10.24425/acs.2018.125488

Archives of Control Sciences
Volume 29(LXV), 2019
No. 1*, pages 5–23*

A new D-stability area for linear discrete-time systems

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The paper addresses the problem of constrained pole placement in discrete-time linear systems. The design conditions are outlined in terms of linear matrix inequalities for the *D*-stable ellipse region in the complex $\mathcal{Z}$ plain. In addition, it is demonstrated that the *D*-stable circle region formulation is the special case of by this way formulated and solved pole placement problem. The proposed principle is enhanced for discrete-lime linear systems with polytopic uncertainties.

Key words: discrete-time linear systems, state control, pole placement constraints, $\mathcal{D}$-stability region, linear matrix inequalities, polytopic uncertainties

1. Introduction

The first closed-loop pole constraint placement schemes have exploited the properties of linear-quadratic (LQ) control theory to assign poles to prescribed regions. The problem of locating the poles in a specified disk, for both continuous and discrete-time systems by using the Riccati equation, was so solved in [6]. More complete description of a variety of pole clustering regions, using also Riccati equations, was presented in [8], characterizing the matrices whose eigenvalues lie in a given sector in terms of modified Lyapunov equations and, beyond, on linear matrix inequalities (LMI).

The design of H_∞ controllers that satisfy additional constraints on the closed-loop poles was proposed in [4]. Since stability conditions, quadratic constraints and additional coupling objectives were formulated in terms of a common Lyapunov function, the control design was reduced to solving a system of LMIs, and the proposed characteristic functions have became mainly an LMI-based representation of $\mathcal{D}$-stability circle regions [5, 17]. Some adaptations to dynamic controller design, cascade reconfiguration control design, uncertain descriptor

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The work presented in this paper was supported by VEGA, the Grant Agency of the Ministry of Education and the Academy of Science of Slovak Republic, under Grant No. 1/0608/17. This support is very gratefully acknowledged.

Rceived 05.02.2018. Revised 13.02.2019.

linear system control, continuous-time singular systems, as well as to systems with real convex polytopic uncertainties, are given in [3, 7, 18–20], respectively. Other specific forms can be found in the active vibration controller design [13].

The nonlinear H_∞ control design is based either on the dissipative theory or on the nonlinear version of bounded real lemma (BRL). An alternative to this principle is the technique based on Takagi-Sugeno (TS) fuzzy models, which employ linear controllers, corresponding to the local linear system models, connected by the fuzzy membership functions. Motivated by the above mentioned LMI based design of the conventional state control with pole placement constraints, the fuzzy controller adjustment formulas, imposing H_∞ objectives for guaranteeing global stability and transient behaviors [11], were adapted to closed-loop pole location in the conic sectors [2, 10, 14], or circular regions [8, 12]. Note, de facto, most of these algorithms are concerned with the continuous-time systems.

Since of the closed form of BRL for discrete-time systems, requiring in its beginnings the use of Finsler's theorem to solve the associated LMIs, the problem leads to a relatively small number of references to this field [15]. The modern concept prefers the methodology given in [6], relaxing a circular region as a restriction for closed-loop pole placement, while the region parameters are considered in accordance with [1]. It is proven that the prescription of a D-stable circle region is equivalent to the specific quadratic boundary on the system state variables [17].

This paper proposes to use the more generalized D-stable region, defined by the ellipse parameters. Exploiting the Lypunov-Krasovskii theorem, the closed-loop system pole clustering is tailored via a finite set of LMIs with expansion of the Lyapunov matrix inequality by one sub-block, comparing to its structure for the D-stable circle region. Using the slack matrix approach, the method is enhanced to ensure stability for linear discrete-time systems with polytopic uncertainties. The proposed approach seems to be reasonable also when the control problem involves further additive quadratic performance constraints.

The content and scope of the paper are as follows. Following the problem formulation in Sec. 2, the basic concept used in the paper for the ellipse eigenvalue clustering in discrete-time linear systems is given in Sec. 3. Section 4 derives the control law parameter design conditions in the framework of LMIs and, orienting on linear discrete-time systems with polytopic uncertainties, enhanced pole placement is introduced in Sec. 5 The relevance of the proposed conditions is illustrated by a numerical example in Sec. 6 and in Sec. 7 are drawn some concluding remarks.

Throughout this paper, the used notations are narrowly standard in such way that $\boldsymbol{x}^T$, $\boldsymbol{X}^T$ denotes the transpose of the vector $\boldsymbol{x}$ and the matrix $\boldsymbol{X}$, respectively, $\boldsymbol{X} = \boldsymbol{X}^T > 0$ means that $\boldsymbol{X}$ is a symmetric positive definite matrix, the symbol $\boldsymbol{I}_n$ indicates the n-th order unit matrix, $\mathbb{R}$ denotes the set of real numbers and $\mathbb{R}^{n\times r}$ refers to the set of all $n\times r$ real matrices.

2. Problem Formulation

In the paper, the task is concerned with design of the state feedback control law matrix gain for controlling the discrete-time linear dynamic systems given by the state equations

$$\boldsymbol{q}(i+1) = \boldsymbol{F}\boldsymbol{q}(i) + \boldsymbol{G}\boldsymbol{u}(i), \tag{1}$$

$$\boldsymbol{y}(i) = \boldsymbol{C}\boldsymbol{q}(i), \tag{2}$$

where $\boldsymbol{q}(i) \in \mathbb{R}^n$, $\boldsymbol{u}(i) \in \mathbb{R}^r$, and $\boldsymbol{y}(i) \in \mathbb{R}^m$ are vectors of the system state, input and output variables, respectively, and $\boldsymbol{F} \in \mathbb{R}^{n\times n}$, $\boldsymbol{G} \in \mathbb{R}^{n\times r}$, $\boldsymbol{C} \in \mathbb{R}^{m\times n}$ are real matrices. Evidently, the autonomous system part of (1) is

$$\boldsymbol{q}(i+1) = \boldsymbol{F}\boldsymbol{q}(i)\,. \tag{3}$$

Considering the linear state feedback controller of the form

$$\boldsymbol{u}(i) = -\boldsymbol{K}\boldsymbol{q}(i), \tag{4}$$

where $\boldsymbol{K} \in \mathbb{R}^{r\times n}$ is the feedback gain matrix, then substituting (4) in (1) it can get that

$$\boldsymbol{q}(i+1) = \boldsymbol{F}_c\boldsymbol{q}(i), \tag{5}$$

where the closed-loop system matrix is

$$\boldsymbol{F}_c = \boldsymbol{F} - \boldsymbol{G}\boldsymbol{K}\,. \tag{6}$$

These equations can be viewed as the dynamics of the closed-lop system.

3. Closed-loop Eigenvalue Clustering

For simplicity, it is assumed in this section that there exists no multiple eigenvalue of the system matrix. This assumption is not a restriction, the key idea remaining the same. Moreover, is enough to consider the model of the type (7) [17].

Theorem 1 *If a stable discrete-time linear autonomous system is described as*

$$\boldsymbol{p}(i+1) = \frac{\boldsymbol{F} - m\boldsymbol{I}}{a}\boldsymbol{p}(i), \tag{7}$$

where with the positive scalars $m, a \in \mathbb{R}$, $y \leqslant b < a$, $m + a < 1$, and for $h = 1, 2, \ldots, n$ it yields

$$\sigma(\boldsymbol{F}) = \{z_h,\ z_h \in \mathcal{Z} : z^{\star} z < 1\}, \tag{8}$$

then with the positive tuning parameters $y, b \in \mathbb{R}$, $y \leqslant b < a$,

$$\sigma_{abmy}(\boldsymbol{F}) = \left\{ z_h, z_h \in \mathcal{Z} : (z^\star - a)(z - a) < a^2 - y^2 \left(\frac{a^2}{b^2} - 1 \right) \right\}. \tag{9}$$

Hereafter, $\sigma(\cdot)$ denotes the eigenvalue spectrum of a real square matrix and the eigenvalue $z_h^\star$ is complex conjugated to the eigenvalue z_h.

Proof Defining the Lyapunov function as

$$v(\boldsymbol{p}(i)) = \boldsymbol{p}^T(i)\boldsymbol{P}\boldsymbol{p}(i) > 0, \tag{10}$$

where $\boldsymbol{P} \in \mathbb{R}^{n \times n}$ is positive definite matrix, to solve stability condition of the autonomous system (7), the forward difference of (10) has to satisfy the inequality

$$\Delta v(\boldsymbol{p}(i)) = \boldsymbol{p}^T(i+1)\boldsymbol{P}\boldsymbol{p}(i+1) - \boldsymbol{p}^T(i)\boldsymbol{P}\boldsymbol{p}(i) < 0. \tag{11}$$

Considering the Lyapunov-Krasovskii theorem [9], it can be prescribed

$$\begin{aligned} \Delta v(\boldsymbol{p}(i)) &= \boldsymbol{p}^T(i+1)\boldsymbol{P}\boldsymbol{p}(i+1) - \boldsymbol{p}^T(i)\boldsymbol{P}\boldsymbol{p}(i) \\ &\leqslant -\frac{y^2}{a^2}\left(\frac{a^2}{b^2} - 1 \right) \boldsymbol{p}^T(i)\boldsymbol{P}\boldsymbol{p}(i) < 0, \end{aligned} \tag{12}$$

where the positive tuning parameters $b, y \in \mathbb{R}$ and $y \leqslant b < a$ are setting.

Substituting (7) in (12) implies

$$\boldsymbol{p}^T(i)\left(\frac{(\boldsymbol{F} - m\boldsymbol{I})^T}{a}\boldsymbol{P}\frac{(\boldsymbol{F} - m\boldsymbol{I})}{a} - \boldsymbol{P} + \frac{y^2}{a^2}\left(\frac{a^2}{b^2} - 1 \right)\boldsymbol{P} \right)\boldsymbol{p}(i) < 0, \tag{13}$$

which predefines that matrix inequality implying from (13) takes the form

$$\boldsymbol{F}^T\boldsymbol{P}\boldsymbol{F} - m\boldsymbol{P}\boldsymbol{F} - m\boldsymbol{F}^T\boldsymbol{P} + \left(m^2 - a^2 + y^2\left(\frac{a^2}{b^2} - 1 \right) \right)\boldsymbol{P} < 0. \tag{14}$$

If z_h is the h-th eigenvalue of $\boldsymbol{F}$ and $\boldsymbol{n}_h$ is the associated right eigenvector, then for $z_h^\star$ complex conjugate to z_h and $\boldsymbol{n}_h^\star$ complex conjugate to $\boldsymbol{n}_h$ it is true that

$$\boldsymbol{F}\boldsymbol{n}_h = z_h\boldsymbol{n}_h, \quad \boldsymbol{F}\boldsymbol{n}_h^\star = z_h^\star\boldsymbol{n}_h^\star, \quad \boldsymbol{n}_h^{\star T}\boldsymbol{F}^T = z_h^\star\boldsymbol{n}_h^{\star T}. \tag{15}$$

Pre-multiplying the left side by $\boldsymbol{n}_h^{\star T}$ and the right side by $\boldsymbol{n}_h$ then (14) states the following stability condition

$$\boldsymbol{n}_h^{\star T}\boldsymbol{F}^T\boldsymbol{P}\boldsymbol{F}\boldsymbol{n}_h - m\boldsymbol{n}_h^{\star T}\boldsymbol{F}^T\boldsymbol{P}\boldsymbol{n}_h - m\boldsymbol{n}_h^{\star T}\boldsymbol{P}\boldsymbol{F}\boldsymbol{n}_h + \\ + \left(m^2 - a^2 + y^2\left(\frac{a^2}{b^2} - 1\right)\right)\boldsymbol{n}_h^{\star T}\boldsymbol{P}\boldsymbol{n}_h < 0 \tag{16}$$

and inserting (15) into (16) it must be satisfied that

$$\left(z_h z_h^\star - m(z_h + z_h^\star) + m^2 - a^2 + y^2\left(\frac{a^2}{b^2} - 1\right)\right)\boldsymbol{n}_h^{\star T}\boldsymbol{P}\boldsymbol{n}_h < 0\,. \tag{17}$$

Then, since the matrix $\boldsymbol{P}$ is positive definite, the inequality (17) brings the stability region description

$$z_h z_h^\star - m\left(z_h + z_h^\star\right) + m^2 + y^2\left(\frac{a^2}{b^2} - 1\right) < a^2, \tag{18}$$

$$(z_h^\star - m)(z_h - m) < a^2 - y^2\left(\frac{a^2}{b^2} - 1\right), \tag{19}$$

respectively, while (19) predefines (9).

For given (18) the problem is illustrated by setting $z_h = x + \mathrm{i}y$. Then the inequality (18) gives

$$-2mx + x^2 + y^2 + m^2 + y^2\left(\frac{a^2}{b^2} - 1\right) < a^2 \tag{20}$$

and, subsequently, this follows from the inequality (20)

$$(x - m)^2 + \frac{a^2}{b^2}y^2 < a^2. \tag{21}$$

Because the analytical equation of the shifted ellipse is

$$\frac{(x - m)^2}{a^2} + \frac{(y - n)^2}{b^2} < 1, \tag{22}$$

it is evident that (21) represents the ellipse equation with the center $z_o = (m + 0\mathrm{j})$ in the plane $\mathcal{Z}$, the ellipse axes are parallel to the x- and jy-axes, while a is the semi-major axe and b is the semi-minor axe of the ellipse. This concludes the proof.

□

Remark 1 *Setting $a = b$, then (21) implies*

$$(x - m)^2 + y^2 < a^2, \tag{23}$$

that means the eigenvalues of $\boldsymbol{F}$ are inside the circle with radius a and the origin $m = m + 0i$ in the plane $\mathcal{Z}$.

The following results show the central importance of this description.

Definition 1 *(LMI region) A subset $\mathcal{D}$ of the complex plane $\mathcal{Z}$ is the stable LMI region if*

$$\mathcal{D} = \{z \in \mathcal{Z} : \; f_{\mathcal{D}}(z) < 0\}, \tag{24}$$

where $f_{\mathcal{D}}(z)$ is the region characteristic function.

Theorem 2 *The matrix $\boldsymbol{F} \in \mathbb{R}^{n \times n}$ is $\mathcal{D}$-stable if and only if for given positive scalars $a, b, m, y \in \mathbb{R}$, $y \leqslant b < a$, $m + a < 1$, there exists a symmetric positive definite matrix $\boldsymbol{Q} \in \mathbb{R}^{n \times n}$ such that*

$$\boldsymbol{Q} = \boldsymbol{Q}^T > 0, \tag{25}$$

$$\boldsymbol{M}_{\mathcal{D}}(\boldsymbol{F}, \boldsymbol{Q}) = \begin{bmatrix} -a\boldsymbol{Q} & * & * \\ (\boldsymbol{F} - m\boldsymbol{I})\boldsymbol{Q} & -a\boldsymbol{Q} & * \\ c\boldsymbol{Q} & \boldsymbol{0} & -a\boldsymbol{Q} \end{bmatrix} < 0, \tag{26}$$

where

$$c = y\sqrt{\frac{a^2}{b^2} - 1}\,. \tag{27}$$

Marking the LMI region by the characteristic function

$$f_{\mathcal{D}}(z) = \begin{bmatrix} -a & z^{\star} - m & c \\ z - m & -a & 0 \\ c & 0 & -a \end{bmatrix}, \tag{28}$$

then $\boldsymbol{M}_{\mathcal{D}}(\boldsymbol{F}, \boldsymbol{Q})$ and $f_{\mathcal{D}}(z)$ are related by the substitution

$$(\boldsymbol{Q}, \boldsymbol{F}\boldsymbol{Q}, \boldsymbol{Q}\boldsymbol{F}^T) \longleftrightarrow (1, z, z^{\star})\,. \tag{29}$$

Hereafter, $*$ denotes the symmetric item in a symmetric matrix.

Proof Since $a > 0$, multiplying by a then (13) implies

$$a^{-1}(\boldsymbol{F} - m\boldsymbol{I})^T \boldsymbol{P}(\boldsymbol{F} - m\boldsymbol{I}) - a\boldsymbol{P} + \frac{y^2}{a}\left(\frac{a^2}{b^2} - 1\right)\boldsymbol{P} < 0\,. \tag{30}$$

Using the Schur complement property, (30) can be written as

$$\begin{bmatrix} -a\boldsymbol{P} + \dfrac{y^2}{a}\left(\dfrac{a^2}{b^2} - 1\right)\boldsymbol{P} & (\boldsymbol{F} - m\boldsymbol{I})^T \\ \boldsymbol{F} - m\boldsymbol{I} & -a\boldsymbol{P}^{-1} \end{bmatrix} < 0\,. \tag{31}$$

Thus, defining the transform matrix

$$\boldsymbol{T} = [\boldsymbol{Q} \ \ \boldsymbol{I}], \quad \boldsymbol{Q} = \boldsymbol{P}^{-1} \tag{32}$$

and pre-multiplying the left side and post-multiplying the right side by the above defined $\boldsymbol{T}$, then (31) gives

$$\begin{bmatrix} -a\boldsymbol{Q} + \dfrac{y^2}{a}\left(\dfrac{a^2}{b^2} - 1\right)\boldsymbol{Q} & \boldsymbol{Q}(\boldsymbol{F} - m\boldsymbol{I})^T \\ (\boldsymbol{F} - m\boldsymbol{I})\boldsymbol{Q} & -a\boldsymbol{Q} \end{bmatrix} < 0. \tag{33}$$

Since it yields

$$\begin{bmatrix} \dfrac{y^2}{a}\left(\dfrac{a^2}{b^2} - 1\right)\boldsymbol{Q} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} = \begin{bmatrix} y\sqrt{\dfrac{a^2}{b^2} - 1} \\ \boldsymbol{0} \end{bmatrix} a^{-1}\boldsymbol{Q}\boldsymbol{Q}^{-1}\boldsymbol{Q}\left[y\sqrt{\dfrac{a^2}{b^2} - 1} \ \ \boldsymbol{0} \right], \tag{34}$$

using the Schur complement property then (33), (34) implies (26). This concludes the proof. □

The results of the previous theorem yield a new representation for D-stability region.

Remark 2 *In the limit case where $y = b$ the D-stability region is strictly given by the area inside the ellipse. In the last case*

$$\boldsymbol{M}_{\mathcal{D}}(\boldsymbol{F}, \boldsymbol{Q}) = \begin{bmatrix} -a\boldsymbol{Q} & * & * \\ (\boldsymbol{F} - m\boldsymbol{I})\boldsymbol{Q} & -a\boldsymbol{Q} & * \\ c_o\boldsymbol{Q} & \boldsymbol{0} & -a\boldsymbol{Q} \end{bmatrix} < 0, \tag{35}$$

where

$$c_o = \sqrt{a^2 - b^2}. \tag{36}$$

Corollary 1 *Considering $a = b$ then (26) takes the form*

$$\boldsymbol{M}_{\mathcal{D}}(\boldsymbol{F}, \boldsymbol{Q}) = \begin{bmatrix} -a\boldsymbol{Q} & \boldsymbol{Q}(\boldsymbol{F} - m\boldsymbol{I})^T & \boldsymbol{0} \\ (\boldsymbol{F} - m\boldsymbol{I})\boldsymbol{Q} & -a\boldsymbol{Q} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & -a\boldsymbol{Q} \end{bmatrix} < 0 \tag{37}$$

and using the Schur complement property, then (37) can be reduced as

$$\boldsymbol{M}_{\mathcal{D}}(\boldsymbol{F}, \boldsymbol{Q}) = \begin{bmatrix} -a\boldsymbol{Q} & \boldsymbol{Q}(\boldsymbol{F} - m\boldsymbol{I})^T \\ (\boldsymbol{F} - m\boldsymbol{I})\boldsymbol{Q} & -a\boldsymbol{Q} \end{bmatrix} < 0, \tag{38}$$

while, consequently, the characteristic function takes the form

$$f_{\mathcal{D}}(z) = \begin{bmatrix} -a & z^* - m \\ z - m & -a \end{bmatrix}, \tag{39}$$

where $\boldsymbol{M}_{\mathcal{D}}(\boldsymbol{F},\boldsymbol{Q})$ *and* $f_{\mathcal{D}}(z)$ *are tied by the relation (29). Evidently, (38), (39) define D-circle stability region [5, 17].*

If $a = 1$, $m = 0$, *the relations (38), (39) are reduced as*

$$\boldsymbol{M}_{\mathcal{D}}(\boldsymbol{F},\boldsymbol{Q}) = \begin{bmatrix} -\boldsymbol{Q} & \boldsymbol{Q}\boldsymbol{F}^T \\ \boldsymbol{F}\boldsymbol{Q} & -\boldsymbol{Q} \end{bmatrix} < 0, \tag{40}$$

$$f_{\mathcal{D}}(z) = \begin{bmatrix} -a & z^* \\ z & -a \end{bmatrix} \tag{41}$$

and it corresponds to the entire stability area with the open unit disc located in the complex $\mathcal{Z}$ *plane.*

In the theory of linear systems with polytopic uncertainties the slack matrix approach is preferred, which leads to enhanced forms of the stability area description.

Theorem 3 *The matrix* $\boldsymbol{F} \in \mathbb{R}^{n \times n}$ *is* $\mathcal{D}$*-stable if and only if for given positive scalars* $m, a, b, y \in \mathbb{R}$, $y \leqslant b < a$, $m + a < 1$, *there exists symmetric positive definite matrices* $\boldsymbol{R}, \boldsymbol{S} \in \mathbb{R}^{n \times n}$ *such that*

$$\boldsymbol{S} = \boldsymbol{S}^T > 0, \qquad \boldsymbol{R} = \boldsymbol{R}^T > 0, \tag{42}$$

$$\begin{bmatrix} -a^2\boldsymbol{S} & * & * \\ a(\boldsymbol{F} - m\boldsymbol{I})\boldsymbol{R} & \boldsymbol{S} - 2a\boldsymbol{R} & * \\ c\boldsymbol{S} & \boldsymbol{0} & -\boldsymbol{S} \end{bmatrix} < 0. \tag{43}$$

The region characteristic function is

$$f_{\mathcal{D}}(z) = f_{\mathcal{D}S}(z) + f_{\mathcal{D}R}(z) < 0, \tag{44}$$

where

$$f_{\mathcal{D}S}(z) = \begin{bmatrix} -a^2 & 0 & c \\ 0 & 1 & 0 \\ c & 0 & -1 \end{bmatrix}, \quad c = y\sqrt{\frac{a^2}{b^2} - 1}, \quad (\boldsymbol{S}) \longleftrightarrow (1), \tag{45}$$

$$f_{\mathcal{D}R}(z) \begin{bmatrix} 0 & az^\star - am & 0 \\ az - am & -2a & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad (\boldsymbol{R}, \boldsymbol{F}\boldsymbol{R}, \boldsymbol{R}\boldsymbol{F}^T) \longleftrightarrow (1, z, z^\star). \tag{46}$$

Proof Since the difference equation (7) can be written as

$$\boldsymbol{F}\boldsymbol{p}(i) - a\boldsymbol{p}(i+1) = \boldsymbol{0}, \tag{47}$$

with a positive definite symmetric matrix $\boldsymbol{U} \in \mathbb{R}^{n \times n}$, it yields

$$\boldsymbol{p}^T(i+1)\boldsymbol{U}(\boldsymbol{F}\boldsymbol{p}(i) - a\boldsymbol{p}(i+1)) = \boldsymbol{0}. \tag{48}$$

Therefore, adding of (48) and its transposition to (12) gives

$$\Delta v(\boldsymbol{p}(i)) = \boldsymbol{p}^T(i+1)\boldsymbol{P}\boldsymbol{p}(i+1) - \boldsymbol{p}^T(i)\boldsymbol{P}\boldsymbol{p}(i) + \frac{y^2}{a^2}\left(\frac{a^2}{b^2} - 1\right)\boldsymbol{p}^T(i)\boldsymbol{P}\boldsymbol{p}(i) +$$

$$+ (\boldsymbol{F}\boldsymbol{p}(i) - a\boldsymbol{p}(i+1))^T\boldsymbol{U}\boldsymbol{p}(i+1) + \boldsymbol{p}^T(i+1)\boldsymbol{U}(\boldsymbol{F}\boldsymbol{p}(i) - a\boldsymbol{p}(i+1)) < 0. \tag{49}$$

Using the following notation

$$\boldsymbol{p}^{\diamond T}(i) = \left[\boldsymbol{p}^T(i) \;\; \boldsymbol{p}^T(i+1)\right], \tag{50}$$

the difference of the Lyapunov function can be rewritten as

$$\Delta v(\boldsymbol{p}^{\diamond}(i)) = \boldsymbol{p}^{\diamond T}(i)\boldsymbol{J}^{\diamond}\boldsymbol{p}^{\diamond}(i) < 0, \tag{51}$$

where

$$\boldsymbol{J}^{\diamond} = \begin{bmatrix} -\boldsymbol{P} + \frac{y^2}{a^2}\left(\frac{a^2}{b^2} - 1\right)\boldsymbol{P} & (\boldsymbol{F} - a\boldsymbol{I})^T\boldsymbol{U} \\ \boldsymbol{U}(\boldsymbol{F} - a\boldsymbol{I}) & \boldsymbol{P} - 2a\boldsymbol{U} \end{bmatrix} < 0. \tag{52}$$

Then, using (34), the inequality (52) can be rewritten as follows

$$\begin{bmatrix} -\boldsymbol{P} & (\boldsymbol{F} - m\boldsymbol{I})^T\boldsymbol{U} & \frac{y}{a}\sqrt{\frac{a^2}{b^2} - 1}\boldsymbol{P} \\ \boldsymbol{U}(\boldsymbol{F} - m\boldsymbol{I}) & \boldsymbol{P} - 2a\boldsymbol{U} & \boldsymbol{0} \\ \frac{y}{a}\sqrt{\frac{a^2}{b^2} - 1}\boldsymbol{P} & \boldsymbol{0} & -\boldsymbol{P} \end{bmatrix} < 0. \tag{53}$$

Defining the transform matrix

$$\boldsymbol{T}^{\diamond} = [a\boldsymbol{R} \;\; \boldsymbol{R} \;\; \boldsymbol{R}], \quad \boldsymbol{R} = \boldsymbol{U}^{-1} \tag{54}$$

and pre-multiplying the left side and post-multiplying the right side by $\boldsymbol{T}^{\diamond}$ then (53) implies

$$\begin{bmatrix} -a^2\boldsymbol{R}\boldsymbol{P}\boldsymbol{R} & a\boldsymbol{R}(\boldsymbol{F} - m\boldsymbol{I})^T & y\sqrt{\frac{a^2}{b^2} - 1}\boldsymbol{R}\boldsymbol{P}\boldsymbol{R} \\ a(\boldsymbol{F} - m\boldsymbol{I})\boldsymbol{R} & \boldsymbol{R}\boldsymbol{P}\boldsymbol{R} - 2a\boldsymbol{R} & \boldsymbol{0} \\ y\sqrt{\frac{a^2}{b^2} - 1}\boldsymbol{R}\boldsymbol{P}\boldsymbol{R} & \boldsymbol{0} & -\boldsymbol{R}\boldsymbol{P}\boldsymbol{R} \end{bmatrix} < 0. \tag{55}$$

Denoting

$$S = RPR, \tag{56}$$

then (55) implies (43) with the same parameter c as defined above. This concludes the proof. □

The following relations are also immediate in view of (43), (44).

Corollary 2 *Considering $a = b$ then (43) implies*

$$\begin{bmatrix} -a^2 S & * \\ a(F - mI)R & S - 2aR \end{bmatrix} < 0, \tag{57}$$

which represents the enhanced D-circle stability region, and

$$f_{\mathcal{D}}(z) = f_{\mathcal{D}S}(z) + f_{\mathcal{D}R}(z) < 0, \tag{58}$$

where

$$f_{\mathcal{D}S}(z) = \begin{bmatrix} -a^2 & 0 \\ 0 & 1 \end{bmatrix}, \quad S \longleftrightarrow 1, \tag{59}$$

$$f_{\mathcal{D}R}(z) \begin{bmatrix} 0 & az^\star - am \\ az - am & -2a \end{bmatrix}, \quad (R, FR, RF^T) \longleftrightarrow (1, z, z^\star). \tag{60}$$

In addition, if $a = 1$, $m = 0$, the enhanced full stability region is obtained, where

$$\begin{bmatrix} -S & * \\ FR & S - 2R \end{bmatrix} < 0 \tag{61}$$

and

$$f_{\mathcal{D}}(z) = f_{\mathcal{D}S}(z) + f_{\mathcal{D}R}(z) < 0, \tag{62}$$

$$f_{\mathcal{D}S}(z) = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}, \quad S \longleftrightarrow 1, \tag{63}$$

$$f_{\mathcal{D}R}(z) \begin{bmatrix} 0 & z^\star \\ z & -2 \end{bmatrix}, \quad (R, FR, RF^T) \longleftrightarrow (1, z, z^\star). \tag{64}$$

4. Control Law Parameter Design

Considering the control problem to find an r-dimensional vector $u(i)$ in the relation given in (3), the corresponding solutions are presented.

Theorem 4 *The system (1) under influence of the control (3) is stable if for given positive scalars $m, a, b, y \in \mathbb{R}$, $y \leqslant b < a$, $m + a < 1$, there exist a symmetric positive definite matrix $\boldsymbol{Q} \in \mathbb{R}^{n\times n}$ and a matrix $\boldsymbol{Y} \in \mathbb{R}^{r\times n}$ such that*

$$\boldsymbol{Q} = \boldsymbol{Q}^T > 0, \tag{65}$$

$$\begin{bmatrix} -a\boldsymbol{Q} & * & * \\ \boldsymbol{F}\boldsymbol{Q} - \boldsymbol{G}\boldsymbol{Y} - m\boldsymbol{Q} & -a\boldsymbol{Q} & * \\ c\boldsymbol{Q} & \boldsymbol{0} & -a\boldsymbol{Q} \end{bmatrix} < 0, \quad c = y\sqrt{\frac{a^2}{b^2} - 1}. \tag{66}$$

When the above conditions hold, the control law gain matrix can be found as

$$\boldsymbol{K} = \boldsymbol{Y}\boldsymbol{Q}^{-1}. \tag{67}$$

Proof Inserting (6) then the inequality (26) gives

$$\begin{bmatrix} -a\boldsymbol{Q} & * & * \\ (\boldsymbol{F} - \boldsymbol{G}\boldsymbol{K} - m\boldsymbol{I})\boldsymbol{Q} & -a\boldsymbol{Q} & * \\ c\boldsymbol{Q} & \boldsymbol{0} & -a\boldsymbol{Q} \end{bmatrix} < 0 \tag{68}$$

and denoting

$$\boldsymbol{Y} = \boldsymbol{K}\boldsymbol{Q} \tag{69}$$

(68) implies (66). This concludes the proof. □

Theorem 5 *The system (1) under influence of the control (3) is stable if for given positive scalars $m, a, b, y \in \mathbb{R}$, $y \leqslant b < a$, $m + a < 1$, there exist symmetric positive definite matrices $\boldsymbol{R}, \boldsymbol{S} \in \mathbb{R}^{n\times n}$ and a matrix $\boldsymbol{Z} \in \mathbb{R}^{r\times n}$ such that*

$$\boldsymbol{R} = \boldsymbol{R}^T > 0, \qquad \boldsymbol{S} = \boldsymbol{S}^T > 0 \tag{70}$$

$$\begin{bmatrix} -a^2\boldsymbol{S} & * & * \\ a\boldsymbol{F}\boldsymbol{R} - a\boldsymbol{G}\boldsymbol{Z} - am\boldsymbol{R} & \boldsymbol{S} - 2a\boldsymbol{R} & * \\ c\boldsymbol{S} & \boldsymbol{0} & -\boldsymbol{S} \end{bmatrix} < 0, \quad c = y\sqrt{\frac{a^2}{b^2} - 1}. \tag{71}$$

When the above conditions hold, the control law gain matrix is given by the relation

$$\boldsymbol{K} = \boldsymbol{Z}\boldsymbol{R}^{-1} \tag{72}$$

Proof Inserting (6) then (43) gives

$$\begin{bmatrix} -a^2\boldsymbol{S} & * & * \\ a((\boldsymbol{F} - \boldsymbol{G}\boldsymbol{K} - m\boldsymbol{I})\boldsymbol{R} & \boldsymbol{S} - 2a\boldsymbol{R} & * \\ c\boldsymbol{S} & \boldsymbol{0} & -\boldsymbol{S} \end{bmatrix} < 0 \tag{73}$$

and denoting

$$\boldsymbol{Z} = \boldsymbol{K}\boldsymbol{R} \tag{74}$$

then (73) implies (71). This concludes the proof. □

Remark 3 *Since the deductions and proofs are completely routine, the standard synthesis conditions resulting from inequalities (38), (40) or (57), (61) are left to the readers.*

5. Uncertain Discrete-Time Systems

The importance of Theorem 5 is that it separates the Lyapunov matrix $\boldsymbol{S}$ from $\boldsymbol{F}$ and $\boldsymbol{G}$. This enables to derive design conditions for a system with polytopic uncertainties.

Assuming that the matrices $\boldsymbol{F}$, $\boldsymbol{G}$ of (1) are not precisely known but belong to a polytopic uncertainty domain

$$\mathcal{O} := \left\{ \boldsymbol{d} \in \mathcal{Q}, (\boldsymbol{F}, \boldsymbol{G})(\boldsymbol{d}) : (\boldsymbol{F}, \boldsymbol{G})(\boldsymbol{d}) = \sum_{l=1}^{s} d_l (\boldsymbol{F}_l, \boldsymbol{G}_l) \right\}, \tag{75}$$

$$\mathcal{Q} = \left\{ (d_1, \cdots, d_s) : \sum_{l=1}^{s} d_l = 1;\ d_l > 0,\ i = 1, 2, \ldots, s \right\}, \tag{76}$$

where $\mathcal{Q}$ is the unit simplex, $\boldsymbol{F}_l$ and $\boldsymbol{G}_l$ are constant matrices with appropriate dimensions, and d_l, $l = 1, 2, \ldots, s$, are time-invariant uncertainties.

Since $\boldsymbol{d}$ is constrained to the unit simplex as (76), the matrices $(\boldsymbol{F}, \boldsymbol{G})(\boldsymbol{d})$ are affine functions of the uncertain parameter vector $\boldsymbol{d} \in \mathbb{R}^n$, described by the convex combination of the vertex matrices $(\boldsymbol{F}_l, \boldsymbol{G}_l)$, $l = 1, 2, \ldots, s$.

The following theorem and corollary describe the control design conditions for uncertain linear discrete-time systems.

Theorem 6 *The uncertain system (75) under influence of the control (3) is stable if for given positive scalars $m, a, b, y \in \mathbb{R}$, $y \leqslant b < a$, $m + a < 1$, there exist symmetric positive definite matrices $\boldsymbol{R}, \boldsymbol{S} \in \mathbb{R}^{n \times n}$ and a matrix $\boldsymbol{Z} \in \mathbb{R}^{r \times n}$ such that for $l = 1, 2, \ldots, s$*

$$\boldsymbol{R} = \boldsymbol{R}^T > 0, \qquad \boldsymbol{S} = \boldsymbol{S}^T > 0, \tag{77}$$

$$\begin{bmatrix} -a^2\boldsymbol{S} & * & * \\ a\boldsymbol{F}_l\boldsymbol{R} - a\boldsymbol{G}_l\boldsymbol{Z} - am\boldsymbol{R} & \boldsymbol{S} - 2a\boldsymbol{R} & * \\ c\boldsymbol{S} & \boldsymbol{0} & -\boldsymbol{S} \end{bmatrix} < 0, \qquad c = y\sqrt{\frac{a^2}{b^2} - 1}\,. \tag{78}$$

If the existence is affirmative, the control law gain matrix can be found using (74).

Proof To solve this problem it can rely on the expression where the difference equation (7) takes the form

$$a\boldsymbol{p}(i+1) = \sum_{l=1}^{s} d_l (\boldsymbol{F}_l - m\boldsymbol{I})\boldsymbol{p}(i) \tag{79}$$

and, considering that $\sum_{l=1}^{s} d_l = 1$, an alternative form of (79) is

$$\sum_{l=1}^{s} d_l\, \boldsymbol{p}(i+1) = \sum_{l=1}^{s} d_l (\boldsymbol{F}_l - m\boldsymbol{I})\boldsymbol{p}(i)\,. \tag{80}$$

Observing that (80) takes on the suggestive form

$$\sum_{l=1}^{s} d_l \left((\boldsymbol{F}_l - m\boldsymbol{I})\boldsymbol{p}(i) - \boldsymbol{p}(i+1)\right) = \boldsymbol{0} \tag{81}$$

and using a positive definite matrix $\boldsymbol{U} \in \mathbb{R}^{n\times n}$, it can get

$$\sum_{l=1}^{s} d_l \boldsymbol{p}^T(i+1)\boldsymbol{U}\left((\boldsymbol{F}_l - m\boldsymbol{I})\boldsymbol{p}(i) - \boldsymbol{p}(i+1)\right) = \boldsymbol{0}\,. \tag{82}$$

Thus, following the way of proof to Theorem 3, then for $l = 1,2,\ldots,s$ the new result is

$$\begin{bmatrix} -a^2\boldsymbol{S} & * & * \\ a(\boldsymbol{F}_l - m\boldsymbol{I})\boldsymbol{R} & \boldsymbol{S} - 2a\boldsymbol{R} & * \\ c\boldsymbol{S} & \boldsymbol{0} & -\boldsymbol{S} \end{bmatrix} < 0, \qquad c = y\sqrt{\frac{a^2}{b^2} - 1}\,. \tag{83}$$

Plugging $\boldsymbol{F}_l$ by $\boldsymbol{F}_{cl} = \boldsymbol{F}_l - \boldsymbol{G}_l\boldsymbol{K}$ then (83) can be rewritten as

$$\begin{bmatrix} -a^2\boldsymbol{S} & * & * \\ a(\boldsymbol{F}_l - \boldsymbol{G}_l\boldsymbol{K} - m\boldsymbol{I})\boldsymbol{R} & \boldsymbol{S} - 2a\boldsymbol{R} & * \\ c\boldsymbol{S} & \boldsymbol{0} & -\boldsymbol{S} \end{bmatrix} < 0 \tag{84}$$

and with the notation (74) then (84) implies (78). This concludes the proof. □

Corollary 3 *Considering $a = b$ then (77), (78) imply*

$$\boldsymbol{R} = \boldsymbol{R}^T > 0, \qquad \boldsymbol{S} = \boldsymbol{S}^T > 0, \tag{85}$$

$$\begin{bmatrix} -a^2\boldsymbol{S} & * \\ a\boldsymbol{F}_l\boldsymbol{R} - a\boldsymbol{G}_l\boldsymbol{Z} - am\boldsymbol{R} & \boldsymbol{S} - 2a\boldsymbol{R} \end{bmatrix} < 0 \tag{86}$$

and the uncertain system (75) under influence of the control (3) is stable if for given positive scalars $m, a, b, y \in \mathbb{R}$, $y \leqslant b < a$, $m + a < 1$, there exist symmetric positive definite matrices $\boldsymbol{R}, \boldsymbol{S} \in \mathbb{R}^{n\times n}$ and a matrix $\boldsymbol{Z} \in \mathbb{R}^{r\times n}$ such that for $l = 1,2,\ldots,s$ the inequalities (85), (86) are satisfied.

In addition, if $a = 1$, $m = 0$, then (85), (86) are reduced to

$$\boldsymbol{R} = \boldsymbol{R}^T > 0, \qquad \boldsymbol{S} = \boldsymbol{S}^T > 0, \tag{87}$$

$$\begin{bmatrix} -\boldsymbol{S} & * \\ \boldsymbol{F}_l\boldsymbol{R}-\boldsymbol{G}_l\boldsymbol{Z} & \boldsymbol{S}-2a\boldsymbol{R} \end{bmatrix} < 0 \tag{88}$$

and the uncertain system (75) under influence of the control (3) is stable if for given positive scalars $m, a, b, y \in \mathbb{R}$, $y \leqslant b < a$, $m + a < 1$, there exist symmetric positive definite matrices $\boldsymbol{R}, \boldsymbol{S} \in \mathbb{R}^{n\times n}$ and a matrix $\boldsymbol{Z} \in \mathbb{R}^{r\times n}$ such that for $l = 1, 2, \ldots, s$ the inequalities (87), (88) are satisfied.

In both cases, the solution is given by the relationship (74).

6. Illustrative Example

The proposed $\mathcal{D}$-pole assignment design approach was applied to the system described by (1), (2) with the following matrix parameters

$$\boldsymbol{F} = \begin{bmatrix} 0.9993 & 0.0987 & 0.0042 \\ -0.0212 & 0.9612 & 0.0775 \\ -0.3875 & -0.7187 & 0.5737 \end{bmatrix}, \quad \boldsymbol{G} = \begin{bmatrix} 0.0051 & 0.0050 \\ 0.1029 & 0.0987 \\ 0.0387 & -0.0388 \end{bmatrix}, \quad \boldsymbol{C}^T = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix},$$

with the sampling period $t_s = 0.2$ s.

To illustrate the results, the $\mathcal{D}$-stability circle region is the disk with the radius $a = 0.3$ and the center $m = 0.5$ on the real axe of the complex plane $\mathcal{Z}$, and the $\mathcal{D}$-stability ellipse region is defined by the ellipse center $m = 0.5$ on the real axe of the complex plane $\mathcal{Z}$, the semi-major axe of the ellipse $a = 0.3$, the semi-minor axe of the ellipse $b = 0.1$ and the tuning parameter $y = 0.95\,b$. The LMI-based synthesis is performed for the design conditions (65), (66) and for he design conditions derived from (38) with a positive definite matrix $\boldsymbol{Q}$.

Solving (65), (66) with respect to the matrix variables $\boldsymbol{R}$, $\boldsymbol{Z}$, using Self–Dual–Minimization (SeDuMi)package for Matlab [21], the design task is feasible with the LMI matrix variables

$$\boldsymbol{Q} = \begin{bmatrix} 0.0066 & -0.0372 & -0.0121 \\ -0.0372 & 0.2372 & 0.0662 \\ -0.0121 & 0.0662 & 0.9512 \end{bmatrix}, \qquad \boldsymbol{Y} = \begin{bmatrix} 0.1482 & -0.8654 & 0.7037 \\ -0.3160 & 2.0288 & 0.2749 \end{bmatrix}.$$

The corresponding feedback gain matrix $\boldsymbol{K}$, closed-loop system matrix $\boldsymbol{F}_c$ as well as the eigenvalue spectrum of the closed-loop system matrix $\sigma(\boldsymbol{F}_c)$ are

$$\boldsymbol{K} = \begin{bmatrix} 19.3925 & -0.8992 & 1.0486 \\ 2.2349 & 8.9884 & -0.3081 \end{bmatrix}, \qquad \boldsymbol{F}_c = \begin{bmatrix} 0.8892 & 0.0583 & 0.0004 \\ -2.2373 & 0.1666 & 0.0000 \\ -1.0513 & -0.3352 & 0.5212 \end{bmatrix},$$

$$\sigma(\boldsymbol{F}_c) = \{0.5760,\ 0.5005 \pm 0.0477\,\mathrm{i}\}\,.$$

Solving the design task with the $\mathcal{D}$-stability circle region, it is found that

$$\boldsymbol{Q} = \begin{bmatrix} 0.0431 & -0.1316 & 0.0393 \\ -0.1316 & 0.6060 & -0.1496 \\ 0.0393 & -0.1496 & 1.9861 \end{bmatrix}, \qquad \boldsymbol{Y} = \begin{bmatrix} 0.3019 & -0.6942 & 0.8197 \\ -0.5523 & 2.8147 & 0.0864 \end{bmatrix}.$$

and the control law gain matrix $\boldsymbol{K}$, closed-loop system matrix $\boldsymbol{F}_c$ and the eigenvalue spectrum of the closed-loop system matrix $\sigma(\boldsymbol{F}_c)$ are computed as

$$\boldsymbol{K} = \begin{bmatrix} 10.2476 & 1.1530 & 0.2967 \\ 3.8646 & 5.5793 & 0.3873 \end{bmatrix}, \qquad \boldsymbol{F}_c = \begin{bmatrix} 0.9277 & 0.0649 & 0.0008 \\ -1.4571 & 0.2919 & 0.0088 \\ -0.6341 & -0.5468 & 0.5772 \end{bmatrix},$$

$$\sigma(\boldsymbol{F}_c) = \{0.7249,\ 0.5360 \pm 0.1017\,\mathrm{i}\}.$$

Evidently, all solutions obtained reflect the associated $\mathcal{D}$-stability region constraints.

Working with nonzero system outputs, the forced mode feedback control

$$\boldsymbol{u}(i) = -\boldsymbol{K}\boldsymbol{q}(i) + \boldsymbol{W}\boldsymbol{w}_o$$

is applied in simulation to force the desired steady-state of the system output, where $\boldsymbol{W}$ is the signal gain matrix and $\boldsymbol{w}_o$ is desired output values vector. Constructing the signal gain matrices as [16]

$$\boldsymbol{W} = (\boldsymbol{C}(\boldsymbol{I}_n - (\boldsymbol{F} - \boldsymbol{G}\boldsymbol{K})^{-1}\boldsymbol{G})^{-1},$$

then, with the associated signal gain matrices

$$\boldsymbol{W}_\diamond = \begin{bmatrix} 6.0206 & 18.3539 \\ -6.3182 & 3.5469 \end{bmatrix}, \quad \boldsymbol{W}_\circ = \begin{bmatrix} 5.2990 & 9.9504 \\ -5.6356 & 4.4855 \end{bmatrix},$$

the system output steady-state vector $\boldsymbol{w}_o^T = [0.5\ 1.0]$ is forced in the simulations, while the initial conditions stay as $\boldsymbol{q}(0) = \boldsymbol{0}$.

The time profiles of the resulting system states variables and system output variables are illustrated in Fig. 1 and Fig. 2, where the control law is designed using the LMI conditions implying from the prescribed $\mathcal{D}$-stability ellipse region. The simulation results, reflecting the LMI control design conditions implying from the prescribed $\mathcal{D}$-stability circle region are presented in Fig. 3 and Fig. 4.

Although using the same center value in definition of both stability regions and the circle radius equal to the semi-major axe of the ellipse, it is evident that the dynamics, setting time of the responses and the overshoots in the closed-loop structure with the control law designed for the circular area of stability are positively worse than those in the closed-loop structure designed using the ellipse-like criterion.

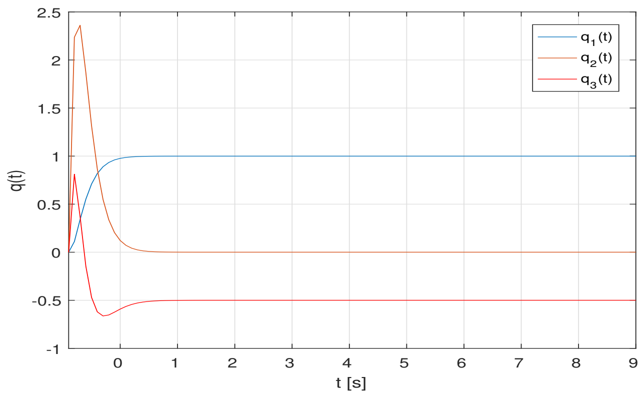

Figure 1: State variables response

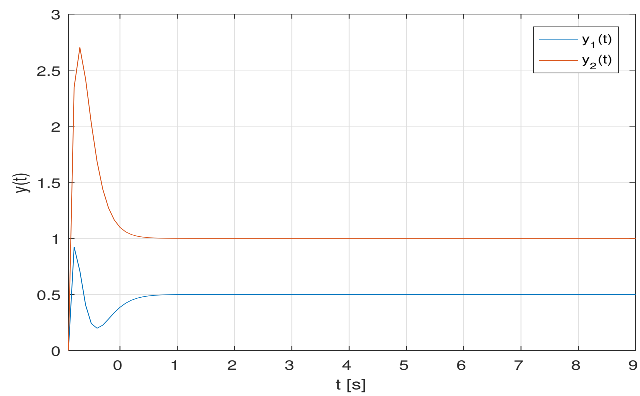

Figure 2: Output variables response

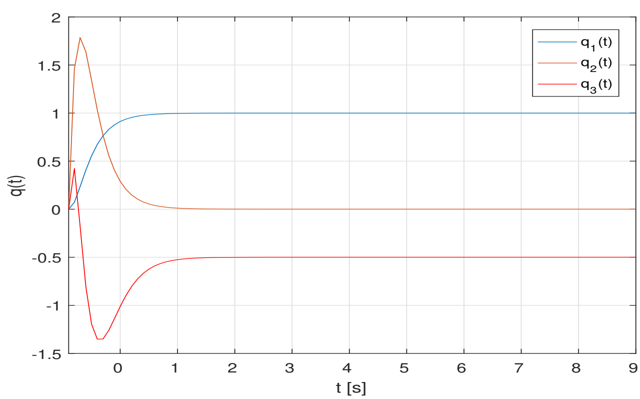

Figure 3: State variables response

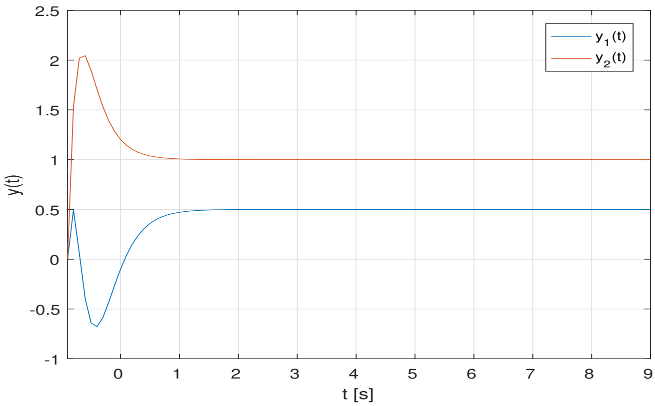

Figure 4: Output variables response

7. Concluding Remarks

In this paper, new sufficient conditions for the discrete-time linear systems to be $\mathcal{D}$-stable are obtained. These conditions are given in terms of strict LMIs, based on which, the control law gain matrices are given in explicit expressions. The proposed approach involves allows to include in the resolution further additive quadratic performance constraints but, in general, not involving state variables of the system. Presented numerical example adequately illustrates the efficiency of the approach.

Note, here the algorithms have been derived under the assumption that the system matrix $\boldsymbol{A}$ has no multiple eigenvalues. This assumption is for convenience only, and the given algorithms hold without this preliminary condition. This is a fundamental generalization of the methodology, while the basic structure of the algorithms had already been presented in [4, 5] and [17].

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