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Mathematical model of Coriolis effect

Introduction

The *Coriolis* effect occurs in the rotating reference systems. For an observer remaining in such a system it manifests as deflection of a path of an object moving within a rotating system. This deflection seems to be generated by a force, the so-called *Coriolis* force. The *Coriolis* force is an apparent effect that occurs only in rotating non-inertial systems. For an external observer such force does not exist. For him the system changes its location while a moving body maintains its motional state in agreement with the first principle of dynamics.

A French engineer and mathematician *Gaspard Gustave Coriolis* was an inventor of this effect whilst its first experimental confirmation for the Earth has been demonstrated by *Jean Bernard Léon Foucault* who used his pendulum [*Bogusz et al., 2010; Skorko, 1982*].

The *Foucault* pendulum [*Bogusz et al., 2010*] that was used to prove the existence of the *Coriolis* force, changes its plane of oscillations influenced by the effect of the horizontal component of the *Coriolis* force (F_c), resulting in its rotation in an opposite direction with respect to the Earth rotation while at the same time the pendulum weight draws a rosette. Under the action of the *Coriolis* force the plane of oscillations of the mathematical pendulum z changes with the angular velocity $\omega = \omega_z \sin\varphi$ with a period

$$T = \frac{T_z}{\sin\varphi} \tag{1}$$

The rotation of the plane of oscillations is the fastest at the poles (T = 24 h), and disappears at the equator ($T = \infty$). In Poland the period T equals from 30 to 32 hours.

Treating the pendulum motion [*Bogusz et al., 2010*] of length $l = 28$ m at small deflections as harmonic, the equations of motion with accounting components of the *Coriolis* force are as follows

$$m\frac{d_2 x}{dt^2} = -kx + 2m\omega_z\frac{dy}{dt} \tag{2}$$

$$m\frac{d_2 y}{dt^2} = -ky - 2m\omega_z\frac{dx}{dt} \tag{3}$$

The *Coriolis* force is given as

$$F_c = 2m\,(\boldsymbol{\omega}\times\mathbf{v}) \tag{4}$$

while the acceleration involved with this force is as follows

$$a_c = -2\,(\boldsymbol{\omega}\times\mathbf{v}) \tag{5}$$

where:
m – body mass,
v – body velocity
ω – angular velocity of the system.

A simple proof on the *Coriolis* acceleration can be offered [*Skorko, 1982*] a distance passed by a sphere with linear velocity v along the radius of the spinning disc is equal to $\Delta r = v\ t$, while a motionless observer will state that the path *made* along an arc of the spinning disc during its rotation by an angle: $\varphi = \omega$ t will be

$$\Delta S = \varphi\Delta r = (\omega t)vt = \frac{1}{2}a_c t^2 \tag{6}$$

Hence the *Coriolis* acceleration, a_c, of a sphere is equal to

$$a_c = 2\omega v \tag{7}$$

An example of the existence of the *Coriolis* acceleration for a falling sphere onto a spinning Earth is discussed in [*Bogusz et al., 2010*]. To determine the acceleration

$$a = \frac{d^2 r}{dt^2} = \frac{1}{m}(F + F_{od} + F_c) = g - \omega^2 r - 2\omega v \tag{8}$$

the gravity force, $F = mg$, the centrifugal force $F_{cen} = m\omega^2 r$ and the *Coriolis force,* $F_c = 2m\omega v$. The effect of the *Coriolis* force on spinning single stars and a system of binary stars has been derived by [*Lal et al., 2008; 2009*] of the *Coriolis* force on thermal convection and impurity segregation during crystal growth under

$$\frac{\partial v}{\partial t} + v\nabla v = -\frac{1}{\rho}\nabla P + v\nabla\cdot(v\nabla F + \nabla v) - 2(\omega\times v) + \beta g(T - T_m) \tag{9}$$

microgravity [*Müller et al., 1992; Kohno i Tanahashi, 2002*].

Investigations of the effect of the *Coriolis* force on hydrodynamics of the liquid flow have been carried out by [*Martinez et al., 2012*] and on erosion of impellers and spinning cylindrical mixers – by *Krupicz* [*2000*] and *Gałkowski et al*., [*1999*] and on the separation effectiveness of suspensions [*Łagutkin, 2003*], *Choi and Yoo* [*2015*] utilizing the *Coriolis* effect $f_{Coriolis} = -2\omega v\rho$ and the different values of the ρ density of these particles the separation for DNA purification.

Investigations of the effect *Coriolis* on the astrophysics problems shapes of rotating stars or secondary circulation around headlands and islands and the problem of the physical nature of the East-Antarctic atmospheric boundary layer and regional climate model and other the problem meteorology results of a it by using the following relationship (Eq. 4), which does not agree with definition of acceleration of an object moving with a velocity $v = dr/dt$ along a radius of a spinning disk with the angular velocity ω

$$a_c = \frac{dv_r}{dt} = \frac{d(\omega r)}{dt} = \omega\frac{dr}{dt} = \omega v \tag{10}$$

Investigations of the effect of the *Coriolis* force on hydrodynamics of the liquid flow and on the thermal convection and segregation during crystal growth and on the separation effectiveness of suspensions and on the separation for DNA purification and other chemical engineering and bioengineering process results by using the Eqs (4) and (5).

New mathematical model of Coriolis effect

The object that moves along a meridian of the Earth globe with a linear velocity v covers a distance dt during the time ds and changes its location (geographical latitude) with respect to the equator plane by an angle $d\varphi = \frac{dS}{R}$ and a distance $r = R\cos\varphi$ measured from the axis of the Earth rotation and thus changes linear spinning velocity around the axis $v_r = \frac{2\pi r}{T} = \omega r = \omega R\cos\varphi$, hence

$$v = \frac{dS}{dt} = \frac{Rd\varphi}{dt} \qquad \varphi\in(-\frac{\pi}{2},\frac{\pi}{2}) \tag{11}$$

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Making use of the definition of acceleration (tangential to a latitude of the radius *r*)

$$a_c = \frac{d\upsilon_r}{dt} = \frac{d(\omega R\cos\varphi)}{dt} = \omega R(-\sin\varphi)\frac{d\varphi}{dt} \tag{12}$$

and the object linear velocity along a meridian (Eq. 11) one will get an expression for the *Coriolis* acceleration

$$a_c = \omega(-\sin\varphi)v \tag{13}$$

or

$$a_c = -(\boldsymbol{\omega}\times\mathbf{v}) \tag{14}$$

In agreement with the *Newton's* inertia principle ($F_c = -ma_c$) an equation for the *Coriolis* force takes the form

$$F_c = m(\omega\times v) \tag{15}$$

If the object at the same time moves along a meridian with a velocity *v* and along a parallel of latitude with a velocity $v_s = s\omega R$, assuming $s \in (-1,+1)$, its angular velocity of circulation around the Earth globe will change by $\Delta\omega = \frac{\upsilon_s}{r}$, therefore the expression for the *Coriolis acceleration* (13) will take the following form

$$a_c = (\omega + \frac{v_s}{r})(-\sin\varphi)v = -(\omega v\sin\varphi)\left(1+\frac{s}{\cos\varphi}\right) \tag{16}$$

or

$$a_c = -(\omega\times v)(1+\frac{s}{\cos\varphi}) \quad s = \frac{v_s}{\omega R} \tag{17}$$

$$s \in (-1,+1) \qquad \omega = \frac{2\pi}{T},$$

and according to the *Newton's* inertia principle ($F = -ma$) the *Coriolis* force will be

$$F_c = m(\omega\times v)\left(1+\frac{s}{\cos\varphi}\right) \tag{18}$$

After substituting the positive components $\{v_s, v\} > 0$ directed eastwards $v_s \rightarrow (+)$ and northwards $v\overset{(+)}{\uparrow}$ as well as the negative $\{v_s, v\} < 0$ directed westwards $(-) \leftarrow v_s$ and southwards $v\underset{(-)}{\downarrow}$ (according to a scheme $(-) \leftarrow v_s \rightarrow (+)$ and $v \updownarrow^{+}_{-}$) with a reduced object velocity along the parallel of latitude $s = \frac{v_s}{\omega R}$, $s \in (-1,+1)$ as well as meridian $k_s = \frac{v}{R\omega}$, $k_s \in (-1,+1)$, for $k_s = \pm s$ a modified form of Eq. (16) has been obtained

$$a_c = -R\omega^2\sin\varphi(1+\frac{s}{\cos\varphi})k_s \tag{19}$$

for $|v| = |v_s| \rightarrow v = \pm v_s$ and sign(v)=sign(v_s) (k_s=s)

$$a_c = -R\omega^2\sin\varphi(1+\frac{s}{\cos\varphi})s \tag{20}$$

for sign(v) = -sign(v_s) (k_s=-s)

$$a_c = R\omega^2\sin\varphi\ (1+\frac{s}{\cos\varphi})\ s \qquad s = \frac{v_s}{\omega R} \tag{21}$$

From the graphs drawn for *a* dependence of the *Coriolis* acceleration on the reduced object velocity, *s*, (for $v = \pm v_S = \pm s\omega R$) and increase in the absolute value of the *Coriolis* acceleration at $v_S > 0$ has been observed while at $v_S < 0$ a similar decrease in the absolute value obtained (Eqs (15) or (18) and Figs. 1÷4.

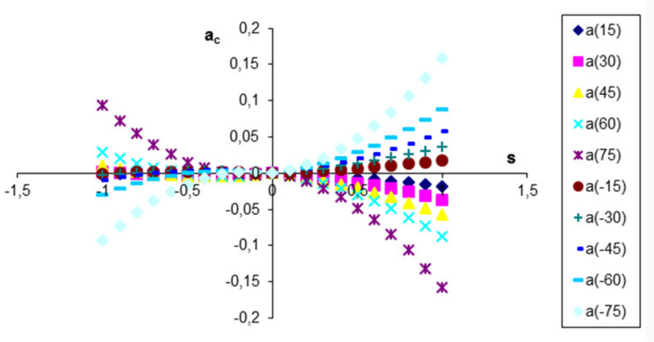

Fig. 1. Dependence of *Coriolis* acceleration on the velocity *v* of object motion simultaneously along meridian within the range ϕ = (-75°, +75°) and along the parallel of latitude at velocity v_S ($|v| = |v_S|$, $s = v_S/\omega R$) for (-) ← v_S and $v\downarrow$ (-)} and for {v_S → (+) and $v\uparrow$ (+)}

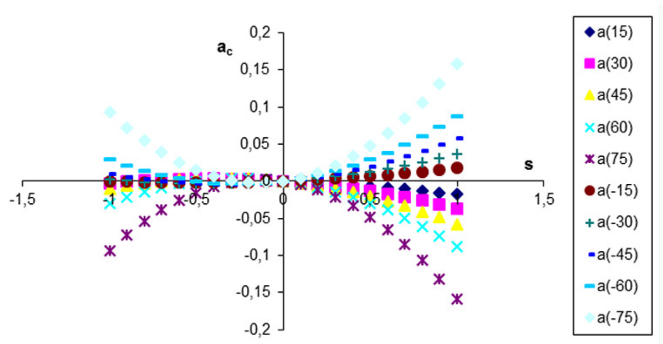

Fig. 2 Dependence of *Coriolis* acceleration on the velocity *v* of object motion simultanously along meridian within the range ϕ = (-75°, +75°) and along the parallel of latitude at velocity v_S ($|v| = |v_S|$, $s = v_S/\omega R$) for { v_S → (-) and $v\uparrow$ (+)} and for {v_S → (+) and $v\uparrow$ (+)}

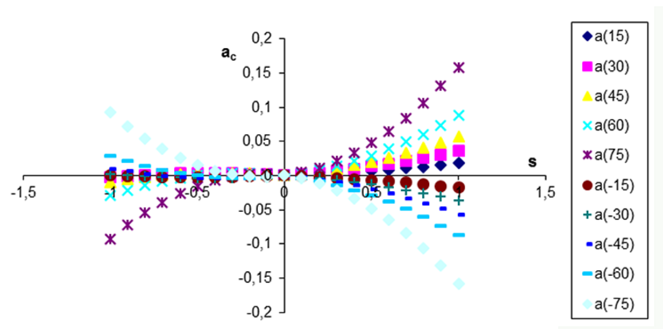

Fig. 3 Dependence of *Coriolis* acceleration on the velocity *v* of object motion simultanouly along meridian within the range ϕ = (-75°, +75°) and along the parallel of latitude at velocity v_S ($|v| = |v_S|$, $s = v_S/\omega R$) for {(-) ← v_S and $v\uparrow$ (+)} and for {v_S → (+) and $v\downarrow$ (-)}

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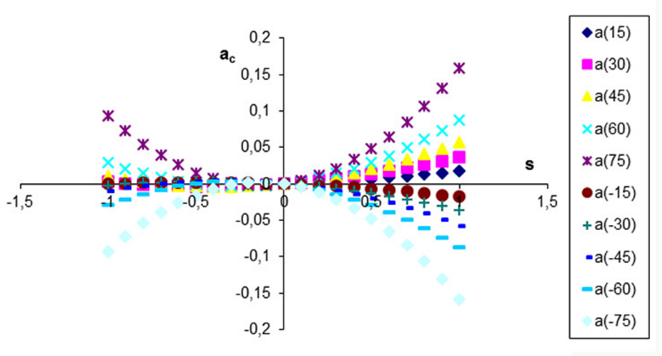

Fig. 4 Dependence of *Coriolis* acceleration on the velocity v of object motion simultanouly along meridian within the range $\phi = (-75^o, +75^o)$ and along the parallel of latitude at velocity v_S ($|v| = |v_S|$, $s = v_S/\omega R$) for $\{(-) \leftarrow v_S$ and $v\downarrow (-)\}$ and for $\{v_S \rightarrow (+)$ and $v\uparrow (-)\}$

Discussion

As follows from Eq. (15) modified to the forms (18) and graphically presented in Figs. 1÷4 for the direction of the component $v_S \rightarrow (+)$ being in accord with that of the Earth surface $v_r = \omega r$, i.e. from west to east, ($v_S > 0$ for $s \in (0,+1)$ an increase of the angular velocity of the Earth globe cycle ($\omega + v_S/r$) and an increase of the absolute value of the *Coriolis* acceleration $|a_c|$.

The derived universal mathematical model (Eq.15) or (Eq.16) enables calculation of the *Coriolis* acceleration for a moving object simultaneously along a meridian or a parallel of latitude (northwards with a velocity ↑ ($v > 0$) or southwards with a velocity ↓ ($v < 0$) and eastwards with a velocity → ($v_s > 0$) or westwards with a velocity ← ($v_s < 0$) for geographical latitude $\phi = (-\pi/2, \pi/2)$.

For a given object velocity along a meridian v ↑ (+) or v ↓ (-) and a positive velocity along the parallel of latitude $v_s > 0$ (eastwards), being in accord with a scheme (-) ← v_s → (+), v (+) ↔ (-) an increase in the angular velocity of the Earth globe cycle by a value of $\Delta\omega = \frac{v_s}{r}$ as well as of an absolute value of the *Coriolis* force, while for $v_s < 0$ (eastwards) the opposite effect will occur. Acceleration and *Coriolis* force will not arise at latitude $\varphi = 0$ or for the velocity component along the meridian $v = 0$ or for the parallel component: $v_s = -\omega r = -\omega R cos\phi$ directed westwards (Eq. 15).

In the case launching rocket from the point at latitude $\varphi_0 > 0$ with a velocity ($+v_s$,$+ v$), in the result of the *Coriolis* effect track deviation bullet from the meridian $(\Delta S_s > 0)$ in the east direction has been obtained:

$$\Delta S_s = v_s \Delta t + \omega v (\sin\varphi_s)(\Delta t)^2 \tag{22}$$

where

$$\Delta t = \frac{R(\varphi - \varphi_o)}{v} \qquad \varphi_s = \frac{1}{2}(\varphi + \varphi_o)$$

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