

**A study on the effect of harmonic components
of the current supplying an induction motor
with reactive power compensation with the use
of capacitors on the power loss and voltage variations**

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The paper presents an analysis of the effect of voltage harmonics on the power loss in an LV line supplying an induction motor with reactive power compensation, with the use of capacitors, and analysis of the values of output voltage of an MV/LV transformer depending on the load type and value.

KEYWORDS: induction motor, current and voltage harmonics, power loss compensation

1. Introduction

The power loss in an electric power line, including the ones providing the induction motor supply, depends on the conductor effective resistance and square of the current, i.e. $\Delta P = R \cdot I^2$. The induction motors draw from the line not only the active power P but also reactive power Q , often referred to as a reactive magnetizing power. Value of the reactive (induction) power of the motor depends not only on its characteristic parameters but also on the load, i.e. rotational speed. Both the active and reactive power values are decisive for a characteristic parameter of the machine, namely the power coefficient ($\cos\phi$). In order to keep possibly low power loss during the electric power transmission the power coefficient value should be as high as possible, but at least corresponding to tangent of the angle below 0.4. This maximum tangent value is determined by the regulations. The power coefficient value corresponding to this tangent value is equal to 0.928. Since the natural value of the power coefficient of induction motors is less than the one suggested by the regulations, the reactive power absorbed from the network may be reduced using a so-called compensation of the induction reactive power by capacitive power. The reactive capacitive power is usually obtained from the capacitors. The capacitors as sources of reactive power are convenient parts of an electric power system. On the other hand, their disadvantage is due to the dependence of their reactance (X/f) on the frequency. The disadvantage becomes particularly important, when the so-called higher order harmonic components of the network voltage must be taken into account.

2. Harmonic components of the induction motor supply voltage and current

Consideration of voltage harmonic components is at present basically always necessary. According to allowable voltage harmonic contents the proper classes are distinguished (i.e. the first, second, and third one). For the first class a lower harmonic contents is admitted, a larger for the second one, and the highest for the third one. Allowable harmonic contents is determined based on a so-called THD coefficient and allowable values of particular voltage harmonics. None of these values must not be exceeded. According to binding regulations up to 40th order voltage harmonics should be taken into account, nevertheless, what concerns the capacitors, up to 17th order harmonics are sufficient. In some cases only the 5th and 7th harmonics may be considered, as giving the highest contribution to the network voltage. Values of the THD coefficient of particular classes amount to 5%, 8%, and 10% for the first, second, and third class, respectively. The THD coefficient is calculated as the root of the sum of squared allowable values of particular harmonics. The allowable values of particular harmonics are specified in Table 1.

Table 1. Allowable values of voltage harmonics for the classes 1, 2, 3

Harmonic order	1	3	5	7	9	11	13	15	17
Class 1 The voltage value (%)	1	3	3	3	1.5	3	3	0.3	2
Class 2 The voltage value (%)	1	5	6	5	1.5	3.5	3	0.4	2
Class 3 The voltage value (%)	1	6	8	7	2.5	5	4.5	2	4

For an assumed value of the active power P the intensity of the current I absorbed from the network depends on the $\cos\varphi$ value. When using the MCAD software the use of an auxiliary function $\cos\alpha \cdot 2\pi/3$ instead of cosine is more convenient, where α takes the values from the range ensuring $\cos\varphi$ variation in due range.

The current intensity is given by the formula (1)

$$I(\alpha) = \frac{P}{\sqrt{3}U_n \cos\left(\frac{2}{3}\alpha\pi\right)\eta} \quad (1)$$

where U is the line-to-line voltage. In LV network the rated line-to-line voltage is equal to 400V.

Figure 1 presents illustrative relationship between the current absorbed by a 10kW receiver as a function of the angle α .

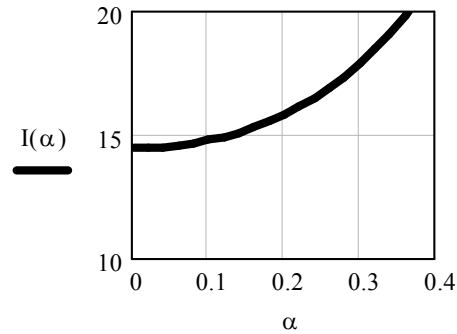


Fig. 1. The effect of the angle α on the current intensity

Figure 1 clearly depicts remarkable impact of the power coefficient on the current intensity. For $\alpha = 0.3$, when the power coefficient $\cos\varphi = 0.81$, the current intensity amounts to 20.3 A, while for $\alpha = 0.1$ and $\cos\varphi = 0.978$, the current intensity drops to 16.76 A. Since the power loss depends on the square of the current intensity (I^2), it is easy to notice that such a change of the power coefficient induces the power loss variation by about 20 percent. Similar effect occurs in case of power losses caused by the current harmonics. The current intensity is a consequence of all the harmonic components, that is described by the equation (2).

$$I = I_{h1} + I_{h3} + I_{h5} + I_{h7} + I_{h9} + I_{h11} + I_{h13} + I_{h15} + I_{h17} \quad (2)$$

In case of the circuits supplying the three-phase motors the third and ninth harmonics may be omitted.

The voltage harmonic components significantly affect the current intensity in the circuits including the capacitors, since the higher the harmonic order, the lower the reactance. In consequence, for a definite voltage the current intensity is higher. Therefore, the regulations stipulate lower allowable voltage values for particular higher-order harmonics. The voltage harmonic components that cause the current flow induced by these harmonics, induce not only the growth of the current in the supplying conductors but also, or even first of all, in the capacitors, thus causing their heating. Therefore, the capacitors of improved heat resistance are available, and of course for higher price, that may be charged even to 1.3 or even 1.5 of their rated current value. In case of higher harmonic currents special chokes should be used. The capacitor rated current is defined for the rated frequency (50 Hz). In order to calculate the current intensity in the circuit, with consideration of the selected harmonic components of the currents induced by the voltage harmonics (the first, third, fifth, seventh, ninth, eleventh, thirteenth, and seventeenth) one may make use of the formula (3)

$$I = \sqrt{I_{1h}^2 + I_{3h}^2 + I_{5h}^2 + I_{7h}^2 + I_{9h}^2 + I_{11h}^2 + I_{13h}^2 + I_{15h}^2 + I_{17h}^2} \quad (3)$$

The current corresponding to the first voltage harmonic component is given by the equation (4)

$$I_{1h} = \frac{U_f}{X_{c1}} \quad (4)$$

where: U_f is the rated phase voltage, while X_{c1} is the reactance of the first voltage harmonics of the capacitor having C_n capacitance. Capacitor reactance of the consecutive harmonic component is described by the formula (5)

$$X_{hn} = \frac{1}{2\pi f_n C_n} \quad (5)$$

in which n is a multiple of the basic harmonic component.

The currents of particular harmonic components are calculated from the formula (6)

$$I_{hn} = \frac{U_{hn}}{X_{hn}} \quad (6)$$

where: U_{hn} is allowable value of the definite voltage harmonic component, according to Table 1.

3. Power loss in supply conductors

As it was mentioned above, in the electric power networks attention should be paid to additional power losses caused by higher current harmonics and by growth of the current intensity due to transmission of the reactive power, related to active power transmission resulting from operation of electric equipment, especially electric motors. In consequence of these additional power losses the efficiency of the electric power system decreases. The paper presents the values of the additional power losses on the example of an induction motor of the power 10 kW.

In the case of a motor of rated power 10 kW and efficiency $\eta = 0.88$, the active power absorbed from mains amounts to 11360 W. It was assumed that the rated power coefficient of the motor is equal to 0.89. For these values of the power coefficient and efficiency the current drawn from mains by the motor, i.e. rated current, is given by the relationship (7)

$$I_n = \frac{P_n}{\sqrt{3}U_n \cdot 0.809 \cdot 0.88} \quad (7)$$

The rated intensity of the motor current $I_n = 20.27$ A.

This level of active power corresponds to reactive power $Q_n = 8256$ var absorbed by the motor from mains. In order to decrease the current intensity the reactive power should be reduced. It was assumed that the power coefficient is enlarged to the level recommended by the regulations. For the value $\tan\varphi = 0.4$

one obtains $\cos\varphi = 0.929$. The reactive power is compensated by a capacitor of reactive power $Q_k = 4000$ var. In this case the reactive power absorbed from mains drops to the level $Q_1 = Q_{in} - Q_k$, i.e. $Q_1 = 4.257$ kvar. Capacitor capacitance $C_k = 79.6$ μ F. In result of reactive power compensation the current intensity absorbed by the motor is reduced to $I = 17.5$ A. The rate of the current before and after the compensation is equal to 0.864. In result of the compensation the power loss in the supply conductors of the motor drops by 25 percent.

The reduction of power loss resulting from reactive power compensation is not sufficient to compensate the power loss caused by the harmonics occurring in the current supplied to the capacitor in result of the voltage harmonic components. The current values caused by the voltage harmonics were calculated with assumption of allowable voltage values for the Class 2, according to Table 1.

The capacitor rated current is determined for its rated capacitance, rated voltage, and rated frequency. Hence, the rated capacitor current $I_{nk} = 5.77$ A. It is the current corresponding to the first harmonic component of the current supplied to the capacitor. The fifth harmonic component of the current supplied to the capacitor is determined by the formula (8)

$$I_{h5} = u_{h5} \frac{U_f}{X_{ck5}} \quad (8)$$

where u_{h5} is the allowable value of the fifth voltage harmonic component. The capacitor reactance for the fifth harmonic (9)

$$X_{ck5} = \frac{1}{2\pi 5 f_n C_k} \quad (9)$$

the current of the fifth harmonics $I_{h5} = 1.73$ A.

The currents of consecutive harmonic components are calculated similarly. The current of the seventh harmonic component calculated from the formula (10)

$$I_{h7} = u_{h7} \frac{U_f}{X_{ck7}} \quad (10)$$

amounts to $I_{h7} = 2.02$ A.

Similar calculation of the current of the eleventh, thirteenth, and seventeenth harmonics give $I_{h11} = 2.22$ A, $I_{h13} = 2.25$ A, and $I_{h17} = 1.96$ A, respectively. The current arising in the circuit, with consideration of the above mentioned harmonic components is presented by the relationship (11)

$$I = \sqrt{I_{h1}^2 + I_{h5}^2 + I_{h7}^2 + I_{h11}^2 + I_{h13}^2 + I_{h17}^2} \quad (11)$$

$I = 7.37$ A.

The ratio of the current value calculated as above to the current without the harmonic components, i.e. to the first harmonic current, is equal to $7.37/5.77 = 1.28$. Taking into account that the loss depends on the square of the ratio, the

ratio grows to $1.28^2 = 1.63$. It means that due to the current harmonic components the power loss grows up to 63 percent. It is easy to notice that the additional power loss induced by the current harmonic components, equal to 63 percent, exceeds the power loss reduced in result of reactive power compensation equal to 25 percent.

4. Voltage drops across the transformers

Power transformers (also referred to as circuit-separation transformers), used to convert the medium voltage to low voltage, operate without voltage adjustment. They are usually provided with three taps adjusted in voltage-less state. They are used under various load conditions, which means that the receivers of various types and frequent variations of load impedance are connected to them. In the low voltage networks, composed usually of four conductors, inclusive of a neutral one, the asymmetric two-phase loads also occur. The circuit-separation transformers are designed for asymmetric single-phase loads since they are provided with the circuit configuration that remains insensitive to such loads. The values of output (secondary) voltage depend on the input (primary) voltage and voltage drop. The voltage drops depend on the load current intensity and the power coefficient. Under symmetrical load the value of the voltage drop may be calculated from a simple formula (12)

$$\Delta U(I, \alpha) = I(R \cos(\alpha) + X \sin(\alpha)) \quad (12)$$

While calculating the functions e.g. $\cos\varphi$ and $\sin\varphi$ with the MCAD software, one must make use of an auxiliary function of the α angle, i.e. instead of $\cos\varphi$ we write

$$\cos\left(2\alpha \frac{\pi}{3}\right)$$

Figure 2 presents the voltage drop as a function of the angle for a selected example, for which $I = 5$ A, $R = 2 \Omega$ and $X_L = 5 \Omega$.

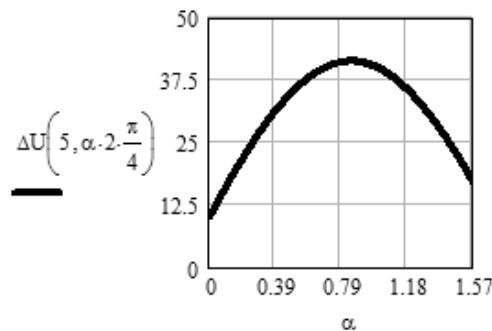


Fig. 2. The effect of the angle on the voltage drop

Based on Figure 2 a significant effect of the load angle on the voltage drop can be observed.

For a similar example (Fig. 3) the effect of the load angle on the output voltage is shown. In this case the output voltage of the 231 V transformer is considered as the input voltage. The output voltage is presented by the equation (13) $U_{wyj} = U_{we} - \Delta U$

$$U_{wyj}(\alpha) = 231 - (R \cos(\alpha) + X \sin(\alpha)) \quad (13)$$

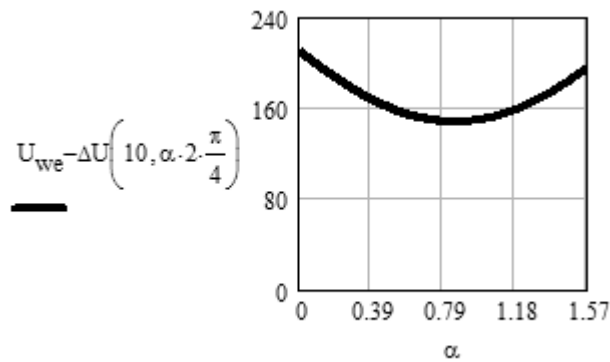


Fig. 3. The effect of the angle on the output voltage

Influence of the load angle on the transformer output voltage is illustrated by the curves shown in Fig. 4.

Analysis of the output voltage under asymmetric load is more complicated. The method of symmetrical components is a convenient way of analyzing the problem with the use of Mathcad Computation Software. The method enables also easy calculation of the asymmetry coefficient of the transformer output voltage, as being equal to the ratio of symmetrical component of negative- to positive-sequence voltage or zero- to positive-sequence voltage.

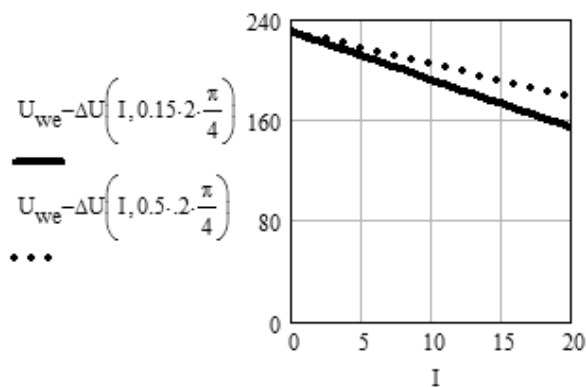


Fig. 4. The effect of the current intensity on the output voltage for two various values of the load angle

Values of the coefficients belong to characteristic parameters determining the mains voltage quality. Frequent checking of the ratio of symmetrical component of negative- to positive-sequence voltage during operation of motors is very important. Exceeding the allowable value of the factor equal to 0.2 may result in overheating of a three-phase motor.

5. Basic formulas for analysis of the output voltage as a function of the load

In order to carry out analysis of the problem a preliminary assumption should be made. The most convenient one consists in adopting the impedance values of particular phases. The impedances may be defined in the following form (14)

$$Z_{\text{odb } u, v, w} = k_{1,2,3} \cdot Z_{\text{odn}} \exp j0.1072\pi / 3 \quad (14)$$

where u, v, w denote consecutive phases.

Based on these impedances the impedance of the symmetrical components are calculated in the form (15)

- positive-sequence: $Z_1(k_1, k_2, k_3) = 1/3(Z_u(k_1) + a \cdot Z_v(k_2) + a^2 \cdot Z_w(k_3))$
- negative-sequence: $Z_2(k_1, k_2, k_3) = 1/3(Z_u(k_1) + a^2 \cdot Z_v(k_2) + a \cdot Z_w(k_3))$
- zero-sequence: $Z_0(k_1, k_2, k_3) = 1/3(Z_u(k_1) + Z_v(k_2) + Z_w(k_3))$.

In the matrix form the impedances of the symmetrical components take a form (16):

$$\begin{pmatrix} Z_1 \\ Z_2 \\ Z_0 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 & 1 \\ a^2 & a & 1 \\ a & a^2 & 1 \end{pmatrix} \cdot \frac{1}{3} \cdot \begin{pmatrix} Z_u \\ Z_v \\ Z_w \end{pmatrix} \quad (16)$$

Decomposition of the supply voltages, currents and receiver impedances to symmetrical components and transformation of the equations of the type $U = I \cdot Z$ into the inverse one $I = Y \cdot U$ give the equations of the current symmetrical components in the form (17):

$$\begin{pmatrix} I_1 \\ I_2 \\ I_0 \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} & M_{10} \\ M_{21} & M_{22} & M_{20} \\ M_{01} & M_{02} & M_{00} \end{pmatrix} \frac{1}{D} \begin{pmatrix} U_1 \\ U_2 \\ U_0 \end{pmatrix} \quad (17)$$

where

$$\begin{aligned} D_1(k_1, k_2, k_3) &= (Z_0(k_1, k_2, k_3) + Z_z)(Z_0(k_1, k_2, k_3) + Z_z)(Z_0(k_1, k_2, k_3) + Z_{\mu 0}) \\ D_2(k_1, k_2, k_3) &= -Z_1(k_1, k_2, k_3)Z_2(k_1, k_2, k_3)[3Z_0(k_1, k_2, k_3) + (Z_z + Z_z + Z_{\mu 0})] \\ D_3(k_1, k_2, k_3) &= Z_1(k_1, k_2, k_3)^3 + Z_2(k_1, k_2, k_3)^3 \\ D(k_1, k_2, k_3) &= D_1(k_1, k_2, k_3) + D_2(k_1, k_2, k_3) + D_3(k_1, k_2, k_3) \end{aligned}$$

$$\begin{aligned}
 M_{11}(k_1, k_2, k_3) &= (Z_z + Z_0(k_1, k_2, k_3))(Z_0(k_1, k_2, k_3) + Z_{\mu 0}) - Z_1(k_1, k_2, k_3)Z_2(k_1, k_2, k_3) \\
 M_{12}(k_1, k_2, k_3) &= Z_1(k_1, k_2, k_3)^2 - Z_2(k_1, k_2, k_3)(Z_{\mu 0} + Z_0(k_1, k_2, k_3)) \\
 M_{10}(k_1, k_2, k_3) &= Z_2(k_1, k_2, k_3)^2 - Z_1(k_1, k_2, k_3)(Z_0(k_1, k_2, k_3) + Z_z) \\
 M_{21}(k_1, k_2, k_3) &= Z_2(k_1, k_2, k_3)^2 - Z_1(k_1, k_2, k_3)(Z_0(k_1, k_2, k_3) + Z_{\mu 0}) \\
 M_{20}(k_1, k_2, k_3) &= Z_1(k_1, k_2, k_3)^2 - Z_2(k_1, k_2, k_3)(Z_0(k_1, k_2, k_3) + Z_z) \\
 M_{01}(k_1, k_2, k_3) &= Z_1(k_1, k_2, k_3)^2 - Z_2(k_1, k_2, k_3)(Z_0(k_1, k_2, k_3) + Z_z) \\
 M_{02}(k_1, k_2, k_3) &= Z_2(k_1, k_2, k_3)^2 - Z_1(k_1, k_2, k_3)(Z_0(k_1, k_2, k_3) + Z_z) \\
 M_{00}(k_1, k_2, k_3) &= (Z_z + Z_0(k_1, k_2, k_3))^2 - Z_1(k_1, k_2, k_3)Z_2(k_1, k_2, k_3) \\
 M_{22}(k_1, k_2, k_3) &= (Z_0(k_1, k_2, k_3) + Z_z)(Z_0(k_1, k_2, k_3) + Z_{\mu 0}) - Z_1(k_1, k_2, k_3)Z_2(k_1, k_2, k_3)
 \end{aligned}$$

Assuming that only the positive-sequence component is taken into account in the supply voltage, the current symmetrical components at secondary transformer side are given by the formulas (18):

– positive-sequence current (18a):

$$I_1(k_1, k_2, k_3) = M_{11}(k_1, k_2, k_3)U_{\text{ntf}} \frac{1}{D(k_1, k_2, k_3)} \quad (18a)$$

– negative-sequence current (18b):

$$I_2(k_1, k_2, k_3) = M_{21}(k_1, k_2, k_3)U_{\text{ntf}} \frac{1}{D(k_1, k_2, k_3)} \quad (18b)$$

– zero-sequence current (18c):

$$I_0(k_1, k_2, k_3) = M_{01}(k_1, k_2, k_3)U_{\text{ntf}} \frac{1}{D(k_1, k_2, k_3)} \quad (18c)$$

The phase currents are calculated from the formulas (19):

$$\begin{pmatrix} I_a(k_1, k_2, k_3) \\ I_b(k_1, k_2, k_3) \\ I_c(k_1, k_2, k_3) \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ a^2 & a & 1 \\ a & a^2 & 1 \end{pmatrix} \begin{pmatrix} I_1(k_1, k_2, k_3) \\ I_2(k_1, k_2, k_3) \\ I_3(k_1, k_2, k_3) \end{pmatrix} \quad (19)$$

The neutral conductor current is given by the formula (20):

$$I_{p0}(k_1, k_2, k_3) = I_a(k_1, k_2, k_3) + I_b(k_1, k_2, k_3) + I_c(k_1, k_2, k_3) \quad (20)$$

Phase voltages are described by the relationships (21):

$$\begin{aligned}
 U_a(k_1, k_2, k_3) &= I_a(k_1, k_2, k_3)Z_{zu}(k_1) \\
 U_b(k_1, k_2, k_3) &= I_a(k_1, k_2, k_3)Z_{zv}(k_2) \\
 U_c(k_1, k_2, k_3) &= I_c(k_1, k_2, k_3)Z_{zw}(k_3)
 \end{aligned} \quad (21)$$

Voltage symmetrical components at the secondary transformer side may be calculated from the formulas (22):

$$\begin{aligned}
 U_1(k_1, k_2, k_3) &= \frac{1}{3}(U_a(k_1, k_2, k_3) + aU_b(k_1, k_2, k_3) + a^2U_c(k_1, k_2, k_3)) \\
 U_2(k_1, k_2, k_3) &= \frac{1}{3}(U_a(k_1, k_2, k_3) + a^2U_b(k_1, k_2, k_3) + aU_c(k_1, k_2, k_3)) \quad (22) \\
 U_0(k_1, k_2, k_3) &= \frac{1}{3}(U_a(k_1, k_2, k_3) + U_b(k_1, k_2, k_3) + U_c(k_1, k_2, k_3))
 \end{aligned}$$

Voltage asymmetry coefficients of the secondary transformer side, that are very important in case of an asymmetrical load, are given by the relationships (23):

$$\begin{aligned}
 K_u(k_1, k_2, k_3) &= \frac{|U_2(k_1, k_2, k_3)|}{|U_1(k_1, k_2, k_3)|} \\
 K_{u0}(k_1, k_2, k_3) &= \frac{|U_0(k_1, k_2, k_3)|}{|U_1(k_1, k_2, k_3)|}
 \end{aligned} \quad (23)$$

6. Numerical example

Parameters of the transformer of the power equal to 630 kVA, the voltages 15000 V/20-242.5 V and short-circuit voltage 5.6% served as a basis for calculating characteristic data important for the objective of the present paper. The figures provide graphical illustration of some calculation results. The calculation was carried out on the example of an asymmetrical receiver of the following parameters:

$$\begin{aligned}
 Z_{zu}(k_1) &= \left(k_1 Z_{odn} e^{j \cdot 0.107 \cdot 2 \frac{\pi}{3}} \right) \\
 Z_{zv}(k_2) &= \left(Z_{odn} 1.2 e^{j k_2 \cdot 0.307 \cdot 2 \frac{\pi}{3}} \right) \\
 Z_{zw}(k_3) &= \left(k_3 \cdot 0.8 Z_{odn} e^{-j \cdot 0.207 \cdot 2 \frac{\pi}{3}} \right)
 \end{aligned}$$

In this case of asymmetrical load and under various values of k_1 , k_2 , and k_3 coefficients the voltage asymmetry coefficients are equal to:

$$\begin{aligned}
 K_u(1,1,1) &= 0.073 \\
 K_u(1,0.85,0.9) &= 0.067 \\
 K_u(0.9,1,1) &= 0.076
 \end{aligned}$$

For example, in case of other load cases the results are as follows:

$$K_u(1,1,1) = 0.022$$

$$K_u(1,0.85,0.9) = 0.025$$

$$K_u(0.9,1,1) = 0.025$$

It is easy to notice that in the considered load case the asymmetry coefficients exceed the allowable level 0.02. Variation of the asymmetry coefficient induced by the k_1 value is shown in Fig. 5.

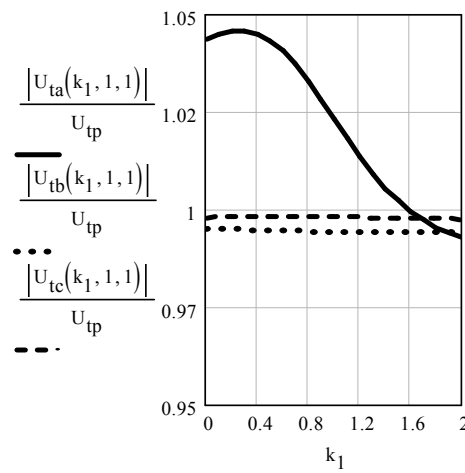


Fig. 5. Variation of the phase voltages as a function of the coefficient k_1

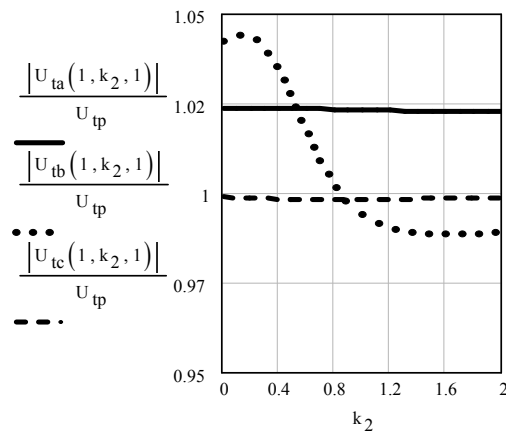


Fig. 6. Variation of the phase voltages as a function of the coefficient k_2

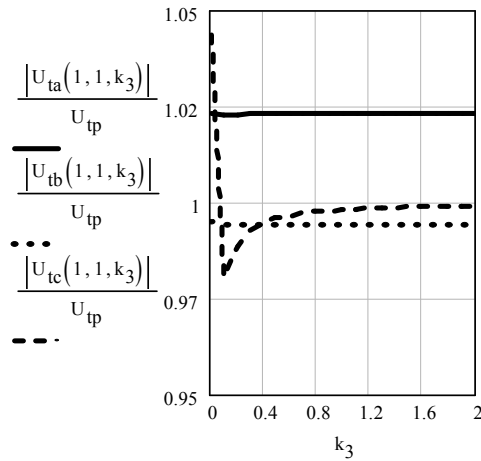


Fig. 7. Variation of the phase voltages as a function of the coefficient k_3

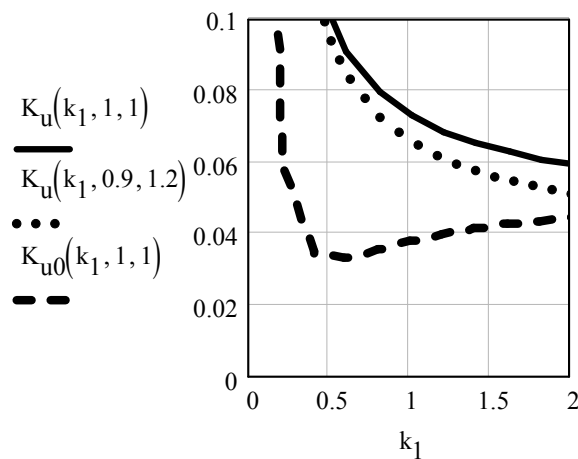


Fig. 8. Variation of the voltage asymmetry coefficients as a function of the coefficient k_1

7. Summary and final conclusions

The current flowing through the transformer winding induces the voltage drops. Values of the voltage drops depend not only on the current intensity but also on the load type, i.e. the value of the power coefficient. In case of a leading load the output voltages may exceed the input ones. In case of an asymmetric load the output voltages in particular phases are not equal. While supplying three-phase motors the value of the asymmetry coefficient must be monitored, since in case of its high level the motor winding may overheat.

It can be easily noticed that in the considered case of voltage harmonic components of the Class 2 the circuit currents exceed the rated values of current intensity. In case of the Class 1 voltage harmonic components the current intensity does not exceed the current multiple of 1.3 and, therefore, the use of a capacitor of allowable current 1.3 is sufficient. In the circuits in which the harmonic components count among the Class 3, usually the chokes should be used. The power loss in the conductor supplying the considered circuit caused by the voltage harmonic components of the second class increased 1.63 times. The power loss in the conductor supplying the considered circuit caused by the voltage harmonic components of the third class would increase 2.22 times.

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