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SERVICE LEVEL IN MODEL OF INVENTORY LOCATION WITH STOCHASTIC DEMAND

Milena Bieniek
Maria Curie-Sklodowska University, Statistics and Econometrics Department, Lublin, Poland
e-mail: milena.bieniek@umcs.lublin.pl

Abstract: *In this paper we consider the influence of the safety factor on the decision of the inventory location. This decision is done based on the model which centralizes or decentralizes the safety stock. In this model we have to choose between the location of the inventory in the regional warehouses or the location in the central warehouse. The decision is made due to the minimizing the holding costs and the supply costs of the safety stock from the central warehouse to the customer. The main assumption is that the customers have the stochastic demands on the inventory items. Moreover, the customers' demands have the known distribution with the known parameters. The complex analysis of the influences of possible probabilistic demands' distributions on the safety factors is conducted. The numerical computations for the safety factors used in the facility location model are also presented. In numerical examples we take into considerations the demands' distributions the most often used in practice like the normal, the Poisson, the Gamma and the exponential distribution. Some graphs for the safety factors of these distributions are also drawn. Moreover for the mentioned demands' distributions the model of the safety stock location depends on the specific factors. Among other things these factors are the mean and the variance of the demand, the number of the regional warehouses, the assumed service level, and some cost factors like the holding costs and the transportation costs. Some graphs which illustrated the dependence of the model elements on some listed before parameters are presented and their influence on the location decision is studied also.*

Key words: *inventory location, stochastic demand, safety stock, service level, safety factor*

1. Introduction

The key of an appropriate strategy of a firm is a good inventory management. In the inventory control they have to choose the best strategy. Here "the best" means "the best in a particular sense". Some companies have the cheaper products and some fulfil the orders very quickly. But all of them want to satisfy all of the customers' requirements. The firms want to make the orders on the right time in the right place. To satisfy this aim the company has to keep the sufficient amount of the inventory. The significant role of holding the inventory is keeping the appropriate service level and also the protection of the results of the unexpected events like wars, disasters, hesitation of rating expense, demand and supply.

Mainly, the inventory management focuses on the uncertainties in the demands of products and also of the sufficient service levels. The demand can be treated as a random variable with a known distribution, with a known or an unknown parameters, or an unknown distribution. Another way to describe the stochastic demand is choosing one scenario from a set of a few of them. Each scenario happens with a specific probability. In such case only limited values of the demand can be considered. Hence there are very small number of possibilities of the values of the demand.

The problem of the inventory management is strictly connected also with the problem where to locate the stock. This is a type of a facility location problem, sometimes with the stochastic demands. The decision of the location of the inventory is mainly based on minimization of the total costs (see [13]). The cost taken into account can be the holding costs, the transportation costs, the service costs and so on. In [10] the scenario approach to the inventory location with the stochastic demand is considered.

While the paper considering the demands with some specific distributions is for instance [1]. In the last mentioned paper the uncertain customers' demands have the Bernoulli distribution. The parameter of this distribution is known in advance. The problem is to decide where to locate the facilities and how to assigned the customers to the operating facilities. The facilities can be also treated as the warehouses. The decision of the location is also based on the minimization of the overall costs here. Moreover, in [7] the authors study robust optimization for the facility location problem with the stochastic demands.

The distribution of the demand must have the specific features to describe the real demand properly. The possible distributions of the demand is widely discussed in [6]. The most often used demand probability distributions satisfied the given conditions are the normal, the Poisson and the Gamma distribution. In particular the exponential distribution can be also used to model the random demand in the inventory management.

The assumed service level is influenced much by the uncertain demand. A very important aim of the strategy of a firm is to keep the high service level. There are some measures of the service level. In one of them the measure of the service level influences the amount of the safety stock. Using the service level we can calculate the safety factor and using the safety factor we can calculate the safety stock. The mentioned safety stock is used to minimize the stockouts due to the uncertainties, especially in the demand.

For more information on the inventory management we refer to the book [8] or [11].

In this paper we try to show the connections between the service level, the distributions of the demands and the location of the safety stock. The computations of the safety factor is presented if the service level and the specific demand's distribution are known. We also use these safety factors in the model of the stock location with the uncertain demand proposed in [9] and extended in [4].

In the mentioned model the decision of the location of the safety stock has to be made. We can choose between the option of locating it in the central warehouse or in a few of the regional warehouses. Additionally we assume the stochastic demands environment. The regional warehouses are close to the customer so there are no costs for the supply. If the inventory is placed in the central warehouse some transportation costs have to be paid. The distribution of the demand is given by the cumulative distribution function with known parameters. In our note the first kind of the service levels is used. This service level is defined as the probability of not going out of stock. We present the formulas for the service levels in case of the the specific distributions of the demand. More precisely, the service levels for the normal, the Poisson and the Gamma distribution are studied. Also the considerations for the popular exponential distribution is given as a special case of the Gamma distribution. This is the complementation of the work [9]. The additional analysis of the service levels is presented. Some computational examples of the safety factors for the specific distributions of the demand are also given. The plots illustrated the connections of the safety factor with the specific parameters are made. We also try to show which strategy of the safety stock location choose in case of the variability of the mean of the weekly customers' demands. Furthermore the dependence of the location decision on the costs and the number of the regional warehouses is considered. To complement the considerations we make some plots which illustrate the mentioned dependence for used demands' distributions..

The paper is organized as follows. The Section 2 is devoted to the definition of the service levels. The primary notation is presented also. Moreover, the calculation of the exact formulas for the safety factors in case of the typical distributions of the demands like the normal, the Gamma and the Poisson distribution are derived also. In the Section 3 we compute the safety factors for the specific parameter of the distributions of the demand and assumed service level. We also draw the plots of the service level dependence on some parameters. The next section is devoted to the reminding of the solution of the inventory location model which was given in [4]. Moreover the graphs illustrating the computational examples considering the specific demands' distributions are presented. In the computations the various parameters like the costs, the number of the warehouses, the distribution's parameters are considered. Finally the last section concludes the paper.

2. Safety factors in general

The problem considered in this section is the calculation of the exact formulas for the safety factors for various distributions of the customers' demands. These expressions are calculated using one of the definitions of the service level. To this aim in the beginning of this section we define the service level in the ways presented in the literature. Next we give some notation which will be used later in our inventory model. Then we present the probabilistic features of the most typical distributions of the customer's demand, namely the normal, the Poisson and the Gamma distribution. Moreover, from the definition of the service level we calculate the expressions for the safety factors for the above distributions. In the end of the section we present some computational examples for the most popular demands' distributions. In these computations we assume various parameters on the demands' distributions, various service levels and so on.

The measurement of the service level can be done in many ways. The most popular measurements are the following. The first considered measure of the service level is denoted by S_1. This is understood as the probability of no stockout per order cycle. The second measure of the service level denoted by S_2 is "a fill rate" which means a fraction of the demand that can be satisfied immediately from the stock on hand. Finally, the third kind S_3 and it is called "a ready rate" and it is defined as a fraction of the time with the positive stock on hand. For more information on the service levels we refer to the book [2] and the reference therein. In this paper the first kind of the service level will be considered in our model of inventory location, precisely of the safety stock location. The service level S_1 is directly connected with the safety stock and the distribution of the lead time demand. The mathematical definition of S_1 is as follows

The service level type $S_1$is given by the equality

$$P(D_T < R) = S_1,$$

where R is the reorder point and D_T is the random variable with the cumulative distribution function $F_{D_T}(.)$, the mean μ' and the standard deviation σ'. The random variable D_T is the customer's demand in the lead time T. Equivalent definition is given by

$$F_{D_T}(R) = S_1 \tag{1}$$

where the reorder point is equal to

$$R = \mu' + k\sigma'.$$

Here k is a safety factor. The safety factor is used in the safety stock definition and depends on the distribution of the demand in the lead time. Let us denote the safety stock by SS. Then the safety stock is a multiplication of the safety factor and the standard deviation of the demand in the lead time, which can be written as

$$SS = k\sigma',$$

and it is the average stock on hand just before the arriving of the order. This kind of stock can be interpreted as an additional stock that is used as a protection against the demand variations including the uncertainties.

Now let us remind the main ideas of the mentioned model of the stock location. Obviously we are interested only in the location of the safety stock. The possibilities of the inventory locations are studied. The safety stock can be located in n regional warehouses or in the central warehouse. The customers can be served by the regional warehouses located close to them and then there is no transportation costs. Alternatively, for some payment for delivery, they can be served by the central warehouse. The main question is: which of these two locations costs less. Precisely, one have to solve the problem for which parameters centralize the safety stock is cheaper than decentralize it. The decision is based on minimization of the sum of holding costs of the safety stock and the cost of the direct transportation to the customer.

The following notation is used in our considerations. First, assume that in the following computations the random variables X_i which represent the weekly demands in the regional warehouses, are independent and identically distributed. Hence the means of these variables are equal to each other for all customers and also the standard deviations are equal to each other for every regional warehouse. Consequently the safety factors for the regional warehouses are equal to each other, too. Now let us present the main notation.

- X_i denotes the random variable, with the cumulative distribution functions $F(.)$, which means the weekly demand in the regional warehouse i. The demands X_i are the independent and identically distributed random variables and have the common distribution with the mean μ and the standard deviation σ, $i = 1, \ldots, n$;
- X_c denotes the random variable with the cumulative distribution function $F_c(.)$, the mean

μ_c and the standard deviation σ_c, X_c models the weekly demand in the central warehouse;
- T denotes the lead time in the regional warehouses;
- X_{iT} denotes the lead time demand in the regional warehouse i, $i = 1,2,\dots,n$, with the cumulative distribution function $F_{X_{iT}}(.)$ with the mean given by μ_T and the standard deviation equal to σ_T;
- Y_T denotes the lead time demand in the central warehouse with the cumulative distribution function $F_{Y_T}(.)$ with the mean μ_{Tc} and the standard deviation equal to σ_{Tc};
- ω denotes the safety factor in the regional warehouse i, $i = 1,\dots, n$;
- ω_c is used to denote the safety factor in the central warehouse;
- S_1 from now denotes the service level of the first type in the warehouses.

When determining the safety stock one should first specify a service level. Assume that the service levels in the regional warehouses and in the central warehouse have the same value. Note that the service level is given in terms of the safety factor and the parameters of the customers' demands in the lead time.

For determining the safety stocks the suitable demand model have to be chosen. First of all, the demand has to be the nonnegative random variable. It has to have some other features. The most often used demand probability distributions are the normal, the Poisson and the Gamma distribution. For some parameters the Gamma distribution reduces to the exponential distribution. Usually the normally distributed demand is a good model for high moving items, while the Poisson distribution and the Gamma distribution for slow moving ones. Sometimes the Gamma distribution is a better choice than the normal distribution because it has the smaller probability of the negative demand which is obviously inappropriate (see [5]). Most often in practise the demand in certain time is discrete nonnegative random variable. Provided that the demand is reasonably low the discrete demand is used to model the real demand. In the case when the demand is large the continuous demand is better choice.

Our problem is to calculate the exact formulas for the safety factor given the service level and the distribution of the customers' demands. Now, let us note that the weekly demand in the central warehouse is equal to the sum of the weekly demands in the regional warehouses, which can be written in terms of the random variables as

$$X_c = \sum_{i=1}^{n} X_i .$$

Since the weekly demands X_i are the independent and identically distributed random variables then the mean and the variance of the customer's demand in the central warehouse are equal to

$$\mu_c = n\mu$$

and

$$\sigma_c{}^2 = n\sigma^2,$$

respectively. Similarly, the means and the variances for the lead time demands are equal to

$$\mu_T = \mu T, \qquad \mu_{Tc} = n\mu T,$$

and

$$\sigma_T{}^2 = \sigma^2 T, \qquad \sigma_{Tc}{}^2 = n\sigma^2 T.$$

Note that even for the independent and identically distributed random variables X_i, the random variables X_{iT} have a different distribution then the random variable Y_T. In other words, the distribution of the lead time demand in the regional warehouses is different to the distribution of the lead time demand in the central warehouse. Provided that the safety factor depends on the distribution of the demand, the safety factors of the regional warehouses ω are different than the safety factor of the central warehouse ω_c in general. Now using the definition of the service level and the above remarks we calculate the exact expressions for the safety factors in case of the uncertain demand.

From the definition of the service level (1) we get the expressions for the safety factor in the regional warehouses

$$\omega = \frac{1}{\sigma\sqrt{T}}\left(F_{X_{iT}}^{-1}(S_1) - \mu T\right) \tag{2}$$

and in the central warehouse

$$\omega_c = \frac{1}{\sigma\sqrt{nT}}\left(F_{Y_T}^{-1}(S_1) - n\mu T\right) \tag{3}$$

where $F_{X_{iT}}^{-1}(.)$ and $F_{Y_T}^{-1}(.)$ are the inverse functions of the cumulative distribution functions of the lead time demands.

3. Safety factors for specific distributions

In this section we consider the safety factors for the specific distributions. In the formulas of the safety factors given in the previous section we take into account the typical demand distributions like the normal distribution, the Poisson distribution and the Gamma distribution which includes the exponential distribution, too. The proofs of the probabilistic statements used in this section can be found in [12]. Mainly this theorems are concerned with the distributions of the sums of the independent and identically distributed random variables.

3.1. Safety factors for the normally distributed demands

For higher demand the normal distribution is usually used to model the demand over the time period. There are many reasons for the usage of the normal distribution. First of all, by the central limit theorem the sum of large number of independent random variables with an arbitrary distribution is approximately normally distributed.
For instance if the demand comes from many customers with the independent demands (also for the discrete Poisson distributed demands) than the overall demand can be approximated by the normal distribution. But there is some problem with the normal distribution. It is a fact that it has small but a positive probability of a negative demand.
If the weekly demands in the regional warehouses X_i are the normally distributed random variables with the mean μ and the variance σ then the lead time demands are also normally distributed with the respective values of the mean and the variances. Given that the service level is known and equal to S_1 by the equation (1) we get

$$F_{X_{iT}}\left(\mu T + \omega\sigma\sqrt{T}\right) = S_1$$

and

$$F_{Y_T}\left(n\mu T + \omega_c\, \sigma\sqrt{nT}\right) = S_1.$$

Hence the safety factor in the regional warehouses ω is equal to

$$\omega = \phi^{-1}(S_1),$$

where $\phi(.)$ is the cumulative distribution function of the standard normal distribution and $\phi^{-1}(.)$ is the inverse function of this cumulative distribution function.
Since the safety factor in this case does not depend on the mean and the variance of the demand, we state that for the normally distributed lead time demands the safety factors in the regional warehouses are equal to the safety factor in the central warehouse

$$\omega_c = \omega.$$

The cumulative distribution function of the standard normal distribution is tabled and we can easily get the values of the safety factors.

Numerical computations for the safety factors
For the service level $S_1 = 0.95$ the safety factors $\omega = \omega_c = 1.64$.
For $S_1 = 0.975$ the safety factors $\omega = \omega_c = 1.96$.
For $S_1 = 0.99$ the safety factors $\omega = \omega_c = 2.58$.

3.2. Safety factors for the Poisson distributed demands

In practice the Poisson distribution is fitted to describe a low demand. Especially it is a good approximation for the demand of slow moving items. For small number of the customers we cannot use the normal distribution to model their demand.
For the Poisson distribution the form of the cumulative distribution function is not so simple, but still the values of the safety factors can be calculated by the computer. Since the Poisson distribution is discrete the cumulative distribution function is a sum of components and can be written in a close form.
If the weekly demands in the regional warehouses X_i are the Poisson distributed random variables with the mean μ and the variance σ^2 then the mean is equal to the variance of the distribution $\mu = \sigma^2$. The demands in the lead time are also Poisson distributed.
Now assume that the service level S_1 is given. Then the safety factors can be calculated from (2) and (3). Namely, by the property of the Poisson distribution the safety factor in the regional warehouses is given by

$$\omega = (F_{X_{iT}}^{-1}(S_1) - \mu T)/\sqrt{\mu T}$$

and in the central warehouse has the form

$$\omega_c = (F_{Y_T}^{-1}(S_1) - n\mu T)/\sqrt{n\mu T}.$$

Here $F_{X_{iT}}^{-1}(S_1)$ is the interval and we take here the left end of this interval. Similarly we treat $F_{Y_T}^{-1}(S_1)$. It is worth noting here that for the Poisson distributed demands the values of the safety factors depend on the mean of the demand's distribution. It is contrary to the statements of the normal distribution where the safety factor depends on the service level only. Hence, for the Poisson distributed demand the values of the safety factor in the central warehouse is different to the value of the safety factor in the regional warehouses except of the situation when we

have only one regional warehouse which implies $n = 1$.
Now let us present the numerical examples of the values of the safety factor. Assume the specific service levels and the means of the customers' demands.

Numerical computations for the safety factors

For the service level $S_1 = 0.90$, $n = 10$ and
– T = 1, μ = 1 we get ω = 1.0, $\omega_c = 1.265$;
– T = 1, μ = 5 we have ω = 1.342, $\omega_c = 1.273$;
– T = 1, μ = 10 we get ω = 1.265, $\omega_c = 1.3$;
– T = 12, μ = 1 we have ω = 1.225, $\omega_c = 1.291$;
– T = 12, μ = 5 we get ω = 1.291, $\omega_c = 1.266$;
– T = 12, μ = 10 we have ω = 1.278, $\omega_c = 1.27$.

For the service level $S_1 = 0.99$, $n = 10$ and
– T = 1, μ = 1 we get ω = 3, $\omega_c = 2.53$;
– T = 1, μ = 5 we have ω = 2.683, $\omega_c = 2.404$;
– T = 1, μ = 10 we get ω = 2.53, $\omega_c = 2.4$;
– T = 12, μ = 1 we get ω = 2.598, $\omega_c = 2.373$;
– T = 12 μ = 5 we have ω = 2.453, $\omega_c = 2.368$;
– T = 12, μ = 10 we get ω = 2.373, $\omega_c = 2.338$.

Now we make some plots which describes how much the safety factors are explained by the lead time, the mean or the number of warehouses.
First assume that $S_1 = 0.99$ and μ = 1. Then the chart describing the dependence of the safety factor on the lead time T is as follows

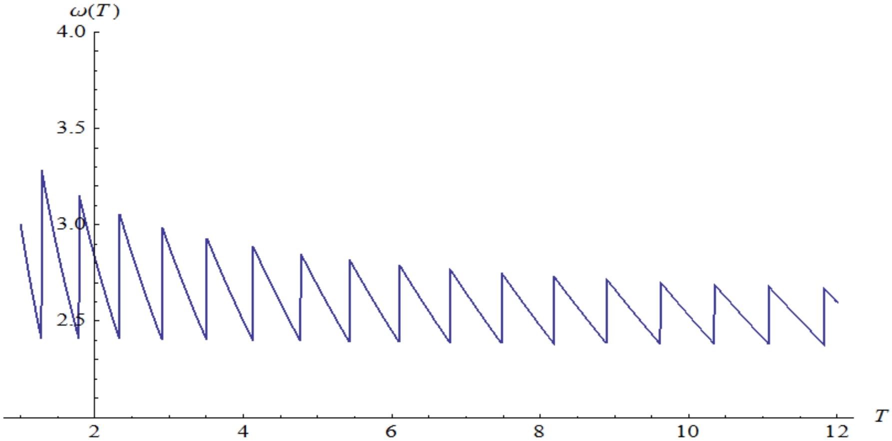

Fig. 1 The dependence of the safety factor on the lead time

Furthermore if T = 12 the dependence of ω on the mean μ is illustrated by the graph:

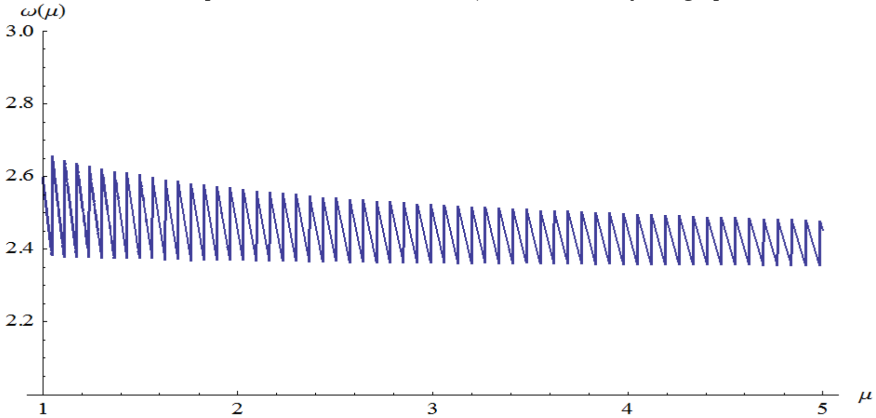

Fig. 2 The dependence of the safety factor of the demand's mean

If the independent random variable is n than assuming $S_1 = 0.99$, $\mathrm{T} = 1$ and $\mu = 1$ we get the plot:

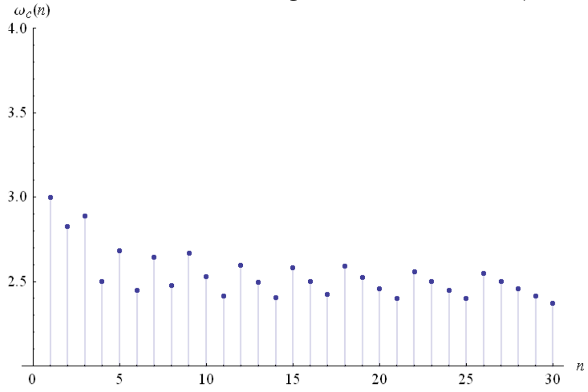

Fig. 3 The dependence of the safety factor on the number of the warehouses

The above graphs imply that the safety factors generally decrease when the lead time T or the mean μ or n increase.

3.3. Safety factors for the Gamma distributed demands

If the ratio of the standard deviation and the mean of the lead time distribution $\sigma/(\mu\sqrt{T})$ is not considerably less than 1 than the occurrence of the negative demand is relatively high. It may be better in such cases to use the Gamma distribution receiving only positive values than the normal distribution. The Gamma distribution has two parameters λ and r which are both positive, the density function given by:

$$f_{r,\lambda}(x) = \frac{\lambda(\lambda x)^{r-1}e^{-\lambda x}}{\Gamma(r)},$$

where $\Gamma(x)$ is the gamma function and the cumulative distribution function $F_{r,\lambda}(.)$. The gamma function is defined by

$$\Gamma(z) = \int_0^\infty x^{z-1}e^{-x}dx,$$

and in particular for integer z

$$\Gamma(z) = (z-1)!.$$

The Gamma distribution has the mean equal to r/λ and the standard deviation equal to $\sqrt{r}/\lambda$, which makes the computations much more simpler. The values of the cumulative distribution function can be calculated by the computer programs.

We get many advantages using the Gamma distribution as the distribution of the demand. First of all the demand is always positive then. But it should be pointed here that the probability of very high demand is larger here than for the normal distribution.

Assume that the weekly demands in the regional warehouses are Gamma distributed with the mean μ and the standard deviation σ. Then the lead time demand in the regional warehouses has the Gamma distribution with the parameters

$$r = T\frac{\mu^2}{\sigma^2}$$

and

$$\lambda = \frac{\mu}{\sigma^2}.$$

In the central warehouse the lead time demand is also Gamma distributed but with the parameters

$$r = nT\frac{\mu^2}{\sigma^2}$$

and

$$\lambda = \frac{\mu}{\sigma^2}.$$

By the formulas (2) and (3) the service levels can be conducted. Thus the service level in the regional warehouses is given by

$$\omega = (F^{-1}_{T\frac{\mu^2}{\sigma^2},\frac{\mu}{\sigma^2}}(S_1) - \mu T)/(\sigma\sqrt{T})$$

and in the central warehouse can be written by

$$\omega_c = (F^{-1}_{nT\frac{\mu^2}{\sigma^2},\frac{\mu}{\sigma^2}}(S_1) - n\mu T)/(\sigma\sqrt{nT})$$

Note, that in this case the safety factors depend on the mean, the variance, the lead time and the number of the regional warehouses. Here also the values of the safety factors in the regional and in the central warehouse are different to each other assuming the same parameters. Using the Mathematica software below we present some numerical examples.

Numerical computations for the safety factors

Assume that $n = 30$.

For the service level $S_1 = 0.9$ and

– $T = 1$, $\mu = 8$, σ=7 we get $\omega = 1.321$ and $\omega_c = 1.311$;

– $T = 1$, $\mu = 10$, σ=6 we have $\omega = 1.34$ and $\omega_c = 1.3$;

– $T = 12$, $\mu = 8$, σ=7 we get $\omega = 1.323$ and $\omega_c = 1.291$;

– $T = 12$, $\mu = 10$, σ=6 we get $\omega = 1.313$ and $\omega_c = 1.288$.

For the service level $S_1 = 0.99$ and

– $T = 1$, $\mu = 8$, σ=7 we have $\omega = 3.372$ and $\omega_c = 2.558$;

– $T = 1$, $\mu = 10$, σ=6 we get $\omega = 3.149$ and $\omega_c = 2.486$;

– $T = 12$, $\mu = 8$, σ=7 we have $\omega = 2.689$ and $\omega_c = 2.394$;

– $T = 12$, $\mu = 10$, σ=6 we get $\omega = 2.577$ and $\omega_c = 2.373$.

Now let us make some plots in which the dependences of the safety factor on the lead time, on the mean and on the number of warehouses are presented. First assume that

- $S_1 = 0.99$;
- σ=6.

Then if the mean $\mu = 10$ and $n = 30$ then the dependence of the safety factor ω on T is described by the plot

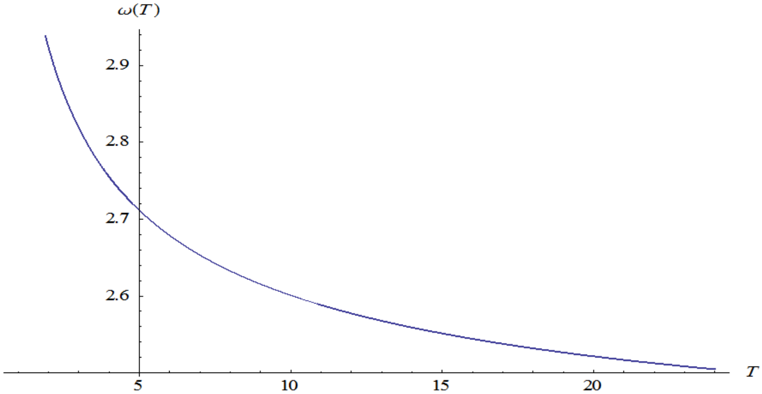

Fig. 4 The dependence of the safety factor on the lead time

Moreover if the lead time $T = 12$ and $n = 30$ the following chart says how much μ explains the safety factor ω.

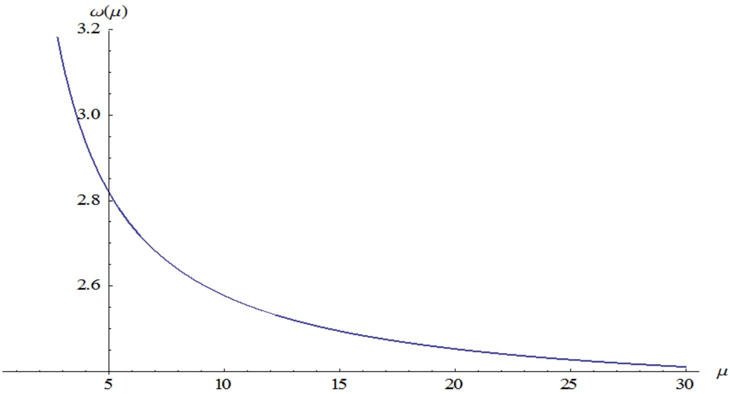

Fig. 5 The dependence of the safety factor of the demand's mean

Finally for $\mathrm{T} = 12$ and $\mu = 1$ the influence of the number of the regional warehouses n on the safety factor ω_c is described by the plot

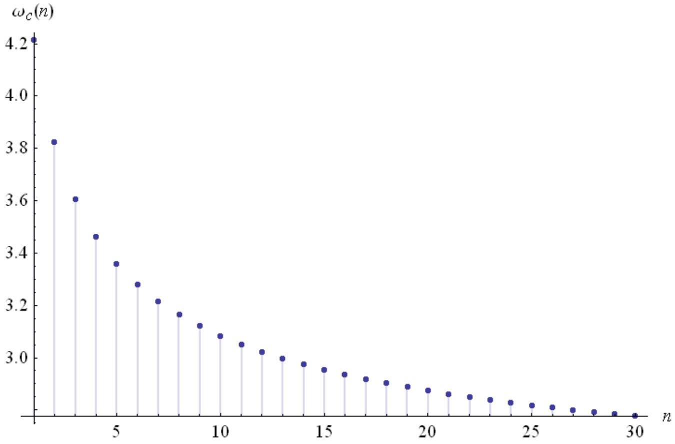

Fig. 6 The dependence of the safety factor on the number of the warehouses

The conclusion is as follows: the safety factors decrease if the lead time T or the mean μ or n increase.

3.4. Safety factors for the exponentially distributed demands

Note that for $r = 1$ the Gamma distribution simplifies to the exponential distribution with the positive parameter λ for which the density function is given by

$$f(x) = \lambda e^{-\lambda x}.$$

For this kind of distribution the mean is equal to $\mu = 1/\lambda$ and it is equal to the standard deviation σ. The exponential distribution is also a good approximation of the demand, especially for the demand on slow moving items. In this case the lead time demands have the Gamma distribution. Since $\mu = \sigma$ the distribution of the lead time demands in the regional warehouses has the parameters $r = T$ and $\lambda = \frac{1}{\mu}$, while in the central warehouse these parameters are given by $r = nT$ and $\lambda = \frac{1}{\mu}$. Moreover the equation (2) and (3) reduces to

$$\omega = (F^{-1}_{T,\frac{1}{\mu}}(S_1) - \mu T)/(\mu\sqrt{T})$$

for the regional warehouses and

$$\omega_c = (F^{-1}_{nT,\frac{1}{\mu}}(S_1) - n\mu T)/(\mu\sqrt{nT}).$$

for the central warehouse. Then the safety factors of the regional warehouses ω are not equal in general to the safety factor of the central warehouse

$$\omega \neq \omega_c.$$

Additionally note that the safety factors do not depend on the mean μ of the weekly demand because from (1) we get

$$F_{T,\frac{1}{\mu}}(\mu T + \omega\mu\sqrt{T}) = S_1 = F_{T,1}(T + \omega\sqrt{T}).$$

As a result of the above fact the safety factors for the exponentially distributed weekly demands depends on the lead time T and the service level S_1 only in the regional warehouses. While in the central warehouse it depends additionally on the number of the regional warehouses n.

Now we present some numerical examples received by the computer program Mathematica.

Numerical computations for the safety factors

For the service level $S_1 = 0.9$ and

- $n = 5$, $\mathrm{T} = 1$ we get $\omega = 1.302$ and $\omega_c = 1.339$;
- $n = 5$, $\mathrm{T} = 4$ we have $\omega = 1.34$ and $\omega_c = 1.32$;
- $n = 5$, T=12 we get $\omega = 1.327$ and $\omega_c = 1.306$;
- $n = 15$, $\mathrm{T} = 1$ we have $\omega = 1.302$ and $\omega_c = 1.324$;
- $n = 15$, $\mathrm{T} = 4$ we have $\omega = 1.34$ and $\omega_c = 1.306$;
- $n = 15$, T=12 we get $\omega = 1.327$ and $\omega_c = 1.296$.

For the service level $S_1 = 0.99$ and

- $n = 5$, $\mathrm{T} = 1$ we get $\omega = 3.605$ and $\omega_c = 2.954$;
- $n = 5$, $\mathrm{T} = 4$ we get $\omega = 3.023$ and $\omega_c = 2.649$;
- $n = 5$, $\mathrm{T} = 12$ we get $\omega = 2.74$ and $\omega_c = 2.514$;
- $n = 15$, $\mathrm{T} = 1$ we get, $\omega = 3.605$ and $\omega_c =$

2.697;

– $n = 15$, T = 4 we get ω = 3.023 and $\omega_c =$ 2.514;

– $n = 15$, T = 12 we get ω = 2.74 and $\omega_c =$ 2.435

Now for the service level $S_1 = 0.99$ the safety factor ω is explained by the lead time T in the following way

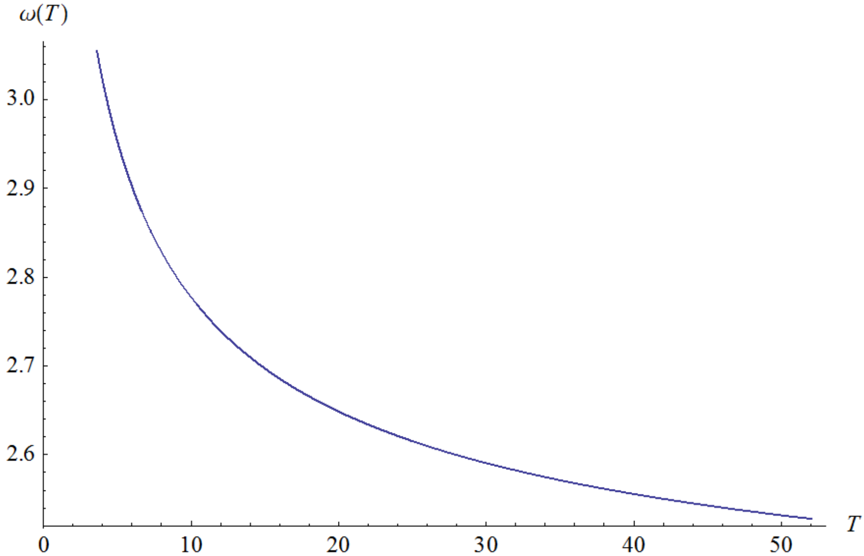

Fig. 7 The dependence of the safety factor on the lead time

Moreover if T = 12 then the dependence of the safety factor ω_c on n is describing by the graph

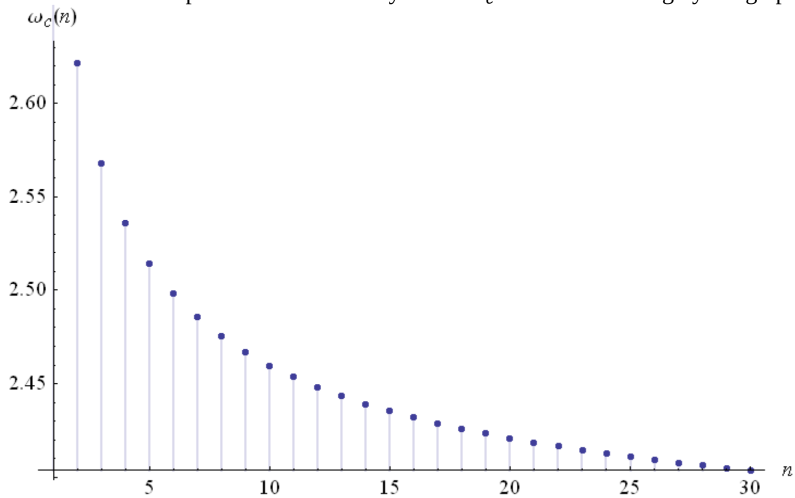

Fig. 8 The dependence of the safety factor on the number of the warehouses

We conclude that the safety factors decrease if T or n increase.

4. Inventory location model for specific distributions

The model introduced in [9] was extended in [4] and now holds for any distribution function of the demand. Below we remind the used notation in the model and also we present main formulas of the model proved in the mentioned papers. The novelty of this chapter is to show how the formulas telling when it is worth to decentralize the stock (see [4]) work on the numbers. Moreover we try to analyse the influence of the modification of the parameters for the decision of the safety stock location.

First let us present the notation used in the inventory location model with stochastic demands. In the studied model

- p denotes the price of the supply of the unit of the inventory;
- h_i denotes the weekly cost factor in the regional warehouse i, i =1,…, n;
- T denotes the lead time in the regional warehouses;
- k denotes the transportation costs of the unit of the safety stock from the central warehouse to the customer;
- HC_i is equal to the holding cost of the safety stock in the regional warehouse i, i =1,…, n;
- HC denotes the overall holding costs of the safety stock in the regional warehouses;
- HC_c is equal to the holding cost of the safety stock in the central warehouse;
- SC denotes the cost of the direct supply from the central warehouse to the customer.

Additionally, assume that the lead time in the central warehouse is equal to αT, where α can be less than, equal to or greater than 1. But in practice usually the lead time in the central warehouse is shorter than in the regional warehouse, which implies that α is usually less than 1.

Moreover assume that the weekly cost factor in the central warehouse is the mean of the cost factors in the regional warehouses and equal to

$$\frac{1}{n}\sum_{i=1}^{n} h_i \, .$$

First we should remember that the specific demands do not influence to each other which means that the random variables X_i are the independent random variables. Due to this condition let us calculate the overall weekly cost of holding the safety stock in all of the regional warehouses. Thus the overall holding cost is equal to

$$HC = \sum_{i=1}^{n} HC_i = p\omega\sigma\sqrt{T}\sum_{i=1}^{n} h_i$$

which can be found in the paper [4].

Furthermore let us remind that the demand in the central warehouse has the same distribution as the sum of the random variables which represent the demands in the regional warehouses. Thus the holding cost in the central warehouse is given by

$$HC_c = \frac{1}{n}\, p\omega_c\sigma\sqrt{n\alpha T}\sum_{i=1}^{n} h_i$$

Moreover the cost of the supply from the central warehouse to the customer is equal to

$$SC = kn\mu.$$

Consequently it is less costly not to centralize the safety stock if the overall holding cost in the regional warehouses is less than the sum of the holding cost in the central warehouse and the cost of the direct supply to the costumer, which can be written as

$$HC < HC_c + SC$$

or equivalently

$$p\omega\sigma\sqrt{T}\sum_{i=1}^{n} h_i < \frac{1}{n}\, p\omega_c\sigma\sqrt{n\alpha T}\sum_{i=1}^{n} h_i \; + \; kn\mu.$$

Hence we obtain the following inequality:

$$\frac{k}{p\sum_{i=1}^{n} h_i} > \frac{\sigma\sqrt{T}}{n\mu}\left(\omega - \sqrt{\frac{\alpha}{n}}\omega_c\right) \quad (4)$$

in which on the left hand side are the cost factors.

Let us remind how these equality works for the most often used demands' distributions like the normal, the Poisson, the Gamma distribution and also the exponential one.

In case of the normally distributed demands the condition of decentralization the inventory reduces to

$$\frac{k}{p\sum_{i=1}^{n} h_i} > \frac{\sqrt{T}\omega\sigma}{n\mu}\left(1 - \sqrt{\frac{\alpha}{n}}\right)$$

since the safety factors are equal to each other in this case $\omega_c = \omega$ (see [9]).

For the Poisson distributed weekly demands the following equation holds $\sigma^2 = \mu$ and it is worth to decentralize the safety stock if

$$\frac{k}{p\sum_{i=1}^{n} h_i} > \frac{\sqrt{T}}{n\sqrt{\mu}}\left(\omega - \omega_c\sqrt{\frac{\alpha}{n}}\right).$$

For the Gamma distribution the value of the safety factor in the central warehouse ω_c and in the regional warehouses ω can be calculated from the cumulative distribution function. In this case the decision on the decentralization of the inventory is made on the condition given by the main formula (4).

But for the exponentially distributed weekly demands the expression (4) is much more simpler because the mean is equal to the standard deviation here $\mu = \sigma$. Hence it is better not to centralize the inventory if

$$\frac{k}{p\sum_{i=1}^{n} h_i} > \frac{\sqrt{T}}{n}\left(\omega - \omega_c\sqrt{\frac{\alpha}{n}}\right).$$

Below some graphs on the model are presented. Denote by R(.) the function of one argument representing the right hand side of the inequality (4). For simplicity we are going to treat this function as the function of one argument. Further this argument can be the mean μ or the number of the warehouses n. Then another parameters are constant and they are given in advance. Assume that

- $S_1 = 0.99$;
- $T = 4$;
- $\alpha = 0.8$.

In the consecutive subsections in the graphs we consider the specific demands' distributions.

The graphs for the normally distributed demands

First assume that the standard deviation of that weekly demand is known and the number of the regional warehouses is known also. For these assumptions the graph which ilustrate the connection between the cost factors and the mean is the linear function which was presented in the article [9]. It was stated there that the greater the mean of the weekly demands is the more reasonable is the decision of the safety stock decentralization.

On the other hand assuming that the mean and the standard deviation of weekly demands are known we treat the number of the regional warehouses n as the argument of the function $R(.)$. Then from the formula (4) we state that the function $R(.)$ is decreasing which implies that the inventory decentralization is more justified for large number of the regional warehouses.

The graphs for the Poisson distributed demands

Now assume that the mean of the weekly demand in the regional warehouses is the argument of the function R(.) and the mean varies from 1 to 7. Additionally let the numer of the regional warehouses be equal to $n = 10$. For these assumptions the plot which ilustrate the influence of the variability of the mean on $R(.)$ is as follows:

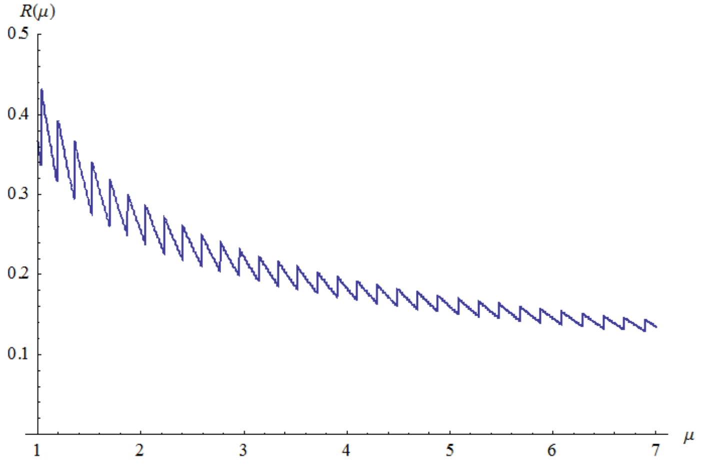

Fig. 9 The dependence of the dispersion of the safety stock on the demand's mean

We note that in general for the grater mean of the weekly demands the better decision could be not to centralize the safety stock.

On the other hand assume that the mean of the weekly demands in the regional warehouses is equal to $\mu = 2$. Then the graph of the function $R(.)$ for the independent variable equal to the number of the regional warehouses n looks like

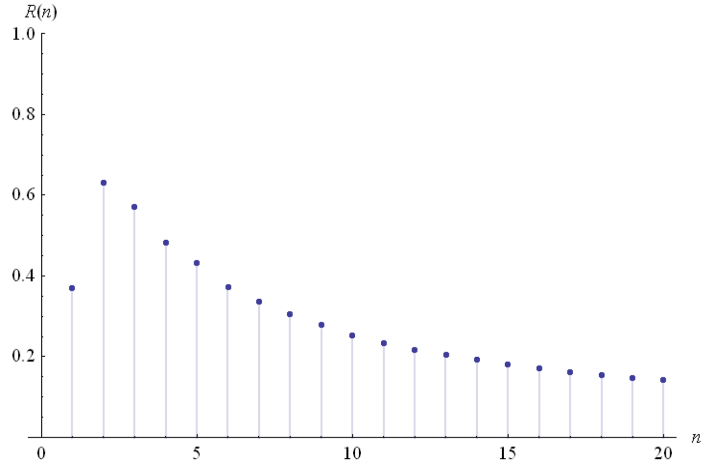

Fig. 10 The dependence of the dispersion of the safety stock on the number of the warehouses

We state that in general if the number of the warehouses is larger than the decentralization the stock is much more justified.

The graphs for the Gamma distributed demands

First assume that the mean of the weekly demand in the regional warehouses μ is the independent variable which varies from 1 to 10. Additionally assume that the standard deviation of that weekly demand is equal to $\sigma = 6$ and the number of the regional warehouses is equal to $n = 20$. For these assumptions the graph of the function $R(.)$ which ilustrate the dependence of our inventory location decision on the mean is as follows:

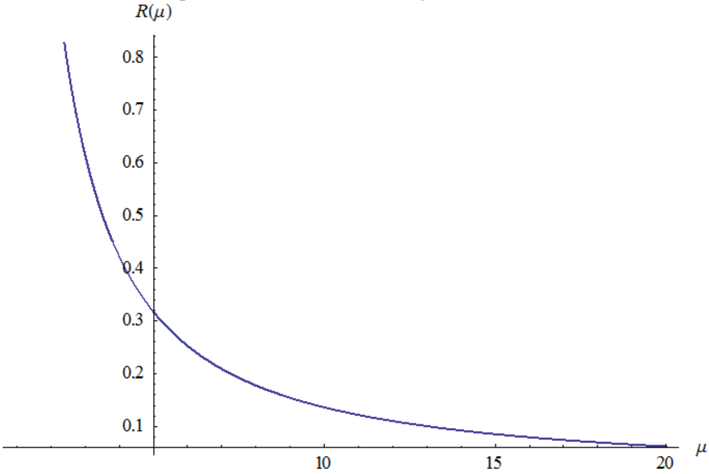

Fig. 11 The dependence of the dispersion of the safety stock on the demand's mean

The similar colclusion as the previous one holds. Namely, if the mean is larger the decentralization of the safety stock is more defended.

On the other hand assume that the mean and the standard deviation of the weekly demands in the regional warehouses are equal to $\mu = 2$ and $\sigma = 6$. Then the graph of the function $R(.)$ for the independent variable equal to the number of the regional warehouses n looks like

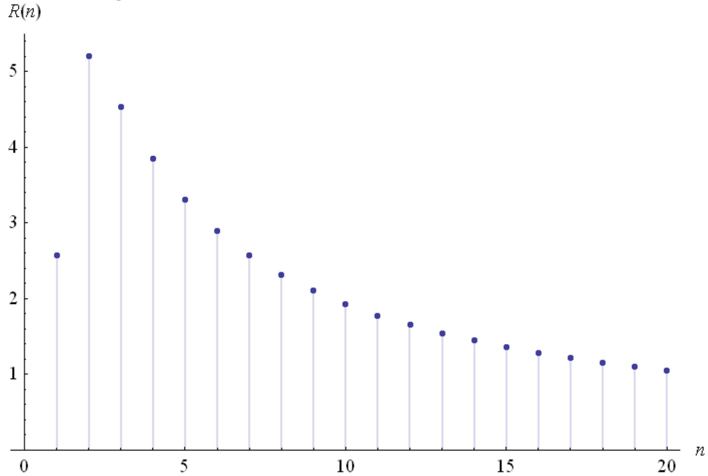

Fig. 12 The dependence of the dispersion of the safety stock on the number of the warehouses

Here also rather it is better to bring the inventory in the regional warehouses if the number of the regional warehouses is larger.

The graphs for the exponentially distributed demands

In this case the safety factors do not depend on the mean of the weekly demands. Hence below we present only the graph of the function $R(.)$ in which the independent variable is the number of the regional warehouses n.

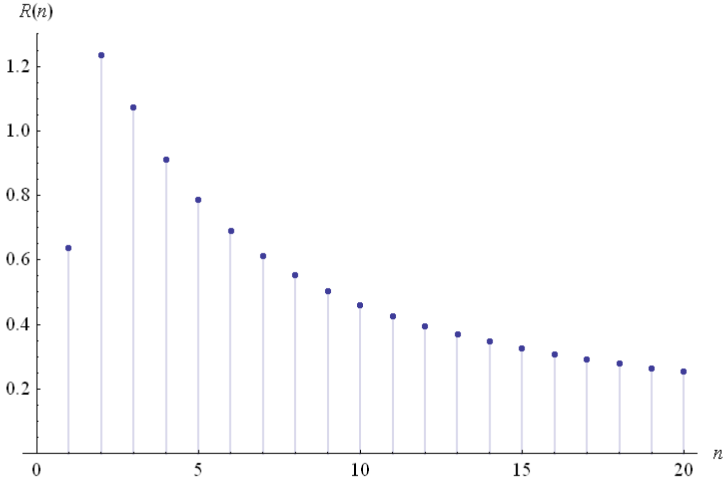

Fig. 13 The dependence of the dispersion of the safety stock on the number of the warehouses

In this case also it is better not to centralize the stock if the number of the regional warehouses is relatively high.

5. Conclusions

In the paper we derive the formulas for the safety factors used in the problem of the safety stock location. In this facility location problem we assume the stochastic demands of the customers. In case of the most used in practice distributions of the demand the numerical examples for the safety factors are given. Additionally some graphs illustrated the dependence of the safety factors on the parameters of the model are drown. Also the analysis of the influence of the model parameters on the decision of inventory location is presented. Namely, the variability on the mean, the standard deviation and the number of the warehouses are considered. The normally, the exponentially, the Poisson and the Gamma distributed demands are widely studied. For the mentioned distributions the plots illustrating the dependence the location decision on the mean, the lead time and the number of the warehouses are presented.

The conclusions are the following:

1) The formulas for the safety factors for the specific demands' distributions can simplify. As a consequence also the investigated inventory location model is getting simpler in some cases.
2) In general the safety factors in the regional warehouses and in the central warehouse are different to each other but can be calculated using the probabilistic theorems.
3) The safety factors do not depend on the mean and the variance in case of the exponentially distributed customer's demand.
4) The safety factors in general decrease if the lead time T or the mean μ or the number of the regional warehouses n increase for every kind of the demand's distribution.
5) The inventory location in the model always depends on the cost factors, the service level and the number of the regional warehouses.
6) The demand's mean influences on the decision of the dispersion of the warehouses.
7) In general the decision of the stock decentralization is much more justified for the greater demand's mean for every studied distribution.
8) Similarly, it is better not to centralize the inventory if the number of the warehouses increases in all cases.

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