ATTRIBUTE np CONTROL CHARTS USING RESAMPLING SYSTEMS FOR MONITORING NON-CONFORMING ITEMS UNDER EXPONENTIATED HALF-LOGISTIC DISTRIBUTION

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An attribute np control chart has been designed using resampling systems for monitoring non--conforming items under exponentiated half-logistic distribution. We suppose that lifetime follows exponentiated half-logistic distribution. For the proposed control charts, the optimal parameters and control limits have been obtained. The operational formulas for in-control and out of control average run lengths (*ARLs*) have been derived. Control constants are established by considering the target in-control *ARL* at a normal process. The extensive *ARL* tables are reported for various parameters and shifted values of process parameters. The performance of the proposed control chart is evaluated with several existing charts with regard to *ARLs*, which empower the presented chart and prove far better for timely detection of assignable causes. A wide range of tables, a real-life example, and simulation study for RGS and MDS are given for a better understanding of the problem.

Keywords: *attribute chart, time truncation, process, statistical distributions, sampling schemes*

1. Introduction

Quality is the main important feature for production growth and standing. Statistical techniques can be used to improve the quality of the product. The control chart is the

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primary instrument to monitor the process and to finally develop high-quality products. Control charts provide a fast signal when the process is out of control. The control chart enables to monitor the deviation of the method over time and discovers the foundation for differences. Amongst the two types of control charts, i.e., attribute control charts and variable control charts, the former is used to track the quality characteristic of an attribute. Examples of attribute control charts include p, np, and u control charts. The variable control charts, e.g., $\overline{X}$, R, and S control charts are used to monitor the characteristics of the issue in question. For the construction of control charts, different schemes, e.g., single sampling scheme, double sampling scheme, repetitive group sampling scheme, and multiple dependent state sampling (MDS) schemes are used.

Repetitive group sampling (RGS) scheme is introduced by Sherman [1]. RGS is the expansion of a single sampling scheme. The control chart under RGS has two pairs of control limits. If a point lies within the inner control limits, the procedure is said to be in-control, whereas if a point lies beyond the outer control limits, the process is out of control. The sampling is repeated if a point is identified between inner and outer control limits until a decision is made. Balamurali and Jun [2] introduce the RGS plan for variable inspection. They reveal that the average sample number (ASN) of the RGS plan is lesser than both single and double sampling plans. Ahmad et al. [3] Design the $\overline{X}$ control chart for RGS based on the process capability index. Aslam et al. [4], using RGS, present attributes and variables control charts. Aslam et al. [5] introduce a t-chart using RGS. Rao [6] introduces a control chart based on exponentiated half logistic distribution with known shape parameters. The wide range of tables based on a simulated study is given.

Multiple dependent state (MDS) sampling scheme was created by Wortham and Baker [7]. The conclusion will be made not only on current but also on a previous or upcoming lot. The plan parameters are established by meeting the vendor and customer hazards at a number of acceptable levels of quality and by restricting the amount of quality. Later, Aslam et al. [8] propose a new loss consideration process under multiple dependent state variable sampling plans. Aslam et al. [9] propose multiple dependent state control chart based on two pairs of control limits. The proposed control chart is compared with the existing $\overline{X}$ chart which shows that the proposed control chart performs better than Shewart X bar control chart. Rao and Naidu [10] Recommend a fresh sampling plan for exponentiated half logistic distribution for group acceptance. The scheme implemented by them shows that the percentile ratio rises for all parameters and decreases the number of associations. Aslam [11] proposes a control chart by using the Weibull distribution based on MDS sampling which is more efficient than the existing control charts.

In this study, an attribute np control chart for process monitoring using exponentiated half logistic distribution (EHLD) is provided as based on RGS and MDS. Both of the control charts are compared with the existing control charts.

RGS chart is compared with the existing chart on the basis of exponentiated half logistic distribution provided by Jeyadurga et al. [12]. We acquire optimal parameters, control limits, and also show the respective *ARL* and *ASN* values of the control chart using RGS. We compare the table values for the shape parameters $\alpha = 2$ and 3 and the specified *ARL* is $r_0 = 300$ and 370, respectively. We can infer from our data that the proposed chart identifies the process shift quicker than the existing chart under RGS sampling scheme. The control chart performance in RGS is also compared with the control chart presented by Rao [6] based on EHLD. Results are presented in form of tables using different shape and scale parameters, respectively.

The efficiency of the proposed MDS control chart is compared to the control chart results by Aslam [11]. We calculate the optimal parameters, control limits, and present the respective *ARL* values of the control chart, using dependent state sampling. The results are shown in the form of a table which indicates that the proposed control chart is more responsive than the existing control chart to recognize a shift in the process. The efficiency of the proposed MDS control chart is also compared with the results of Rao [6] and concluded in the form of tables, using different ARL_0 (200, 300, 370, $i = 1, 2, 3$).

Moreover, using RGS and MDS, we find the *ARL*s when scale and shape parameters are shifted separately, and also when both parameters are shifted at once. A real-life data example is also used to demonstrate the proposed np control chart by using simulated data.

2. The exponentiated half-logistic distribution (EHLD)

EHLD is a simplification of half-logistic distribution proposed by Mudholkar and Srivastava [13]. Cordeiro et al. [14] study mathematical properties of the exponentiated half logistic family of distributions. Seo and Kang [15] derive moment estimators and maximum likelihood estimators of unknown parameters of exponentiated half logistic distribution along with an entropy. Elgarhy et al. [16] present exponentiated extended family of distributions along with its mathematical properties and applications. Usman et al. [17] study the Kumaraswamy half-logistic distribution and explore the properties and applications of the presented model. Anwar and Bibi [18] propose half logistic generalized Weibull (HLGW) distribution and investigate its properties.

The cumulative distribution function ($F(t)$ and the probability density function ($f(t)$ of EHLD are given as

$$F\left(t\right) = \left(\frac{1 - \mathrm{e}^{-t/\sigma}}{1 + \mathrm{e}^{-t/\sigma}}\right)^{\alpha}, \quad t \geq 0,\ \alpha > 0,\ \sigma > 0 \tag{1}$$

$$f(t) = \frac{2\alpha e^{-t/\sigma} \left(1 - e^{-t/\sigma}\right)^{\alpha-1}}{\sigma \left(1 + e^{-t/\sigma}\right)^{\alpha+1}}, \quad t \geq 0, \alpha > 0, \sigma > 0 \tag{2}$$

Here α is the shape parameter, and σ is the scale parameter. The arithmetic mean of EHLD distribution is specified as

$$\mu = \sigma \ln\left(\frac{1 + 0.5^{1/\alpha}}{1 - 0.5^{1/\alpha}}\right), \quad \eta = \ln\left(\frac{1 + 0.5^{1/\alpha}}{1 - 0.5^{1/\alpha}}\right), \quad \mu = \sigma\eta, \quad \sigma = \frac{\mu}{\eta} \tag{3}$$

3. Design of the proposed control chart under RGS

It is important to note that the attribute's RGS plan is a generalization of the ordinary single sampling plan. In RGS, the sampling is repeated until a decision is made. RGS is more efficient than the single sampling in terms of the ASN which is needed for deciding on the disposition of the lot.

Following are the steps of the np control chart based on the RGS scheme.

Step 1. From the manufacturing process, choose a subgroup as a random sample of size n and place it on the life test for the particular time t_0. Count number of failures says D during t_0.

Step 2. If $LCL_1 \leq D \leq UCL_1$, then the process is said to be in control. If either $D \geq UCL_2$, or $D \leq LCL_2$, then the process is said to be out of control.

Step 3. Repeat Steps 1 and 2 if $LCL_2 < D < LCL_1$ or $UCL_1 < D < UCL_2$, until a decision is made.

The control limits for an np control chart are as follows:

$$UCL_1 = np_0 + k_1\sqrt{np_0\left(1 - p_0\right)} \tag{4a}$$

$$LCL_1 = \max\left(0, np_0 - k_1\sqrt{np_0\left(1 - p_0\right)}\right) \tag{4b}$$

$$UCL_2 = np_0 + k_2\sqrt{np_0\left(1 - p_0\right)} \tag{4c}$$

$$LCL_2 = \max\left(0, np_0 - k_2\sqrt{np_0\left(1 - p_0\right)}\right) \tag{4d}$$

The likelihood p_0 is generally unknown. So, the control limits to be used in practical applications will be as given by [19]:

$$UCL_1 = \overline{d} + k_1 \sqrt{\overline{d}\left(1 - \frac{\overline{d}}{n}\right)} \tag{5a}$$

$$LCL_1 = \max\left(0, \overline{d} - k_1 \sqrt{\overline{d}\left(1 - \frac{\overline{d}}{n}\right)}\right) \tag{5b}$$

$$UCL_2 = \overline{d} + k_2 \sqrt{\overline{d}\left(1 - \frac{\overline{d}}{n}\right)} \tag{5c}$$

$$LCL_2 = \max\left(0, \overline{d} - k_2 \sqrt{\overline{d}\left(1 - \frac{\overline{d}}{n}\right)}\right) \tag{5d}$$

where $\overline{d}$ is the average number of failures in a subgroup over a preliminary sample. However, in this paper, we will consider the control limits in the form of equation (4) to investigate the performance of the proposed chart analytically in terms of the average run length.

The method is stated to be controlled if $t = \alpha\mu_0$, $\mu = \mu_0$ (scale parameter is $\sigma = \sigma_0$ and shape parameter is $\alpha = \alpha_0$). If the process is in control, the failure likelihood p_0 is given by equation (1)

$$p = \left(\frac{1 - e^{-t/\sigma}}{1 + e^{-t/\sigma}}\right)^{\alpha}, \quad p_0 = \left(\frac{1 - e^{-a\mu_0/\sigma_0}}{1 + e^{-a\mu_0/\sigma_0}}\right)^{\alpha_0}$$

$$= \left(\frac{1 - e^{-a\sigma_0\eta_0/\sigma_0}}{1 + e^{-a\sigma_0\eta_0/\sigma_0}}\right)^{\alpha_0} = \left(\frac{1 - e^{-a\eta_0}}{1 + e^{-a\eta_0}}\right)^{\alpha_0} \tag{6}$$

3.1. Shift in the scale parameter

We suppose that the scale parameter is shifted from σ_0 to $\sigma_1 = c\sigma_0$, where c is a shift constant (or scale parameter is $\sigma = \sigma_1$). So, if the process is out of control, the failure likelihood of an item p_1 is specified as

$$p = \left(\frac{1-e^{-t/\sigma}}{1+e^{-t/\sigma}}\right)^{\alpha}, \quad p_1 = \left(\frac{1-e^{-\alpha\mu_0/\sigma_1}}{1+e^{-\alpha\mu_0/\sigma_1}}\right)^{\alpha}$$

$$= \left(\frac{1-e^{-a\sigma_0\eta_0/c\sigma_0}}{1+e^{-a\sigma_0\eta_0/c\sigma_0}}\right)^{\alpha} = \left(\frac{1-e^{-a\eta_0/c}}{1+e^{-a\eta_0/c}}\right)^{\alpha} \tag{7}$$

3.2. Shift in shape and both shape and scale parameters

If shape parameter is shifted from α_0 to $\alpha_1 = f\alpha_0$ (or the shape parameter is $\alpha = \alpha_1$). The probability that an item is failed p_2 is specified as

$$p_2 = \left(\frac{1-e^{-a\eta_1}}{1+e^{-a\eta_1}}\right)^{f\alpha_0} \quad \text{where} \quad \eta_1 = \ln\left(\frac{1+0.5^{1/f\alpha_0}}{1-0.5^{1/f\alpha_0}}\right) \tag{8}$$

Suppose that both the shape and scale parameters are shifted to $\alpha_1 = fa_0$ and $\sigma_1 = c\sigma_0$ (now the shape parameter is α_1 and the scale parameter is σ_1). The likelihood that an item is failed p_3 is specified as

$$p_3 = \left(\frac{1-e^{-a\eta_1/c}}{1+e^{-a\eta_1/c}}\right)^{fa_0} \quad \text{where} \quad \eta_1 = \ln\left(\frac{1+0.5^{1/f\alpha_0}}{1-0.5^{1/f\alpha_0}}\right) \tag{9}$$

When the probability that the process is likely to be in-control, $L_1(p)$ is specified as

$$L_1(p) = P\left(LCL_1 \le d \le UCL_1\right), \quad d = 0, 1, 2, ..., n$$

$$L_1(p) = \sum_{d=[LCL_1]+1}^{[UCL_1]} \binom{n}{d} p^d (1-p)^{n-d}, \quad d = 0, 1, 2, ..., n \tag{10}$$

Similarly, when the process is likely to be stated out of control, $L_2(p)$ is specified as

$$L_2(p) = P(d > UCL_2) + P(d < LCL_2)$$

$$L_2(p) = \sum_{d=[UCL_2]+1}^{[LCL_2]} \binom{n}{d} p^d (1-p)^{n-d} \tag{11}$$

The likelihood of repeating the sampling until the choice is made, $L_3(p)$ is as illustrated

$$L_3(p) = P\left(LCL_2 \le d < LCL_1\right) + P\left(UCL_1 \le d < UCL_2\right)$$

$$L_3(p) = \sum_{d=[LCL_2]+1}^{[LCL_1]} \binom{n}{d} p^d (1-p)^{n-d} + \sum_{d=[UCL_1]+1}^{[UCL_2]} \binom{n}{d} p^d (1-p)^{n-d} \tag{12}$$

The likelihood that the process will be controlled the ASN and average run length (ARL) using RGS are given below:

$$p_{in}(p) = \frac{L_1(p)}{L_3 - (p)} \tag{13}$$

$$ASN(p) = \frac{n}{1 - L_3(p)} \tag{14}$$

$$ARL = \frac{1}{1 - p_{in}(p)} \tag{15}$$

The likelihood that the process will be controlled when it is under control using RGS, the scale parameter is shifted, shape parameter is shifted and both shape and scale parameters are shifted is given in equations (13a)–(13d), respectively:

$$p_{in}(p_0) = \frac{L_1(p_0)}{1 - L_3(p_0)} \tag{13a}$$

$$p_{in}(p_1) = \frac{L_1(p_1)}{1 - L_3(p_1)} \tag{13b}$$

$$p_{in}(p_2) = \frac{L_1(p_2)}{1 - L_3(p_2)} \tag{13c}$$

$$p_{in}(p_3) = \frac{L_1(p_3)}{1 - L_3(p_3)} \tag{13d}$$

The *ASN* for the proposed chart when the process is in control, the scale parameter is shifted, the shape parameter is shifted and both scale and shape parameters are shifted is given in equation (14a)–(14d), respectively:

$$ASN(p_0) = \frac{n}{1 - L_3(p_0)} \tag{14a}$$

$$ASN(p_1) = \frac{n}{1 - L_3(p_1)} \tag{14b}$$

$$ASN(p_2) = \frac{n}{1 - L_3(p_2)} \tag{14c}$$

$$ASN\left(p_3\right)=\frac{n}{1-L_3\left(p_3\right)} \tag{14d}$$

The *ARL* for the proposed chart when the process is in control, the scale parameter is shifted, the shape parameter is shifted and both scale and shape parameters are shifted is given in equation (15a)–(15d), respectively:

$$ARL_0=\frac{1}{1-p_{in}\left(p_0\right)} \tag{15a}$$

$$ARL_1=\frac{1}{1-p_{in}\left(p_1\right)} \tag{15b}$$

$$ARL_2=\frac{1}{1-p_{in}\left(p_2\right)} \tag{15c}$$

$$ARL_3=\frac{1}{1-p_{in}\left(p_3\right)} \tag{15d}$$

3.3. Optimization conditions

The following are the optimization conditions:

Minimize ASN(p_0)

Subject to

$ARL_0 \geq r_0$

$k_2 > k_1$

Algorithm

Step 1. Set the values of α_0 (shape parameter), a (test termination ratio), r_0 (in-control target *ARL*), and set a greatest value for *ASN* as ASN_m.

Step 2. Set the values of n, k_1 and k_2.

Step 3. Put the values of a, α_0 in equation (6) to find the failure probability (p_0).

Step 4. Discover the control limits of the proposed control chart using the values of n, p_0, k_1, and k_2 in equations (4a)–(4d).

Step 5. Find the likelihood that the process will be declared in-control when the process is in-control by replacing the values of p_0 and control limits in equations (10), (12), and (13a); also use the equation (15a) to locate ARL_0. By using equation (14a), compute the *ASN* at p_0.

Step 6. Check parameters n, k_1 and k_2 such that the in-control ARL_0 is as near as to the final value r_0. Make the comparison of $ASN(p_0)$ value with ASN_m. If $ASN(p_0) \leq ASN_m$, then set $ASN_m = ASN(p_0)$.

Step 7. Continue Steps 4–6 for various combinations of n, k_1 and k_2 until you get an ARL_0 as near as to r_0 and obtain the least value of ASN as ASN_m.

Step 8. Replace the values n, k_1 and k_2 in equations (7)–(9), (13b)–(13d), (15b)–(15d) for discovering the out of control ARL_1 for various shift values. Use equations (14b)–(14d) to discover the respective $ASNs$.

3.4. Control charts performance comparison

Results are simulated using the proposed control chart and algorithm and are presented in tables. Tables A1–A5 present optimal parameters such as control coefficients k_2 and k_1 for fixed sample size (n). These parameters are calculated to get ARL value as close as the specified in-control $ARL = 200$, 300, and 370. Shape parameter $\alpha = 1.5$, 2.0, 2.5, 3.0, shift constant f from 1.0 to 0.1, and sample sizes $n = 10$, 15, 20, 25 are chosen. It is found that ARL and ASN are decreasing if the shift constant is decreasing.

Tables A6–A8 present the optimal parameters for the shift in scale parameter, using shift c from 1.0 to 0.1.

Table A9 gives the optimal parameters for the shift in both shape and scale parameters for $\alpha = 1.5$, 2.0, 2.5, and 3.0, using shift c and f from 1.0 to 0.1.

In this segment, the control chart performance in RGS is contrasted with the available control chart presented by Jeyadurga et al. [12] as based on EHLD. Tables A10 and A11 demonstrate the optimum parameters n, a, k_1, k_2, higher and lower control boundaries along with ARL and ASN values, using RGS scheme for the shape parameter $\alpha = 2$ and 3 and specified $r_0 = 300$ and 370. These tables show that the proposed chart identifies the change quicker than the existing one. For example, when $r_0 = 300$, $\alpha = 2$ and $n = 25$, the proposed control chart identifies the shift at 41th observation and $ASN = 29.64$. However, the shift is identified by the existing control chart at 116th observation.

The control chart performance in RGS is also contrasted with another available control chart presented by Rao [6] as based on EHLD. Tables A12 and A13 demonstrate the optimum parameters, higher and lower control boundaries, ARL values using RGS scheme, and single sampling scheme for shape parameter $\alpha = 2$ and $r_0 = 370$, and for the scale parameter $\alpha = 2.5$, 3, and $r_0 = 370$. The proposed control chart identifies the change quicker than the existing one. For example, when $r_0 = 370$, $\alpha = 2$ and $n = 25$, the proposed chart identifies the shift at 63rd observation. However, the shift is identified by the existing control chart at 247th observation. The shift in scale parameter is also shifted earlier, using the proposed control chart.

3.5. Illustrative example

We consider Hinkley's [20] real dataset. This dataset for Saint Paul, Minneapolis, is 30 consecutive rainfall values (in inches) in March over a period of 30 years. The observed values are: 0.77, 1.74, 0.81, 1.20, 1.95, 1.20, 0.47, 1.43, 3.37, 2.20, 3.00, 3.09, 1.51, 2.10, 0.52, 1.62, 1.31, 0.32, 0.59, 0.81, 2.81, 1.87, 1.18, 1.35, 4.75, 2.48, 0.96, 1.89, 0.90, 2.05.

Seo et al. [21], Azimi and Sarikhanbaglu [22], Torabi and Bagheri [23], Seo and Kang [15], and others evaluated this dataset. The exponentiated half logistic distribution suits the information well, see Seo and Kang [15]. Here, $n = 15$, $\alpha = 2$, $a = 0.77$, and $r_0 = 200$. The value of p_0 from equation (6) becomes 0.35. The value of μ from equation (3) is calculated as 1.7627 with test duration $t_0 = a\mu_0 = 0.77 \times 1.7627 = 1.3572$. The control limits under RGS are calculated as $UCL_1 = 2.52$, $UCL_2 = 2.79$, $LCL_1 = 0.8$, $LCL_2 = 0.57$ for the parameters $k_1 = 0.69$, $k_2 = 0.92$.
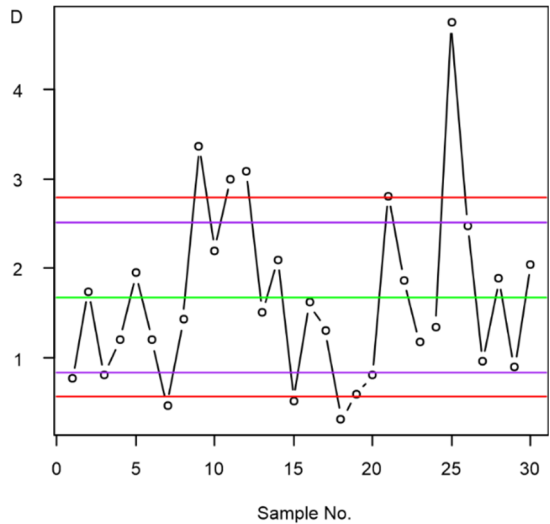
Fig. 1. Proposed RGS control chart for a real dataset

It can be seen from Fig. 1. That out of control signal is generated seven times while some points fall in the repetitive region.

3.6. Simulation study

Using simulated information, we address the implementation of the proposed control chart. The data is produced from an exponential half logistic distribution when

$\alpha = 3$ and $\sigma = 1$. Let $n = 25$, $a = 0.76$, and $r_0 = 370$. The process is declared to be in-control when $\alpha = 3$ and $\sigma = 1$. The value of p_0 from equation (6) is calculated as 0.3082, $\mu = 2.16$, the test duration is therefore calculated as $t_0 = a\mu_0 = 0.76 \times 2.16 = 1.64$. The value of $\bar{d} = \sum d_i/l = 6.6$ and $l = 20$. We find out the control limits coefficients $k_1 = 1.06173089$ and $k_2 = 1.131890$ and control limits are $UCL_1 = 8.94$, $UCL_2 = 9.09$, $LCL_1 = 4.25$, $LCL_2 = 4.10$, the amount of failures per subgroup is reported before the specified time.

Table 1. The simulated data

Subgroup	1	2	3	4	5	6	7	8	9	10
D	7	8	6	7	8	6	8	3	5	7
Subgroup	11	12	13	14	15	16	17	18	19	20
D	8	6	7	6	8	8	5	6	5	8
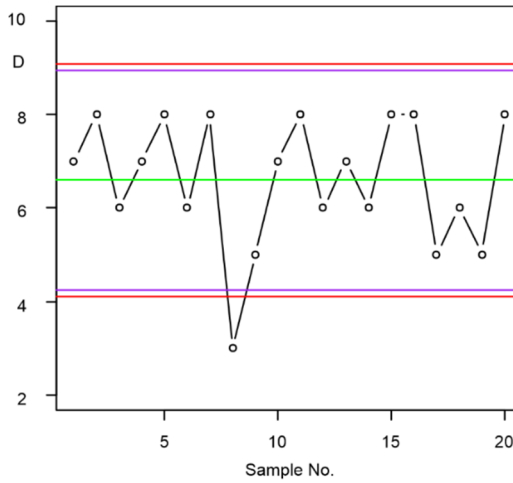
Fig. 2. The proposed RGS control chart for simulated data.

We can see from Fig. 2 that if the number of fail items in the subgroup is 5, 6, 7, or 8, the process is said to be in-control. The sampling will be repeated if the subgroup includes 9 failed items. If the number of failures is less than or equal to 4, the process is said to be out of control.

4. Design of the proposed control chart under MDS

Under the MDS scheme, the decision about the in-control or the out of control process is made, considering the results of the previous samples. If we select a sample from

the online process and post it on the control chart, then it may fall in any of three mutu-
ally exclusive states, i.e., in-control state, out of control state, or in the state in which
the decision depends on the previous samples. Following are the steps of np control
chart based on the MDS scheme.

Step 1. Choose from the production process a random sample of size n. Count failed
items D for a specified time $t = a\mu_0$, where μ_0 is the target mean when the process is in-
control and a is constant.

Step 2. If $LCL_2 \leq D \leq UCL_2$, declare the process as in-control. If $D > UCL_1$ or $D < LCL_1$,
declare the process as out of control. For all the other cases, repeat the process until ith
sample and declare the process as in-control if $LCL_2 \leq D \leq UCL_2$ for all i samples.

The method is stated to be controlled when $t = a\mu_0$, $\mu = \mu_0$. We express the scale
parameter σ in terms of μ using equation (3b), and then equation (1) can be rewritten as

$$p_0 = \left(\frac{1 - e^{-a\eta}}{1 + e^{-a\eta}} \right)^{\alpha} \tag{16}$$

4.1. Shift in scale and shape parameters

We are designing a chart for monitoring mean shift. Assume that the process mean
is shifted from $\mu = \mu_0$ and $\mu_1 = c\mu_0$, then equation (1) becomes

$$p_1 = \left(\frac{1 - e^{-a\eta/c}}{1 + e^{-a\eta/c}} \right)^{\alpha} \tag{17}$$

If the shape parameter is shifted from α_0 to $\alpha_1 = f\alpha_0$ (or the shape parameter is $\alpha = \alpha_1$),
then equation (1) becomes

$$p_2 = \left(\frac{1 - e^{-a\eta_1}}{1 + e^{-a\eta_1}} \right)^{f\alpha_0} \quad \text{where} \quad \eta_1 = \ln \frac{1 + 0.5^{1/f\alpha_0}}{1 - 0.5^{1/f\alpha_0}} \tag{18}$$

4.2. Shift in both shape and scale parameters

Suppose that the shape and scale of both parameters are shifted to $\alpha_1 = f\alpha_0$ and $\sigma_1 = c\sigma_0$
(now the shape parameter is α_1 and the scale parameter is σ_1). The likelihood that an
item is failed p_3 is specified as

$$p_3 = \left(\frac{1 - e^{-a\eta_1/c}}{1 + e^{-a\eta_1/c}}\right)^{f\alpha_0} \quad \text{where} \quad \eta_1 = \ln\frac{1 + 0.5^{1/f\alpha_0}}{1 - 0.5^{1/f\alpha_0}} \tag{19}$$

The probability that the process is stated in-control if the process is in-control is

$$P_{in} = P\left(LCL_2 \le D \le UCL_2\right) + \left(P\left(LCL_1 \le D \le LCL_2\right)\right.$$
$$\left. + P\left(UCL_2 \le D \le UCL_1\right)\right) \times \left(P\left(LCL_2 \le D \le UCL_2\right)\right)^i$$

$$\frac{P_{in}^0}{p_0} = P\left(d \ge LCL_2\right) + P\left(d \le UCL_2\right) + \left(P\left(d \ge LCL_1\right) + P\left(d \le LCL_2\right)\right.$$
$$\left. + P\left(d \ge UCL_2\right) + P\left(d \le UCL_1\right)\right) \times \left(P\left(d \ge LCL_2\right) + P\left(d \le UCL_2\right)\right)^i$$

$$\frac{P_{in}^0}{p_0} = \sum_{d=[LCL_2]+1}^{n}\binom{n}{d}p_0^d\left(1-p_0\right)^{n-d} + \sum_{d=0}^{[UCL_2]}\binom{n}{d}p_0^d\left(1-p_0\right)^{n-d}$$
$$+ \left[\sum_{d=[LCL_1]+1}^{n}\binom{n}{d}p_0^d\left(1-p_0\right)^{n-d} + \sum_{d=0}^{[LCL_2]}\binom{n}{d}p_0^d\left(1-p_0\right)^{n-d}\right.$$
$$+ \sum_{d=[UCL_2]+1}^{n}\binom{n}{d}p_0^d\left(1-p_0\right)^{n-d} + \sum_{d=0}^{[UCL_1]}\binom{n}{d}p_0^d\left(1-p_0\right)^{n-d}\Bigg]$$
$$\times \left[\sum_{d=[LCL_2]+1}^{n}\binom{n}{d}p_0^d\left(1-p_0\right)^{n-d} + \sum_{d=0}^{[UCL_2]}\binom{n}{d}p_0^d\left(1-p_0\right)^{n-d}\right]^i$$

$$\frac{P_{in}^0}{p_0} = \sum_{d=[LCL_2]+1}^{[UCL_2]}\binom{n}{d}p_0^d\left(1-p_0\right)^{n-d}$$
$$+ \left[\sum_{d=[LCL_1]+1}^{[LCL_2]}\binom{n}{d}p_0^d\left(1-p_0\right)^{n-d} + \sum_{d=[UCL_2]+1}^{[UCL_1]}\binom{n}{d}p_0^d\left(1-p_0\right)^{n-d}\right]$$
$$\times \left[\sum_{d=[LCL_2]+1}^{[UCL_2]}\binom{n}{d}p_0^d\left(1-p_0\right)^{n-d}\right]^i \tag{20}$$

Likewise, the probability that the process will be stated in-control when the process is shifted to μ_1 is specified as

$$\frac{P_{in}^1}{p_1} = \sum_{d=[LCL_2]+1}^{[UCL_2]} \binom{n}{d} p_1^d (1-p_1)^{n-d}$$

$$+ \left[\sum_{d=[LCL_1]+1}^{[LCL_2]} \binom{n}{d} p_1^d (1-p_1)^{n-d} + \sum_{d=[UCL_2]+1}^{[UCL_1]} \binom{n}{d} p_1^d (1-p_1)^{n-d} \right]$$

$$\times \left[\sum_{d=[LCL_2]+1}^{[UCL_2]} \binom{n}{d} p_1^d (1-p_1)^{n-d} \right]^i \tag{21}$$

Likewise, the probability when the shape parameter is shifted is specified as

$$\frac{P_{in}^2}{p_2} = \sum_{d=[LCL_2]+1}^{[UCL_2]} \binom{n}{d} p_2^d (1-p_2)^{n-d}$$

$$+ \left[\sum_{d=[LCL_1]+1}^{[LCL_2]} \binom{n}{d} p_2^d (1-p_2)^{(n-d)} + \sum_{d=[UCL_2]+1}^{[UCL_1]} \binom{n}{d} p_2^d (1-p_2)^{n-d} \right]$$

$$\times \left[\sum_{d=[LCL_2]+1}^{[UCL_2]} \binom{n}{d} p_2^d (1-p_2)^{n-d} \right]^i \tag{22}$$

Likewise, the probability when both shape and scale parameters are shifted is

$$\frac{P_{in}^3}{p_3} = \sum_{d=[LCL_2]+1}^{[UCL_2]} \binom{n}{d} p_3^d (1-p_3)^{n-d}$$

$$+ \left[\sum_{d=[LCL_1]+1}^{[LCL_2]} \binom{n}{d} p_3^d (1-p_3)^{n-d} + \sum_{d=[UCL_2]+1}^{[UCL_1]} \binom{n}{d} p_3^d (1-p_3)^{n-d} \right]$$

$$\times \left[\sum_{d=[LCL_2]+1}^{[UCL_2]} \binom{n}{d} p_3^d (1-p_3)^{n-d} \right]^i \tag{23}$$

The average run length (*ARL*) is used to evaluate the efficiency of a control chart. The in-control *ARL* and out of control *ARL* are given in equations (24), (25), (26), and (27), respectively.

$$ARL_0 = \frac{1}{1-P_{in}^0} \tag{24}$$

$$ARL_1 = \frac{1}{1-P_{in}^1} \tag{25}$$

$$ARL_2 = \frac{1}{1 - P_{in}^2} \qquad (26)$$

$$ARL_3 = \frac{1}{1 - P_{in}^3} \qquad (27)$$

4.3. Contrast of the proposed control chart

Tables A14–A16 show the *ARL* values for $n = 30$, $ARL_0 = 200, 300$, and 370 with $i = 1, 2, 3$ for shift in the scale parameter.

Tables A17 and A18 show the *ARL* values for $n = 30$, $ARL_0 = 300$, and 370 with $i = 1, 2, 3$ for shift in shape parameter, and for both shift parameters, respectively. It can be concluded from these tables that the *ARL* values decrease as the shift size decreases ($c = 1$ to 0.1).

The proposed MDS control chart based on EHLD is compared with the existing control chart presented by Aslam [11] at $ARL_0 = 300, 370$, $i = 1$ and $n = 30$, and the results are shown in Table A19. It can be seen that when $c = 0.99$, $\alpha = 1$ and $ARL_0 = 300$, the *ARL* value of the proposed control chart is 100.42, while it is 280.62 from the existing control chart. So, the proposed control is more responsive to identify a shift in the process as compared to the existing control chart.

The proposed control chart is also compared with the one proposed by Rao [6] at $ARL_0 = 200, 300, 370$, $i = 1, 2, 3$, and results are shown in Table A20. It can be seen that for $c = 0.9$, $\alpha = 1$ and $ARL_0 = 200$, the *ARL* value of the proposed control chart is 16.51, while it is 64.48 from the previous control chart. So, the proposed control is preferable in earlier detection of shifts.

Algorithm

Step 1. Set the values of α_0 (shape parameter), a (test termination ratio), and r_0 (in control target *ARL*).

Step 2. Set the values of n, k_1, k_2.

Step 3. Put the values of a, α_0 in equation (16) to find the failure probability (p_0).

Step 4. Discover the control limits of the proposed control chart, using the values of n, p_0, k_1 and k_2 in equations (4a)–(4d).

Step 5. The probability that the process is stated in-control if the process is actually in control, by replacing the values of p_0 and control limits in equations (18), and also use equation (24) to locate ARL_0.

Step 6. Check parameters n, k_1, and k_2 such that the in-control ARL_0 is as near as to the final value r_0.

Step 7. Continue Steps 4–6 for various combinations of n, k_1, and k_2 until you get an ARL_0 as near as to r_0.

Step 8. Replace the values n, k_1, and k_2 in equations (17)–(23), (25)–(27) for discovering the out of control ARL_1 for various shift values.

4.4. Illustrative example

We consider Hinkley's [20] real dataset. This dataset for Saint Paul, Minneapolis, is 30 consecutive rainfall values (in inches) in March over a period of 30 years. The observed values are: 0.77, 1.74, 0.81, 1.20, 1.95, 1.20, 0.47, 1.43, 3.37, 2.20, 3.00, 3.09, 1.51, 2.10, 0.52, 1.62, 1.31, 0.32, 0.59, 0.81, 2.81, 1.87, 1.18, 1.35, 4.75, 2.48, 0.96, 1.89, 0.90, 2.05.

Seo et al. [21], Azimi and Sarikhanbaglu [22], Torabi and Bagheri [23], Seo and Kang [15], and others evaluated this dataset. The exponentiated half logistic distribution suits the information well [15]. Here, $n = 30$, $\alpha = 2$, $i = 1$, $a = 0.32$, and $r_0 = 370$. The value of p_0 from equation (16) becomes 0.075. The value of μ from equation (3) is calculated as 1.7627 with test duration $t_0 = a\mu_0 = 0.32 \times 1.7627 = 0.564$. The control limits under MDS calculated are $UCL_1 = 3.15$, $UCL_2 = 2.64$ $LCL_1 = 0.21$, $LCL_2 = 0.72$ for the parameters $k_1 = 1.17$, $k_2 = 0.76$.
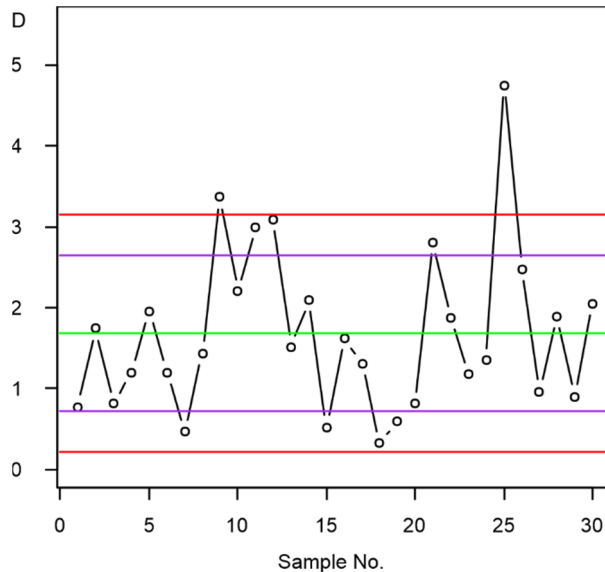
Fig. 3. The proposed MDS control chart for a real dataset

It can be seen from Fig. 3 that out of control signal is generated twice, while some points fall in the repetitive region.

4.5. Simulation study

In this study, using simulated information, we address the implementation of the proposed control chart. The data is produced from an exponential half logistic distribution when $\alpha = 3$ and $\sigma = 1$. Let $n = 30$, $a = 0.88$, and $r_0 = 370$. The process is declared to be in-control when $\alpha = 3$ and $\sigma = 1$. The value of p_0 from equation (16) is calculated as 0.4092, $\mu = 2.16$, the test duration is therefore calculated as $t_0 = a\mu_0 = 0.88 \times 2.16 = 1.90$. The value of $\bar{d} = \sum d_i / l = 12.275$ and $l = 40$. We find out the control limits coefficients $k_1 = 2.735302$, and $k_2 = 2.324929$, and control limits are $UCL_1 = 19.64$, $UCL_2 = 18.54$, $LCL_1 = 4.91$, $LCL_2 = 6.01$, the number of failures per subgroup is reported before the specified time. If the number of fail items in the subgroup is 7–18, the process is said to be in-control. The sampling will be repeated if the subgroup includes 5, 6 or 19 failed items. If the number of failures is less than or equal to 4, the process is stated to be out of control (Fig. 4). By comparing Figs. 4 and 5, we can conclude that the proposed control chart has the ability to detect a small shift in the process earlier than the existing control chart.

Table 2. The simulated data

Subgroup	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
D	11	11	15	11	13	13	9	11	16	14	14	13	17	19	16	8	16	15	11	11
Subgroup	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
D	13	12	11	13	12	10	6	13	10	12	11	15	11	10	15	7	11	14	12	11
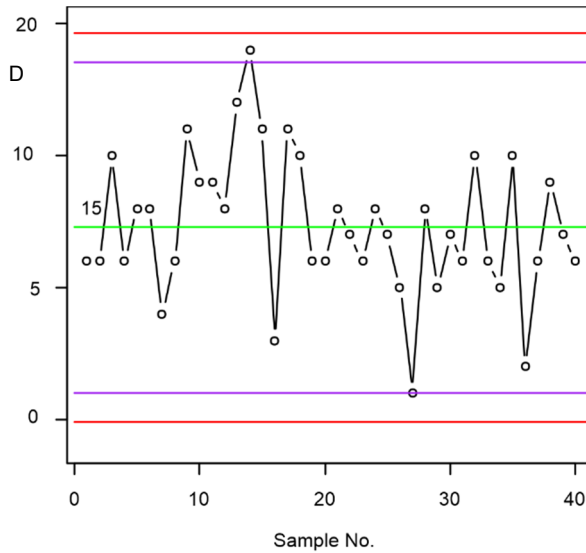
Fig. 4. The proposed MDS control chart for simulated data
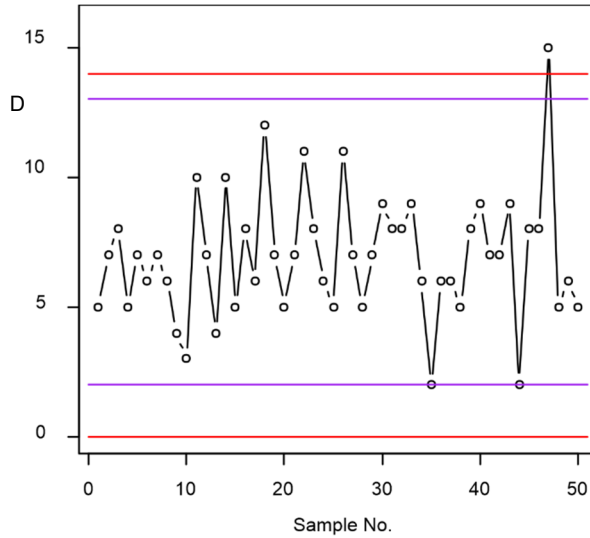
Fig. 5. Aslam's [11] chart for simulated data

5. Conclusion

We propose an attribute np control chart under RGS and MDS based on time truncated life tests for EHLD. The performance of the proposed control chart is evaluated in terms of ARL. The optimal parameters used for constructing the chart are determined such that the in-control ARL is as close as to the specified value of ARL. The performance of the proposed chart is compared with the performance of the chart developed by Jeyadurga et al. [12]. The control chart performance in RGS is also contrasted with the control chart presented by Rao [6], based on EHLD. Results are presented in the form of tables, using different scales and shape parameters. These tables show that the proposed chart identifies the change quicker than the existing chart. The efficiency of the proposed MDS control chart is compared to the control chart results by Aslam [11]. The results indicate that the proposed control chart is more responsive than the existing one to recognize a shift in the process. The efficiency of the proposed MDS control chart is also compared to Rao's [6] results and concluded in the form of tables using different ARL_0 values. It is concluded that the proposed control is more responsive to identify a shift in the process as compared to the existing control chart. For future work, the proposed control chart might be applied to other distributions.

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Appendix

Table A1. *ARLs* while shape parameter is shifted for specified sample sizes as $r_0 = 200$

f	$n = 10$ $\alpha = 1.5$ $a = 0.5512680$ $k_1 = 0{,}05528892$ $k_2 = 0.5023516$ $UCL_1 = 3.09, LCL_1 = 2.93$ $UCL_2 = 3.74, LCL_2 = 2.28$		$n = 15$ $\alpha = 2.0$ $a = 0.7989646$ $k_1 = 0.6462318$ $k_2 = 0.8760140$ $UCL_1 = 7.26, LCL_1 = 4.81$ $UCL_2 = 7.70, LCL_2 = 4.37$		$n = 20$ $\alpha = 2.5$ $a = 0.7397315$ $k_1 = 0.7845172$ $k_2 = 0.8709434$ $UCL_1 = 8.22, LCL_1 = 4.92$ $UCL_2 = 8.39, LCL_2 = 4.74$		$n = 25$ $\alpha = 3$ $a = 0.9177807$ $k_1 = 0.3937093$ $k_2 = 0.8649713$ $UCL_1 = 9.55, LCL_1 = 7.68$ $UCL_2 = 10.67, LCL_2 = 6.55$	
	ASN	*ARL*	*ASN*	*ARL*	*ASN*	*ARL*	*ASN*	*ARL*
1.0	19.95	200.42	21.52	200.79	26.32	200.22	53.62	200.83
0.9	19.19	12.47	21.43	42.05	26.25	37.21	53.29	53.94
0.8	18.32	5.95	21.32	21.46	26.17	18.37	52.92	29.86
0.7	17.35	3.77	21.19	13.58	26.07	11.37	32.51	20.09
0.6	16.33	2.72	21.04	9.52	25.95	7.84	52.08	14.86
0.5	15.30	2.13	20.87	7.11	25.80	5.78	51.62	11.63
0.4	14.31	1.76	20.67	5.55	25.62	4.47	51.14	9.46
0.3	13.41	1.52	20.46	4.48	25.40	3.58	50.63	7.91
0.2	12.61	1.35	20.22	3.72	25.14	2.95	50.10	6.76
0.1	11.93	1.24	19.96	3.15	24.84	2.49	49.55	5.88

Table A2. *ARLs* while the shape parameter is shifted for specified sample sizes as $r_0 = 300$ and $\alpha = 2$

f	$n = 10$ $a = 0.5937813$ $k_1 = 0.097755169$ $k_2 = 0.3993260$ $UCL_1 = 3.15, LCL_1 = 2.87$ $UCL_2 = 3.58, LCL_2 = 2.43$		$n = 15$ $a = 0.5822424$ $k_1 = 0.88124085$ $k_2 = 0.8820722$ $UCL_1 = 5.99, LCL_1 = 2.87$ $UCL_2 = 5.99, LCL_2 = 2.87$		$n = 20$ $a = 0.6472815$ $k_1 = 1.0221351$ $k_2 = 1.0785852$ $UCL_1 = 8.70, LCL_1 = 4.40$ $UCL_2 = 8.82, LCL_2 = 4.29$		$n = 25$ $a = 0.6912889$ $k_1 = 1.23808879$ $k_2 = 1.2789334$ $UCL_1 = 11.69, LCL_1 = 5.79$ $UCL_2 = 11.79, LCL_2 = 5.69$	
	ASN	*ARL*	*ASN*	*ARL*	*ASN*	*ARL*	*ASN*	*ARL*
1.0	19.97	300.32	21.44	300.51	26.32	300.92	29.55	300.48
0.9	19.27	13.68	21.17	15.05	26.23	29.03	29.64	41.37
0.8	18.46	6.50	20.83	7.13	26.10	13.05	29.76	19.05
0.7	17.57	4.12	20.43	4.46	25.94	7.65	29.88	11.16
0.6	16.65	2.98	19.96	3.17	25.72	5.08	29.10	7.31
0.5	15.71	2.33	19.44	2.43	25.42	3.66	30.07	5.14
0.4	14.81	1.93	18.86	1.96	25.04	2.79	30.06	3.80
0.3	13.96	1.66	18.26	1.66	24.57	2.22	29.95	2.93
0.2	13.19	1.47	17.66	1.44	24.04	1.85	29.72	2.35
0.1	12.51	1.34	17.09	1.30	23.46	1.58	29.36	1.94

Table A3. *ARLs* while shape parameter is shifted for specified sample sizes as $r_0 = 370$ and $\alpha = 2$

f	$n = 10$ $a = 0.9424639$ $k_1 = 0.52002428$ $k_2 = 0.559912$ $UCL_1 = 5.54, LCL_1 = 3.90$ $UCL_2 = 5.61, LCL_2 = 3.84$		$n = 15$ $a = 0.7967768$ $k_1 = 0.71566975$ $k_2 = 0.9685180$ $UCL_1 = 7.38, LCL_1 = 4.66$ $UCL_2 = 7.86, LCL_2 = 4.18$		$n = 20$ $a = 0.6466634$ $k_1 = 0.9532185$ $k_2 = 1.048110$ $UCL_1 = 8.55, LCL_1 = 4.55$ $UCL_2 = 8.75, LCL_2 = 4.35$		$n = 25$ $a = 0.7581569$ $k_1 = 1.2570453$ $k_2 = 1.3277907$ $UCL_1 = 12.62, LCL_1 = 6.51$ $UCL_2 = 12.79, LCL_2 = 6.34$	
	ASN	*ARL*	*ASN*	*ARL*	*ASN*	*ARL*	*ASN*	*ARL*
1.0	16.29	370.21	21.53	370.01	26.32	370.56	29.59	370.24
0.9	16.25	89.99	21.44	46.93	26.23	29.70	29.64	63.08
0.8	16.19	49.86	21.33	22.72	26.11	13.20	29.71	29.64
0.7	16.15	33.86	21.21	14.07	25.94	7.71	29.79	17.50
0.6	16.09	25.29	21.06	9.75	25.72	5.11	19.89	11.53
0.5	16.03	19.99	20.88	7.23	25.43	3.67	29.98	8.11
0.4	15.97	16.42	20.68	5.61	25.05	2.79	30.05	5.99
0.3	15.90	13.86	20.46	4.51	24.58	2.23	30.08	4.59
0.2	15.84	11.95	20.22	3.73	24.05	1.85	30.05	3.63
0.1	15.78	10.48	19.96	3.16	23.46	1.58	29.95	2.95

Table A4. *ARLs* while shape parameter is shifted for specified sample sizes as $r_0 = 370$ and $\alpha = 3$

f	$n = 10$ $a = 0.8008269$ $k_1 = 0.18693262$ $k_2 = 0.5685758$ $UCL_1 = 3.28, LCL_1 = 2.74$ $UCL_2 = 3.83, LCL_2 = 2.18$		$n = 15$ $a = 0.4902479$ $k_1 = 0.20165683$ $k_2 = 0.2444121$ $UCL_1 = 3.07, LCL_1 = 2.47$ $UCL_2 = 3.14, LCL_2 = 2.40$		$n = 20$ $a = 0.6632994$ $k_1 = 0.738043$ $k_2 = 0.8295848$ $UCL_1 = 6.42, LCL_1 = 3.56$ $UCL_2 = 6.59, LCL_2 = 3.38$		$n = 25$ $a = 0.7582928$ $k_1 = 1.2195140$ $k_2 = 1.2490227$ $UCL_1 = 9.88, LCL_1 = 4.37$ $UCL_2 = 9.94, LCL_2 = 4.31$	
	ASN	*ARL*	*ASN*	*ARL*	*ASN*	*ARL*	*ASN*	*ARL*
1.0	19.97	370.47	29.96	370.16	28.69	370.92	30.90	370.52
0.9	19.67	30.48	29.07	16.16	28.45	27.79	30.89	49.91
0.8	19.34	15.25	27.84	6.95	28.15	12.80	30.89	24.16
0.7	18.99	9.94	26.36	4.12	27.80	7.77	30.89	14.92
0.6	18.62	7.26	24.75	2.86	27.39	5.34	30.88	10.30
0.5	18.23	5.67	23.11	2.18	26.94	3.97	30.86	7.59
0.4	17.84	4.63	21.56	1.78	16.43	3.11	30.83	5.86
0.3	17.44	3.89	20.14	1.52	25.88	2.54	30.76	4.67
0.2	17.03	3.36	18.90	1.35	25.29	2.13	30.65	3.82
0.1	16.63	2.96	17.86	1.23	24.69	1.85	30.51	3.20

Table A5. *ARLs* while shape parameter is shifted for specified sample sizes as $r_0 = 300$ and $\alpha = 3$

f	$n = 10$ $a = 0.8011999$ $k_1 = 0.43761073$ $k_2 = 0.65635916$ $UCL_1 = 3.64, LCL_1 = 2.38$ $UCL_2 = 3.96, LCL_2 = 2.06$		$n = 15$ $a = 0.6557643$ $k_1 = 0.5344432$ $k_2 = 0.7220106$ $UCL_1 = 4.59, LCL_1 = 2.81$ $UCL_2 = 4.91, LCL_2 = 2.49$		$n = 20$ $a = 0.8727061$ $k_1 = 1.0366778$ $k_2 = 1.1551780$ $UCL_1 = 8.73, LCL_1 = 4.38$ $UCL_2 = 8.98, LCL_2 = 4.13$		$n = 25$ $a = 0.6611102$ $k_1 = 0.8529688$ $k_2 = 0.9595490$ $UCL_1 = 8.06, LCL_1 = 4.37$ $UCL_2 = 8.29, LCL_2 = 4.14$	
	ASN	ARL	ASN	ARL	ASN	ARL	ASN	ARL
1.0	19.97	300.76	24.39	300.84	26.32	300.02	32.98	300.03
0.9	19.67	29.92	23.91	17.53	26.29	79.29	32.86	44.87
0.8	19.34	15.12	23.39	8.52	26.26	43.24	32.72	19.65
0.7	18.99	9.88	22.81	5.46	26.23	28.65	32.54	11.17
0.6	18.62	7.24	22.19	3.96	26.19	20.86	32.31	7.24
0.5	18.23	5.66	21.56	3.08	26.15	16.07	32.03	5.09
0.4	17.84	4.62	20.91	2.52	26.10	12.86	31.68	3.81
0.3	17.43	3.89	20.26	2.13	26.05	10.58	31.27	2.98
0.2	17.03	3.36	19.63	1.86	25.99	8.89	30.78	2.42
0.1	16.62	2.96	19.01	1.65	25.94	7.61	30.23	2.02

Table A6. *ARLs* while scale parameter is shifted for specified sample sizes as $r_0 = 200$

c	$n = 10$ $\alpha = 1.5$ $a = 0.9500074$ $k_1 = 0.47716305$ $k_2 = 0.7500152$ $UCL_1 = 5.48, LCL_1 = 3.98$ $UCL_2 = 5.91, LCL_2 = 3.55$		$n = 15$ $\alpha = 2.0$ $a = 0.7703821$ $k_1 = 0.6959164$ $k_2 = 0.9184962$ $UCL_1 = 6.52, LCL_1 = 3.95$ $UCL_2 = 6.93, LCL_2 = 3.54$		$n = 20$ $\alpha = 2.5$ $a = 0.7660026$ $k_1 = 0.8074456$ $k_2 = 1.0804421$ $UCL_1 = 8.27, LCL_1 = 4.87$ $UCL_2 = 8.84, LCL_2 = 4.30$		$n = 25$ $\alpha = 3.0$ $a = 0.9847301$ $k_1 = 1.16254630$ $k_2 = 1.2692900$ $UCL_1 = 15.12, LCL_1 = 9.30$ $UCL_2 = 15.38, LCL_2 = 9.04$	
	ASN	ARL	ASN	ARL	ASN	ARL	ASN	ARL
1.0	16.28	200.31	21.49	200.19	26.32	200.00	29.67	200.08
0.9	14.99	4.33	20.39	4.30	25.61	4.43	30.17	5.75
0.8	13.36	1.97	18.60	1.82	23.71	1.68	28.95	1.65
0.7	11.78	1.32	16.65	1.22	21.33	1.13	26.07	1.08
0.6	10.66	1.09	15.40	1.04	20.17	1.01	25.06	1.00
0.5	10.13	1.01	15.03	1.00	20.00	1.00	25.00	1.00
0.4	10.01	1.00	15.00	1.00	20.00	1.00	25.00	1.00
0.3	10.00	1.00	15.00	1.00	20.00	1.00	25.00	1.00
0.2	10.00	1.00	15.00	1.00	20.00	1.00	25.00	1.00
0.1	10.00	1.00	15.00	1.00	20.00	1.00	25.00	1.00

Table A7. *ARLs* while scale parameter is shifted for specified sample sizes as $r_0 = 300$

c	n = 10 α = 1.5 a = 0.6523552 k_1 = 0.30734632 k_2 = 0.4879256 UCL_1 = 3.46, LCL_1 = 2.56 UCL_2 = 3.72, LCL_2 = 2.30		n = 15 α = 2.0 a = 0.9315071 k_1 = 0.8401192 k_2 = 0.9259327 UCL_1 = 8.47, LCL_1 = 5.23 UCL_2 = 8.63, LCL_2 = 5.06		n = 20 α = 2.5 a = 0.8560754 k_1 = 0.5869916 k_2 = 0.9236466 UCL_1 = 9.20, LCL_1 = 6.63 UCL_2 = 9.94, LCL_2 = 5.89		n = 25 α = 3.0 a = 0.9948439 k_1 = 1.3612363 k_2 = 1.4057785 UCL_1 = 15.81, LCL_1 = 8.99 UCL_2 = 15.92, LCL_2 = 8.89	
	ASN	ARL	ASN	ARL	ASN	ARL	ASN	ARL
1.0	19.97	303.99	21.55	300.35	31.45	300.19	28.68	300.89
0.9	17.79	4.53	20.69	5.49	27.20	3.05	29.85	4.84
0.8	15.18	2.08	18.88	1.97	23.30	1.41	28.68	1.56
0.7	12.82	1.39	16.76	1.23	20.85	1.07	25.92	1.07
0.6	11.14	1.13	15.41	1.04	20.07	1.00	25.05	1.00
0.5	10.28	1.03	15.03	1.00	20.00	1.00	25.00	1.00
0.4	10.02	1.00	15.00	1.00	20.00	1.00	25.00	1.00
0.3	10.00	1.00	15.00	1.00	20.00	1.00	25.00	1.00
0.2	10.00	1.00	15.00	1.00	20.00	1.00	25.00	1.00
0.1	10.00	1.00	15.00	1.00	20.00	1.00	25.00	1.00

Table A8. *ARLs* while scale parameter is shifted for specified sample sizes as $r_0 = 370$

c	n = 10 α = 1.5 a = 0.6521354 k_1 = 0.0482576 k_2 = 0.59935838 UCL_1 = 3.08, LCL_1 = 2.94 UCL_2 = 3.88, LCL_2 = 2.14		n = 15 α = 2.0 a = 0.8480571 k_1 = 0.7641154 k_2 = 0.9392819 UCL_1 = 7.47, LCL_1 = 4.57 UCL_2 = 7.81, LCL_2 = 4.24		n = 20 α = 2.5 a = 0.6590831 k_1 = 0.5823619 k_2 = 0.7271074 UCL_1 = 6.62, LCL_1 = 3.86 UCL_2 = 6.39, LCL_2 = 3.58		n = 25 α = 3.0 a = 0.7644676 k_1 = 1.06173089 k_2 = 1.131890 UCL_1 = 10.28, LCL_1 = 5.35 UCL_2 = 10.44, LCL_2 = 5.19	
	ASN	ARL	ASN	ARL	ASN	ARL	ASN	ARL
1.0	19.97	370.12	21.53	370.57	28.69	370.69	30.99	370.60
0.9	17.79	4.54	20.54	4.85	27.02	4.16	30.75	4.81
0.8	15.19	2.08	18.73	1.89	24.22	1.67	28.85	1.59
0.7	12.82	1.39	16.69	1.23	21.51	1.14	26.03	1.08
0.6	11.15	1.13	15.39	1.04	20.22	1.02	25.06	1.00
0.5	10.28	1.03	15.03	1.00	20.01	1.00	25.00	1.00
0.4	10.02	1.00	15.00	1.00	20.00	1.00	25.00	1.00
0.3	10.00	1.00	15.00	1.00	20.00	1.00	25.00	1.00
0.2	10.00	1.00	15.00	1.00	20.00	1.00	25.00	1.00
0.1	10.00	1.00	15.00	1.00	20.00	1.00	25.00	1.00

Table A9. *ARLs* while both shape and scale parameters are shifted for specified sample sizes as $r_0 = 200$

c	f	$n = 10$ $\alpha = 1.5$ $a = 0.3186226$ $k_1 = 0.4739333$ $k_2 = 0.6542452$ $UCL_1 = 5.48$ $LCL_1 = 3.98$ $UCL_2 = 5.76$ $LCL_2 = 3.69$		$n = 15$ $\alpha = 2.0$ $a = 0.4129923$ $k_1 = 0.7344990$ $k_2 = 0.8054793$ $UCL_1 = 8.28$ $LCL_1 = 5.45$ $UCL_2 = 8.42$ $LCL_2 = 5.31$		$n = 20$ $\alpha = 2.5$ $a = 0.95929883$ $k_1 = 0.607698$ $k_2 = 8363810$ $UCL_1 = 10.15$ $LCL_1 = 7.45$ $UCL_2 = 10.66$ $LCL_2 = 6.95$		$n = 25$ $\alpha = 3.0$ $a = 0.9702668$ $k_1 = 0.9089739$ $k_2 = 0.9865484$ $UCL_1 = 11.55$ $LCL_1 = 7.15$ $UCL_2 = 11.73$ $LCL_2 = 6.96$	
		ASN	ARL	ASN	ARL	ASN	ARL	ASN	ARL
1.0	0.1	16.28	200.01	21.54	200.12	31.64	200.14	35.72	200.00
0.9	0.9	13.36	1.91	18.27	1.61	29.99	9.62	34.22	9.88
0.8	0.8	14.71	3.33	19.66	2.50	27.46	3.25	32.69	3.63
0.7	0.7	16.07	17.18	20.86	5.95	25.34	1.96	30.80	2.19
0.4	0.4	16.05	21.79	20.96	7.89	22.24	1.23	28.09	1,34
0.3	0.3	14.72	3.58	20.06	3.31	22.26	1.23	28.14	1.23
0.2	0.2	13.65	2.17	19.37	2.34	22.99	1.35	28.89	1.50
0.1	0.1	13.32	1.95	19.42	2.39	25.14	1.89	30.82	2.20

Table A10. *ARLs* of the proposed control chart with the existing control chart
while shape parameter is shifted as $r_0 = 300$ and $\alpha = 2$

f	Proposed	Existing	Proposed	Existing	Proposed	Existing	Proposed	Existing
	$a = 0.582424$ $n = 15$ $k_1 = 0.88124085$ $k_2 = 0.8820722$ $UCL_1 = 5.99, LCL_1 = 2.87$ $UCL_2 = 5.99, LCL_2 = 2.87$		$a = 0.3389691$ $n = 15$ $k_1 = 1.369041$ $k_2 = 2.790004$ $UCL_1 = 9, LCL_1 = 3$ $UCL_2 = 12, LCL_2 = 1$		$a = 6912889$ $n = 25$ $k_1 = 1.23808879$ $k_2 = 1.2789334$ $UCL_1 = 11.69, LCL_1 = 5.79$ $UCL_2 = 11.79, LCL_2 = 5.69$		$a = 0.618004$ $n = 25$ $k_1 = 1.872718$ $k_2 = 2.695981$ $UCL_1 = 20, LCL_1 = 10$ $UCL_2 = 22, LCL_2 = 8$	
	ASN	ARL	ASN	ARL	ASN	ARL	ASN	ARL
1.0	21.44	300.51	16.92	302.19	29.55	300.48	25.88	300.01
0.9	21.17	15.05	19.95	190.22	29.64	41.37	26.35	116.59
0.8	20.83	7.12	17.47	94.14	29.76	19.05	27.72	37.70
0.7	20.43	4.46	18.62	43.09	29.88	11.16	30.37	12.56
0.6	19.96	3.17	20.70	19.02	29.10	7.31	34.19	4.57
0.5	19.44	2.43	23.98	8.19	30.07	5.14	26.69	2.01
0.4	18.86	1.96	28.21	3.57	30.06	3.80	33.90	1.24
0.3	18.26	1.66	30.54	1.76	29.95	2.93	28.45	1.04
0.2	17.66	1.44	26.29	1.15	29.72	2.35	25.50	1.00
0.1	17.09	1.30	18.73	1.01	29.36	1.94	25.01	1.00

Table A11. *ARLs of the proposed control chart with the existing control chart while shape parameter is shifted as $r_0 = 370$ and $\alpha = 3$*

f	Proposed	Existing	Proposed	Existing	Proposed	Existing	Proposed	Existing
	$a = 0.4902479$ $n = 15$ $k_1 = 0.20165683$ $k_2 = 0.2444121$ $UCL_1 = 3.07, LCL_1 = 2.47$ $UCL_2 = 3.14, LCL_2 = 2.40$		$a = 1.029771$ $n = 15$ $k_1 = 1.890701$ $k_2 = 3.0315$ $UCL_1 = 14, LCL_1 = 7$ $UCL_2 = 16, LCL_2 = 5$		$a = 0.6632994$ $n = 20$ $k_1 = 0.738043$ $k_2 = 0.8295848$ $UCL_1 = 6.42, LCL_1 = 3.56$ $UCL_2 = 6.59, LCL_2 = 3.38$		$a = 0.5122036$ $n = 20$ $k_1 = 1.830755$ $k_2 = 2.725221$ $UCL_1 = 14, LCL_1 = 3$ $UCL_2 = 16, LCL_2 = 3$	
	ASN	*ARL*	*ASN*	*ARL*	*ASN*	*ARL*	*ASN*	*ARL*
1.0	29.96	370.16	15.68	370.06	28.69	370.92	20.81	370.01
0.9	29.07	16.16	16.20	130.42	28.45	27.79	21.06	223.80
0.8	27.84	6.95	17.23	46.69	28.15	12.80	21.82	92.05
0.7	26.36	4.12	19.03	17.00	27.80	7.77	23.33	35.32
0.6	24.75	2.86	21.74	6.43	27.39	5.34	25.97	13.51
0.5	23.11	2.18	24.41	2.70	26.94	3.97	29.87	5.30
0.4	21.56	1.78	24.10	1.46	26.43	3.11	33.38	2.31
0.3	20.14	1.52	20.06	1.09	25.88	2.54	31.85	1.32
0.2	18.90	1.35	16.37	1.00	25.29	2.13	25.41	1.05
0.1	17.86	1.23	15.07	1.00	24.69	1.85	20.79	1.00

Table A12. *ARLs of the proposed control chart with the existing control chart while shape parameter is shifted as $r_0 = 370$ and $\alpha = 2$*

f	Proposed	Existing	Proposed	Existing	Proposed	Existing	Proposed	Existing
	$a = 0.6466634$ $n = 20$ $k_1 = 0.9532185$ $k_2 = 1.048110$ $UCL_1 = 8.55, LCL_1 = 4.55$ $UCL_2 = 8.75, LCL_2 = 4.35$		$a = 0.1491$ $n = 38$ $k = 3.1600$ $UCL = 36, LCL = 20$		$a = 0.7581569$ $n = 25$ $k_1 = 1.2570453$ $k_2 = 1.3277907$ $UCL_1 = 12.62, LCL_1 = 6.51$ $UCL_2 = 12.79, LCL_2 = 6.34$		$a = 0.703$ $n = 47$ $k = 3.0885$ $UCL = 23, LCL = 4$	
	ARL							
1.0	370.56		370.04		370.24		370.01	
0.9	29.70		238.54		63.08		247.22	
0.8	13.20		144.36		29.64		154.76	
0.7	7.71		83.31		17.50		93.13	
0.6	5.11		46.17		11.53		54.52	
0.5	3.67		24.68		8.11		31.22	
0.4	2.79		12.80		5.99		17.54	
0.3	2.23		6.56		4.59		9.73	
0.2	1.85		3.47		3.63		5.41	
0.1	1.58		2.05		2.95		3.13	

Table A13. *ARLs* of the proposed control chart with the existing control chart
while scale parameter is shifted as $r_0 = 370$

c	Proposed	Existing	Proposed	Existing	Proposed	Existing	Proposed	Existing
	$a = 0.6590831$ $n = 20$ $\alpha = 2.5$ $k_1 = 0.9532185$ $k_2 = 1.048110$		$a = 0.841$ $n = 23$ $\alpha = 2.5$ $k = 3.1600$		$a = 0.7644676$ $n = 23$ $\alpha = 3$ $k_1 = 1.2570453$ $k_2 = 1.3277907$		$a = 0.957$ $n = 46$ $\alpha = 3$ $k = 3.0885$	
	$UCL_1 = 6.62, LCL_1 = 3.86$ $UCL_2 = 6.39, LCL_2 = 3.58$		$UCL = 15, LCL = 1$		$UCL_1 = 10.28, LCL_1 = 5.35$ $UCL_2 = 10.44, LCL_2 = 5.19$		$UCL = 31, LCL = 11$	
	ARL							
1.0	370.69		37.02		370.60		370.00	
0.9	4.16		59.92		4.81		34.96	
0.8	1.67		11.16		1.59		4.01	
0.7	1.14		2.99		1.08		1.30	
0.6	1.02		1.34		1.00		1.01	
0.5	1.00		1.03		1.00		1.00	
0.4	1.00		1.00		1.00		1.00	
0.3	1.00		1.00		1.00		1.00	
0.2	1.00		1.00		1.00		1.00	
0.1	1.00		1.00		1.00		1.00	

Table A14. *ARL* values of the proposed control chart for the shifted scale parameter
as $ARL_0 = 200$ and $n = 30$

i	1			2			3		
k_1	1.21	3.83	3.91	3.13	1.72	2.87	3.24	3.44	2.81
k_2	1.15	3.49	2.94	2.91	1.55	2.42	2.73	2.75	2.35
α	1	2	3	1	2	3	1	2	3
a	0.35	0.27	0.72	0.29	0.73	0.96	0.21	0.96	0.87
c	ARL								
1	200.17	200.04	200.05	200.01	200.03	200.07	200.01	200.05	200.02
0.99	114.62	182.13	152.94	185.17	96.87	124.76	187.19	151.90	128.95
0.95	36.50	124.64	59.90	135.48	23.40	40.02	142.99	63.27	42.76
0.93	25.48	102.85	39.09	115.61	14.85	26.13	124.68	43.35	28.01
0.91	18.88	84.75	5.99	98.50	10.11	17.71	108.52	30.33	18.99
0.9	16.51	76.89	21.33	90.87	8.50	14.73	101.18	25.54	15.79
0.88	12.92	63.22	14.58	77.24	6.20	10.37	87.84	18.33	11.09
0.85	9.34	47.03	8.55	60.37	4.11	6.40	70.83	11.52	6.81
0.8	5.90	28.59	3.97	39.75	2.39	3.23	49.06	5.86	3.39
0.75	3.99	17.33	2.21	25.98	1.63	1.92	33.63	3.39	1.98
0.7	2.85	10.54	1.51	16.88	1.27	1.35	22.83	2.19	1.37
0.6	1.67	4.06	1.07	7.10	1.03	1.02	10.26	1.20	1.03
0.5	1.19	1.82	1.00	3.10	1.00	1.00	4.57	1.01	1.00
0.4	1.02	1.12	1.00	1.56	1.00	1.00	2.14	1.00	1.00
0.3	1.00	1.00	1.00	1.06	1.00	1.00	1.22	1.00	1.00
0.2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table A15. *ARL* values of the proposed control chart for shifted scale parameter as $ARL_0 = 300$ and $n = 30$

i	1			2			3		
k_1	3.38	3.09	2.74	1.19	2.73	2.78	3.38	2.69	3.60
k_2	0.12	2.82	2.21	1.18	2.45	2.39	2.95	2.28	3.09
α	1	2	3	1	2	3	1	2	3
a	0.02	0.77	0.88	0.39	0.79	0.82	0.24	0.94	0.52
c					ARL				
1	300.10	300.17	300.22	300.26	300.10	300.01	300.07	300.02	300.50
0.99	100.42	221.96	133.85	146.68	192.57	175.88	278.81	177.36	250.52
0.95	26.82	90.37	34.42	41.19	67.14	53.03	206.88	56.84	121.92
0.93	19.43	62.16	22.29	28.26	45.69	34.22	177.72	38.20	85.54
0.91	15.15	43.67	15.18	20.69	32.19	22.93	152.400	26.70	60.31
0.9	13.62	36.81	12.69	17.99	27.24	18.96	141.03	22.54	50.75
0.88	11.29	26.39	9.04	13.96	19.76	13.18	120.60	16.29	36.10
0.85	8.93	16.38	5.69	9.97	12.52	7.96	95.05	10.32	21.97
0.8	6.54	7.84	2.96	6.18	6.26	3.84	63.34	5.21	10.08
0.75	5.08	4.11	1.81	4.11	3.43	2.16	41.74	2.92	5.00
0.7	4.10	2.40	1.30	2.89	2.10	1.45	27.23	1.86	2.76
0.6	2.88	1.22	1.01	1.68	1.17	1.03	11.32	1.12	1.27
0.5	2.16	1.01	1.00	1.19	1.01	1.00	4.69	1.00	1.01
0.4	1.68	1.00	1.00	1.03	1.00	1.00	2.09	1.00	1.00
0.3	1.37	1.00	1.00	1.00	1.00	1.00	1.18	1.00	1.00
0.2	1.15	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.1	1.02	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table A16. *ARL* values of the proposed control chart for shifted scale parameter as $ARL_0 = 370$ and $n = 30$

i	1			2			3		
k_1	2.95	1.48	3.83	1.56	3.95	3.78	2.57	3.26	2.73
k_2	2.43	1.32	2.86	1.51	3.02	3.49	2.10	2.61	2.32
α	1	2	3	1	2	3	1	2	3
a	0.69	0.84	0.71	0.92	0.87	0.39	0.51	0.77	0.88
c					ARL				
1	370.37	370.26	370.28	370.03	370.13	370.05	370.01	370.16	370.14
0.99	262.69	92.25	264.98	208.03	278.90	318.39	192.94	261.80	148.44
0.95	111.27	17.17	94.12	55.05	110.81	174.32	61.81	100.37	36.14
0.93	81.88	10.84	60.28	35.11	73.65	129.01	44.44	68.35	23.28
0.91	62.34	7.46	39.50	23.89	49.79	95.54	33.84	47.71	15.81
0.9	54.85	6.32	32.20	20.06	41.15	82.25	29.94	40.11	13.19
0.88	42.95	4.69	21.70	14.52	28.39	61.03	23.90	28.63	9.38
0.85	30.32	3.22	12.44	9.42	16.72	39.17	17.61	17.64	5.88
0.8	17.45	1.98	5.50	5.09	7.54	19.02	11.13	8.35	3.03
0.75	10.29	1.42	2.87	3.09	3.91	9.56	7.28	4.32	1.83
0.7	6.21	1.16	1.79	2.07	2.38	5.06	4.87	2.49	1.32
0.6	2.55	1.01	1.10	1.25	1.31	1.83	2.37	1.24	1.02
0.5	1.35	1.00	1.00	1.03	1.02	1.10	1.38	1.01	1.00
0.4	1.03	1.00	1.00	1.00	1.00	1.00	1.05	1.00	1.00
0.3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table A17. *ARL* values of the proposed control chart for shifted shape parameter
as $ARL_0 = 300$ and $n = 30$

i	1			2			3		
k_1	3.86	2.72	3.46	3.48	2.85	3.29	1.52	1.36	3.53
k_2	2.98	2.30	2.93	2.68	2.47	2.93	1.51	1.31	2.49
α	1	2	3	1	2	3	1	2	3
a	0.82	0.79	0.97	0.95	0.44	0.83	0.43	0.23	0.94
f					*ARL*				
1	300.14	300.07	300.01	300.07	300.00	300.05	300.03	300.05	300.36
0.99	292.91	275.64	298.38	295.73	242.27	293.41	178.81	90.14	292.73
0.95	265.98	206.08	292.00	279.21	131.88	268.44	62.63	21.60	265.46
0.93	253.60	182.05	288.88	271.42	105.11	256.87	45.39	15.04	253.50
0.91	241.87	162.48	285.79	263.93	86.11	245.85	34.80	11.31	242.47
0.9	236.23	154.00	284.27	260.29	78.53	240.53	30.92	10.00	237.28
0.88	225.38	139.10	281.24	253.21	66.06	230.28	24.93	8.03	227.47
0.85	210.09	120.78	276.79	243.05	52.08	215.80	18.79	6.07	214.05
0.8	186.96	97.68	269.56	227.26	36.38	193.83	12.60	4.18	194.59
0.75	166.38	80.66	262.58	212.75	26.17	174.27	8.96	3.11	178.05
0.7	147.99	67.58	255.83	199.35	19.19	156.82	6.64	2.44	163.83
0.6	116.69	48.87	243.04	175.44	10.77	127.34	4.00	1.70	140.64
0.5	91.41	36.26	231.12	154.73	6.36	103.76	2.66	1.33	122.58
0.4	71.01	27.36	220.03	136.70	3.98	84.84	1.93	1.14	108.14
0.3	54.76	20.92	209.69	121.02	2.65	69.60	1.51	1.05	96.32
0.2	42.09	16.18	200.04	107.52	1.90	57.30	1.26	1.01	86.48
0.1	32.44	12.65	191.01	96.09	1.46	47.33	1.12	1.00	78.15

Table A18. *ARL* values of the proposed control chart for shifted scale and shape parameters
as $ARL_0 = 370$ and $n = 30$

i		1			2			3		
k_1		3.93	3.13	3.36	3.00	3.95	3.28	2.68	3.81	3.15
k_2		2.93	2.68	3.02	2.51	2.97	2.89	2.12	2.91	2.75
α		1	2	3	1	2	3	1	2	3
a		0.67	0.56	0.74	0.73	0.70	0.75	0.44	0.77	0.93
c	f					*ARL*				
1	1.0	370.01	370.11	370.01	370.00	370.08	370.12	370.08	370.28	370.59
0.9	0.2	11.70	4.08	25.24	15.00	11.09	24.56	1.75	19.07	35.78
0.8	0.3	10.95	4.44	21.89	12.88	10.05	21.03	1.98	15.46	59.83
0.7	0.4	7.78	3.97	14.69	8.39	7.01	13.95	2.01	9.41	28.13
0.6	0.5	4.32	2.86	7.54	4.29	3.87	7.12	1.79	4.52	10.03
0.5	0.6	2.15	1.75	3.13	2.00	1.92	2.97	1.42	1.97	3.01
0.4	0.7	1.24	1.12	1.34	1.15	1.08	1.31	1.12	1.05	1.17
0.3	0.8	1.00	1.00	1.00	1.00	1.00	1.00	1.01	1.00	1.00
0.2	0.9	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.1	1.0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table A19**. Comparison in *ARLs* of the proposed control chart and the existing chart at *ARL$_0$* = 300, 370, *i* = 1, and *n* = 30

ARL$_0$	300						370					
α	1		2		3		1		2		3	
c	Pro	Exi	Pro	Exi	Pro	Exi	Pro	Exi	Pro	Exi	Pro	Exi
1	300.10	300.12	300.17	300.36	300.22	300.03	370.37	370.0	370.26	370.0	370.28	370.03
0.99	100.42	280.62	221.96	255.62	133.85	243.42	262.69	351.75	92.25	326.03	264.98	322.11
0.95	26.82	204.05	90.37	131.96	34.42	88.14	111.27	260.99	17.17	166.03	94.12	158.25
0.93	19.43	170.13	62.16	94.21	22.29	51.12	81.88	214.54	10.84	112.28	60.28	106.07
0.91	15.15	140.41	43.67	67.13	15.18	29.75	62.34	172.37	7.46	74.90	39.50	70.40
0.9	13.62	127.17	36.81	56.65	12.96	22.80	54.85	153.46	6.32	61.04	32.20	57.27
0.88	11.29	103.85	26.39	40.37	9.04	13.62	42.95	120.34	4.69	40.52	21.70	37.91
0.85	8.93	75.97	16.34	24.40	5.69	6.67	30.32	81.96	3.22	22.10	12.44	20.56
0.8	6.54	44.50	7.84	10.86	2.96	2.54	17.45	42.06	1.98	8.52	5.50	7.80
0.75	5.08	25.84	4.11	5.18	1.81	1.39	10.29	21.46	1.42	3.73	2.87	3.35
0.7	4.10	14.99	2.40	2.75	1.30	1.07	6.21	11.16	1.16	1.97	1.79	1.77
0.6	2.88	5.22	1.22	1.24	1.01	1.00	2.55	3.49	1.01	1.08	1.10	1.04
0.5	2.16	2.12	1.01	1.01	1.00	1.00	1.35	1.53	1.00	1.00	1.00	1.00
0.4	1.68	1.20	1.00	1.00	1.00	1.00	1.03	1.06	1.00	1.00	1.00	1.00
0.3	1.37	1.01	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.2	1.15	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.1	1.02	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Pro – stands for the proposed control charts, Exi – stands for the existing control chart.

Table A20. Comparison in *ARL*s of the proposed control chart and the existing chart (single sampling plan) at *ARL$_0$* = 200, 300, 370

	ARL$_0$ = 200, *i* = 1				ARL$_0$ = 300, *i* = 2				ARL$_0$ = 370, *i* = 3			
α	1		3		1		3		2		3	
c	Pro	Exi	Pro	Exi	Pro	Exi	Pro	Exi	Pro	Exi	Pro	Exi
1	200.17	200.01	200.05	200.00	300.26	300.00	300.01	300.00	370.16	370.00	370.14	370.00
0.9	16.51	64.48	21.33	23.28	17.99	175.23	18.96	21.66	40.11	60.94	13.19	34.96
0.8	5.90	15.02	3.97	3.16	6.18	45.01	3.84	2.92	8.35	11.01	3.03	4.01
0.7	2.85	4.21	1.51	1.21	2.89	11.51	1.45	1.16	2.49	2.87	1.32	1.30
0.6	1.67	1.68	1.07	1.00	1.68	3.51	1.03	1.00	1.24	1.29	1.02	1.0
0.5	1.19	1.08	1.00	1.00	1.19	1.50	1.00	1.00	1.01	1.01	1.00	1.00
0.4	1.02	1.00	1.00	1.00	1.03	1.04	1.00	1.00	1.00	1.00	1.00	1.00
0.3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Pro – stands for the proposed control charts, Exi – stands for the existing control chart.