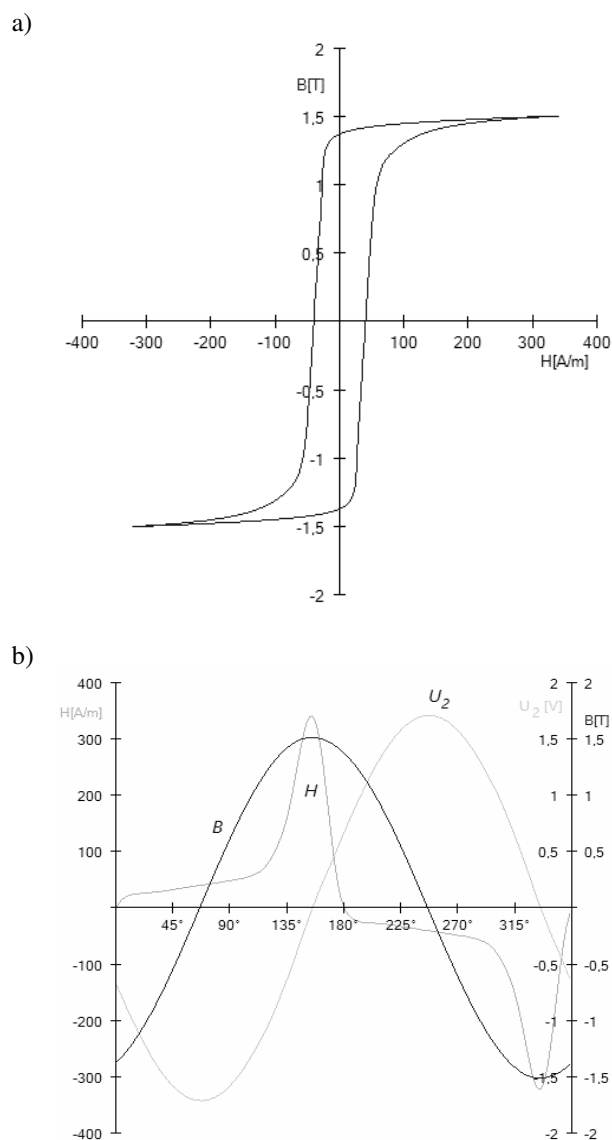


Fig. 5. a) JH loop for  $f = 100 \text{ Hz}$ , b) signals  $U_2(t)$ ,  $H(t)$ ,  $B(t)$  as a function of time (angle)

The following data are specifications of the MJC yoke:  $N_1 = 100$  turns,  $N_2 = 200$  turns and  $l_m = 0.058$  m. As in the case 1, we can estimate:  $u_{rel\_N1} = 1/N_1/\sqrt{3} = 0.0029$  and  $u_{rel\_N2} = 1/N_2/\sqrt{3} = 0.0015$ . The resolution of indication of  $l_m$  is  $\Delta l_m = 0.1$  mm and  $u_{rel\_lm} = \Delta l_m/\sqrt{12}/l_m = 0.005$ . The uncertainty components related to  $U_1$  and  $U_2$  are  $u_{rel\_U1} = 0.8/2/100 = 0.004$ ,  $u_{rel\_R} = 0.002$  and  $u_{rel\_U2} = 0.2/2/100 = 0.001$ , respectively. The sample dimensions are: 30 mm width and 0.3 mm thickness. These parameters were measured with resolution 0.01 mm. Therefore, the calculated cross-section of the sample is equal to  $A = 9 \cdot 10^{-6}$  m<sup>2</sup>. The relative standard uncertainty related to the width is  $u_{rel\_w} = 0.01/\sqrt{12}/30$  and the component related to the thickness is  $u_{rel\_t} = 0.01/\sqrt{12}/0.3$ . Therefore the  $u_{rel}$  of  $A$  is:

$$u_{rel\_A} = \sqrt{u_{rel\_w}^2 + u_{rel\_t}^2 + 2 \cdot r \cdot u_{rel\_w} \cdot u_{rel\_t}} = 0.01 \quad (8)$$

where  $r$  is a correlation factor  $\neq 0$  since the same digital thickness caliper was used to measure both dimensions. In the calculation its maximum value  $r = 1$  was taken (and is therefore probably overestimated) due to the lack of precise information on it. The  $u_{rel\_A}$  factor is the dominant part in the budget of  $u_{rel\_H}$ . The variance of integrator output signal is the same as in previous case, i.e.  $u_{rel\_intU2}^2 = 0.000015$ . Finally we obtain that:

$$U_{rel\_H} = k \cdot u_{rel\_H} = 0.0177 \approx 2\% \quad (9)$$

and

$$U_{rel\_B} = k \cdot u_{rel\_B} = 0.0203 \approx 2\% \quad (10)$$

#### 4. CONCLUSION

In the both cases considered in this paper, the relative expanded uncertainty of the hysteresis loop measurements was about 2%. These values seem to be satisfactory for technical measurements. It was revealed that the dominating contribution to the uncertainty budget was due to the inaccuracy in measuring of the sample dimensions.

Future research will be carried out in order to estimate the uncertainty of the power loss, which corresponds to the area of the hysteresis loop.

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## NIEPEWNOŚĆ WYZNACZANIA PĘTLI HISTEREZY MATERIAŁÓW MAGNETYCZNYCH

W artykule opisano system pomiarowy REMACOMP® C-200 służący do wyznaczania dynamicznych pętli histerezy dla materiałów magnetycznie miękkich oraz zaproponowano metodę wyznaczania dokładności takich pomiarów. Oszacowano niepewność względną wyznaczania natężenia pola magnetycznego  $H$  i indukcji magnetycznej  $B$ . Analizę niepewności przeprowadzono dla pomiarów pętli histerezy magnetycznej dla blach elektrotechnicznych w dwóch przypadkach: dla próbki wykonanej z blachy o ziarnie orientowanym (pomiarzy za pomocą aparatu Epsteina) i dla próbki z blachy nieorientowanej (pomiar na pojedynczym pasku za pomocą jarzma MJC). Na podstawie opisu metody pomiarowej zidentyfikowano wielkości wejściowe wpływające na niepewność natężenia pola  $H$  oraz niezależnie na indukcję magnetyczną  $B$ . Wprowadzono je do budżetu niepewności na podstawie dostępnej dokumentacji, założeń konstrukcyjnych cewek, pomiaru wymiarów próbek oraz symulacji komputerowej do oceny propagacji niepewności zmiennej losowej przez układ całkujący. W tym celu przeprowadzono analizę Monte Carlo. W obu analizowanych przypadkach uzyskano niepewność rozszerzoną względną dla pomiaru pętli histerezy na poziomie 2%. Uzyskane wyniki można uznać za akceptowalne w przypadku pomiarów technicznych. Dalsze prace będą prowadzone w kierunku szacowania niepewności pomiaru strat w blachach elektrotechnicznych, wyznaczonych z powierzchni pętli histerezy.

**Słowa kluczowe:** pętla histerezy, niepewność pomiaru.