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MODELLING OF THE COORDINATE MEASURING SYSTEMS ACCURACY

Coordinate metrology determines nowadays the relevant directions of development in automated measuring systems and quality management in the field of machine industry. The essential problem of coordinate measuring technique application is the issue of accuracy assessment of performed measurements. This paper describes the development of coordinate systems modelling as a new field of assessing the accuracy of measurements carried out in a quasi-real time. The practical solutions of the so-called virtual machines and virtual measuring systems were described along with the results of their evaluation and validation methodology based on a comparison of obtained results with the results produced by the typical methods of coordinate measurements accuracy assessment.

1. INTRODUCTION

The assessment of dimensional compliance with geometrical product specification (GPS) is a crucial task for the quality control. The wrong decisions may cause product complaints and what is connected to this, large financial losses for the companies. The proper estimation of measurement accuracy is important here, as the bare result given without its accuracy estimation is useless from the practical point of view. Also the accuracies assessed in wrong way may be the cause for wrong decisions during assessing of the compliance with the GPS requirements. Overestimated uncertainty (which may be regarded as a quantitative estimation of measurement accuracy) may be the reason for rejecting a properly manufactured parts, while the underestimated one, may lead to acceptance of faulty products.

Accuracy assessment of measurements done using coordinate measuring techniques (CMT) is particularly difficult and not always straightforward [7],[8],[9]. Therefore, users of this technique, as well as manufacturers of these measuring systems often overlook the problem of measurement accuracy giving in exchange the accuracy of measuring devices. This accuracy is determined for all measuring tasks as if they were a measurements of distance and is usually given as the maximum permissible error (MPE) of the measuring

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system. In described case, the accuracy assessment is significantly different from the real accuracy of considered task, and as mentioned above, may lead to bad decisions in the determination of compliance with the specifications of the product [7],[8],[9]. Moreover, the methodology of accuracy assessment based on the usage of MPE is not consistent with the coordinate measuring technique nature, because it brings the CMT to measurements of distances while its nature may be described as measuring the values of coordinates of single measuring points.

Therefore, from the practical point of view, it is extremely important to implement new, correct and metrologically validated methods. The ones that are currently in use, are usually difficult and require pretty much knowledge and experience in the field of measurement. Since they require multiple repetitions of measurements, they are also hugely time-consuming and cost-intensive (it is also one of the reasons for simplifying the measurement accuracy assessment and replacing it with the accuracy of measuring system). From couple of years, new trend in accuracy estimation can be seen and is connected with evolution of simulative methods [1],[5],[6]. These methods are implemented by creation of so-called virtual measuring machines which are used for on-line accuracy assessment [11],[12],[13],[14]. On-line means here that the accuracy is determined during the measurements are performed, and it is possible to get its result given with the corresponding accuracy instantly after the measurements. Simulative methods are as far, the most accurate [15],[16],[17] because they are based on the idea of measuring point reproducibility, which makes them consistent with coordinate measuring technique nature.

In next few chapters the practical applications of virtual machines for different measuring systems (coordinate measuring machines and articulated arms) are presented.

2. PRACTICAL APPLICATIONS OF COORDINATE MEASURING SYSTEMS MODELLING

This chapter presents examples of virtual models of the coordinate measuring systems that were developed in Laboratory of Coordinate Metrology (LCM) at Cracow University of Technology.

2.1. VIRTUAL CMM BUILT USING NEURAL NETWORKS

Neuro CMM PK is a virtual CMM model that uses artificial neural networks for the simulation of errors of measuring machine. The functioning of artificial neural networks is based on the learning process. It is very important because the coordinate measurement process is complex and difficult, and its modelling requires construction of complicated mathematical models. Here, the strict mathematical model was partially replaced by the implementation of neural networks and learning data for them, gathered experimentally. Prof. Sładek has formulated a number of assumptions that motivate the use of the above method [7],[8],[9]:

- rejection of analysis of the sources of CMM errors and focusing only on the identification and location of the error in the selected area of the measuring volume of machine,
- possibility of model creation basing on set of errors (deviations) in chosen points,
- finite and possibly small number of data points required for the construction of the CMM model,
- usage of standards to create the grid of reference points,
- the possibility of prediction on the (expected) errors occurring outside the reference points based solely on a small set of results of standard measurements.

As for the majority of CMM virtual models, also this one is based on determination and modelling of kinematic system errors and probe head errors. The kinematic system errors were determined in reference points (arranged in regular 3d grid in CMM's measuring volume) by measurements of ball plate standard (Fig. 1).

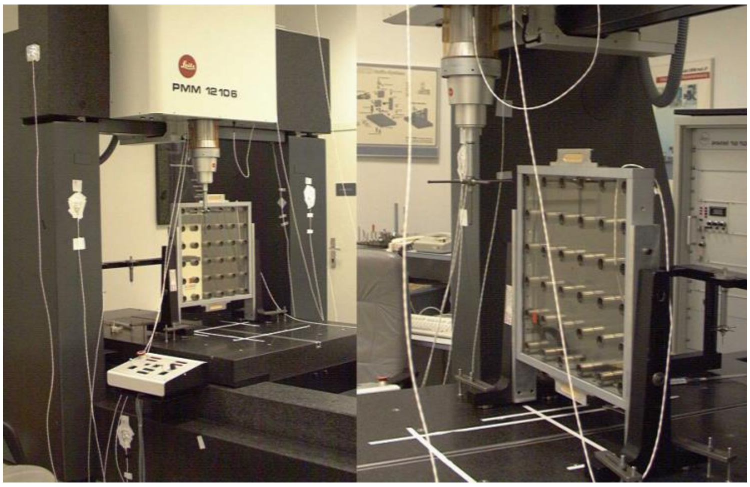


Fig. 1. Measurement of a hole plate standard [7]

The measurements of the plate were done in five positions parallel to the XY plane and the four positions parallel to the YZ and ZX planes. The difference between the coordinates of the holes/balls obtained during calibration and those received as a result of measurement on CMM gives spatial grid of errors, which is a base for the functioning of the module responsible for kinematic errors simulation. Measurements of the individual holes are made on both sides of the plate. Measurements are done using following strategy: starting from the bottom, from hole number 1 through 5 and then the next line of holes, which is measured in the opposite direction, and so on until the last hole (the 25-hole-plate was used for measurements described here). Each hole was measured as a circle using 4 points. Thanks to this procedure 325 reference points were obtained and taken as nodes of grid. Nominal coordinates of hole centres are an input for the learning set, while the errors obtained by measurement are an output. After many tests, it was decided that for the simulation of the systematic errors of the CMM's kinematic system the best solution would be a network made up of three layers, working on the principle of backpropagation.

All of the artificial neural networks presented here were made in NeuroShell v4.0 software and they all have three layers but differs regarding the number of points in each layer [7],[8],[9].

Data needed for simulation of probe head errors were also gathered experimentally. The spherical standard and the standard ring, both with nominal diameter close to 25 mm, were measured using all of the 5 styli (Fig. 2). The standard elements were measured using 64 points. The Probe Error Function (PEF), which was described in [2],[7],[8],[10], was adapted for description of probe head errors. The results of presented measurements were the base for creating 10 neural networks for simulation of systematic errors (5 for internal and 5 for external measurements) and 10 neural networks for simulation of random errors. For simulation of systematic error the value of error was determined by taking the mean value of 32 repetitions for each of the 64 positions of the probe head (described in polar system). 5 sets of data consisting of 64 pairs: angle - deviation (which is the value of PEF) were obtained for internal measurements, and the corresponding 5 sets for external measurements. For the random errors, the experiment were the same as for systematic errors, but this time, for all of 64 positions the probability distribution were determined. It turned out to be the most similar to a normal distribution. Then, a simulator based on the Monte Carlo method is used for generating the normal distribution for all positions. Concluding, the operation of a random error simulator is based on the 64 reference points assigned to the stylus (each of 5 styli) and the probability distributions obtained for them.

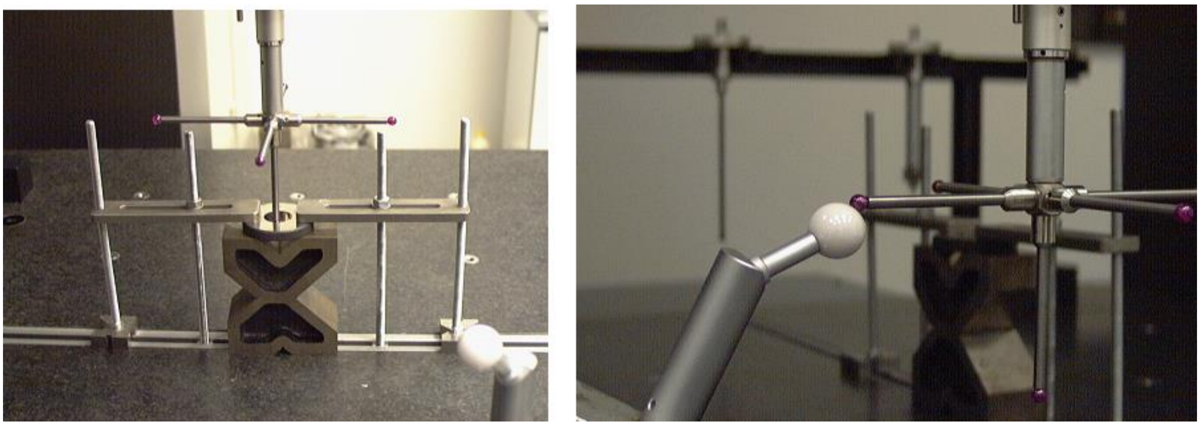


Fig. 2. Identification of the probe head systematic and random errors - measurements of the spherical standard and the standard ring [7]

Basing on presented neural networks and the data from the real measurements (coordinates of measuring points and approach vectors assigned to them), the simulation of measurement is performed in QUINDOS software. It is repeated 32 times, resulting in a set of parameters or relations between simulated features, as well as the standard uncertainty corresponding to them.

In order to check the proper functioning of the virtual CMM the measurements of spherical and cylindrical (internal and external cylinder) standards were performed. The example of obtained results is given in Fig. 3 (for measurements of standard ring diameter).

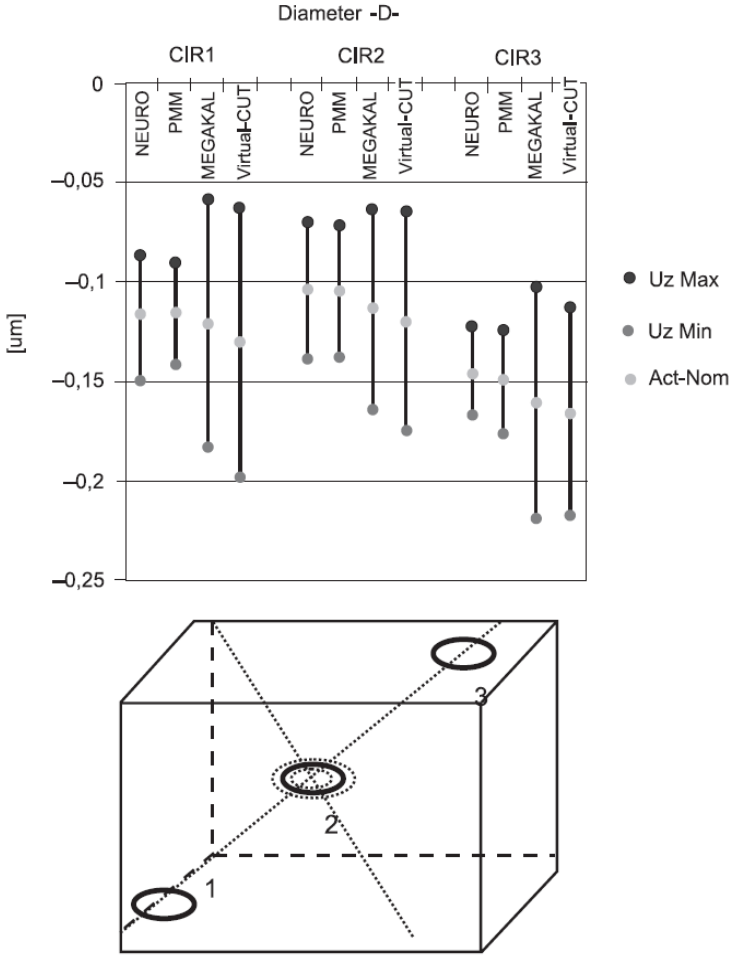


Fig. 3. Results of the Virtual Neuro CMM model verification based on comparative measurements: Neuro – tested Virtual Neuro CMM model, PMM – the results of measurements on real CMM, MegaKal – virtual CMM model developed in PTB, Virtual CUT – other virtual CMM model developed in LCM [7]

2.2. VIRTUAL MODEL OF ARTICULATED ARM COORDINATE MEASURING MACHINE

The implementation of virtual models of coordinate measuring systems is not limited only to CMMs. The area of its implementation was recently extended by an articulated arm coordinate measuring machines (AACMMs), also the research on preparing the virtual model for laser tracker system is currently run in PTB.

The virtual model for AACMM is built using the kinematic description of this device given in the form of forward kinematics task expressed in Denavit-Hartenberg convention. This convention associates a local coordinate system with every joint of the machine. Thus, the position and orientation of the end effecter of CMA are to be determined through mentioned task followed by a string of transformations of adjacent coordinate systems. Understanding the processes of the metrological model, as well as identifying any possible errors affecting the accuracy of measurement allows for simulating multiple measurements with the application of the Monte Carlo method [4]. The functioning of virtual AACMM may be described by 7 main stages depicted on Fig. 4.

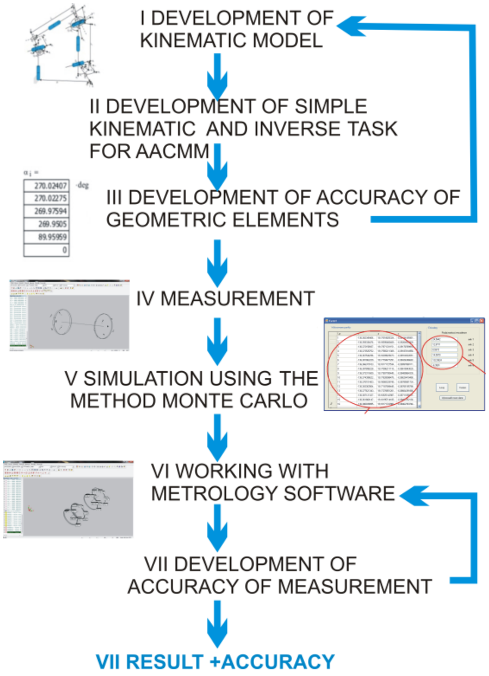


Fig. 4. Main stages of virtual AACMM functioning [4]

In the first and second stages of virtual AACMM implementation the well-known equations that describe the kinematics of the arm basing on the relations between its rotary joints has to be developed. It is usually done basing on the technical documentation of the arm and other manufacturer's data.

In the third step, the real values of arm's geometric parameters have to be determined. It is the most important step of implementation of virtual AACMM. This is obvious that data given by the manufacturer are just some kind of design assumption and that the parameters for the real CMA would be different. The significance of this difference may be of a great importance for developed method and due to this, the real values for CMA parameters have to be determined. There are 22 parameters (for 6-axis-AACMM) that include: the length of the segments, as well as eccentricities, the angles between the axes, “zero shifts,” that is, the difference between the real indications of the encoders, and the initial assumptions. Those parameters are irrespective to the CMA configuration as they are constant in all CMA positions [4].

From the practical point of view, in order to determine real CMA parameters a series of point measurements has to be accomplished. Each of the series has to contain at least 22 measurements in order to create the equation system that allows the determining of 22 unknown variables and the measured points have to be spread across the whole measuring space of the arm. Then, the configuration coordinates, as well as the Cartesian coordinates have to be input first. Then, after receiving data from the measuring device, that data has to be substituted into the system of equations formed previously, which would allow to calculate the exact geometrical parameters. So, in this step, the data that are usually constant during the computation of forward kinematics task are taken as unknown while data concerning the position and orientation of last joint are taken as constant and read from the

CMA software (those are coordinates measured by arm in each point, unit vectors of stylus orientation and the angles indicated by the encoders during each measurement) [4].

In order to simulate chosen measuring task, in the next stage, the considered measurement has to be performed on the real system and the data from encoders has to be recorded during measurement of each point included in certain measuring task. Using these data, the reproduction of each point is then simulated (it is the 5th stage) utilizing the Monte Carlo Method. The probability density functions associated to functioning of each encoder are the Gaussian distributions. For each point, the recorded indications of encoders are taken as a mean value of this distribution, while the accuracy of the encoders (given by the manufacturer or obtained experimentally) is taken as its standard deviation. *N* samples from these distributions are then simulated for all encoders and corresponding results of simulations are then put into the forward kinematics equations, giving *n* simulated values of all points included in simulated measurement.

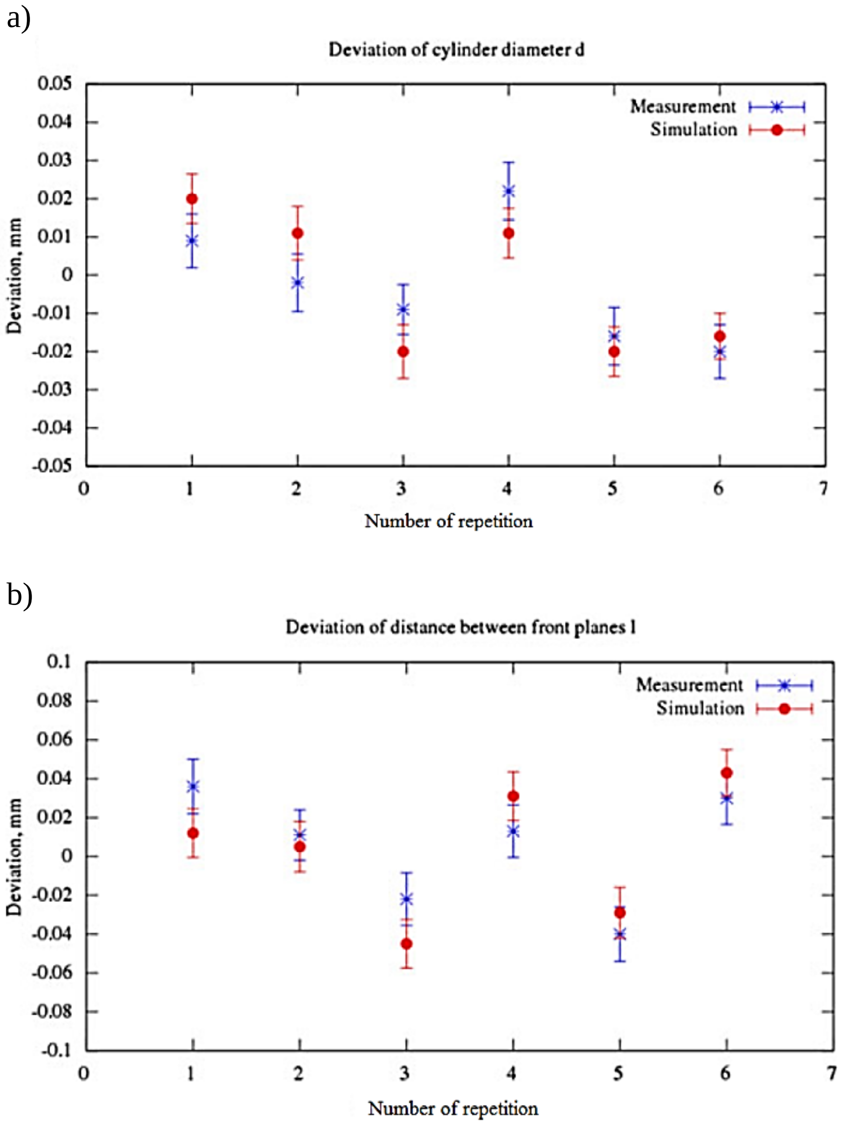


Fig. 5. Results of virtual AACMM verification: a) measurement of the standard cylinder diameter, b) measurement of distance between front planes of the standard cylinder [4]

In last two steps, the simulated points are sent back to the metrological software, where the geometrical features that are described by them are calculated. Then the estimated relations are determined *n*-times and the statistical analysis of simulation results is done in order to evaluate the uncertainty of the measurements.

The virtual CMM presented here was developed for standard 6-axis-AACMM. Its proper functioning was verified by performing the real measurements of standard cylinder and comparing their results with the results of virtual AACMM. The example of results is shown on Fig.5. Analysis of performed comparisons proved that the virtual AACMM gives comparable results to the real measurements thus it should be regarded as working properly.

2.3. VIRTUAL CMM BASED ON THE MODELLING OF RESIDUAL ERRORS AND USAGE OF MONTE CARLO METHOD

Model described in this subsection is the newest development of LCM researchers. It is predisposed for the CMMs that use the software error correction systems (so for the great majority of the coordinate machines used nowadays). On CMMs like that, the great portion of systematic errors is corrected and there is no need for including their impact in the virtual model of the machine. In this case, the idea of system modelling is translated to the area of random errors that are represented by the residual errors of the CMM.

Described model uses two main modules for simulation of coordinate measurement. The first one is responsible for simulating of the residual errors of CMM's kinematic system and the second one is responsible for simulation of probe head errors.

The first module of CMM model is built by describing each point on the grid of reference points with the probability distribution (*t* distribution in case of this model) with which it is reproduced on a machine. The LaserTracer (LT) system is used (combined with the multilateration technique) for experimental determination of distribution of errors in reference points [2],[9].

The experiment aim in determination of mentioned reference grid consisted of repeated approaching to the considered points from different directions. In place of the stylus the "cat eye" retroreflector was mounted. Position of the retroreflector was tracked in the dynamic mode by LaserTracer installed in the measuring volume of the machine. After a sequence of approaches at the point machine reached the next one and the cycle was repeated. Whole measurement sequence was repeated five times, each in different position of LT, in order to determine the coordinates of points using multilateration method. The experiments were performed with geometric errors compensation map switched on so the resulting standard deviations of point coordinates reproduction should be interpreted as values of residual errors.

The second module forming part of the described virtual machine is a module responsible for the simulation of probe head of CMM. To describe this system Probe Errors Function (PEF) described in [2],[7],[8],[10] has been used. The module gets the values of the individual errors of the PEF through Monte Carlo simulations. Data used to build this module was collected by multiple measurement of spherical standard. Standard was measured each time in 163 points that create the reference grid for values of PEF. Proper operation of the model is provided by using a spherical standard of suitably small form errors (its diameter should be smaller than 30 mm) [9].

The problem of interpolating values of errors between nods of the reference grid arises for both described modules. It is obvious that in real measurements majority of measuring points would lay between nods of reference grid. In order to get the variability of errors in these points, authors used different interpolation methods. In case of kinematic system errors the b-spline and “nearest-neighbor” interpolation methods were used, while in case of probe head errors interpolation, the bilinear interpolation was used [9].

The verification of presented virtual model was done using two different methodologies. The first one was similar to that described for two previous virtual models. The measurements of different material standards were done according to calibrated and non-calibrated object methods, and the results were compared to the ones provided by the virtual model. The measuring tasks for which the results was compared were the point-to-point distance, the plane-to-plane distance, the diameter of the sphere, the form deviation of the sphere and the distance between the centers of two spheres. The second method was based on the guidelines of [14]. The cylindrical standard was measured in three different positions using different distribution of measuring points on the cylinder. In order to prove the proper functioning of the model that is under verification the following condition has to be satisfied (1):

$$|y_k - y| \leq U_k + U \qquad (1)$$

where y_k - the value obtained in the calibration process of standard, y - the measured value, U_k - expanded uncertainty of the calibration of standard, U - expanded uncertainty of measurement (determined using simulation model that is being checked).

The results for chosen position of cylindrical standard were given in Fig. 6.

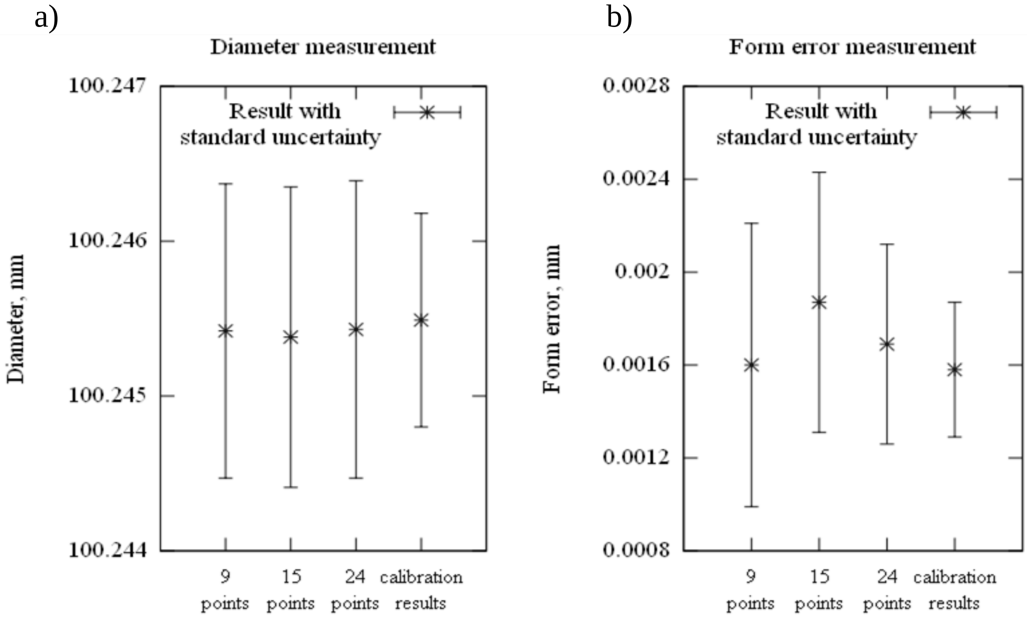


Fig. 6. Results of virtual model verification: a) results of diameter measurements, b) results of form error measurements [8]

The relationship described in equation (1) was satisfied for all obtained results (in all positions), so the described model is consistent with the recommendations of [14], and this is why it should be considered as working properly.

3. METHODOLOGY OF VIRTUAL CMM MODELS VALIDATION

Basing on the extensive experience with development and usage of the virtual CMM models, the team of researchers from LCM has also developed a universal model of CMM validation. The accredited calibration laboratories have to use validated methods to perform their measurements and assess its uncertainty. This is why they usually use methods based on internationally recognized normative documents. However some of them, due to the complexity of performed measurements, have to modify the methods that may be regarded as already validated or has to develop their own, new methods. In this case, currently there is no universal method for proving the proper functioning of developed methods and validating them. This is why the authors decided that such a methodology should be developed and started research on that matter. As a result, the methodology was conceived, which is based on the comparison of the results provided by the new (or modified) methods with the ones obtained using method that is already validated.

The key activity that has to be done during described procedure, is the measurement of multi-feature check (Fig. 7) done according to methodologies described for method using non-calibrated workpiece (multiple measurement method) and method using calibrated workpiece (called here the comparative method).

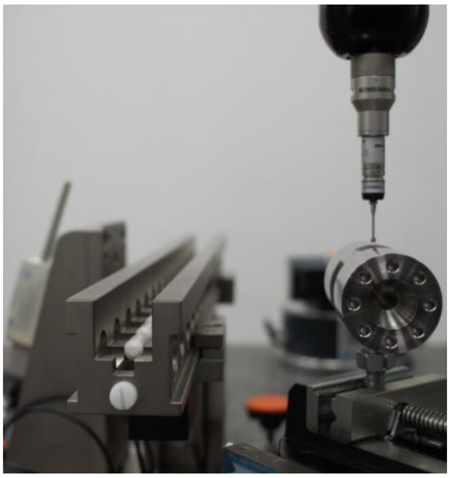


Fig. 7. Multi-feature check during measurements on CMM (on the right side of the picture)

After the measurements, all features and relations between them have to be evaluated according to mentioned methods and virtual CMM method. Next, the chi-squared test known from the concept of consistency control have to be checked for all of the results. If it passes, the validation acceptance interval has to be calculated. If it has a common part with the intervals containing the true value of a measurement (for all performed measuring tasks), obtained using methods being under validation, then this methods may from now on be regarded as validated.

More details on presented validation method would be given soon in the following publications of the authors.

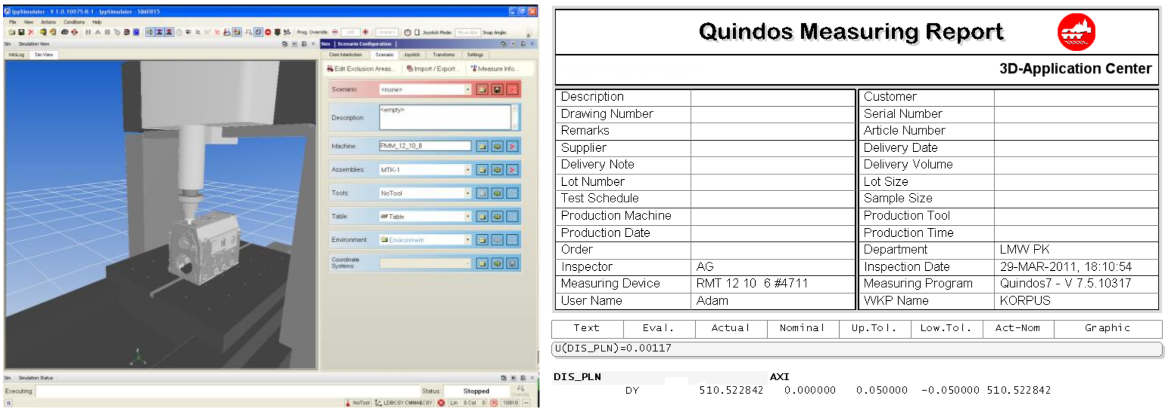
4. CONCLUSION AND DIRECTIONS FOR FURTHER DEVELOPMENT

As was presented in the paper, all of described virtual models of coordinate measuring systems are able to faithfully simulate the results and uncertainties of performed measurements. They were all verified by the comparison of results of real measurements and/or the guidelines of normative documents devoted to verification of virtual systems [14]. Additionally, the model based on modelling of residual errors was validated using methodology presented in section 3 of this paper. The rest of presented models would also be validated using this methodology soon.

Also the third of presented models was successfully implemented in industrial conditions what was the first implementation of this type in Poland. The implementation took place in one of companies from automotive industry near Cracow and the installed virtual model is now used daily for determination of the uncertainty of performed measurements and is especially helpful when the assessed tolerances are at the border of tolerance zones and the uncertainty plays the main role during deciding whether the geometrical specifications are met or not.

The virtual model of AACMM was verified in laboratory conditions and the next step of works connected with its development is its adaptation for use in industrial conditions. The main problem that has to be solved here is the influence of temperature on the working of measuring arm. This device is usually made of carbon fibre and thus the influence of temperature is reduced, however a significant loss of measurement accuracy may be observed when measurements are taken in high temperatures (higher than 25°C). Also the influence of operator's body heat on functioning of AACMM was observed by several researchers. The works on including these influences are now conducted on LCM and the model will soon be tested in industrial conditions.

Another direction for further development of described virtual models is connecting them with the Simulator I++ software (presented in [3]) or any other software that is used for off-line programming and simulating of the coordinate measurements. Using this combination it would be possible to predict the uncertainty of the measurements before they are performed on real measuring system. Such a system may be helpful when preparing the measurement strategy and may give an answer to questions like: how many points on each feature should be measured, what is the best possible localization and orientation of measured workpiece and if the resulting accuracy of the measurement is enough for performing considered tasks? The initial experiments on this matter were already undertaken and the Virtual MMC PK model was connected with the Simulator I++ which in result gives a system for measurement uncertainty forecasting. The measurements were programmed in Simulator I++ software and the uncertainty was simulated basing on the input quantities given by the simulator. Hence, the measurement uncertainties had been estimated before the real measurement was done (Fig. 8).



a) b)

Quindos Measuring Report

3D-Application Center

Description		Customer	
Drawing Number		Serial Number	
Remarks		Article Number	
Supplier		Delivery Date	
Delivery Note		Delivery Volume	
Lot Number		Lot Size	
Test Schedule		Sample Size	
Production Machine		Production Tool	
Production Date		Production Time	
Order		Department	LMW PK
Inspector	AG	Inspection Date	29-MAR-2011, 18:10:54
Measuring Device	RMT 12 10 6 #4711	Measuring Program	Quindos7 - V 7.5.10317
User Name	Adam	WKP Name	KORPUS

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   Text       Eval.      Actual     Nominal    Up.Tol.    Low.Tol.   Act-Nom       Graphic
U(DIS_PLN)=0.00117

DIS_PLN                             AXI
              DY         510.522842   0.000000    0.050000  -0.050000 510.522842
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Fig. 8. Simulation of the measurement of distance between frontal planes of the engine body: a) simulation in I++ Simulator, b) measurement report with estimated uncertainty of measurement [2]

The real measurement was done afterwards and its uncertainty was estimated using calibrated workpiece method. The differences in uncertainty values were negligible which may be the good prognostic for the future development of the uncertainty forecasting system.

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