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RELATIVISTIC EFFECTS IN THE ROTATION OF DWARF PLANETS AND ASTEROIDS

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ABSTRACT. The effect of the geodetic rotation (which includes two relativistic effects: geodetic precession and geodetic nutation) is the most significant relativistic effect in the rotation of the celestial bodies. For the first time in this research, this relativistic effect is determined in the rotation of dwarf planets (Ceres, Pluto, and Charon) and asteroids (Pallas, Vesta, Lutetia, Europa, Ida, Eros, Davida, Gaspra, Steins, and Itokawa) in the Solar System with known values of their rotation parameters. Calculations of the values of their geodetic rotation are made by a method for studying any bodies in the Solar System with a long-term ephemeris. Values of geodetic precession and geodetic nutation for all these celestial bodies were calculated in ecliptic Euler angles relative to their proper coordinate systems and in their rotational elements relative to the fixed equator of the Earth and the vernal equinox (at the epoch J2000.0). The obtained analytical values of the geodetic rotation for the celestial bodies can be used to numerically investigate their rotation in the relativistic approximation, and also used to estimate the influence of relativistic effects on the orbital–rotational dynamics for the bodies of exoplanetary systems.

Keywords: relativistic effects, geodetic rotation, Solar System bodies, the rotation of the dwarf planets and asteroids, exoplanetary systems bodies

1. INTRODUCTION

The effect of the geodetic rotation is the most significant relativistic effect in the rotation of the celestial bodies. This effect includes two relativistic effects: geodetic precession (De Sitter, 1916) and geodetic nutation (Fukushima, 1991), which are secular and periodical changes in the direction of the axis of rotation of a celestial body as a result of the parallel transfer of the angular momentum vector of the body along its orbit in curved space–time, respectively.

This article is a continuation of our previous investigations (Eroshkin and Pashkevich, 2007, 2009), (Pashkevich, 2016), (Pashkevich and Vershkov, 2019, 2020) of the geodetic rotation for the Solar System bodies. As a result, foregoing study showed that the values of the geodetic precession can be significant not only for objects orbiting around super-massive central relativistic bodies but also for bodies with a short distance to the central body, for example, close satellites of giant planets. The Solar System is a good model for studying the rotational dynamics of exoplanetary systems. Based on studies of the Solar System bodies with well-known parameters of motion, it is possible to reveal patterns in the distribution and influence of relativistic effects on the orbital–rotational dynamics of exoplanetary systems bodies. Thus, a more detailed study of the relativistic effects in the rotation of the bodies in the Solar System becomes relevant and interesting.

The main purpose of this investigation is to study the relativistic effect of the geodetic rotation for dwarf planets and asteroids (Appendix A: Table 1) in the Solar System with known values of their rotation parameters. Applying the method of studying the geodetic rotation of any bodies in the Solar System with long-term ephemeris (Pashkevich, 2016), for the first time, the secular and periodic terms of the geodetic rotation of these celestial bodies in ecliptic Euler angles relative to their proper coordinate systems and in their rotational elements relative to the fixed equator of the Earth and the vernal equinox (at the epoch J2000.0) are calculated.

2. MATHEMATICAL MODEL

This research is devoted to the study of the most significant relativistic effect in the rotational motion of dwarf planets (Ceres, Pluto with its satellite Charon) and asteroids (Pallas, Vesta, Lutetia, Europa, Ida, Eros, Davida, Gaspra, Steins, and Itokawa) in the Solar System with known values of their rotation parameters (Archinal et al., 2018). It is a geodetic rotation effect, which includes two relativistic effects: geodetic precession and geodetic nutation. The first of these is a systematic or secular effect, and the second is a periodic effect. The problem of geodetic (relativistic) rotation of the investigated bodies was studied with respect to their proper coordinate system (Archinal et al., 2018).[1]

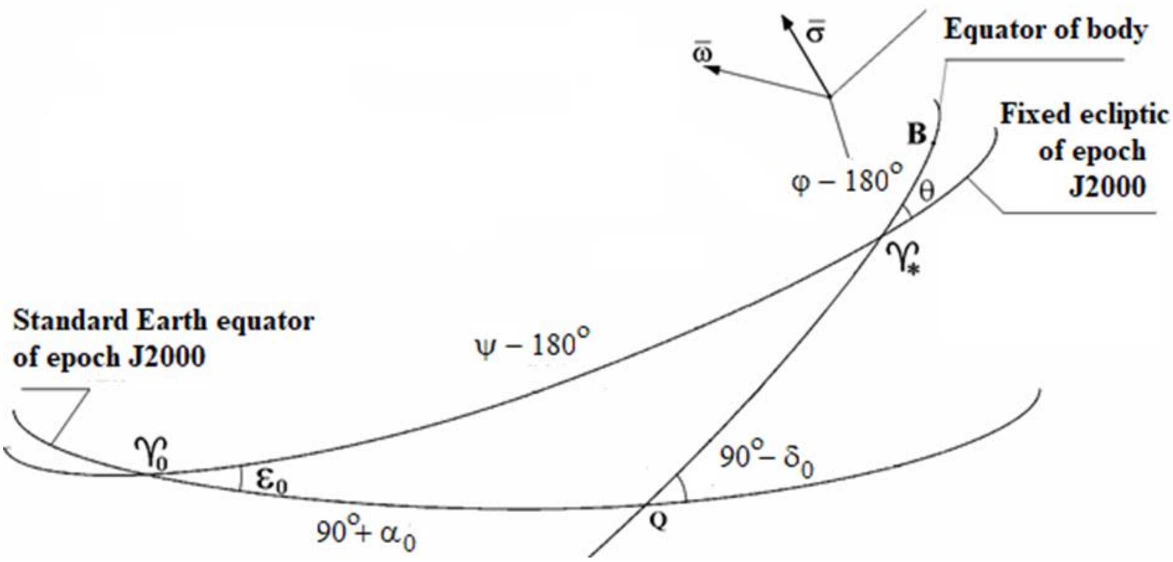

Figure 1. Triangle used to define the direction of the angular velocity vector of the geodetic rotation

It is well known that the angular velocity vector of the geodetic rotation of the body under study, which is the most essential relativistic component of the body rotational motion around the proper center of mass, is defined by the following expression (Eroshkin, 2005):

$$\bar{\sigma} = \frac{1}{c^2} \sum_j \frac{Gm_j}{\left|\bar{R} - \bar{R}_j\right|^3} \left(\bar{R} - \bar{R}_j\right) \times \left(\frac{3}{2}\dot{\bar{R}} - 2\dot{\bar{R}}_j\right), \quad (1)$$

[1] Thus, in this study, the Euler angles (see Figure 1) refer to the equator of rotation of the body under investigation, as defined in Archinal et al. (2018) and may not coincide with the equator of the body figure (as in Classical Mechanics (Suslov 1946)). Except when the equator of the body figure coincides with the equator of the body rotation.

Here c is the velocity of light; G is the gravitational constant; m_j is the mass of a perturbing body j; $\bar{R}$ and $\dot{\bar{R}}$ are the vectors of the barycentric position and velocity of the investigated body, respectively; $\bar{R}_j$ and $\dot{\bar{R}}_j$ are the vectors of the barycentric position and velocity of the perturbing bodies j, respectively. The symbol × means a vector cross product; the subscript j correspond to the perturbing bodies (the Moon, the planets, dwarf planet Pluto, Charon,[2] and the Sun, excluding the body under study from this set). The relativistic angular velocity vector for any Solar System bodies is calculated as follows:

$$\bar{\varpi}_R = \bar{\varpi} + \bar{\sigma}. \tag{2}$$

Here $\bar{\varpi}$ is Newtonian angular velocity vector for any Solar System bodies.

Calculations of the geodetic rotation of each body under study were carried out using data on the positions, velocities, and orbital elements of the bodies of the Solar System from ephemeris at all time intervals of their existence (Appendix A: Table 1). JPL DE431/ LE431 fundamental ephemeris (Folkner et al., 2014) were used for the Sun, the Moon, and the planets of the Solar System. For other investigated bodies of the Solar System with known rotation parameters (Archinal et al., 2018), the ephemeris data samples were formed from the Horizons On-Line Ephemeris System (Giorgini et al., 2001). The rotation parameters for dwarf planets (Ceres, Pluto with its satellite Charon) and asteroids (Appendix A: Table 1) in the Solar System were taken from the article by Archinal et al. (2018). Table 1 (see Appendix A) provides information for each body under study about their rotation and orbital periods and about the step and time interval of the studies.

In this study, the method for studying the geodetic rotation of any bodies in the Solar System (Pashkevich, 2016) using long-time ephemeris was applied. The calculation of the geodetic rotations of dwarf planets and asteroids in the Solar System with known values of their rotation parameters were carried out with respect to the proper coordinate systems of these celestial bodies under study (Archinal et al., 2018), whose origin coincides with their centres of mass:

a) for the parameters of their orientation $(\alpha_0,\ \delta_0,\ W)$ relative to the stationary equator of the Earth of the epoch J2000.0 (ICRF) (Ma et al., 1998) and the vernal equinox of the epoch J2000.0;

b) for ecliptic Euler angles $(\psi,\ \theta,\ \varphi)$ relative to the stationary equator of the Earth of the epoch J2000.0 (ICRF) and the fixed ecliptic of the epoch J2000.0 (see Figure 1).

The expressions of the geodetic rotation velocities of bodies in the Solar System are defined in $\psi,\ \theta,\ \varphi$ angles (left, 1) (Pashkevich, 2016) and in $\alpha_0,\ \delta_0,\ W$ angles (right, 1) (Pashkevich and Vershkov, 2020) as follows:

$$\left.\begin{aligned} \Delta\dot{\psi} &= -\frac{\sigma_1 \sin\varphi + \sigma_2 \cos\varphi}{\sin\theta} \\ \Delta\dot{\theta} &= -\sigma_1 \cos\varphi + \sigma_2 \sin\varphi \\ \Delta\dot{\varphi} &= \sigma_3 - \Delta\dot{\psi}\cos\theta \end{aligned}\right\}, \quad \left.\begin{aligned} \Delta\dot{\alpha}_0 &= \frac{\sigma_1 \sin W + \sigma_2 \cos W}{\cos\delta_0} \\ \Delta\dot{\delta}_0 &= -\sigma_1 \cos W + \sigma_2 \sin W \\ \Delta\dot{W} &= \sigma_3 - \Delta\dot{\alpha}_0 \sin\delta_0 \end{aligned}\right\}. \tag{3}$$

[2] The influence of Charon in this article is investigated only on the geodetic rotation of Pluto and is specifically discussed.

Here ψ is the longitude of the descending node of epoch J2000 of the body equator; θ is the inclination of the body equator to the fixed ecliptic J2000; φ is the proper rotation angle of the body between the descending node of epoch J2000 and point B, where the prime meridian crosses the equator of the body (see Figure 1) (or for the case when the equator of the body figure coincides with the equator of the body rotation: between the descending node of epoch J2000 and the principal axis of the minimum moment of inertia); α_0 is the right ascension of the north pole of rotation of the body; δ_0 is the declination of the north pole of rotation of the body; angle W = QB specifies the location of the prime meridian of the body,[3] which is measured along the equator of the body in easterly direction with respect to the north pole of body from the node Q (located at right ascension $90° + \alpha_0$) of the equator of the body on the standard Earth equator of epoch J2000 to the point B, where the prime meridian crosses the equator of the body (see Figure 1) (recommended values of the constants in the expressions for α_0, δ_0 and W are given by Archinal et al. (2018)); $\Delta\dot{\psi} = \dot{\psi}_r - \dot{\psi}$, $\Delta\dot{\theta} = \dot{\theta}_r - \dot{\theta}$, $\Delta\dot{\varphi} = \dot{\varphi}_r - \dot{\varphi}$, $\Delta\dot{\alpha}_0 = \dot{\alpha}_{0r} - \dot{\alpha}_0$, $\Delta\dot{\delta}_0 = \dot{\delta}_{0r} - \dot{\delta}_0$, $\Delta\dot{W} = \dot{W}_r - \dot{W}$ are the differences of the relativistic and Newtonian angles of rotation of the investigated body, respectively; the dot denotes differentiation with respect to time; $\sigma_1, \sigma_2, \sigma_3$ are projections of the angular velocity vector of the geodetic rotation of the body under study $\bar{\sigma}$ on the coordinate axes in its own body-centric reference frame (Figure 1a), given by Archinal et al. (2018).

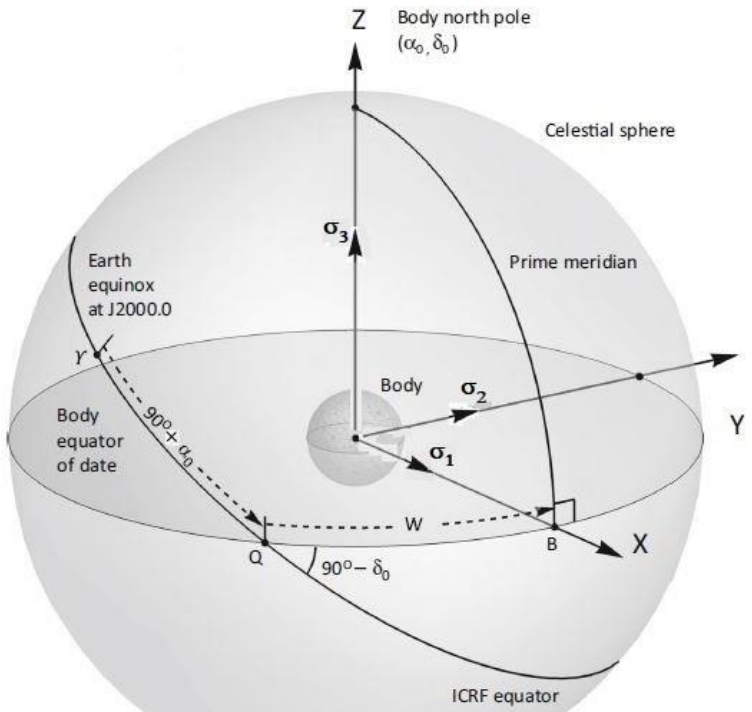

Figure 1a. Reference system used to define orientation of the body under study (Archinal et al., 2018)

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[3] Note from Archinal et al. (2018): "The angle W specifies the ephemeris position of the prime meridian and W_0 is the value of W at J2000.0 (or occasionally, such as for comets, some other specified epoch). For planets or satellites with no accurately observable fixed surface features, the expression for W defines the prime meridian and is not subject to correction for this reason. The rotation rate (authors' note: W_1) may be redefined by some other physical property (e.g., observation of the rotation of the body's magnetic field)." Here $W = W_0 + W_1 d$, d is the time in days from standard epoch is JD 2451545.0, i.e. January 1, 2000 12 hours TDB.

The geodetic rotation velocities for each of the investigated Solar System bodies are determined over various time spans with different time spacing (Appendix A: Table 1) by the least-squares method and spectral analysis (Pashkevich, 2016). The expressions for the secular and periodic terms of the velocities of the geodetic rotation of the body can be represented in the following form:

$$\Delta\dot{x} = \sum_{n=1}^{N} \Delta\dot{x}_n t^{n-1} + \sum_{l}\sum_{k=0}^{M} (\Delta\dot{x}_{Clk} \cos(\nu_{l0} + \nu_{l1}t) + \Delta\dot{x}_{Slk} \sin(\nu_{l0} + \nu_{l1}t))t^k, \qquad (4)$$

where $\Delta\dot{x}_n$ are the coefficients of the secular terms; $\Delta\dot{x}_{Slk}$, $\Delta\dot{x}_{Clk}$ are the coefficients of the periodic terms; $\dot{x} = \dot{\psi},\ \dot{\theta},\ \dot{\varphi},\ \dot{\alpha}_0,\ \dot{\delta}_0,\ \dot{W};\ \nu_{l0}, \nu_{l1}$ are phases and frequencies of the body under study, which are combinations of the corresponding Delaunay arguments (Smart, 1953) and the mean longitudes of the perturbing bodies; the summation index *l* is the number of added periodic terms and its value changes for each body under study; *t* is the time from standard epoch JD 2451545.0, i.e. January 1, 2000 12 hours TDB (Archinal et al., 2018), in Appendix A: Table 1 the time spans and steps for the studies of the geodetic rotation of the bodies are presented; *N* and *M* are approximation parameters.

As a result of calculations by the least squares method, the values of the approximation parameters for providing the best approximation of the geodetic rotation were obtained: $N = 2$ and $M = 1$.

After analytical integration (4) $\Delta x = \int \Delta\dot{x}\ \mathrm{dt}$, the expressions for the secular Δx_{I} and periodic Δx_{II} terms of the body geodetic rotation are obtained:

$$\Delta x = \Delta x_{\mathrm{I}} + \Delta x_{\mathrm{II}} = \sum_{n=1}^{N} \Delta x_n t^n + \sum_{l}\sum_{k=0}^{M} (\Delta x_{Clk} \cos(\nu_{l0} + \nu_{l1}t) + \Delta x_{Slk} \sin(\nu_{l0} + \nu_{l1}t))t^k, \qquad (5)$$

where $x = \psi,\ \theta,\ \varphi,\ \alpha_0,\ \delta_0,\ W;\ \Delta x_n = \dfrac{\Delta\dot{x}_n}{n}$ are the coefficients of the secular terms, and the coefficients for sine Δx_{Slk} and cosine Δx_{Clk} periodic terms are calculated using the "Cascade" method (Pashkevich, 2016).

The absolute value of the angular velocity vector of the geodetic rotation of the body under study is presented by the following expression:

$$|\bar{\sigma}| = \sqrt{\sigma_X^2 + \sigma_Y^2 + \sigma_Z^2} = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2}. \qquad (6)$$

Here $\sigma_X, \sigma_Y, \sigma_Z$ are the components of the geocentric vector of the angular velocity of the geodetic rotation of a body (Pashkevich, 2016); as defined above, $\sigma_1, \sigma_2, \sigma_3$ are reduced (Pashkevich, 2016) components of the body-centric vector of the angular velocity of the geodetic rotation of a body. Equation (6) is true, because the magnitude of the vector does not depend on the coordinate system in which its projections are considered.

The absolute value of the geodetic rotation vector for the Solar System bodies in the parameters of their orientation is presented by the following expression:

$$\Delta\Omega = \sqrt{\Delta\alpha_0^2 + \Delta\delta_0^2 + \Delta W^2}. \qquad (7)$$

3. DWARF PLANETS (CERES, PLUTO, AND CHARON)

In this study, for the first time, the values of the most significant secular (Appendix A: Tables 2 and 3) and periodic (Appendix A: Tables 4 and 5) terms of geodetic rotation were calculated for dwarf planets of the Solar System with known rotation parameters (Ceres, Pluto, and Charon).

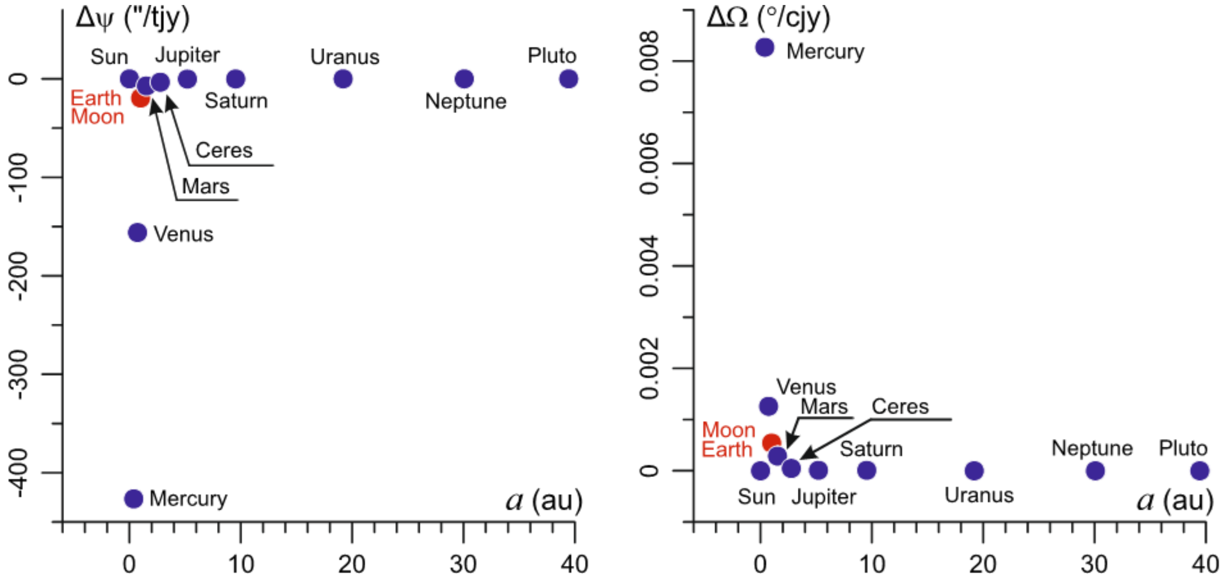

Figure 2. Geodetic rotation of the Sun, the Moon, the planets, and dwarf planets (Ceres and Pluto) of the Solar System in the longitude of the descending node (left side) and in the absolute value of the geodetic rotation vector of the parameters of their orientation (right side)

In Appendix A: Tables 2 and 4 shows the angles of rotation $(\alpha_0,\ \delta_0,\ W)$ of dwarf planets in the Solar System (Archinal et al., 2018) and the most significant secular $(\Delta\alpha_{0\mathrm{I}},\ \Delta\delta_{0\mathrm{I}},\ \Delta W_{\mathrm{I}})$ and periodic $(\Delta\alpha_{0\mathrm{II}},\ \Delta\delta_{0\mathrm{II}},\ \Delta W_{\mathrm{II}})$ terms of their geodetic rotation (Figures 2 and 3, right side) calculated in this study; Tables 3 and 5 shows the most significant secular $(\Delta\psi_{\mathrm{I}},\ \Delta\theta_{\mathrm{I}},\ \Delta\varphi_{\mathrm{I}})$ and periodic $(\Delta\psi_{\mathrm{II}},\ \Delta\theta_{\mathrm{II}},\ \Delta\varphi_{\mathrm{II}})$ terms of geodetic rotation of the dwarf planets in the Solar System for ecliptic Euler angles (Figures 2 and 3, left side) relative to their proper coordinate systems calculated in this study.

As is known from previous articles by the authors Pashkevich (2016) and Pashkevich and Vershkov (2019), the geodetic rotation of the Sun and the major planets of the Solar System in ecliptic Euler angles (Figure 2, left side) have been ranging from –870.02 µas per thousand years to –426".45 per thousand years. The value of the geodetic precession of Ceres (–3".36 per thousand years) obtained in this study is between the values of the geodetic precessions of Mars (–7".11 per thousand years) and Jupiter (–0".21 per thousand years) (Pashkevich and Vershkov, 2019), which is in good agreement with the location of its orbit in the asteroid belt between the orbits of these planets.

Ceres geodetic nutation value (periodic term with the period 4.6049 years) is calculated in ecliptic Euler angles (Appendix A: Table 5) and in their angles of rotation (Appendix A: Table 4).

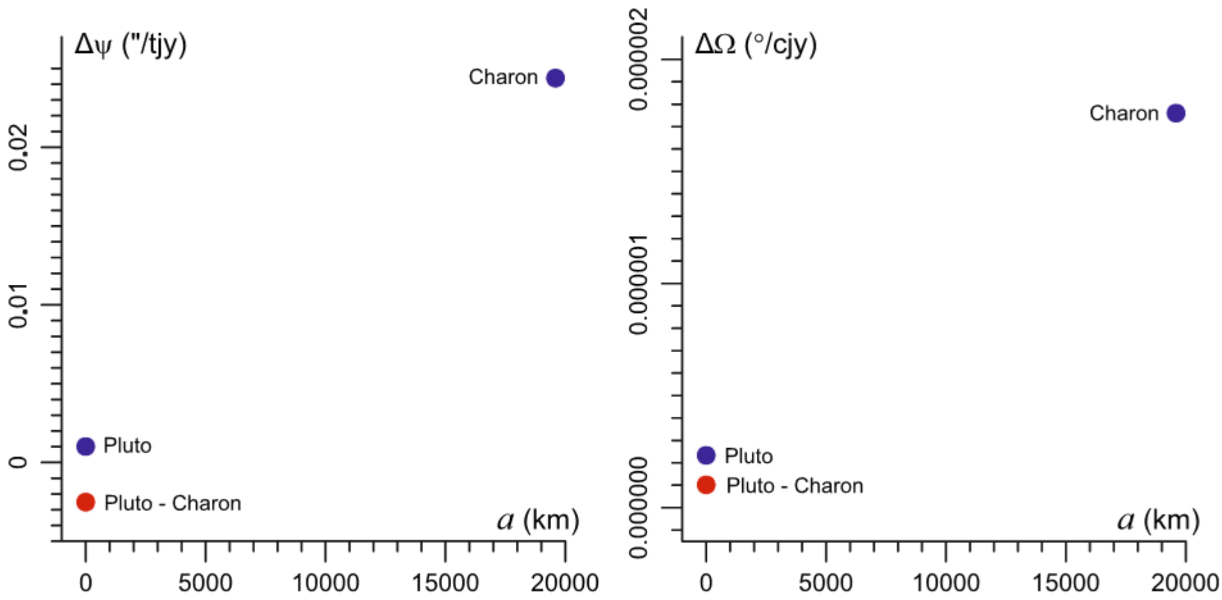

Figure 3. Geodetic rotation of the Pluto–Charon System (without their mutual influence on each other), Pluto (with taking into account the perturbations from Charon) and Charon (with taking into account the perturbations from Pluto) in the longitude of the descending node (left side) and in the absolute value of the geodetic rotation vector of the parameters of their orientation (right side)

The obtained values of the geodetic rotations of Pluto and Charon without mutual influence (that is, with the influence only from the Sun, Moon, and planets) are quite close (Figure 3, Pluto–Charon) (Figure 3a, Pluto, Charon) (Appendix A: Tables 2, 3, 4, and 5). In this case, the difference between Pluto and Charon values of the geodetic precessions[4] $\Delta\psi_{\mathrm{I}}$ is 0.0003 μas per thousand years (Appendix A: Table 3). This is due to the same distance between Pluto and Charon from the Sun, which in this case has the greatest effect on the dwarf planets under study. In this case, the negative value of the geodetic precession corresponds to their prograde rotation along the heliocentric orbit.

The values of geodetic precessions for Pluto and Charon, obtained taking into account the mutual influence of Pluto and Charon on each other, differ significantly from each other (Figure 3, Pluto, Charon) (Appendix A: Tables 2 and 3). This is explained by the fact that Pluto is more massive than Charon (Charon's mass is ~ 0.1 Pluto's mass). This is clearly seen from the values of the periodic harmonics of the geodetic nutation of these bodies (Appendix A: Tables 4 and 5) with the periods 6.3868 days, 6.3877 days, and 6.3872 days, which determine the periodic mutual influences of these celestial bodies on each other. So, the harmonic values (with the periods 6.3868 days, 6.3877 days, and 6.3872 days) contained in Pluto's geodetic nutation are 10 times less than the corresponding harmonic values contained in Charon's geodetic nutation (Figure 3a, Pluto+, Charon+). Thus, Pluto's influence on Charon is significantly greater than Charon's influence on Pluto.

[4] The geodetic precession value given here for Pluto and Charon was calculated in this study (–2196.6 μas per thousand years) using the Horizons On-Line Ephemeris System ephemeris (Giorgini et al., 2001) over a 400-year time interval. Therefore, its value differs from the value (–2091.7 μas per thousand years) obtained for Pluto in our previous research (Pashkevich and Vershkov, 2019), which was calculated using the JPL DE431/LE431 ephemeris (Folkner et al., 2014) at 2000-year time interval.

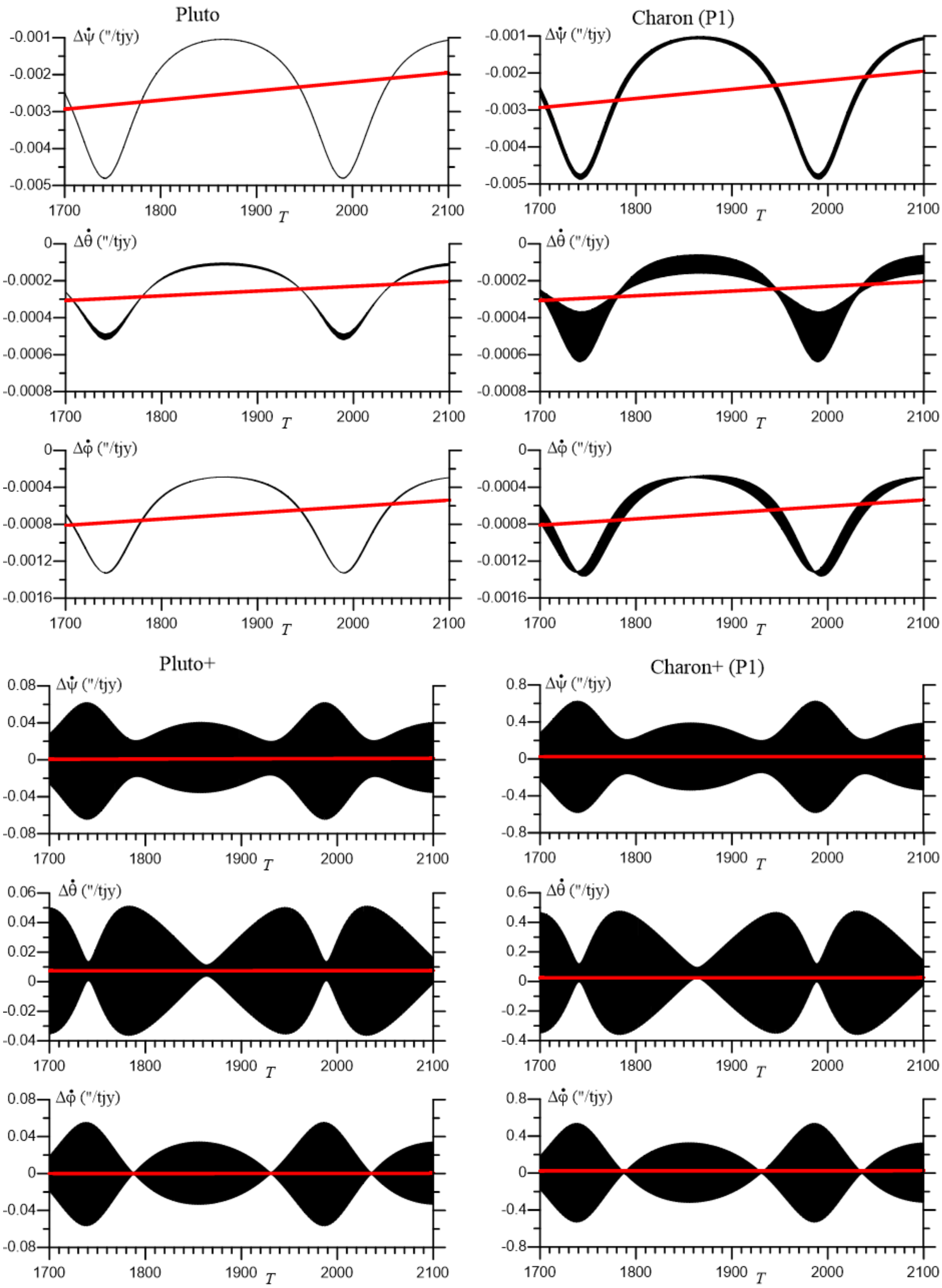

Figure 3a. The values of the velocities of the change in geodetic rotations for Pluto and Charon (without their mutual influence on each other) (top row) and Pluto+ (with taking into account the perturbations from Charon) and Charon+ (with taking into account the perturbations from Pluto) (bottom row) in ecliptic Euler angles (the red line in the graphs shows a secular trend)

The positive value of the geodetic precession corresponds to their retrograde orbital rotation around the common barycenter of the Pluto–Charon system. In absolute value, the geodetic precession of Charon (24690.9 μas per thousand years) (Appendix A: Table 3) exceeds similar values of the geodetic precessions for Pluto (1328.9 μas per thousand years) (Appendix A: Table 3), Neptune (–3903.9 μas per thousand years), and Uranus (–11924.6 μas per thousand years) (Pashkevich and Vershkov, 2019).

The velocities of the geodetic rotation for Pluto and Charon without their mutual influence on each other (Figure 3a, Pluto, Charon) are 30 and 300 times less than corresponding velocities for Pluto and Charon with taking into account their mutual influence on each other (Figure 3a, Pluto+, Charon+) in ecliptic Euler angles. Consequently, in this case, the influence of Pluto and Charon on each other's geodetic rotation turned out to be greater than the influence of the Sun on their geodetic rotation.

4. ASTEROIDS

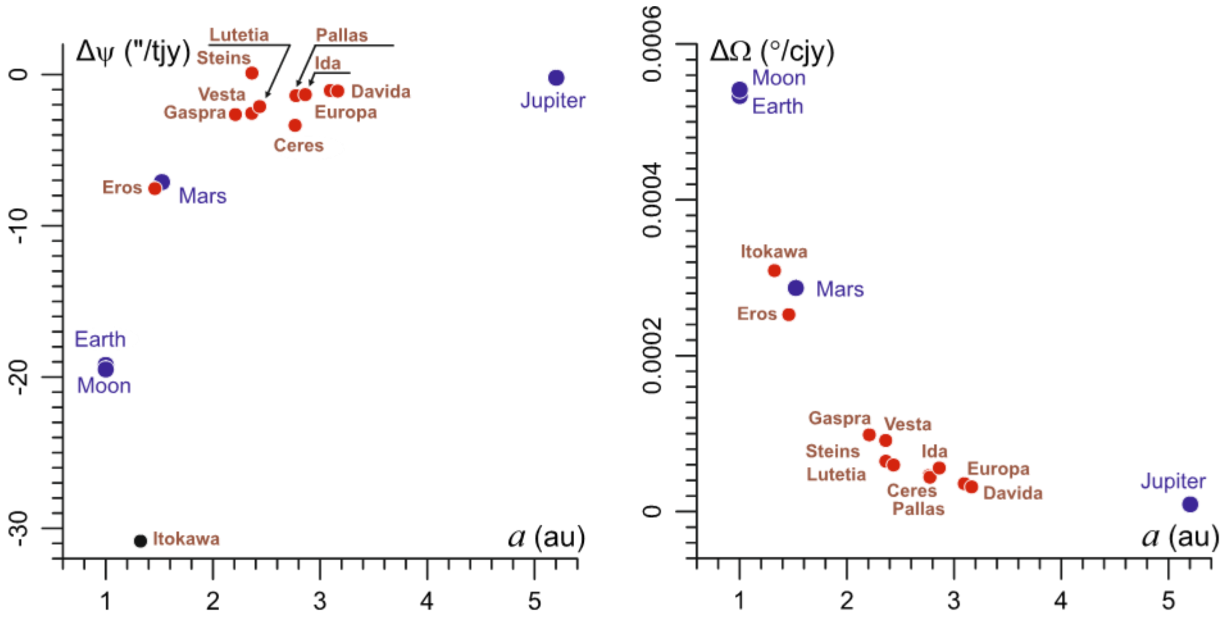

Figure 4. Geodetic rotation of the Earth, the Moon, Mars, Ceres, Jupiter, and asteroids of Solar System in the longitude of the descending node (left side) and in the absolute value of the velocity vector of the geodetic rotation of the parameters of their orientation (right side)

In this study, for the first time, the values of the most significant secular (Appendix A: Tables 2a and 3) and periodic (Appendix A: Tables 4 and 5) terms of geodetic rotation were calculated for asteroids (Pallas, Vesta, Lutetia, Europa, Ida, Eros, Davida, Gaspra, Steins, and Itokawa) of the Solar System with known rotation parameters (Figures 4 and 5).[5]

In Appendix A: Tables 2a and 4 show the angles of rotation $(\alpha_0,\ \delta_0,\ W)$ of asteroids in the Solar System (Archinal et al., 2018) and the most significant secular $(\Delta\alpha_{0\mathrm{I}},\ \Delta\delta_{0\mathrm{I}},\ \Delta W_{\mathrm{I}})$ and periodic $(\Delta\alpha_{0\mathrm{II}},\ \Delta\delta_{0\mathrm{II}},\ \Delta W_{\mathrm{II}})$ terms of their geodetic rotation (Figure 4, right side) calculated in this study; Tables 3 and 5 show the most significant secular $(\Delta\psi_{\mathrm{I}},\ \Delta\theta_{\mathrm{I}},\ \Delta\varphi_{\mathrm{I}})$ and periodic

[5] This figure was obtained using the EPOS software package for studying objects of the Solar System (L'vov et al., 2012).

($\Delta\psi_{II}$, $\Delta\theta_{II}$, $\Delta\varphi_{II}$) terms of geodetic rotation of the asteroids in the Solar System for ecliptic Euler angles (Figure 4, left side) relative to their proper coordinate systems calculated in this study.

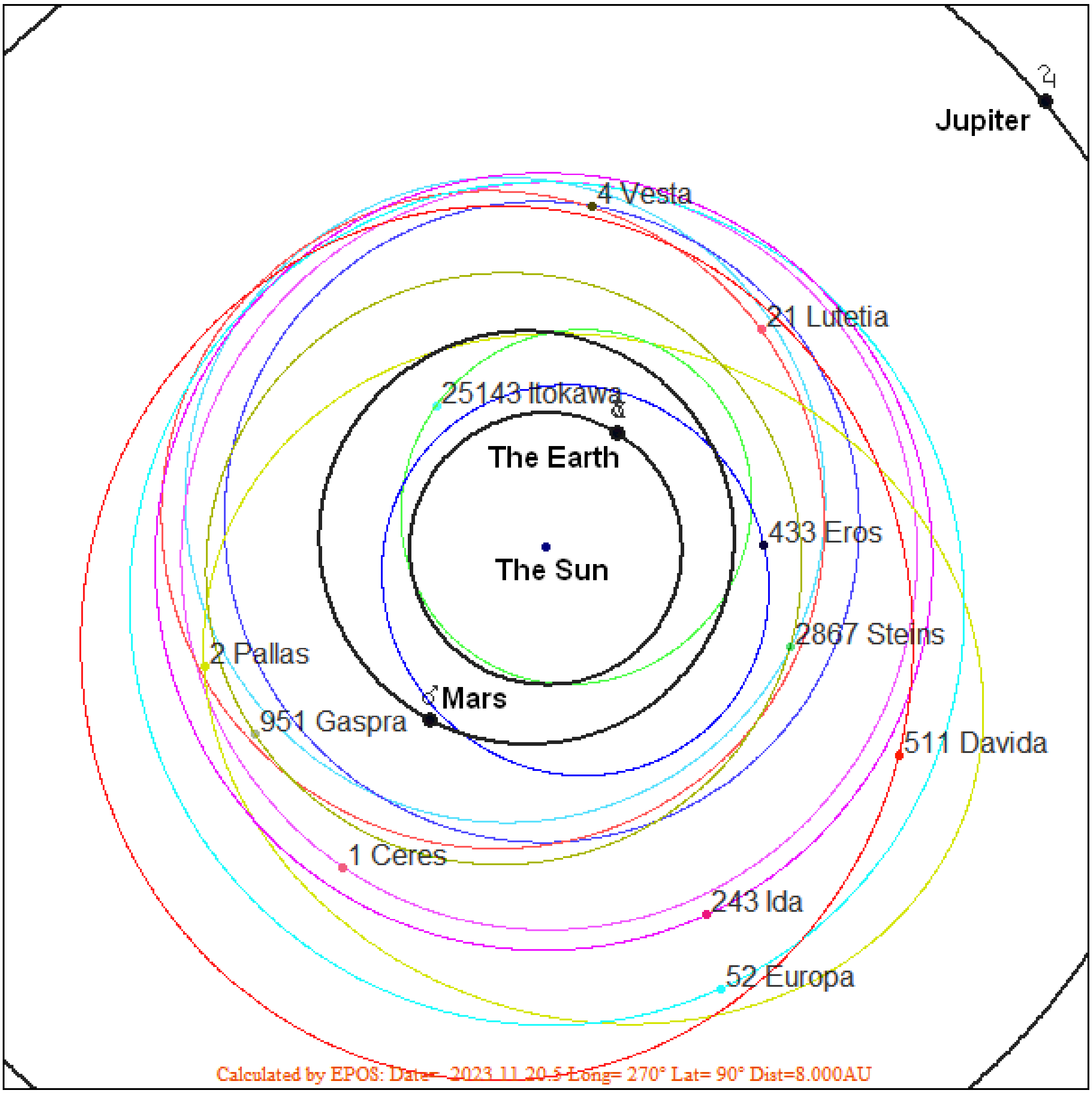

Figure 5. The orbits of the studied asteroids and the dwarf planet Ceres relative to the Sun, and the planets of the Earth, Mars, and Jupiter (L'vov et al., 2012)

The value of the geodetic precession of Itokawa asteroid obtained in this study is –30".89 per thousand years (Appendix A: Table 3); thus, it is 1.6 times higher than similar values (Pashkevich et al., 2019) for the Earth (–19".19 per thousand years) and the Moon (–19".49 per thousand years). This is due to the large elongation of the heliocentric orbit of Itokawa (Figure 5). Indeed, (despite the fact that the value of the semi-major axis of the heliocentric orbit of Itokawa is greater than the corresponding values for the Earth and the Moon, and the aphelion of its orbit is located farther from the Sun than the aphelion of the orbit of Mars) due to the large eccentricity (e = 0.280) (Appendix A: Table 3), the perihelion of its orbit is closer to the Sun than the perihelion of the heliocentric orbits of the Earth and the Moon. Therefore, the Sun has a greater influence on this asteroid than on the Earth and the Moon. The value of the geodetic nutation for Itokawa (Appendix A: Table 5) is the largest among the studied asteroids. This is due to the large lengthening of its orbit (Figure 5) and the influence on it from the planets Mars and the Earth.

The value of the geodetic precession of Eros (–7".54 per thousand years) calculated in this research is between the values of the geodetic precessions of the Earth (–19".19 per thousand years), the Moon (–19".49 per thousand years), and Mars (–7".11 per thousand years) (Pashkevich et al., 2019), which is in good agreement with the location of its orbit in the space between the orbits of these planets (Figure 5).

The orbit of the asteroid Gaspra is between the orbits of Mars and Vesta. The value of its geodetic precession calculated in this study is –2".64 per thousand years.

The asteroid Vesta has a geodetic precession in absolute value slightly less than that of Gaspra and amounts to –2".56 per thousand years.

The value of the geodetic precession of the next Steins asteroid is positive and amounts to 0".10 per thousand years.

Thus, for the asteroids Vesta and Steins, which have similar semimajor axes of their orbits, the values of their geodetic precession $\Delta\psi_{\mathrm{I}}$ (Appendix A: Table 3) differ significantly from each other and have the opposite sign. If you look at the orientations of their rotation axes, you can see significant differences: the ecliptic coordinates of the North Pole of Vesta (λ is the longitude, β is the latitude) λ = 330°.83, β = 57°.73 (Russell et al., 2013), and the North Pole of Steins has λ = 250°.0, β = –89°.0 (Lamy et. al., 2008). Since the ecliptic latitude of the North Pole of Steins is negative, and its rotation around its axis is positive (Appendix A: Table 2a) (Archinal et al., 2018), this means that, unlike Vesta, it has a reverse rotation relative to the North Pole of the ecliptic. This circumstance explains the positive value of the Steins geodetic precession.

Despite the fact that Vesta is the first in mass among asteroids, its relativistic influence on the asteroid Steins, which has a relatively close orbit with it (Appendix A: Table 3), is not too great. Thus, the change in the value of Steins' geodetic rotation from the relativistic influence of Vesta is -1.6×10^{-4} μas per thousand years in the longitude of the node, -1.6×10^{-5} μas per thousand years in the inclination, and 8.0×10^{-5} μas per thousand years in the proper rotation angle.

The next farthest from the Sun is the investigated asteroid Lutetia. The value of its geodetic precession calculated in this study is –2".12 per thousand years.

Further, there are two massive bodies of Ceres and Pallas, their mutual relativistic influence on each other is also small. So, the change in the geodetic rotation of Ceres from the relativistic influence of Pallas is -7.5×10^{-4} μas per thousand years in the longitude of the node, -3.8×10^{-5} μas per thousand years in the inclination, and 6.9×10^{-4} μas per thousand years in the proper rotation angle; and the change in the geodetic rotation of Pallas due to Ceres relativistic influence is equal -5.4×10^{-4} μas per thousand years in the longitude of the node, -6.0×10^{-5} μas per thousand years in the inclination, and 6.0×10^{-4} μas per thousand years in the proper rotation angle.

According to the results of this study, the values of the geodetic precession for the asteroids Pallas, Ida, Europa, and Davida are –1".40 per thousand years, –1".33 per thousand years, –1".06 per thousand years, and –1".09 for a thousand years, respectively.

5. CONCLUSIONS

In the present research, the most significant relativistic effect of geodetic rotation in the rotation motions for dwarf planets (Ceres, Pluto, and Charon) and asteroids (Pallas, Vesta, Lutetia, Europa, Ida, Eros, Davida, Gaspra, Steins, and Itokawa) in the Solar System with known quantities of their rotation parameters was investigated. For the first time, the secular (geodetic precession) and periodic (geodetic nutation) terms of the geodetic rotation of these celestial bodies were calculated in ecliptic Euler angles relative to their proper coordinate systems and in their rotational elements relative to the stationary equator of the Earth and the vernal equinox (for the epoch J2000.0). The mean longitudes for Ceres and for asteroids were computed in this investigation using an algorithm elaborated by Pashkevich (see Appendix B) based on the spectral analysis method (Jenkins, Watts, 1969).

We have shown that the calculated values of geodetic precession are influenced not only by the orbital parameters of the body under study, the distance of these bodies from the disturbing bodies and the values of the masses of the disturbing bodies, but also the spatial nature of their proper rotation, determined by the orientation of the North Pole of the body under study relative to the reference plane (in this study, this plane is the ecliptic of the J2000.0 epoch). As a result, it becomes clear whether the proper rotation is prograde or retrograde relative to the orbital motion, and, accordingly, whether the value of the geodetic precession is negative or positive, and explains some variations in the geodetic precession values of the studied celestial bodies close in position to their orbits.

Pluto’s geodetic nutation value with taking into account the perturbations from Charon is 10 times less than Charon’s geodetic nutation value with taking into account the perturbations from Pluto. Thus, Pluto’s influence on Charon is significantly greater than Charon’s influence on Pluto.

The influence of Pluto and Charon on the geodetic rotation of each other turned out to be greater than the influence of the Sun.

The obtained analytical values for the geodetic rotation of the studied celestial bodies can be used for the numerical study of their rotation in the relativistic approximation, and also used to estimate the influence of relativistic effects on the orbital–rotational dynamics of exoplanetary systems bodies.[6]

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[6] Using the observational data of exoplanets, it is possible to construct their orbits, and hence their ephemerides (the accuracy of which may be sufficient at least to evaluate the studied relativistic effects).

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APPENDIX A

Table 1. The parameters of the investigation of the geodetic rotation for the bodies under study

The body	**The time span (years)**	**Spacing**	**Date of the ephemeris, rotation, and orbital periods**
Itokawa (25143)	900 (from AD1599 12 December 00:00 to AD2500 29 December 23:40)	2 h 20 m	Aug 17 06:26:37 2021 12.13 hrs, 1.52 yrs
Eros (433)	900 (from AD1599 12 December 00:00 to AD2500 30 December 00:00)	2 h 00 m	Aug 26 10:57:04 2021 5.270 hrs, 1.76 yrs
Gaspra (951)	900 (from AD1599 12 December 00:00 to AD2500 30 December 00:00)	2 h 00 m	Sep 2 10:02:44 2021 7.042 hrs, 3.285 yrs
Vesta (4)	900 (from AD1599 12 December 00:00 to AD2500 30 December 00:00)	2 h 00 m	Aug 25 05:22:33 2021 5.342 hrs, 3.63 yrs
Steins (2867)	900 (from AD1599 12 December 00:00 to AD2500 30 December 00:00)	2 h 00 m	Aug 23 00:38:28 2021 6.049 hrs, 3.64 yrs
Lutetia (21)	900 (from AD1599 12 December 00:00 to AD2500 29 December 23:30)	2 h 30 m	Aug 31 12:56:30 2021 8.168 hrs, 3.80 yrs
Ceres (1)	900 (from AD1599 12 December 00:00 to AD2500 29 December 23:30)	2 h 30 m	Aug 19 05:24:26 2021 9.074 hrs, 4.60 yrs
Pallas (2)	900 (from AD1599 12 December 00:00 to AD2500 29 December 23:30)	2 h 30 m	Aug 24 11:27:29 2021 7.813 hrs, 4.61 yrs
Ida (243)	900 (from AD1599 12 December 00:00 to AD2500 30 December 00:00)	2 h 00 m	Aug 29 08:59:22 2021 4.633 hrs, 4.84 yrs
Europa (52)	900 (from AD1599 12 December 00:00 to AD2500 30 December 00:00)	2 h 00 m	Aug 28 13:25:25 2021 5.6304 hrs, 5.451 yrs
Davida (511)	900 (from AD1599 12 December 00:00 to AD2500 30 December 00:00)	2 h 00 m	Aug 30 11:09:54 2021 5.131 hrs, 5.628 yrs
Pluto (134340)	400 (from AD1700 07 January 00:00 to AD2099 31 December 00:00)	1 d	Oct 13 08:08:45 2021 −6.387 days, 247.921 yrs
Charon (Pluto: I)	400 (from AD1700 07 January 00:00 to AD2099 31 December 00:00)	1 d	Oct 13 07:52:41 2021 6.387 days, 6.387 days

Table 2. The rotational elements of dwarf planets of the Solar System (α_0, δ_0, W) and their secular terms of the geodetic rotation

The rotational elements \ Dwarf planets		Ceres (1)[7]	Pluto (134340)[8]	Charon (P I)[9]	Pluto without Charon[10]	Charon without Pluto[11]
α_0 (°)		291.418	132.993	132.993	132.993	132.993
$\Delta\alpha_{0\,\mathrm{I}}$ (°)						
	T	4.54×10^{-6}	-9.40×10^{-8}	-1.09×10^{-6}	5.60×10^{-8}	5.60×10^{-8}
	T^2	2.51×10^{-7}	-3.15×10^{-9}	-3.09×10^{-9}	-3.14×10^{-9}	-3.14×10^{-9}
δ_0 (°)		66.764	−6.163	−6.163	−6.163	−6.163
$\Delta\delta_{0\,\mathrm{I}}$ (°)						
	T	1.36×10^{-5}	-1.88×10^{-7}	-1.37×10^{-6}	-1.04×10^{-8}	-1.04×10^{-8}
	T^2	-1.92×10^{-8}	5.90×10^{-10}	5.71×10^{-10}	5.88×10^{-10}	5.88×10^{-10}
W (°)		170.65	302.695	122.695	302.695	122.695
	d	952.1532	56.3625225	56.3625225	56.3625225	56.362523
ΔW_{I} (°)						
	T	-4.40×10^{-5}	-2.20×10^{-8}	-2.52×10^{-7}	1.28×10^{-8}	1.28×10^{-8}
	T^2	-2.31×10^{-7}	-7.15×10^{-10}	-6.20×10^{-10}	-7.18×10^{-10}	-7.18×10^{-10}

In this table:

T is the time in Julian centuries;
d is the time in days from standard epoch JD 2451545.0, i.e. January 1, 2000 12 hours TDB.

[7] Geodetic Ceres rotation taking into account the perturbations from the planets, dwarf planet Pluto, and the Sun.

[8] Geodetic Pluto rotation taking into account the perturbations from the planets, dwarf planet Charon, and the Sun.

[9] Geodetic Charon rotation taking into account the perturbations from the planets, dwarf planet Pluto, and the Sun.

[10] Geodetic Pluto rotation without taking into account the perturbations from Charon.

[11] Geodetic Charon rotation without taking into account the perturbations from Pluto.

Table 2a. The rotational elements of the asteroids of the Solar System (α_0, δ_0, W) and their secular terms of the geodetic rotation

The rotational elements \ The asteroids		Itokawa (25143)	Eros (433)	Gaspra (951)	Vesta (4)	Steins (2867)
α_0 (°)		90.53	11.35	9.47	309.031	91
$\Delta\alpha_{0\,\mathrm{I}}$ (°)						
	T	2.27×10^{-5}	2.11×10^{-4}	6.99×10^{-5}	4.20×10^{-5}	1.33×10^{-6}
	T^2	-6.08×10^{-7}	-9.81×10^{-8}	-4.29×10^{-8}	6.89×10^{-8}	-1.83×10^{-7}
δ_0 (°)		−66.30	17.22	26.7	42.235	−62
$\Delta\delta_{0\,\mathrm{I}}$ (°)						
	T	3.32×10^{-6}	5.22×10^{-5}	2.86×10^{-5}	2.25×10^{-5}	9.59×10^{-6}
	T^2	1.57×10^{-7}	1.06×10^{-7}	4.91×10^{-8}	-2.16×10^{-8}	-5.34×10^{-8}
W (°)		0	326.07	83.67	285.39	321.76
	d	712.143	1639.38865	1226.91149	1617.33294	1428.09917
ΔW_{I} (°)						
	T	3.08×10^{-4}	-1.30×10^{-4}	-6.32×10^{-5}	-7.78×10^{-5}	6.39×10^{-5}
	T^2	-5.97×10^{-7}	-1.10×10^{-7}	9.82×10^{-9}	-2.07×10^{-8}	-1.41×10^{-7}

The rotational elements \ The asteroids		Lutetia (21)	Pallas (2)	Ida (243)	Europa (52)	Davida (511)
α_0 (°)		52	33	168.76	257	297
$\Delta\alpha_{0\,\mathrm{I}}$ (°)						
	T	5.73×10^{-5}	2.72×10^{-5}	1.59×10^{-5}	2.48×10^{-5}	2.53×10^{-5}
	T^2	-1.45×10^{-9}	-1.49×10^{-7}	-2.63×10^{-7}	1.81×10^{-8}	4.45×10^{-8}
δ_0 (°)		12	−3	−87.12	12	5
$\Delta\delta_{0\,\mathrm{I}}$ (°)						
	T	1.65×10^{-5}	3.47×10^{-5}	-1.45×10^{-5}	-2.51×10^{-7}	1.35×10^{-5}
	T^2	-2.16×10^{-8}	4.88×10^{-8}	-1.89×10^{-8}	5.83×10^{-8}	-1.61×10^{-8}
W (°)		94	38	274.05	55	268.1
	d	1057.7515	1105.8036	1864.62801	1534.64722	1684.41935
ΔW_{I} (°)						
	T	-2.80×10^{-6}	1.53×10^{-6}	5.16×10^{-5}	-2.60×10^{-5}	-1.34×10^{-5}
	T^2	9.85×10^{-9}	1.67×10^{-7}	-2.70×10^{-7}	3.13×10^{-8}	1.40×10^{-7}

In this table:

T is the time in Julian centuries,
d is the time in days from standard epoch JD 2451545.0, i.e. January 1, 2000 12 hours TDB.

Geodetic asteroids rotation taking into account the perturbations from the planets, dwarf planet Pluto, and the Sun.

Table 3. Secular terms of the geodetic rotation for the Solar System bodies under study, calculated for ecliptic Euler angles *(part 1/2)*

	Itokawa (25143) e = 0.280 i = 1.62	**Eros (433)** e = 0.223 i = 10.83	**Gaspra (951)** e = 0.174 i = 4.11	**Vesta (4)** e = 0.088 i = 7.14	**Steins (2867)** e = 0.146 i = 9.93
a (km)	198 094 516	218 138 719	330 494 569	353 354 672	353 580 460
	$\Delta\psi_I$ (μas)	$\Delta\psi_I$ (μas)	$\Delta\psi_I$ (μas)	$\Delta\psi_I$ (μas)	$\Delta\psi_I$ (μas)
t	–30888468.4680	–7539806.8764	–2644643.7323	–2563687.1111	95713.4424
t^2	17712332.5604	16089.7169	5462.4226	–23519.8462	365452.1188
	$\Delta\theta_I$ (μas)	$\Delta\theta_I$ (μas)	$\Delta\theta_I$ (μas)	$\Delta\theta_I$ (μas)	$\Delta\theta_I$ (μas)
t	–299340.7816	1158037.6874 –	9902.5537	–191566.5107	–345887.4810
t^2	11292.2763	48406.3346	–21824.0040	15481.8592	21837.5697
	$\Delta\varphi_I$ (μas)	$\Delta\varphi_I$ (μas)	$\Delta\varphi_I$ (μas)	$\Delta\varphi_I$ (μas)	$\Delta\varphi_I$ (μas)
t	–20532653.8884	–934897.6652	–207819.5424	382841.2435	2351901.1773
t^2	17697440.4989	–53118.6194	–5333.5745	29110.7501	371810.7571

Table 3. Secular terms of the geodetic rotation for the Solar System bodies under study, calculated for ecliptic Euler angles *(part 2/2)*

	Lutetia (21) e = 0.163 i = 3.06	**Ceres (1)** e = 0.078 i =10.59	**Pallas (2)** e = 0.230 i = 34.85	**Ida (243)** e = 0.043 i = 1.13	**Europa (52)** e = 0.111 i = 7.48
a (km)	364 359 304	413 801 038	415 041 593	428 085 277	463 012 430
	$\Delta\psi_{\mathrm{I}}$ (μas)	$\Delta\psi_{\mathrm{I}}$ (μas)	$\Delta\psi_{\mathrm{I}}$ (μas)	$\Delta\psi_{\mathrm{I}}$ (μas)	$\Delta\psi_{\mathrm{I}}$ (μas)
t^2	–2117178.9998	–3360178.0926	–1398863.8492	–1330838.6427	–1057008.1802
t^2	2431.1845	10775.8519	45720.2961	–16354.9403	–4910.3016
	$\Delta\theta_{\mathrm{I}}$ (μas)	$\Delta\theta_{\mathrm{I}}$ (μas)	$\Delta\theta_{\mathrm{I}}$ (μas)	$\Delta\theta_{\mathrm{I}}$ (μas)	$\Delta\theta_{\mathrm{I}}$ (μas)
t	–78659.8041	–8577.2633	–832986.4619	12929.7503	–86060.4985
t^2	7415.8840	36307.2412	–34963.2327	5290.8816	–21571.6021
	$\Delta\varphi_{\mathrm{I}}$ (μas)	$\Delta\varphi_{\mathrm{I}}$ (μas)	$\Delta\varphi_{\mathrm{I}}$ (μas)	$\Delta\varphi_{\mathrm{I}}$ (μas)	$\Delta\varphi_{\mathrm{I}}$ (μas)
t	82943.9848	1891371.5898	–366135.5559	61556.5113	–147420.0367
t^2	3717.5078	–10758.5439	75110.2601	–17648.9762	15432.2955

	Davida (511) e = 0.188 i = 15.94	**Pluto (134340)** e = 0.249 i = 17.12	**Charon (P I)** e = 0.00005 i = 0	**Pluto** (with taking into account the perturbations from Charon)	**Charon** (with taking into account the perturbations from Pluto)
a (km)	473 341 349	5 900 898 409	19 591		
	$\Delta\psi_{\mathrm{I}}$ (μas)	$\Delta\psi_{\mathrm{I}}$ (μas)	$\Delta\psi_{\mathrm{I}}$ (μas)	$\Delta\psi_{\mathrm{I}}$ (μas)	$\Delta\psi_{\mathrm{I}}$ (μas)
t	–1093309.1477	–2196.6026	–2196.6029	1328.9252	24690.9143
t^2	–16061.0721	1233.0913	1233.0910	1235.6518	1214.0777
	$\Delta\theta_{\mathrm{I}}$ (μas)	$\Delta\theta_{\mathrm{I}}$ (μas)	$\Delta\theta_{\mathrm{I}}$ (μas)	$\Delta\theta_{\mathrm{I}}$ (μas)	$\Delta\theta_{\mathrm{I}}$ (μas)
t	–293147.0102	–229.9056	–229.9063	7460.6486	58441.2641
t^2	8887.6360	128.4493	128.4467	128.5626	129.3395
	$\Delta\varphi_{\mathrm{I}}$ (μas)	$\Delta\varphi_{\mathrm{I}}$ (μas)	$\Delta\varphi_{\mathrm{I}}$ (μas)	$\Delta\varphi_{\mathrm{I}}$ (μas)	$\Delta\varphi_{\mathrm{I}}$ (μas)
t	68969.1272	–606.8179	–606.8175	87.6110	4694.8347
t^2	58587.1184	340.9066	340.9119	343.2894	367.0618

In this table:

a is the length of the satellite orbit's semi-major axis;
1 astronomical unit (au) = 149 597 870. 7 km (from Horizons On-Line Ephemeris System (Giorgini et al., 2001));
e is orbit eccentricity;
i is orbit inclination with respect to ecliptic of J2000.0;
t is the time in Julian thousand years from standard epoch JD 2451545.0, i.e. January 1, 2000 12 hours TDB.

Table 4. The periodic terms of the geodetic rotation for the Solar System bodies under study, calculated for the rotational elements *(α_0, δ_0, W)* *(part 1/3)*

Body	Angle	Period	Argument	Coefficient of cos (Argument) (μas)	Coefficient of sin (Argument) (μas)
Pallas (2)	$\Delta\alpha_{0\ \mathrm{II}}$	4.6133 yrs	λ_{Pal}	$-62.3217 \; +20.3863t$	$488.1305 \; -160.8216t$
e = 0.230	$\Delta\delta_{0\ \mathrm{II}}$	4.6133 yrs	λ_{Pal}	$-79.2069 \; +15.0390t$	$621.2777 \; -121.2805t$
i = 34.85	ΔW_{II}	4.6133 yrs	λ_{Pal}	$-3.2991 \; -6.9178t$	$26.3696 \; +52.6044t$
Vesta (4)	$\Delta\alpha_{0\ \mathrm{II}}$	3.6299 yrs	λ_{Ves}	$-79.3188 \; -5.5469t$	$219.4540 \; +15.1126t$
e = 0.088	$\Delta\delta_{0\ \mathrm{II}}$	3.6299 yrs	λ_{Ves}	$-42.5756 \; -0.7573t$	$117.7886 \; +1.9666t$
i = 7.14	ΔW_{II}	3.6299 yrs	λ_{Ves}	$146.9691 \; +6.2171t$	$-406.5890 \; -16.7824t$
Lutetia (21)	$\Delta\alpha_{0\ \mathrm{II}}$	3.8012 yrs	λ_{Lut}	$-423.8069 \; -9.7248t$	$428.7706 \; +8.5707t$
e = 0.163	$\Delta\delta_{0\ \mathrm{II}}$	3.8012 yrs	λ_{Lut}	$-122.1336 \; +0.3414t$	$123.5570 \; -0.7030t$
i = 3.06	ΔW_{II}	3.8012 yrs	λ_{Lut}	$20.7078 \; -0.9744t$	$-20.9469 \; +1.0484t$
Europa (52)	$\Delta\alpha_{0\ \mathrm{II}}$	5.4539 yrs	λ_{Eur}	$177.5560 \; -25.4369t$	$172.7614 \; -24.9603t$
e = 0.111	$\Delta\delta_{0\ \mathrm{II}}$	5.4539 yrs	λ_{Eur}	$-1.8759 \; +8.4951t$	$-1.8435 \; +8.3894t$
i = 7.48	ΔW_{II}	5.4539 yrs	λ_{Eur}	$-186.1907 \; +33.7301t$	$-181.1749 \; +33.0627t$
Ida (243)	$\Delta\alpha_{0\ \mathrm{II}}$	4.8428 yrs	λ_{Ida}	$-53.6870 \; +28.6795t$	$-23.4218 \; +12.6767t$
e = 0.043	$\Delta\delta_{0\ \mathrm{II}}$	4.8428 yrs	λ_{Ida}	$48.4735 \; -9.1614t$	$21.1260 \; -4.0508t$
i = 1.13	ΔW_{II}	4.8428 yrs	λ_{Ida}	$-173.5102 \; +54.8385t$	$-75.6477 \; +24.2165t$
Eros (433)	$\Delta\alpha_{0\ \mathrm{II}}$	1.7609 yrs	λ_{Ero}	$1180.4497 \; -13.8623t$	$742.2291 \; -9.4965t$
e = 0.223	$\Delta\delta_{0\ \mathrm{II}}$	1.7609 yrs	λ_{Ero}	$292.3515 \; +11.1627t$	$183.8167 \; +6.8238t$
i = 10.83	ΔW_{II}	1.7609 yrs	λ_{Ero}	$-726.0456 \; -10.5342t$	$-456.5093 \; -6.1414t$
Davida (511)	$\Delta\alpha_{0\ \mathrm{II}}$	5.6626 yrs	λ_{Dav}	$-67.2810 \; +7.5571t$	$-428.2301 \; +58.3470t$
e = 0.188	$\Delta\delta_{0\ \mathrm{II}}$	5.6626 yrs	λ_{Dav}	$-35.8472 \; +6.1664t$	$-228.2203 \; +44.2293t$
i = 15.94	ΔW_{II}	5.6626 yrs	λ_{Dav}	$35.8691 \; -12.6102t$	$228.2665 \; -85.8956t$
Gaspra (951)	$\Delta\alpha_{0\ \mathrm{II}}$	3.2853 yrs	λ_{Gas}	$673.5156 \; -12.0004t$	$-75.4762 \; +1.3514t$
e = 0.174	$\Delta\delta_{0\ \mathrm{II}}$	3.2853 yrs	λ_{Gas}	$275.1211 \; +7.9210t$	$-30.8416 \; -0.8844t$
i = 4.11	ΔW_{II}	3.2853 yrs	λ_{Gas}	$-608.4609 \; +5.2720t$	$68.1928 \; -0.5941t$
Steins (2867)	$\Delta\alpha_{0\ \mathrm{II}}$	3.6421 yrs	λ_{Ste}	$6.5886 \; -26.7418t$	$-7.6696 \; +13.8217t$
e = 0.146	$\Delta\delta_{0\ \mathrm{II}}$	3.6421 yrs	λ_{Ste}	$5.8465 \; -236.2914t$	$-53.4095 \; +36.2043t$
i = 9.93	ΔW_{II}	3.6421 yrs	λ_{Ste}	$31.9757 -1581.7149t$	$-354.9374 \; +230.5293t$

Table 4. The periodic terms of the geodetic rotation for the Solar System bodies under study, calculated for the rotational elements *(α_0, δ_0, W)* *(part 2/3)*

Body	Angle	Period	Argument	Coefficient of cos (Argument) (μas)	Coefficient of sin (Argument) (μas)
Itokawa (25143) $e = 0.280$ $i = 1.62$	$\Delta\alpha_{0\,\mathrm{II}}$	1.5139 yrs	λ_{Ito}	$41.7577 \quad -75.8886t$	$-70.3291 \quad +153.0315t$
	$\Delta\delta_{0\,\mathrm{II}}$	1.5139 yrs	λ_{Ito}	$4.7516 \quad -4.1915t$	$-8.0811 \quad +5.5740t$
	ΔW_{II}	1.5139 yrs	λ_{Ito}	$508.0721 \quad -548.2598t$	$-876.1647 \quad +1515.4770t$
Ceres (1) $e = 0.078$ $i = 10.59$	$\Delta\alpha_{0\,\mathrm{II}}$	4.6049 yrs	λ_{Cer}	$3.7269 \quad +3.7663t$	$27.5024 \quad +28.9016t$
	$\Delta\delta_{0\,\mathrm{II}}$	4.6049 yrs	λ_{Cer}	$11.0371 \quad -1.1300t$	$82.7509 \quad -6.8295t$
	ΔW_{II}	4.6049 yrs	λ_{Cer}	$-35.7425 \quad -1.0690t$	$-267.3755 \quad -13.4006t$
Pluto (134340) $e = 0.249$ $i = 17.12$	$\Delta\alpha_{0\,\mathrm{II}}$	247.9673 yrs 123.9837 yrs 82.6558 yrs 61.9918 yrs	λ_9 $2\lambda_9$ $3\lambda_9$ $4\lambda_9$	$45.7326 \quad +76.8920t$ $-9.9483 \quad -4.5637t$ $1.0709 \quad -7.1765t$ $-0.0787 \quad +1.2146t$	$-47.1527 \quad -1.6917t$ $3.8725 \quad +35.3736t$ $1.5396 \quad -7.2298t$ $-0.7342 \quad +0.8507t$
	$\Delta\delta_{0\,\mathrm{II}}$	247.9673 yrs 123.9837 yrs 82.6558 yrs 61.9918 yrs	λ_9 $2\lambda_9$ $3\lambda_9$ $4\lambda_9$	$-8.5420 \quad -14.3663t$ $1.8556 \quad +0.8591t$ $-0.1998 \quad +1.3409t$ $-0.0153 \quad -0.2299t$	$8.8011 \quad +0.2718t$ $-0.7277 \quad -6.6002t$ $-0.2840 \quad +1.3547t$ $0.1363 \quad -0.1606t$
	ΔW_{II}	247.9673 yrs 123.9837 yrs 82.6558 yrs 61.9918 yrs	λ_9 $2\lambda_9$ $3\lambda_9$ $4\lambda_9$	$10.4692 \quad +17.6031t$ $-2.2822 \quad -1.0427t$ $0.2481 \quad -1.6437t$ $-0.0188 \quad +0.2775t$	$-10.7952 \quad -0.3926t$ $0.8893 \quad +8.1005t$ $0.3529 \quad -1.6581t$ $-0.1695 \quad +0.1943t$
Charon (P I) $e = 5\times10^{-5}$ $i = 0$	$\Delta\alpha_{0\,\mathrm{II}}$	247.9673 yrs 123.9837 yrs 82.6558 yrs 61.9918 yrs	λ_9 $2\lambda_9$ $3\lambda_9$ $4\lambda_9$	$45.7326 \quad +76.8918t$ $-9.9484 \quad -4.5638t$ $1.0709 \quad -7.1766t$ $-0.0787 \quad +1.2146t$	$-47.1527 \quad -1.6915t$ $3.8725 \quad +35.3736t$ $1.5396 \quad -7.2299t$ $-0.7342 \quad +0.8506t$
	$\Delta\delta_{0\,\mathrm{II}}$	247.9673 yrs 123.9837 yrs 82.6558 yrs 61.9918 yrs	λ_9 $2\lambda_9$ $3\lambda_9$ $4\lambda_9$	$-8.5420 \quad -14.3667t$ $1.8556 \quad +0.8590t$ $-0.1998 \quad +1.3408t$ $-0.0153 \quad -0.2299t$	$8.8012 \quad +0.2719t$ $-0.7277 \quad -6.6002t$ $-0.2840 \quad +1.3546t$ $0.1363 \quad -0.1607t$
	ΔW_{II}	247.9673 yrs 123.9837 yrs 82.6558 yrs 61.9918 yrs	λ_9 $2\lambda_9$ $3\lambda_9$ $4\lambda_9$	$10.4691 \quad +17.6022t$ $-2.2821 \quad -1.0423t$ $0.2481 \quad -1.6440t$ $-0.0188 \quad +0.2776t$	$-10.7951 \quad -0.3932t$ $0.8892 \quad +8.1005t$ $0.3529 \quad -1.6581t$ $-0.1695 \quad +0.1942t$

Table 4. The periodic terms of the geodetic rotation for the Solar System bodies under study, calculated for the rotational elements *(α_0, δ_0, W)* (part 3/3)

Body	Angle	Period	Argument	Coefficient of cos (Argument) (μas)	Coefficient of sin (Argument) (μas)
Pluto (with taking into account the perturbations from Charon)	$\Delta\alpha_{0\ \mathrm{II}}$	6.3868 ^{d}	$\lambda_{Pl} + \lambda_9$	0.0611 –0.0630t	–0.0782 +0.0144t
		6.3877 ^{d}	D_{Pl}	0.0077 +0.0068t	0.0192 –0.0056t
		247.9673 yrs	λ_9	45.7434 +76.9955t	–47.1480 –1.7233t
		123.9837 yrs	$2\lambda_9$	–9.9455 –4.5471t	3.8788 +35.4534t
		6.3872 ^{d}	λ_{Pl}	–0.0001 +0.0089t	0.0010 0.0062t
		82.6558 yrs	$3\lambda_9$	1.0648 –7.2372t	1.5401 –7.2340t
		61.9918 yrs	$4\lambda_9$	–0.0794 +1.2282t	–0.7362 +0.8298t
	$\Delta\delta_{0\ \mathrm{II}}$	6.3868 ^{d}	$\lambda_{Pl} + \lambda_9$	0.0270 +0.0218t	–0.0295 –0.0332t
		6.3877 ^{d}	D_{Pl}	–0.0340 +0.0339t	–0.0466 –0.0012t
		247.9673 yrs	λ_9	–8.5472 –14.4029t	8.8015 +0.2951t
		123.9837 yrs	$2\lambda_9$	1.8521 +0.8428t	–0.7291 –6.6243t
		6.3872 ^{d}	λ_{Pl}	–0.0029 –0.0166t	–0.0024 –0.0025t
		82.6558 yrs	$3\lambda_9$	–0.1992 +1.3479t	–0.2841 +1.3504t
		61.9918 yrs	$4\lambda_9$	0.0147 –0.2367t	0.1359 –0.1591t
	ΔW_{II}	6.3868 ^{d}	$\lambda_{Pl} + \lambda_9$	–0.0294 +0.0237t	–0.0435 +0.0625t
		6.3877 ^{d}	D_{Pl}	0.0439 –0.0003t	–0.0277 +0.0157t
		247.9673 yrs	λ_9	10.4859 +17.6982t	–10.7878 –0.4001t
		123.9837 yrs	$2\lambda_9$	–2.2797 –1.0270t	0.8948 +8.1528t
		6.3872 ^{d}	λ_{Pl}	0.0021 +0.0212t	–0.0018 –0.0181t
		82.6558 yrs	$3\lambda_9$	0.2452 –1.6813t	0.3534 –1.6603t
		61.9918 yrs	$4\lambda_9$	–0.0192 +0.2890t	–0.1704 +0.1846t
Charon (with taking into account the perturbations from Pluto)	$\Delta\alpha_{0\ \mathrm{II}}$	6.3868 ^{d}	$\lambda_{91} + \lambda_9$	0.5835 –0.6002t	–0.7471 +0.1377t
		6.3877 ^{d}	D_{91}	0.0739 +0.0653t	0.1835 –0.0536t
		6.3872 ^{d}	λ_{91}	–0.0012 +0.0851t	0.0094 0.0596t
		247.9673 yrs	λ_9	45.6369 +75.9256t	–47.2173 –1.4465t
		123.9837 yrs	$2\lambda_9$	–9.9400 –4.7545t	3.8009 +34.5981t
		82.6558 yrs	$3\lambda_9$	1.1150 –6.5950t	1.5269 –7.1762t
		61.9918 yrs	$4\lambda_9$	–0.0698 +1.0883t	–0.7105 +1.0422t
	$\Delta\delta_{0\ \mathrm{II}}$	6.3868 ^{d}	$\lambda_{91} + \lambda_9$	0.2579 +0.2086t	–0.2816 –0.3171t
		6.3877 ^{d}	D_{91}	–0.3251 +0.3235t	–0.4451 –0.0118t
		6.3872 ^{d}	λ_{91}	–0.0281 –0.1590t	–0.0231 –0.2377t
		247.9673 yrs	λ_9	–8.4887 –13.8767t	8.7793 +0.0823t
		123.9837 yrs	$2\lambda_9$	1.8674 +1.0673t	–0.7105 –6.3992t
		82.6558 yrs	$3\lambda_9$	–0.1930 +1.2817t	–0.2719 +1.3966t
		61.9918 yrs	$4\lambda_9$	0.0183 –0.1776t	0.1378 –0.1475t
	ΔW_{II}	6.3868 ^{d}	$\lambda_{91} + \lambda_9$	–0.2805 +0.2267t	–0.4154 +0.5972t
		6.3877 ^{d}	D_{91}	0.4190 –0.0028t	–0.2645 +0.1498t
		6.3872 ^{d}	λ_{91}	0.0203 +0.2027t	–0.0171 –0.1727t
		247.9673 yrs	λ_9	10.3847 +16.6751t	–10.7867 –0.2098t
		123.9837 yrs	$2\lambda_9$	–2.2978 –1.2805t	0.8435 +7.5052t
		82.6558 yrs	$3\lambda_9$	0.2783 –1.2621t	0.3346 –1.6827t
		61.9918 yrs	$4\lambda_9$	–0.0156 +0.1888t	–0.1581 +0.3053t

Table 5. The periodic terms of the geodetic rotation for the Solar System bodies under study, calculated for ecliptic Euler angles *(part 1/3)*

Body	Angle	Period	Argument	Coefficient of cos (Argument) (μas)	Coefficient of sin (Argument) (μas)
Pallas (2)	$\Delta\psi_{II}$	4.6133 yrs	λ_{Pal}	88.9576 –25.2003*t*	–697.0760 +199.7455*t*
e = 0.230	$\Delta\theta_{II}$	4.6133 yrs	λ_{Pal}	52.7890 –7.0685*t*	–414.3066 +58.2399*t*
i = 34.85	$\Delta\varphi_{II}$	4.6133 yrs	λ_{Pal}	23.4799 –14.6468*t*	–183.4599 +113.8270*t*
Vesta (4)	$\Delta\psi_{II}$	3.6299 yrs	λ_{Ves}	134.5261 +7.4573*t*	–372.1918 –20.2328*t*
e = 0.088	$\Delta\theta_{II}$	3.6299 yrs	λ_{Ves}	10.0519 –1.2577*t*	–27.8049 +3.5120*t*
i = 7.14	$\Delta\varphi_{II}$	3.6299 yrs	λ_{Ves}	–20.0867 –3.8164*t*	55.6047 +10.4824*t*
Lutetia (21)	$\Delta\psi_{II}$	3.8012 yrs	λ_{Lut}	434.7883 +9.1964*t*	–439.8789 –8.0052*t*
e = 0.163	$\Delta\theta_{II}$	3.8012 yrs	λ_{Lut}	16.1543 –2.6762*t*	–16.3367 +2.7483*t*
i = 3.06	$\Delta\varphi_{II}$	3.8012 yrs	λ_{Lut}	–17.0374 –1.9309*t*	17.2406 +1.9029*t*
Europa (52)	$\Delta\psi_{II}$	5.4539 yrs	λ_{Eur}	–210.3443 +31.2243*t*	–204.6667 +30.6463*t*
e = 0.111	$\Delta\theta_{II}$	5.4539 yrs	λ_{Eur}	–17.0471 –5.7352*t*	–16.5686 –5.6809*t*
i = 7.48	$\Delta\varphi_{II}$	5.4539 yrs	λ_{Eur}	–29.4063 +10.6478*t*	–28.6229 +10.4089*t*
Ida (243)	$\Delta\psi_{II}$	4.8428 yrs	λ_{Ida}	123.9993 –23.6262*t*	54.0423 –10.4464*t*
e = 0.043	$\Delta\theta_{II}$	4.8428 yrs	λ_{Ida}	–1.1923 –0.7028*t*	–0.5184 –0.31056*t*
i = 1.13	$\Delta\varphi_{II}$	4.8428 yrs	λ_{Ida}	–5.7846 +4.4539*t*	–2.5247 +1.9428*t*
Eros (433)	$\Delta\psi_{II}$	1.7609 yrs	λ_{Ero}	–1173.7477 +7.8618*t*	–738.0132 +5.7199*t*
e = 0.223	$\Delta\theta_{II}$	1.7609 yrs	λ_{Ero}	180.2847 –15.5085*t*	113.3614 –9.8689*t*
i = 10.83	$\Delta\varphi_{II}$	1.7609 yrs	λ_{Ero}	–145.5448 –16.1855*t*	–91.5093 –10.0787*t*
Davida (511)	$\Delta\psi_{II}$	5.6626 yrs	λ_{Dav}	80.8186 –9.5533*t*	514.4072 –73.0080*t*
e = 0.188	$\Delta\theta_{II}$	5.6626 yrs	λ_{Dav}	21.6921 –4.5331*t*	138.1255 –31.6875*t*
i = 15.94	$\Delta\varphi_{II}$	5.6626 yrs	λ_{Dav}	–4.9923 –7.8146*t*	–31.8137 –49.1951*t*
Gaspra (951)	$\Delta\psi_{II}$	3.2853 yrs	λ_{Gas}	–707.3531 +6.8524*t*	79.2730 –0.7751*t*
e = 0.174	$\Delta\theta_{II}$	3.2853 yrs	λ_{Gas}	2.6585 –11.6876*t*	–0.2882 +1.3092*t*
i = 4.11	$\Delta\varphi_{II}$	3.2853 yrs	λ_{Gas}	–55.5945 –2.5442*t*	6.2352 +0.2873*t*
Steins (2867)	$\Delta\psi_{II}$	3.6421 yrs	λ_{Ste}	–32.2178 –100.5467*t*	–13.2111 –41.5365*t*
e = 0.146	$\Delta\theta_{II}$	3.6421 yrs	λ_{Ste}	–6.0932 +236.4883*t*	53.5204 –36.6313*t*
i = 9.93	$\Delta\varphi_{II}$	3.6421 yrs	λ_{Ste}	–5.9565–1658.3287*t*	–361.3345 +176.9216*t*

Table 5. The periodic terms of the geodetic rotation for the Solar System bodies under study, calculated for ecliptic Euler angles *(part 2/3)*

Body	Angle	Period	Argument	Coefficient of cos (Argument) (μas)	Coefficient of sin (Argument) (μas)
Itokawa (25143) e = 0.280 i = 1.62	$\Delta\psi_{II}$	1.5139 yrs	λ_{Ito}	-1720.2459 $+4498.8779t$	2888.8876 $-7571.1834t$
	$\Delta\theta_{II}$	1.5139 yrs	λ_{Ito}	-14.2316 $+15.8766t$	24.0302 $-42.9370t$
	$\Delta\varphi_{II}$	1.5139 yrs	λ_{Ito}	-1250.3802 $+4020.0291t$	2077.0710 $-6195.7015t$
Ceres (1) e = 0.078 i = 10.59	$\Delta\psi_{II}$	4.6049 yrs	λ_{Cer}	-75.8046 $+6.1151t$	-568.1727 $+34.5235t$
	$\Delta\theta_{II}$	4.6049 yrs	λ_{Cer}	-0.1761 $+1.6365t$	-1.4920 $+12.2860t$
	$\Delta\varphi_{II}$	4.6049 yrs	λ_{Cer}	42.6647 $-3.6571t$	319.9076 $-20.9923t$
Pluto (134340) e = 0.249 i = 17.12	$\Delta\psi_{II}$	247.9673 yrs	λ_9	-49.8707 $-83.8508t$	51.4174 $+1.8306t$
		123.9837 yrs	$2\lambda_9$	10.8477 $+4.9787t$	-4.2243 $-38.5721t$
		82.6558 yrs	$3\lambda_9$	-1.1678 $+7.8260t$	-1.6777 $+7.8854t$
		61.9918 yrs	$4\lambda_9$	0.0860 $-1.3255t$	0.8004 $-0.9282t$
	$\Delta\theta_{II}$	247.9673 yrs	λ_9	-5.2163 $-8.7663t$	5.3841 $+0.2352t$
		123.9837 yrs	$2\lambda_9$	1.1372 $+0.5141t$	-0.4375 $-4.0414t$
		82.6558 yrs	$3\lambda_9$	-0.1224 $+0.8181t$	-0.1790 $+0.8205t$
		61.9918 yrs	$4\lambda_9$	0.0084 $-0.1356t$	0.0846 $-0.0954t$
	$\Delta\varphi_{II}$	247.9673 yrs	λ_9	-13.7787 $-23.1663t$	14.2049 $+0.\ 4989t$
		123.9837 yrs	$2\lambda_9$	2.9922 $+1.3778t$	-1.1645 $-10.6541t$
		82.6558 yrs	$3\lambda_9$	-0.3196 $+2.1615t$	-0.4629 $+2.1758t$
		61.9918 yrs	$4\lambda_9$	0.0230 $-0.3669t$	0.2197 $-0.2570t$
Charon (P I) $e = 5\times10^{-5}$ i = 0	$\Delta\psi_{II}$	247.9673 yrs	λ_9	-49.8707 $-83.8508t$	51.4173 $+1.8305t$
		123.9837 yrs	$2\lambda_9$	10.8477 $+4.9788t$	-4.2243 $-38.5721t$
		82.6558 yrs	$3\lambda_9$	-1.1678 $+7.8260t$	-1.6777 $+7.8854t$
		61.9918 yrs	$4\lambda_9$	0.0860 $-1.3255t$	0.8004 $-0.9282t$
	$\Delta\theta_{II}$	247.9673 yrs	λ_9	-5.2162 $-8.7658t$	5.3841 $+0.2350t$
		123.9837 yrs	$2\lambda_9$	1.1372 $+0.5143t$	-0.4375 $-4.0414t$
		82.6558 yrs	$3\lambda_9$	-0.1224 $+0.8182t$	-0.1790 $+0.8206t$
		61.9918 yrs	$4\lambda_9$	0.0084 $-0.1356t$	0.0846 $-0.0953t$
	$\Delta\varphi_{II}$	247.9673 yrs	λ_9	-13.7788 $-23.1672t$	14.2049 $+0.4983t$
		123.9837 yrs	$2\lambda_9$	2.9923 $+1.3783t$	-1.1645 $-10.6541t$
		82.6558 yrs	$3\lambda_9$	-0.3197 $+2.1612t$	-0.4629 $+2.1758t$
		61.9918 yrs	$4\lambda_9$	0.0230 $-0.3667t$	0.2197 $-0.2571t$

Table 5. The periodic terms of the geodetic rotation for the Solar System bodies under study, calculated for ecliptic Euler angles *(part 3/3)*

Body	Angle	Period	Argument	Coefficient of cos (Argument) (μas)	Coefficient of sin (Argument) (μas)
Pluto (with taking into account the perturba– tions from Charon)	$\Delta\psi_{II}$	6.3868 ^{d}	$\lambda_{Pl}+\lambda_9$	–0.0544 +0.0719*t*	0.0712 –0.0255*t*
		6.3877 ^{d}	D_{Pl}	–0.0188 +0.0038*t*	–0.0347 +0.0054*t*
		247.9673 yrs	λ_9	–49.8834 –83.9692*t*	51.4126 +1.8707*t*
		6.3872 ^{d}	λ_{Pl}	–0.0008 –0.0145*t*	–0.0018 –0.0144*t*
		123.9837 yrs	$2\lambda_9$	10.8431 +4.9539*t*	–4.2304 –38.6559*t*
		82.6558 yrs	$3\lambda_9$	–1.1614 +7.8894*t*	–1.6786 +7.8874*t*
		61.9918 yrs	$4\lambda_9$	0.0866 –1.3409*t*	0.8022 –0.9077*t*
	$\Delta\theta_{II}$	6.3868 ^{d}	$\lambda_{Pl}+\lambda_9$	–0.0437 –0.0024*t*	0.0511 +0.0275*t*
		6.3877 ^{d}	D_{Pl}	0.0303 –0.0344*t*	0.0389 +0.0028*t*
		247.9673 yrs	λ_9	–5.2145 –8.7615*t*	5.3824 +0.2222*t*
		6.3872 ^{d}	λ_{Pl}	0.0028 +0.0133*t*	0.0020 +0.0220*t*
		123.9837 yrs	$2\lambda_9$	1.1405 +0.5279*t*	–0.4392 –4.0521*t*
		82.6558 yrs	$3\lambda_9$	–0.1210 +0.8317*t*	–0.1787 +0.8270*t*
		61.9918 yrs	$4\lambda_9$	0.0091 –0.1340*t*	0.0857 –0.0882*t*
	$\Delta\varphi_{II}$	6.3868 ^{d}	$\lambda_{Pl}+\lambda_9$	–0.0570 +0.0584*t*	–0.0075 +0.0511*t*
		6.3877 ^{d}	D_{Pl}	0.0357 +0.0004*t*	–0.0432 +0.0184*t*
		247.9673 yrs	λ_9	–13.7681 –23.1282*t*	14.2099 +0.5103*t*
		6.3872 ^{d}	λ_{Pl}	0.0018 +0.0146*t*	–0.0026 –0.0243*t*
		123.9837 yrs	$2\lambda_9$	2.9929 +1.3861*t*	–1.1620 –10.6384*t*
		82.6558 yrs	$3\lambda_9$	–0.3195 +2.1537*t*	–0.4628 +2.1762*t*
		61.9918 yrs	$4\lambda_9$	0.0228 –0.3633*t*	0.2196 –0.2575*t*
Charon (with taking into account the perturba– tions from Pluto)	$\Delta\psi_{II}$	6.3868 ^{d}	$\lambda_{91}+\lambda_9$	–0.5192 +0.6870*t*	0.6803 –0.2432*t*
		6.3877 ^{d}	D_{91}	–0.1800 +0.0360*t*	–0.3312 +0.0516*t*
		6.3872 ^{d}	λ_{91}	–0.0077 –0.1384*t*	–0.0172 –0.1373*t*
		247.9673 yrs	λ_9	–49.7551 –82.7024*t*	51.4769 +1.5174*t*
		123.9837 yrs	$2\lambda_9$	10.8429 +5.2419*t*	–4.1450 –37.7086*t*
		82.6558 yrs	$3\lambda_9$	–1.2110 +7.2077*t*	–1.6608 +7.8435*t*
		61.9918 yrs	$4\lambda_9$	0.0778 –1.1786*t*	0.7764 –1.1215*t*
	$\Delta\theta_{II}$	6.3868 ^{d}	$\lambda_{91}+\lambda_9$	–0.4173 –0.0233*t*	0.4877 +0.2628*t*
		6.3877 ^{d}	D_{91}	0.2891 –0.3283*t*	0.3717 +0.0270*t*
		6.3872 ^{d}	λ_{91}	0.0272 +0.1270*t*	0.0193 +0.2097*t*
		247.9673 yrs	λ_9	–5.2392 –8.9526*t*	5.4239 +0.3446*t*
		123.9837 yrs	$2\lambda_9$	1.1235 +0.3709*t*	–0.4330 –4.0066*t*
		82.6558 yrs	$3\lambda_9$	–0.1418 +0.7045*t*	–0.1869 +0.7648*t*
		61.9918 yrs	$4\lambda_9$	0.0030 –0.1487*t*	0.0761 –0.1640*t*
	$\Delta\varphi_{II}$	6.3868 ^{d}	$\lambda_{91}+\lambda_9$	–0.5445 +0.5577*t*	–0.0714 +0.4881*t*
		6.3877 ^{d}	D_{91}	0.3412 +0.0041*t*	–0.4126 +0.1755*t*
		6.3872 ^{d}	λ_{91}	0.0174 +0.1398*t*	–0.0248 –0.2323*t*
		247.9673 yrs	λ_9	–13.8080 –23.5456*t*	14.2437 +0.5339*t*
		123.9837 yrs	$2\lambda_9$	2.9738 +1.2625*t*	–1.1718 –10.8312*t*
		82.6558 yrs	$3\lambda_9$	–0.3110 +2.2409*t*	–0.4733 +2.1292*t*
		61.9918 yrs	$4\lambda_9$	0.0221 –0.3851*t*	0.2192 –0.2414*t*

In Tables 4 and 5:

the superscripts h and d are the hours and days, respectively;

e is orbit eccentricity;

i is orbit inclination with respect to ecliptic of J2000.0;

t is the time in Julian thousand years from standard epoch JD 2451545.0, i.e. January 1, 2000 12 hours TDB;

λ_{Cer} is the mean longitude of Ceres;

λ_9 is the mean longitude of Pluto (Pluto–Charon system barycenter) relative to its orbital motion around the barycenter of the Solar System;

λ_{Pl} is the mean longitude of Pluto relative to its orbital motion around the barycenter of the Pluto–Charon system;

λ_{91} is the mean longitude of Charon relative to its orbital motion around the barycenter of the Pluto–Charon system;

$D_{\mathrm{Pl}} = \lambda_{\mathrm{Pl}} - \lambda_9 + 180^{\circ}$, $D_{91} = \lambda_{91} - \lambda_9 + 180^{\circ}$, are mean elongations of Pluto and Charon from the Sun, respectively;

λ_{Ito}, λ_{Ste}, λ_{Gas}, λ_{Dav}, λ_{Ero}, λ_{Ida}, λ_{Eur}, λ_{Lut}, λ_{Ves}, and λ_{Pal} are the mean longitudes for Itokawa, Steins, Gaspra, Davida, Eros, Ida, Europa, Lutetia, Vesta, and Pallas, respectively;

$\lambda_{\mathrm{Ito}} = 23780^{\circ}.1890200T$, $\lambda_{\mathrm{Ste}} = 9884^{\circ}.2798905T$, $\lambda_{\mathrm{Gas}} = 10957^{\circ}.7803448T$,

$\lambda_{\mathrm{Dav}} = 6357^{\circ}.5111788T$, $\lambda_{\mathrm{Ero}} = 20444^{\circ}.1322292T$, $\lambda_{\mathrm{Ida}} = 7433^{\circ}.6789177T$,

$\lambda_{\mathrm{Eur}} = 6600^{\circ}.7761088T$, $\lambda_{\mathrm{Lut}} = 9470^{\circ}.5823807T$, $\lambda_{\mathrm{Ves}} = 9917^{\circ}.6680806T$,

$\lambda_{\mathrm{Pal}} = 7803^{\circ}.5291018T$, $\lambda_{\mathrm{Cer}} = 7817^{\circ}.7540574T$, $\lambda_9 = 238^{\circ}.96535011 + 145^{\circ}.18042903T$,

$\lambda_{\mathrm{Pl}} = 302^{\circ}.695 + 2058641^{\circ}.1343125T$, $\lambda_{91} = 122^{\circ}.695 + 2058641^{\circ}.1343125T$;

T is the time in Julian centuries.

The mean longitudes for Ceres and asteroids were computed in this investigation using an algorithm elaborated by Pashkevich (see Appendix B) based on the spectral analysis method (Jenkins, Watts, 1969). The mean longitudes for other Solar System bodies under study were taken from Archinal et al., (2018) and the Horizons On-Line Ephemeris System (Giorgini et al., 2001).

APPENDIX B

Algorithm of Pashkevich for calculating the mean longitudes of celestial bodies

1. The expressions of the geodetic rotation velocities of bodies in the Solar System (3) are used for calculation of the geodetic rotation velocities of celestial body under study.
2. To remove the secular terms from the calculated geodetic rotation velocities of body under study by means of the least-squares method. The expressions for the secular terms of the body's geodetic rotation velocities can be represented as a polynomial in the degree of time:

$$\Delta\dot{x} = \sum_{n=1}^{N} \Delta\dot{x}_n t^{n-1}, \tag{B1}$$

   where $\Delta\dot{x}_n$ are the coefficients of the secular terms; $\dot{x} = \dot{\psi},\ \dot{\theta},\ \dot{\varphi},\ \dot{\alpha}_0,\ \dot{\delta}_0,\ \dot{\mathrm{W}};$ t is the time from standard epoch JD 2451545.0, , i.e. January 1, 2000 12 hours TDB; N is the degree of the approximating polynomial. As a result of calculations by the least squares method, the value of the degree of the approximating polynomial is obtained, which provide the best approximation of the geodetic rotation $N = 2$.
3. The necessary data are set: the confidence interval for refining or finding the period for the harmonic[12] of the mean longitude of the body under study (period search interval in days ($P_{\min}, P_{\max}$)); $Step_n$ is period search step (in days); ε is the accuracy with which the search is performed.
4. After removing the secular terms, the power spectrum (Jenkins, Watts, 1969) of residual periodic data of the geodetic rotation velocities of the studied body is calculated on a given interval ($P_{\min}, P_{\max}$) with a given step ($Step_n$). The amplitude of each harmonic of the power spectrum is calculated by the least squares method. Further, from the obtained spectrum, the period of the maximum harmonic P_n (days) is found. P_n is the period for the harmonic of the mean longitude of the body under study.
5. Accuracy is checked: if $Step_n < |\varepsilon|$ then go to 7.
6. Data redefinition: $P_{\min} = P_n - Step_n;\ P_{\max} = P_n + Step_n;\ Step_n = \dfrac{Step_n}{10}$ and go to 4.
7. $\lambda_{Body} = 36525 \dfrac{360}{P_n} T$ is mean longitude of the body under study in degrees; T is the time in Julian centuries.
8. The end of the procedure.

[12] The period for the harmonic of the mean longitude of the body under study is the orbital body rotation period.