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GOVERNMENTAL COMBAT OF MIGRATION BETWEEN COMPETING TERRORIST ORGANISATIONS

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Each terrorist organisation is modelled with four coupled differential time equations for the evolution of ideologues, criminal mercenaries, captive participants, and capital sponsoring. Emigration of ideologues may cause unbounded growth of the organisation receiving ideologues. The organisation losing ideologues may reach a stationary state where ideologues are supported by capital sponsors and mercenaries. Emigration of mercenaries may cause the organisation losing mercenaries to experience growth. The organisation receiving mercenaries may lose capital sponsors permanently, allowing for the presence of mercenaries, or capital sponsors may rebound deterring mercenaries. Emigration of ideologues from one organisation to another requires more government intervention into the latter to ensure termination. Emigration of mercenaries from one organisation to another may require more government intervention into the latter, since mercenaries support ideologues. Competing terrorist organisations may facilitate their mutual extinction. Various intervention strategies are considered: the most threatening organisation is eliminated first, aided by competition from the least threatening, after which the remaining organisation is eliminated. The government's instantaneous and accumulated utilities are analysed through time and compared, depending on emigration, competition, and government intervention strategies.

Keywords: *terrorism, terrorist organisations, ideologues, mercenaries, captive participants, sponsors, evolution, dynamics, simulation, migration, governments, intervention, differential equations*

1. Introduction

1.1. Background

Terrorist organizations come in all shapes and forms. New organizations are formed. Old organizations die. Participants migrate back and forth among terrorist organizations. For example, migration towards the ISIS terror group was evident in 2014.

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Some migration has occurred from Al-Qaeda. Competition[1] may occur both if their objectives overlap and contradict each other. For example, various factions of the IRA compete with each other. The internal composition of each terrorist organization is essential, i.e., how it is composed of ideologues and captive participants, and whether it receives funding from sponsors, or from criminal mercenaries which may compromise the ideology.

Within this fluid and fluctuating environment through time governments need to assess how to operate. Challenging considerations are whether or not to intervene, how much and when to intervene, and towards which internal parts of which terrorist organizations to intervene. Relevant is also whether some terrorist organizations may compete with each other out of existence, or whether migration patterns may suggest, ignoring some terrorist organizations.

1.2. Contribution

Migration and competition between terrorist organisations are modelled through time with four coupled differential time equations. Each organisation has three labour stocks, i.e., ideologues, criminal mercenaries, and captive participants, and may receive capital sponsoring. Migration and competition are assumed possible for ideologues and mercenaries in different terrorist organisations. Captive participants support the ideologues or the mercenaries. Sponsors provide capital to terrorist organisations which are not too ideologically compromised with support from the criminal mercenaries.

Governments choose labour efforts as strategic choice variables to intervene in a targeted manner towards one or several terrorist organisations. The impact of intervention with different magnitudes and over different time horizons is illustrated. Intervention may be directed towards ideologues, mercenaries, captive participants, or capital sponsoring one or several terrorist organisations. Intervention is shown to alter the composition within and interaction between terrorist organisations through time. For various migration patterns of ideologues and mercenaries, and various degrees of competition, it is shown how various government intervention strategies impact whether terrorist organisations grow, are curtailed, or eliminated.

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[1]This article interprets competition broadly to comprise fighting, war, struggle, conflict, battle, violence, etc., [25] interprets fighting as *falling also into the category of interference struggles are political campaigns, rent-seeking manoeuvres for licenses and monopoly privileges [31], commercial efforts to raise rivals' costs [28] strikes and lockouts, and litigation – all being conflictual activities that need not involve actual violence*.

1.3. Literature

Chamberlain [6] assesses Al-Qaeda's recruitment, training, operations, and reaction to interventions. Saperstein [29] evaluates terrorism and counter terrorism mathematically, with policy implications. Bunn [4] studies terrorism and nuclear theft. Hausken and Zhuang [24] consider resource allocation between attack and defence for a government and a terrorist. Hausken [17] examines when to attack a terrorist. Bier and Hausken [3] endogenise negative and positive incentives towards terrorists. Extinction of organisations through warfare is considered by Hausken and Moxnes [23].

Caulkins et al. [5] evaluate how counter terrorism influences recruitment to terror organisations. They assess counter terrorism that provokes and does not provoke terrorism recruitment. They find two different steady states, one with the near elimination of the terrorist organisation, and one with many terrorists. Feichtinger and Novak [7] consider the terrorists' reaction when determining how to combat terrorism. They illustrate long-run persistent oscillations with nonunique transitory behaviour.

Excluding the time dimension, Hausken [14], Hausken and Gupta [20–22], and Hausken et al. [18] model ideologues and criminal mercenaries. Including the time dimension, Kaminskiy and Ayyub [27] present a model to argue that if a terrorist cell is not disabled after 2–3 half-lives, then a new terrorism intervention policy is needed. Feinstein and Kaplan [8] scrutinise short term and long term attacks for a terrorist organisation. Udwadia et al. [32] evaluate through time interventions against terrorists those susceptible to terrorist and pacifist propaganda, and pacifists. Ignoring migration between terrorist organisations [12, 13, 15, 16], models governments, ideologues, capital sponsors, mercenaries, and captive participants in terrorist organisations. Abbas et al. [1] show methodologies for decreasing terrorism risks and applying homeland security resources efficiently.

In related work, Berman and Gavious [2] study defence against terrorist attacks to minimise the disutility of terrorist attacks. Golany et al. [9] consider the defence of multiple sites against strategic and probabilistic uncertainty. Insua et al. [26] evaluate defence against threats from multiple sites. Further research on attack and defence of multiple sites has been conducted by Zhang and Zhuang [34], assuming multiple attack types, by Zhang et al. [35], accounting for risk preferences, by Shan and Zhuang [30], assuming multiple periods and cumulative defensive resource allocation, and by Guan et al. [10], assuming budget constraints. Hausken and Bier [19] and Xu and Zhuang [33] consider defence against several attackers.

Article organisation: Section 2 presents the model. Section 3 considers emigration. Section 4 considers emigration and competition. Section 5 evaluates government intervention, no competition, and emigration. Section 6 evaluates government intervention, competition, and emigration. Section 7 concludes the study.

2. The model

2.1. The evolution of *N* terrorist organisations

The model consists of four differential equations for the dynamics within each terrorist organisation, government intervention, competition or war between terrorist organisations, and migration between terrorist organisations, i.e.,

$$\dot{I}_i = a_i K_i + e_i M_i + n_i C_i - \theta_i K_i M_i - b_i I_i - s_i \sum_{k=1}^{Q} G_{kIi} - \sum_{j=1,\, j\neq i}^{N} \alpha_{ij} x_{ij} I_j$$
$$- \sum_{j=1}^{N} \max\left(\gamma_{ijt}, 0\right) \mu_{Iijt} I_i + \sum_{j=1}^{N} \max\left(\gamma_{jit}, 0\right) \mu_{Iijt} I_j$$

$$\dot{K}_i = c_i I_i - f_i M_i - d_i K_i - u_i \sum_{k=1}^{Q} G_{kKi}$$

$$\dot{M}_i = g_i I_i + o_i C_i + h_i K_i - \varphi_i I_i K_i - m_i M_i - v_i \sum_{k=1}^{Q} G_{kMi} - \sum_{j=1,\, j\neq i}^{N} \beta_{ij} y_{ij} M_j \qquad (1)$$
$$- \sum_{j=1}^{N} \max\left(\lambda_{ijt}, 0\right) \mu_{Mijt} M_i + \sum_{j=1}^{N} \max\left(\lambda_{jit}, 0\right) \mu_{Mjit} M_j$$

$$\dot{C}_i = p_i I_i + q_i M_i - r_i C_i - w_i \sum_{k=1}^{Q} G_{kCi}$$

$$I_i \geq 0, K_i \geq 0, M_i \geq 0, C_i \geq 0$$

where a dot above a variable means time differentiation d/dt, t is time, subscripts i and j express terrorist organisations i and j, $i, j = 1, 2, ..., N$, $i \neq j$, and all parameters are assumed to be positive or zero.

In (1) ideologue labour $I_i \geq 0$ in terrorist organisation i, $i = 1, ..., N$, increases with the increase of capital $K_i \geq 0$, mercenary labour $M_i \geq 0$, and captive participants labour $C_i \geq 0$, decreases with the product $K_i M_i$, constrained by its own growth. The variables K_i and M_i cannot impact I_i without limits. Criminal mercenaries deter capital sponsors

since they are perceived to dilute or water out the ideological purity of the terrorist organisation. Thus, Gupta [11], Hausken [14], Hausken et al. [18], Hausken and Gupta [20–22] show that terrorist organisations devaluing or debasing themselves to become criminal, such as, e.g., FARC, Abu Sayaaf, and various spinoffs of the Northern Irish IRA, are less successful recruiting ideologically committed sponsors. Hence, jointly high K_i and M_i is detrimental to ideologue labour I_i. Accordingly, we subtract $\theta_i K_i M_i$ from the right hand side of $\dot{I}_i$ in (1). For example, the term $\theta_i K_i M_i$ is large when both K_i and M_i are large, and small when either K_i or M_i is small.

Capital K_i increases with ideologue labour I_i, decreases with mercenary labour M_i, constrained by its own growth. Mercenary labour M_i increases with ideologue labour I_i, captive participants labour C_i, and capital K_i, decreases with the product $I_i K_i$, constrained by its own growth. Subtracting $\varphi_i I_i K_i$ on the right hand side of $\dot{M}_i$ in (1) is analogous to subtracting $\varphi_i I_i K_i$ on the right hand side of $\dot{I}_i$ in (1). The reasoning is that both K_i and I_i cannot impact M_i positively without limits. For example, substantial capital sponsoring K_i benefits ideologue labour I_i, which causes mercenary labour M_i to decrease in practice, so that terrorist organisation i becomes purer. Captive participants labour C_i increases with ideologue labour I_i and mercenary labour M_i, constrained by its own growth. The non-negative parameters a, e, n, c, g, o, h, p, q are growth rates, and θ, b, f, d, φ, m, r are non-negative depreciation rates.

In (1), government k, $k = 1, 2, ..., Q \geq 1$ intervenes in terrorist organisation i, $i = 1, ..., N \geq 1$ with labour efforts G_{kIi}, G_{kKi}, G_{kMi}, G_{kCi} and non-negative unit effort costs s_i, u_i, v_i, w_i against ideologue labour I_i, capital K_i, mercenary labour M_i, and captive participants labour C_i, respectively. Government k's labour efforts G_{kIi}, G_{kKi}, G_{kMi}, G_{kCi} are positive if government k suppresses terrorism, and negative when government k sponsors terrorism. When government k suppresses (sponsors) terrorism, terrorist organisation i is impacted negatively (positively) by $s_i \sum_{k=1}^{Q} G_{kIi}$, $u_i \sum_{k=1}^{Q} G_{kKi}$, $v_i \sum_{k=1}^{Q} G_{kMi}$, $w_i \sum_{k=1}^{Q} G_{kCi}$, respectively, for I_i, K_i, M_i, and C_i.

In (1), terrorist organisation i allocates a fraction x_{ij} of ideologue labour I_i to fight ideologue labour I_j in organisation j, and a fraction y_{ij} of mercenary labour M_i to fight mercenary labour M_j in organisation j, $0 \leq x_{ij}, y_{ij} \leq 1$, $\sum_{j=1, j\neq i}^{N} x_{ij} = \sum_{j=1, j\neq i}^{N} y_{ij} = 1$. Terrorist organisation i is impacted negatively by $\sum_{j=1, j\neq i}^{N} \alpha_{ij} x_{ij} I_j$ for ideologue labour I_i, and negatively by $\sum_{j=1, j\neq i}^{N} \beta_{ij} y_{ij} M_j$ for mercenary labour M_i, where $\alpha_{ij} \geq 0$ and $\beta_{ij} \geq 0$ are loss

rates. Hence, ideologue and mercenary labour in terrorist organisation i suffer a loss rate proportional to the strengths of ideologue and mercenary labour in the competing terrorist organisation j, $j \neq i$, $i, j = 1, ..., N$.

In (1), the negative term $\sum_{j=1}^{N} \max(\gamma_{ijt}, 0)\mu_{Iit} I_i$ expresses emigration of ideologue labour I_i from organisation i to organisation j, $j \neq i$, $i, j = 1, ..., N$. First, γ_{ijt} expresses how organisation j is more attractive than organisation i at time t for ideologue labour I_i when $\gamma_{ijt} > 0$, equally attractive when $\gamma_{ijt} = 0$, and less attractive when $\gamma_{ijt} < 0$, where $\gamma_{ijt} = -\gamma_{jit}$. The attractiveness parameter is also a proportionality parameter which scales the extent to which ideologue labour I_i moves from organisation i to organisation j. The max function $\max(\gamma_{ijt}, 0)$ ensures that the term is only operational when $\gamma_{ijt} > 0$, causing ideologue labour I_i to move from organisation i to organisation j. Hence, $-\infty < \gamma_{ijt} < \infty$ and $\gamma_{iit} = 0$. Second, μ_{Iijt} expresses how organisation i controls the outflow of ideologue labour I_i from organisation i to organisation j at time t. Complete 100% control is expressed as $\mu_{Iijt} = 0$, causing no emigration of ideologue labour I_i from organisation i to organisation j at time t. No control is expressed as $\mu_{Iijt} = 1$, causing emigration of ideologue labour I_i from organisation i to organisation j at time t as determined by $\sum_{j=1}^{N} \max(\gamma_{ijt}, 0)I_i$, where $0 \leq \mu_{Iijt} \leq 1$. Third, proportionality with ideologue labour I_i in $\sum_{j=1}^{N} \max(\gamma_{ijt}, 0)\mu_{Iijt} I_i$ is assumed since more ideologue labour I_i in organisation i at time t can be expected to cause more emigration when organisation j is more attractive than organisation i. The mirror positive term $\sum_{j=1}^{N} \max(\gamma_{jit}, 0)\mu_{Ijit} I_j$ in (1) applies when $\gamma_{ijt} < 0$. Then organisation i is more attractive than organisation j at time t, and immigration to organisation i from organisation j, proportional to ideologue labour I_i in organisation j, and adjusted by organisation j's control parameter μ_{Ijit}, $0 \leq \mu_{Ijit} \leq 1$, can be expected.

In (1), the analogous negative term $\sum_{j=1}^{N} \max(\lambda_{ijt}, 0)\mu_{Mijt} M_i$ applies for emigration of mercenary labour M_i from organisation i to organisation j, $j \neq i$, $i, j = 1, ..., N$. First, λ_{ijt} expresses how organisation j is more attractive than organisation i at time t for mercenary labour M_i when $\lambda_{ijt} > 0$, equally attractive when $\lambda_{ijt} = 0$, and less attractive when $\lambda_{ijt} < 0$, where $\lambda_{ijt} = -\lambda_{jit}$, $-\infty < \lambda_{ijt} < \infty$ and $\lambda_{iit} = 0$. Second, μ_{Mijt} expresses how organisation i controls the outflow of mercenary labour M_i from organisation i to organisation j at time t, $0 \leq \mu_{Mijt} \leq 1$. Third, proportionality with mercenary labour M_i is assumed,

analogously to proportionality with ideologue labour I_i in $\sum_{j=1}^{N} \max(\gamma_{ijt}, 0)\mu_{Iijt} I_i$. The mirror positive term $\sum_{j=1}^{N} \max(\lambda_{jit}, 0)\mu_{Mjit} M_j$ in (1) applies when $\lambda_{ijt} < 0$, where $0 \le \mu_{Mjit} \le 1$.

2.2. The *Q* governments' utilities

Government k's instantaneous utility U_{ki} at time τ due to the presence of terrorist organisation i, $i = 1, ..., N$, $k = 1, 2, ..., Q$, $0 \le \tau \le t$, is

$$U_{ki} = -\left(A_{ki} I_i^{z_{ki}} + B_{ki} K_i^{z_{ki}} + D_{ki} M_i^{z_{ki}} + E_{ki} C_i^{z_{ki}} \right)^{1/z_{ki}} - F_{ki} G_{kIi} - H_{ki} G_{kKi} - J_{ki} G_{kMi} - L_{ki} G_{kCi} \tag{2}$$

where G_{kIi}, G_{kKi}, G_{kMi}, G_{kCi} are government k's labour intervention efforts with non-negative unit costs F_{ki}, H_{ki}, J_{ki}, L_{ki}, respectively, to decrease ideologue labour I_i, capital sponsoring K_i, mercenary labour M_i, and captive participants labour C_i in terrorist organisation i. The non-negative weight parameters A_{ki}, B_{ki}, D_{ki}, E_{ki} express how government k weighs the relative disadvantage of the three labour stocks I_i, M_i, C_i, and capital sponsoring K_i. The elasticity of substitution for government k as impacted by terrorist organisation i is $1/(1 - z_{ki})$, where $z_{ki} = 1$ means perfect substitutes, $-\infty < z_{ki} \le 1$. When z_{ki} approaches minus infinity, perfect complements occur. When z_{ki} approaches zero, the Cobb–Douglas utility function arises.

Government k's utility U_k across the N terrorist organisations is additive, i.e.,

$$U_k = \sum_{i=1}^{N} U_{ki} \tag{3}$$

where U_{ki} is determined by (2). Government k's accumulated utilities U_{kai} and U_{ka} from time $\tau = 0$ to time $\tau = t$, due to terrorist organisation i and all Q terrorist organisations, respectively, with time discount parameter δ_k, $0 \le \delta_k \le 1$, are

$$U_{kai} = \int_{\tau=0}^{\tau=t} \delta_k^{\tau} U_{ki} d\tau, \quad U_{ka} = \int_{\tau=0}^{\tau=t} \delta_k^{\tau} U_k d\tau \tag{4}$$

Table 1 presents an overview of the content in the remainder of the article which focuses on emigration (Section 3), emigration and competition (Section 4), emigration and government intervention and no competition (Section 5), and emigration and competition and government intervention (Section 6).

Table 1. Article overview

Section	Figure	Panels	γ_{jit}	λ_{jit}	$\alpha_{ij} = \alpha_{ji}$	G_{kIi}	G_{kIj}	Characteristic
3	1	a1, a2	0	0	0	0	0	benchmark
		b1, b2	0.125	0	0	0	0	only emigration
		c1, c2	0	1	0	0	0	
		d1, d2	0.125	1	0	0	0	
		e1, e2	0.5	1	0	0	0	
4	2	a1, a2	0	0	0.1	0	0	competition
		b1, b2	0.125	0	0.1	0	0	emigration and competition
		c1, c2	0	1	0.1	0	0	
5	3	a1, a2	0	0	0	0.41	0.41	only government intervention
		b1, b2	0.125	0	0	0.41	0.24	emigration and government intervention
		c1, c2	0	1	0	0.9	0.26	
		d1, d2	0.125	1	0	0.9	0.26	
6	4	a1, a2	0	0	0.1	0.27	0.27	competition, government intervention
		b1, b2	0.125	0	0.1	0.68; 0	0; 0.5	emigration, competition, government intervention
		c1, c2	0	1	0.1	0.69; 0	0; 3	

Emigration (Section 3), emigration and competition (Section 4), emigration and government intervention and no competition (Section 5), and emigration and competition and government intervention (Section 6). γ_{jit} – ideologue emigration from organisation j to organisation i, λ_{jit} – mercenary emigration from organisation j to organisation i, $\alpha_{ij} = \alpha_{ji}$ – ideologue competition between organisation i and organisation j, G_{kIi} – government intervention into organisation i, G_{kIj} – government intervention into organisation j.

3. Analysing emigration for two terrorist organisations

3.1. The evolution of $N = 2$ terrorist organisations

Figure 1 exemplifies (1) for $N = 2$ organisations i and j, i.e., $x_{ij} = x_{ji} = y_{ij} = y_{ji} = 1$, no competition $\alpha_{ij} = \alpha_{ji} = \beta_{ij} = \beta_{ji} = 0$ between the two organisations, and no government intervention, i.e., $G_{kIi} = G_{kKi} = G_{kMi} = G_{kCi} = 0$, $k = 1, 2, ..., Q$, $i, j = 1, 2$, $i \neq j$. The initial values for the variables are $I_i(0) = I_j(0) = 2$, $K_i(0) = K_j(0) = M_i(0) = M_j(0) = C_i(0) = C_j(0) = 0$, to express that ideologues are crucial in starting terrorist organisations.

The benchmark parameter values are chosen to be plausible and simple, while capturing different representative characteristics of the model. Thus many parameter values

are set equal to one. For example, we choose $a_i = 1$ which in (1) means that capital K_i on the right hand side impacts the derivative of ideologue labour $\dot{I}_i$ on the left hand side with a proportionality parameter equal to one, which is assessed to be plausible and simple as a benchmark. Similarly, we set government k's unit effort costs, such as $s_i = 1$ to combat ideologue labour I_i, equal to one, and adjust government k's labour effort G_{kIi} at time t to combat ideologue labour I_i in terrorist organisation k to ensure suitable impact as illustrated in the simulations. Connecting the parameter values to the real world scenarios mentioned in Section 1 (ISIS, Al-Qaeda, etc.) is left for future empirical research. We hypothesise the ratios of the benchmark parameter values chosen in this article may reflect the ratios of the benchmark parameter values determined by empirical research.

The benchmark parameter values are $a_i = a_j = c_i = c_j = d_i = d_j = e_i = e_j = f_i = f_j = h_i = h_j = m_i = m_j = r_i = r_j = s_i = s_j = u_i = u_j = v_i = v_j = w_i = w_j = \mu_{Iijt} = \mu_{Ijit} = \mu_{Mijt} = \mu_{Mjit} = 1$, $b_i = b_j = g_i = g_j = 0.8$, $\theta_i = \theta_j = 0.3$, $\varphi_i = \varphi_j = 0.6$, $n_i = n_j = o_i = o_j = p_i = p_j = q_i = q_j = 0.25$. Growth of I_i and I_j is obtained by $b_i = b_j = 0.8$. That is, ideologue labours I_i and I_j in (1) depreciate 20% less by themselves than M_i, M_j, C_i, C_j, K_i, K_j. Choosing $g_i = g_j = 0.8$ expresses that mercenary labours M_i and M_j in (1) are 20% less positively impacted by I_i and I_j than capital sponsoring K_i and K_j of the two organisations is impacted by I_i and I_j. We assume that organisation i does not control the outflow of ideologue labour I_i and mercenary labour M_i, i.e., $\mu_{Iijt} = \mu_{Ijit} = \mu_{Mijt} = \mu_{Mjit} = 1$. Figure 1 panel a1 assumes the benchmark no emigration $\gamma_{ijt} = \gamma_{jit} = \lambda_{ijt} = \lambda_{jit} = 0$. The ideologue labours I_i and I_j, capital sponsoring K_i and K_j, and captive participants labours C_i and C_j increase without bounds, i.e., $\lim_{t\to\infty} I_i = \lim_{t\to\infty} I_j = \lim_{t\to\infty} K_i = \lim_{t\to\infty} K_j = \lim_{t\to\infty} C_i = \lim_{t\to\infty} C_j = \infty$. This increase is incompatible with mercenary labours M_i and M_j due to $-\varphi_i I_i K_i$ and $-\varphi_j I_j K_j$ in (1), which expresses that mercenary labours M_i and M_j become superfluous and detrimental when ideologue labours I_i and I_j, can rely on capital sponsoring K_i and K_j. Hence, the mercenary labours M_i and M_j in panel 1a first increase to a maximum, and thereafter decrease to $M_i = M_j = 0$ when $t > 14.50$.

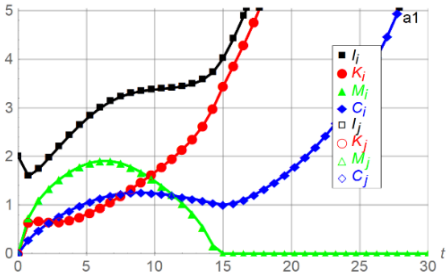
Figure 1 panel b1 assumes attractiveness $\gamma_{jit} = 0.125$ of organisation i relative to organisation j for ideologue labour I_j at time t. That causes ideologue labour I_i in organisation i to increase more than in panel a1, $\lim_{t\to\infty} I_i = \infty$, and consequently $\lim_{t\to\infty} K_i = \lim_{t\to\infty} C_i = \infty$, and causes ideologue labour I_j in organisation j to be lower than in panel a1, $\lim_{t\to\infty} I_j = 2.82$. Limit values are determined numerically. The higher I_i is detrimental to mercenary labour M_i in organisation i which decreases to $M_i = 0$ quicker than in panel a1, i.e., when $t > 8.92$. The lower I_j in organisation j deters sponsors, and thus capital sponsoring K_j is lower than in panel a1, $\lim_{t\to\infty} K_j = 0.87$. The lower I_j and K_j in organisation j allow for the presence of mercenary labour M_j in organisation j which does not vanish, but

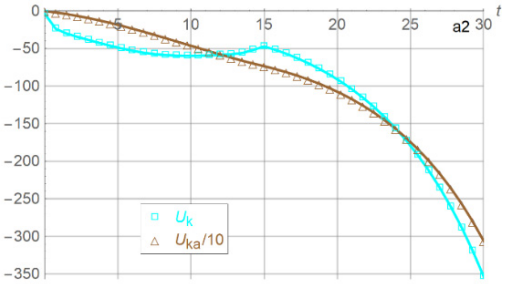
approaches a constant value, $\lim_{t\to\infty} M_j = 1.95$, and consequently $\lim_{t\to\infty} C_j = 1.19$. Continuously losing some ideologue labour I_j prevents organisation j from growing unboundedly. It also does not go extinct. The remaining ideologue labour I_j gets moderately sustained by capital sponsoring K_j which tolerates some presence of mercenary labour M_j.

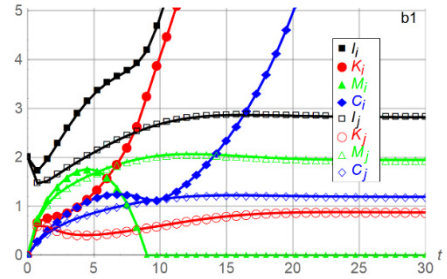
Figure 1 panel c1 assumes attractiveness $\lambda_{jit} = 1$ of organisation i relative to organisation j for mercenary labour M_j at time t. That causes mercenary labour M_j in organisation j to experience a lower and shorter inverse U shape, decreasing to $M_j = 0$ quicker than in panel a1, i.e., when $t > 11.79$, after which emigration from organisation j ceases. It also causes mercenary labour M_i in organisation i initially to increase more than in panel a1, which gives a temporary boost to ideologue labour I_i in organisation i, $\lim_{t\to\infty} I_i = \infty$. The temporarily high M_i is detrimental to capital sponsoring K_i which experiences a temporary dip. Since I_i sustains a high level relative to M_i, capital sponsoring K_i gradually returns, $\lim_{t\to\infty} K_i = \lim_{t\to\infty} C_i = \infty$, which deters mercenary labour M_i which decreases to $M_i = 0$ when $t > 11.99$. That organisation j loses some mercenary labour is beneficial for capital sponsoring K_j, and thus also for ideologue labour I_j, and in this case for captive participants labour C_j, which increases quicker than in panel a1, $\lim_{t\to\infty} K_j = \lim_{t\to\infty} I_j = \lim_{t\to\infty} C_j = \infty$.

Figure 1 panel d1 combines the effects in panels b1 and c1, i.e., both attractiveness $\gamma_{jit} = 0.125$ for ideologue labour I_j and attractiveness $\lambda_{jit} = 1$ for mercenary labour M_j, of organisation i relative to organisation j at time t. That causes an inverse U shaped curve for mercenary labour M_i, reaching $M_i = 0$ when $t > 10.77$. The evolution of organisation i is similar to that of panels b1 and c1, $\lim_{t\to\infty} I_i = \lim_{t\to\infty} K_i = \lim_{t\to\infty} C_i = \infty$. Mercenary labour M_j is lower than in panel b1, since organisation j loses mercenaries through $\lambda_{jit} = 1$. Mercenary labour M_j remains higher for a longer period of time than in panel c1, since

Fig. 1. Ideologue labours I_i and I_j, capital K_i and K_j, mercenary labours M_i and M_j, and captive participants labours C_i and C_j, in $N = 2$ organisations i and j, $i, j = 1, 2$, $i \neq j$, and government k's instantaneous utility U_k and accumulated utility U_{ka}, $k = 1, ..., Q$ as functions of time t with benchmark parameter values $a_i = a_j = c_i = c_j = d_i = d_j = e_i = e_j = f_i = f_j = h_i = h_j = m_i = m_j = r_i = r_j = s_i = s_j = u_i = u_j = v_i = v_j = w_i = w_j = \mu_{Iijt} = \mu_{Ijit} = \mu_{Mijt} = \mu_{Mjit} = 1$, $b_i = b_j = g_i = g_j = 0.8$, $\theta_i = \theta_j = 0.3$, $\varphi_i = \varphi_j = 0.6$, $n_i = n_j = o_i = o_j = p_i = p_j = q_i = q_j = 0.25$, $\alpha_{ij} = \alpha_{ji} = \beta_{ij} = \beta_{ji} = \gamma_{ij} = \gamma_{ji} = \lambda_{ij} = \lambda_{ji} = 0$, $I_i(0) = I_j(0) = 2$, $K_i(0) = K_j(0) = M_i(0) = M_j(0) = C_i(0) = C_j(0) = 0$, $A_{ki} = A_{kj} = B_{ki} = B_{kj} = D_{ki} = D_{kj} = E_{ki} = E_{kj} = \delta_k = 1$, $z_{ki} = z_{kj} = 0.5$, $G_{kIi} = G_{kIj} = G_{kKi} = G_{kKj} = G_{kMi} = G_{kMj} = G_{kCi} = G_{kCj} = 0$. Division of U_{ka} with 10 is for scaling purposes. Panels a1 and a2: benchmark, panels b1 and b2: $\gamma_{jit} = 0.125$, panels c1 and c2: $\lambda_{jit} = 1$, panels d1 and d2: $\gamma_{jit} = 0.125$, $\lambda_{jit} = 1$, panels e1 and e2: $\gamma_{jit} = 0.5$, $\lambda_{jit} = 1$


a1
I_i
K_i
M_i
C_i
I_j
K_j
M_j
C_j
t


a2
U_k
$U_{ka}/10$
t


b1
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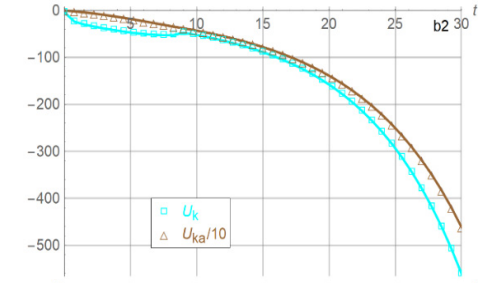
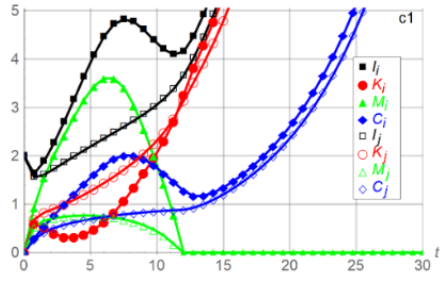
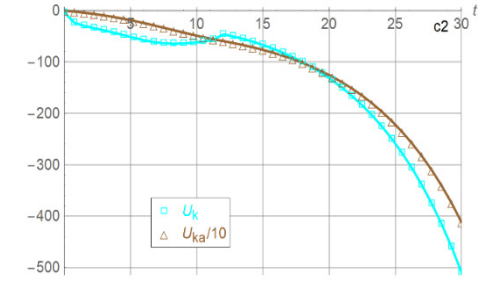
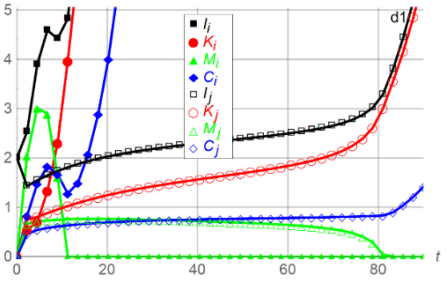
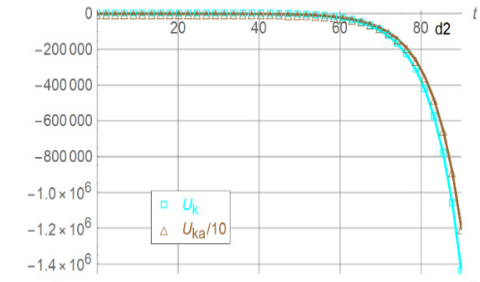
K_i
M_i
C_i
I_j
K_j
M_j
C_j
t

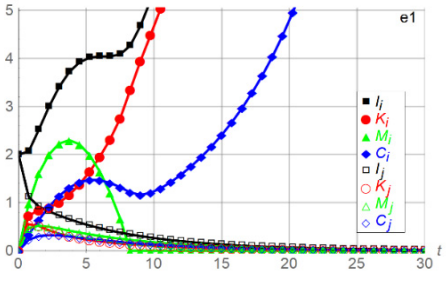

b2
U_k
$U_{ka}/10$
t

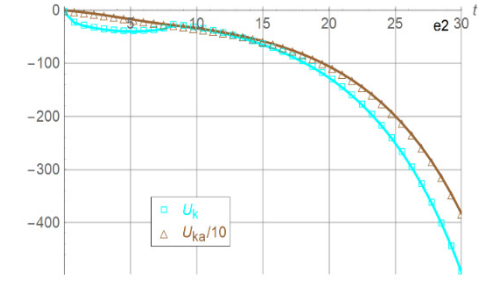

c1
I_i
K_i
M_i
C_i
I_j
K_j
M_j
C_j
t


c2
U_k
$U_{ka}/10$
t


d1
I_i
K_i
M_i
C_i
I_j
K_j
M_j
C_j
t


d2
U_k
$U_{ka}/10$
t


e1
I_i
K_i
M_i
C_i
I_j
K_j
M_j
C_j
t


e2
U_k
$U_{ka}/10$
t

organisation j loses ideologue labour I_j due to $\gamma_{jit} = 0.125$, which partly deters capital sponsoring K_j, allowing some presence of mercenary labour M_j. Eventually, the low presence of mercenary labour M_j gets deterred altogether by I_j and K_j, reaching $M_j = 0$ when $t > 81.23$. That enables ideologue labour I_j, capital sponsoring K_j, and captive participants labour C_j in organisation j eventually to increase towards infinity, $\lim_{t\to\infty} I_j = \lim_{t\to\infty} K_j = \lim_{t\to\infty} C_j = \infty$, although slower than for organisation i due to the drainage of ideologue labour I_j from organisation j to organisation i.

None of the panels b1, c1, d1 cause organisation j to go extinct. Higher attractiveness $\gamma_{jit} = 0.25$ for ideologue labour I_j and equal attractiveness $\lambda_{jit} = 1$ (compared with panel d) for mercenary labour M_j also does not cause extinction, although it causes lower levels of labours and capital sponsoring, i.e., $\lim_{t\to\infty} I_j = 0.66$, $\lim_{t\to\infty} K_j = 0.28$, $\lim_{t\to\infty} M_j = 0.38$, and $\lim_{t\to\infty} C_j = 0.26$, compared with panel b. However, even higher attractiveness $\gamma_{jit} = 0.5$ for ideologue labour I_j and equal attractiveness $\lambda_{jit} = 1$ (compared with panel d) for mercenary labour M_j terminates organisation j, as shown in Fig. 1 panel e1. Mercenary labour M_i is inverse U shaped and reaches $M_i = 0$ when $t > 8.19$. Thereafter, organisation i grows towards infinity without mercenary labour $M_i = 0$, $\lim_{t\to\infty} I_i = \lim_{t\to\infty} K_i = \lim_{t\to\infty} C_i = \infty$, while organisation j goes extinct, $\lim_{t\to\infty} I_j = \lim_{t\to\infty} K_j = \lim_{t\to\infty} M_j = \lim_{t\to\infty} C_j = 0$. Extinction in panel e differs from panels b1, c1, and d1. The reason is that substantial emigration of ideologues and some emigration of mercenaries, as in panel e, is more detrimental for organisation j than a more modest emigration. In contrast, modest emigration of only ideologues may enable organisation j to sustain itself through mercenaries, as in panel b1 and when $\gamma_{jit} = 0.25$, as shown above.

3.2. The *Q* governments' utilities

Figure 1 panels a2, b2, c2, d2, e2 exemplify (2), (3), (4) for the $N = 2$ terrorist organisations in Section 3.1, i.e., no competition $\alpha_{ij} = \alpha_{ji} = \beta_{ij} = \beta_{ji} = 0$ between the two organisations, for Q equivalent governments not intervening, i.e., $G_{kIi} = G_{kKi} = G_{kMi} = G_{kCi} = 0$, $k = 1, 2, ..., Q$, $i = 1, 2$. Government k's benchmark parameter values are $A_{ki} = B_{ki} = D_{ki} = E_{ki} = \delta_k = 1$, $z_{ki} = 0.5$.

In Figure 1 panel a2, the instantaneous utility U_k for government k, $k = 1, ..., Q$, is affected by the inverse U shaped M_i and M_j, and is initially U shaped. After the extinction of mercenary labours M_i and M_j when $t > 14.50$, government k benefits temporarily. Thereafter, I_i, I_j, K_i, K_j, C_i, C_j approach infinity causing government k's instantaneous

utility U_k and accumulated utility $U_{ka}/10$ to decrease towards minus infinity, i.e., $\lim_{t\to\infty} U_k = \lim_{t\to\infty} U_{ka} = -\infty$. Division of U_{ka} with 10 is for scaling purposes.

In Figure 1 panel b2, attractiveness $\gamma_{jit} = 0.125$ of organisation i relative to organisation j for ideologue labour I_j at time t, causes organisation i to grow more substantially. Although organisation j gets confined within bounds, both government k's utilities approach $\lim_{t\to\infty} U_k = \lim_{t\to\infty} U_{ka} = -\infty$ quicker than in panel a2 with no emigration.

In Figure 1 panel c2, attractiveness $\lambda_{jit} = 1$ of organisation i relative to organisation j for mercenary labour M_j at time t causes both organisations i and j to grow unboundedly, causing $\lim_{t\to\infty} U_k = \lim_{t\to\infty} U_{ka} = -\infty$. The evolution until time $t = 30$ is similar (slightly more beneficial for government k) to that of panel b.

In Figure 1 panel d2, attractiveness $\gamma_{jit} = 0.125$ for ideologue labour I_j and attractiveness $\lambda_{jit} = 1$ for mercenary labour M_j of organisation i relative to organisation j at time t causes both organisations i and j to grow unboundedly, causing $\lim_{t\to\infty} U_k = \lim_{t\to\infty} U_{ka} = -\infty$. At time $t = 30$, $U_k = -434.58$ and $U_{ka}/10 = -412.96$, which is similar to panels a, b, and c.

In Figure 1 panel e2, four times higher attractiveness $\gamma_{jit} = 0.5$ for ideologue labour I_j and attractiveness $\lambda_{jit} = 1$ for mercenary labour M_j, of organisation i relative to organisation j at time t, causes organisation i to grow quickly and unboundedly, while organisation j goes extinct, $\lim_{t\to\infty} U_k = \lim_{t\to\infty} U_{ka} = -\infty$. At time $t = 30$, $U_k = -492.70$ and $U_{ka}/10 = -382.82$. Government k prefers the latter, compared with panels b, c, and d.

4. Analysing emigration and ideologue competition for two terrorist organisations

This section assumes competition $\alpha_{ij} = \alpha_{ji} = 0.1$ between ideologue labours I_i and I_j in organisations i and j. For Fig. 2 panels a1 and a2 assume the other assumptions are as in Fig. 1 panels a1 and a2, i.e., $\gamma_{ij} = \gamma_{ji} = \lambda_{ij} = \lambda_{ji} = \beta_{ij} = \beta_{ji} = 0$, $G_{kIi} = G_{kKi} = G_{kMi} = G_{kCi} = 0$, $k = 1, 2, ..., Q$, $i, j = 1, 2$, $i \neq j$, $I_i(0) = I_j(0) = 2$, $K_i(0) = K_j(0) = M_i(0) = M_j(0) = C_i(0) = C_j(0) = 0$, $a_i = a_j = c_i = c_j = d_i = d_j = e_i = e_j = f_i = f_j = h_i = h_j = m_i = m_j = r_i = r_j = s_i = s_j = u_i = u_j = v_i = v_j = w_i = w_j = \mu_{Iijt} = \mu_{Ijit} = \mu_{Mijt} = \mu_{Mjit} = 1$, $b_i = b_j = g_i = g_j = 0.8$, $\theta_i = \theta_j = 0.3$, $\varphi_i = \varphi_j = 0.6$, $n_i = n_j = o_i = o_j = p_i = p_j = q_i = q_j = 0.25$. The competition without emigration prevents the ideologue labours I_i and I_j from growing unboundedly, instead approaching the constant $\lim_{t\to\infty} I_i = \lim_{t\to\infty} I_j = 2.89$, constrained by $-\theta_i K_i M_i$ and $-\theta_j K_j M_j$ in (1). Capital sponsoring approaches $\lim_{t\to\infty} K_i = \lim_{t\to\infty} K_j = 1.07$. The mercenary labours M_i and M_j approach the con-

stant $\lim_{t\to\infty} M_i = \lim_{t\to\infty} M_j = 1.82$, constrained by $-\varphi_i I_i K_i$ and $-\varphi_j I_j K_j$ in (1). Captive participants labours C_i and C_j approach $\lim_{t\to\infty} C_i = \lim_{t\to\infty} C_j = 1.18$. Government k's instantaneous utility U_k benefits from the competition, approaching the constant $\lim_{t\to\infty} U_k = -53.44$. Government k's accumulated utility $U_{ka}/10$ approaches $\lim_{t\to\infty} U_{ka} = -\infty$ slower than in Fig. 1 panel a2.

Figure 2 panels b1 and b2 introduce ideologue competition $\alpha_{ij} = \alpha_{ji} = 0.1$ to the assumptions in Fig. 1 panels b1 and b2, where ideologue labour I_j in organisation j moves to organisation i. That is detrimental to organisation j, which no longer sustains itself within bounds, but approaches extinction. The competition decreases ideologue labour I_j in organisation j, which deters capital sponsoring which decreases to $K_j = 0$ when $t > 10.37$. Increased ideologue labour I_i in organisation i eventually decreases the need for mercenary labour M_i which, after an inverse U shape, decreases to $M_i = 0$ when $t > 13.38$. Thereafter, ideologue labour I_j in organisation i decreases to $I_j = 0$ when $t > 16.66$, after which emigration ceases and $\lim_{t\to\infty} M_j = \lim_{t\to\infty} C_j = 0$. Organisation i is initially hampered by the competition, growing slightly slower than in Fig. 1 panel b1, but eventually unbounded growth occurs, $\lim_{t\to\infty} I_i = \lim_{t\to\infty} K_i = \lim_{t\to\infty} C_i = \infty$. Government k again benefits from the competition, earning higher utilities than in Fig. 1 panel b2, though $\lim_{t\to\infty} U_k = \lim_{t\to\infty} U_{ka} = -\infty$.

Figure 2 panels c1 and c2 introduce ideologue competition $\alpha_{ij} = \alpha_{ji} = 0.1$ to the assumptions in Fig. 1 panels c1 and c2, where mercenary labour M_j in organisation j moves to organisation i. That is also detrimental to organisation j, especially to mercenary labour M_j which after a low and short inverse U shape approaches $\lim_{t\to\infty} M_j = 0$ asymptotically. With limited support of mercenary labour M_j, ideologue labour I_j in organisation j suffers in the competition with organisation i and follows and inverse U shaped form, reaching $I_j = 0$ when $t > 13.78$. Thus, capital sponsoring also dries up, $K_j = 0$ when $t > 13.91$, and $\lim_{t\to\infty} C_j = 0$, as organisation j approaches extinction. That contrasts with Fig. 1 panel c1 without ideologue competition, where ideologue labour I_j increases unboundedly. In contrast, organisation i experiences a temporary boost of mercenary labour M_i, immigrating from organisation j, which temporarily suppresses capital sponsoring K_i in organisation i. However, organisation i eventually prefers the support of sponsoring K_i, causing it to grow unboundedly by surviving the competition, $\lim_{t\to\infty} I_i = \lim_{t\to\infty} K_i = \lim_{t\to\infty} C_i = \infty$, while mercenary labour M_i is inverse U shaped, reaches a maximum higher than in panels a and b, and decreases to $M_i = 0$ when $t > 13.56$. Government

k benefits from the competition, earning higher utilities than in Fig. 1 panel c2, $\lim_{t\to\infty} U_k = \lim_{t\to\infty} U_{ka} = -\infty$.

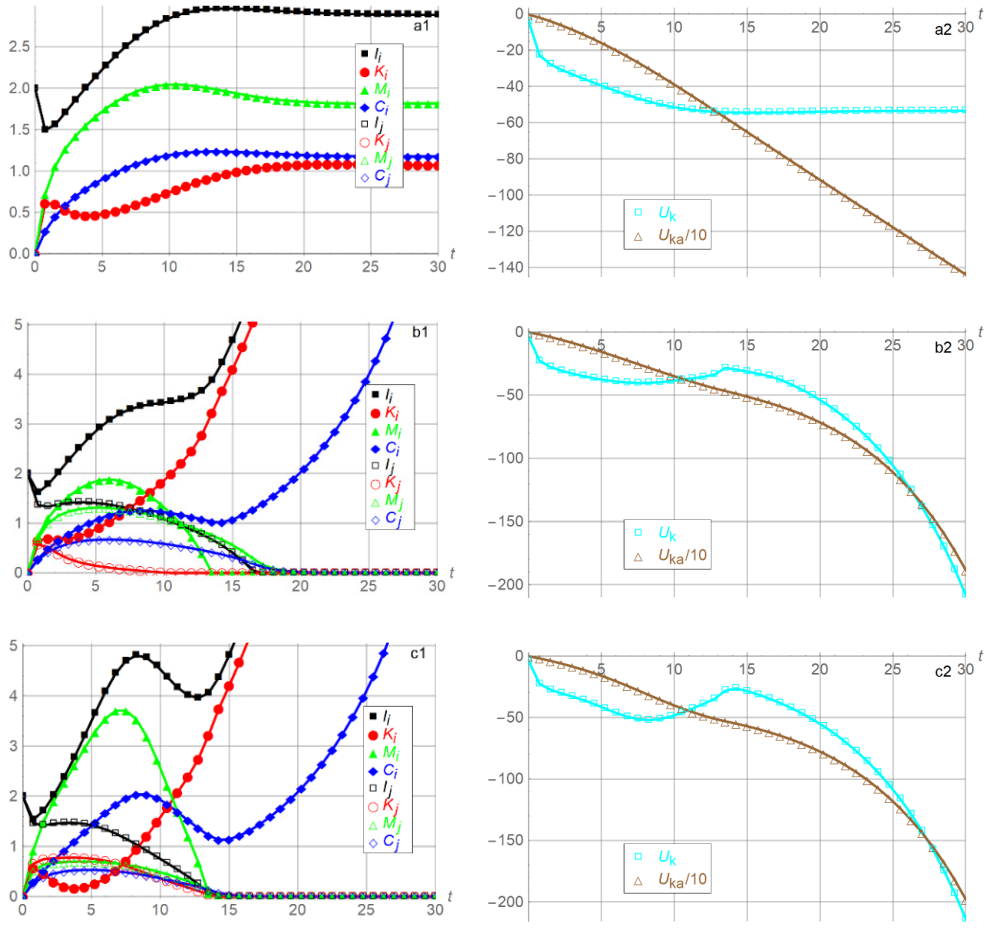


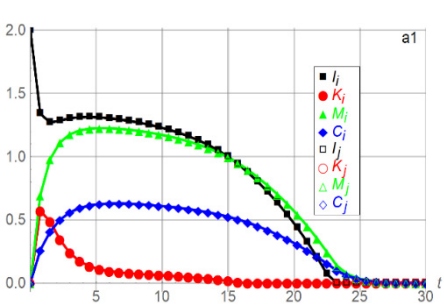
Fig. 2. Ideologue labours I_i and I_j, capital K_i and K_j, mercenary labours M_i and M_j, and captive participants labours C_i and C_j, in $N = 2$ organisations i and j, $i, j = 1, 2$, $i \neq j$, and government k's instantaneous utility U_k and accumulated utility U_{ka}, $k = 1, ..., Q$, as functions of time t with competition $\alpha_{ij} = \alpha_{ji} = 0.1$ and benchmark parameter values $a_i = a_j = c_i = c_j = d_i = d_j = e_i = e_j = f_i = f_j = h_i = h_j = m_i = m_j = r_i = r_j = s_i = s_j = u_i = u_j = v_i = v_j = w_i = w_j = \mu_{Iijt} = \mu_{Ijit} = \mu_{Mijt} = \mu_{Mjit} = 1$, $b_i = b_j = g_i = g_j = 0.8$, $\theta_i = \theta_j = 0.3$, $\varphi_i = \varphi_j = 0.6$, $n_i = n_j = o_i = o_j = p_i = p_j = q_i = q_j = 0.25$, $\beta_{ij} = \beta_{ji} = \gamma_{ij} = \gamma_{ji} = \lambda_{ij} = \lambda_{ji} = 0$, $I_i(0) = I_j(0) = 2$, $K_i(0) = K_j(0) = M_i(0) = M_j(0) = C_i(0) = C_j(0) = 0$, $A_{ki} = A_{kj} = B_{ki} = B_{kj} = D_{ki} = D_{kj} = E_{ki} = E_{kj} = \delta_k = 1$, $z_{ki} = z_{kj} = 0.5$, $G_{kIi} = G_{kIj} = G_{kKi} = G_{kKj} = G_{kMi} = G_{kMj} = G_{kCi} = G_{kCj} = 0$. Division of U_{ka} with 10 is for scaling purposes. Panels a1 and a2: benchmark, panels b1 and b2: $\gamma_{jit} = 0.125$, panels c1 and c2: $\lambda_{jit} = 1$

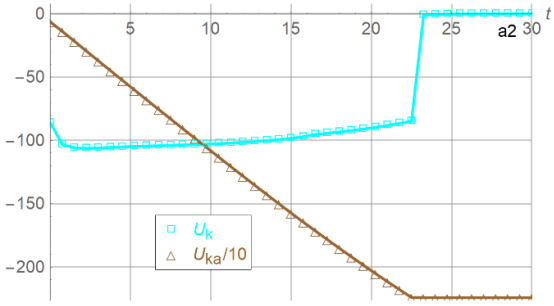
5. Analysing government intervention in two terrorist organisations with emigration and no competition as in Section 3

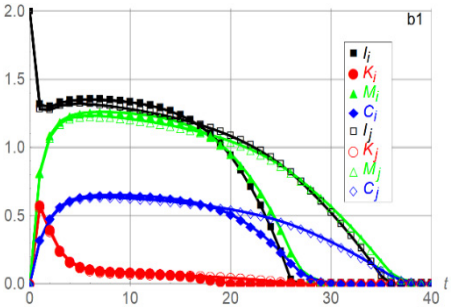
Figure 3 panels a1 and a2 introduce into Fig. 1 panels a1 and a2 government intervention $G_{kIi} = G_{kIj} = 0.41$ unit costs $F_{kIi} = F_{kIj} = 100$ by government k against ideologue labours I_i and I_j, and $G_{kKi} = G_{kKj} = G_{kMi} = G_{kMj} = G_{kCi} = G_{kCj} = 0$, $i, j = 1, 2$, $i \neq j$. This terminates both terrorist organisations. First, capital sponsoring decreases to $K_i = K_j = 0$ when $t > 16.27$. Thereafter, ideologue labours I_i and I_j decrease to $I_i = I_j = 0$ when $t > 22.89$, after which $G_{kIi} = G_{kIj} = 0$. Mercenary labours M_i and M_j vanish when they have no ideologues to support, $\lim_{t\to\infty} M_i = \lim_{t\to\infty} M_j = 0$, and thus also $\lim_{t\to\infty} C_i = \lim_{t\to\infty} C_j = 0$. Government k's instantaneous utility U_k approaches $\lim_{t\to\infty} U_k = 0$, and the accumulated utility U_{ka} approaches $\lim_{t\to\infty} U_{ka}/10 = -224.17$. For comparison, if government k were to intervene $G_{kIi} = G_{kIj} = 0.4$, the organisations would not get terminated, but contained at $\lim_{t\to\infty} I_i = \lim_{t\to\infty} I_j = 2.77$, $\lim_{t\to\infty} K_i = \lim_{t\to\infty} K_j = 0.75$, $\lim_{t\to\infty} M_i = \lim_{t\to\infty} M_j = 2.02$, $\lim_{t\to\infty} C_i = \lim_{t\to\infty} C_j = 1.20$, with $\lim_{t\to\infty} U_k = -130.85$ and $\lim_{t\to\infty} U_{ka} = -\infty$.

Figure 3 panels b1 and b2 introduce into Fig. 1 panels b1 and b2 (where ideologue labour I_i in organisation j moves to organisation i) government intervention $G_{kIi} = 0.57$ and $G_{kIj} = 0.24$ at unit cost $F_{ki} = F_{kj} = 100$ by government k against ideologue labours I_i and I_j, and $G_{kKi} = G_{kKj} = G_{kMi} = G_{kMj} = G_{kCi} = G_{kCj} = 0$, $i, j = 1, 2$, $i \neq j$. This eliminates both terrorist organisations. The intervention $G_{kIi} = 0.57$ is larger than $G_{kIi} = 0.41$ in Fig. 3 panels a1 and a2, since organisation i receives ideologue labour I_j from organisation j, which causes I_i to increase, and which is more challenging for government k to terminate. Capital sponsoring decreases to $K_i = 0$ when $t > 20.02$ and to $K_j = 0$ when $t > 26.65$.

Fig. 3. Ideologue labours I_i and I_j, capital K_i and K_j, mercenary labours M_i and M_j, and captive participants labours C_i and C_j, in $N = 2$ organisations i and j, $i, j = 1, 2$, $i \neq j$, and government k's instantaneous utility U_k and accumulated utility U_{ka}, $k = 1, ..., Q$, as functions of time t with competition $\alpha_{ij} = \alpha_{ji} = 0.1$ and benchmark parameter values $a_i = a_j = c_i = c_j = d_i = d_j = e_i = e_j = f_i = f_j = h_i = h_j = m_i = m_j = r_i = r_j = s_i = s_j = u_i = u_j = v_i = v_j = w_i = w_j = \mu_{Iijt} = \mu_{Ijit} = \mu_{Mijt} = \mu_{Mjit} = 1$, $b_i = b_j = g_i = g_j = 0.8$, $\theta_i = \theta_j = 0.3$, $\varphi_i = \varphi_j = 0.6$, $n_i = n_j = o_i = o_j = p_i = p_j = q_i = q_j = 0.25$, $\alpha_{ij} = \alpha_{ji} = \beta_{ij} = \beta_{ji} = \gamma_{ij} = \gamma_{ji} = \lambda_{ij} = \lambda_{ji} = 0$, $I_i(0) = I_j(0) = 2$, $K_i(0) = K_j(0) = M_i(0) = M_j(0) = C_i(0) = C_j(0) = 0$, $A_{ki} = A_{kj} = B_{ki} = B_{kj} = D_{ki} = D_{kj} = E_{ki} = E_{kj} = \delta_k = 1$, $z_{ki} = z_{kj} = 0.5$, $G_{kIi} = G_{kIj} = G_{kKi} = G_{kKj} = G_{kMi} = G_{kMj} = G_{kCi} = G_{kCj} = 0$, $F_{ki} = F_{kj}$ 100. Division of U_{ka} with 10 is for scaling purposes. Panels a1 and a2: benchmark, panels b1 and b2: $\gamma_{jit} = 0.125$, $G_{kIi} = 0.41$, $G_{kij} = 0.24$, panels c1 and c2: $\lambda_{jit} = 1$, $G_{kIi} = 0.9$, $G_{kIj} = 0.26$, panels d1 and d2: $\gamma_{jit} = 0.125$, $\lambda_{jit} = 1$, $G_{kIi} = 0.9$, $G_{kIj} = 0.26$


a1
I_i
K_i
M_i
C_i
I_j
K_j
M_j
C_j
t


a2
t
U_k
$U_{ka}/10$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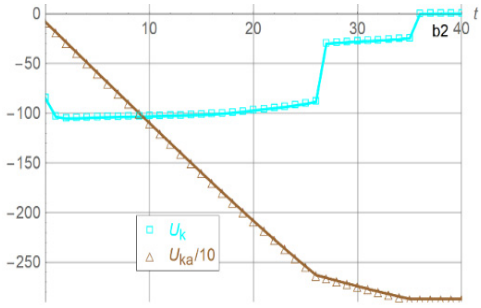
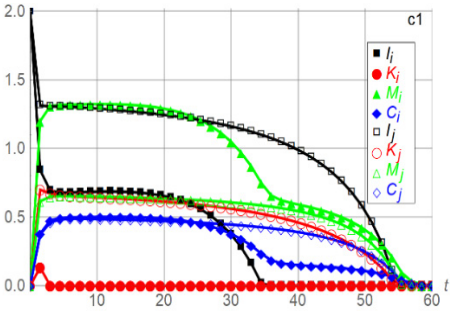
K_i
M_i
C_i
I_j
K_j
M_j
C_j
t


b2
t
U_k
$U_{ka}/10$


c1
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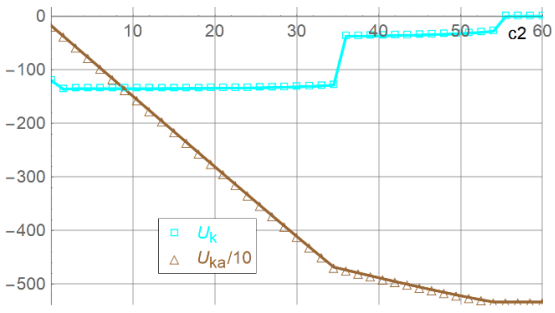
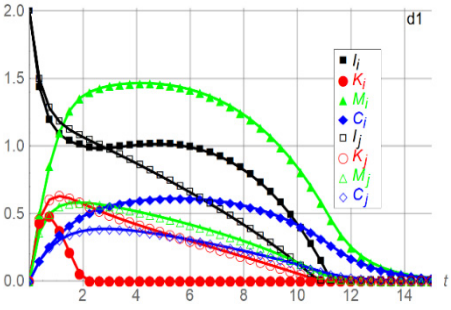
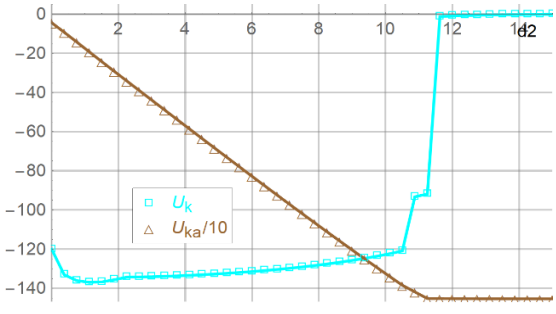
K_i
M_i
C_i
I_j
K_j
M_j
C_j
t


c2
U_k
$U_{ka}/10$


d1
I_i
K_i
M_i
C_i
I_j
K_j
M_j
C_j
t


d2
U_k
$U_{ka}/10$

Ideologue labours decrease to $I_i = 0$ when $t > 26.13$ after which $G_{kIi} = 0$, and to $I_j = 0$ when $t > 35.53$ after which $G_{kIj} = 0$. Thereafter, $\lim_{t\to\infty} M_i = \lim_{t\to\infty} M_j = \lim_{t\to\infty} C_i = \lim_{t\to\infty} C_j = 0.$ Government k's instantaneous utility U_k approaches $\lim_{t\to\infty} U_k = 0,$ and the accumulated utility U_{ka} approaches $\lim_{t\to\infty} U_{ka}/10 = -286.83.$ Intervening with $G_{kIi} = 0.56$ fails to eliminate organisation i. Intervening with $G_{kIj} = 0.23$ fails to eliminate organisation j.

Figure 3 panels c1 and c2 introduce into Fig. 1 panels c1 and c2 (where mercenary labour M_j in organisation j moves to organisation i) government intervention $G_{kIi} = 0.9$ and $G_{kIj} = 0.26$ at unit cost $F_{ki} = F_{kj} = 100$ by government k against ideologue labours I_i and I_j, and $G_{kKi} = G_{kKj} = G_{kMi} = G_{kMj} = G_{kCi} = G_{kCj} = 0$, $i, j = 1, 2$, $i \neq j$. This eliminates both terrorist organisations. Capital sponsoring in organisation i decreases quickly to $K_i = 0$ when $t > 1.78$ due to the drastic decrease in ideologue labour I_i, which reaches $I_i = 0$ when $t > 34.68$. Thereafter, ideologue labour I_j in organisation j decreases to $I_j = 0$ when $t > 54.68$, which decreases capital sponsoring to $K_j = 0$ when $t > 54.78$. Mercenary labours and captive participants labours in both organisations approach zero asymptotically, $\lim_{t\to\infty} M_i = \lim_{t\to\infty} M_j = \lim_{t\to\infty} C_i = \lim_{t\to\infty} C_j = 0.$ Government k's instantaneous utility U_k approaches $\lim_{t\to\infty} U_k = 0$ as the organisations approach extinction. Government k's accumulated utility U_{ka} approaches $\lim_{t\to\infty} U_{ka}/10 = -533.48.$ Intervening with $G_{kIi} = 0.89$ fails to eliminate organisation i. Intervening with $G_{kij} = 0.25$ fails to eliminate organisation j.

Figure 3 panels d1 and d2 introduce into Fig. 1 panels d1 and d2 (where both ideologue labour I_j and mercenary labour M_j in organisation j move to organisation i) the same government intervention as in Fig. 3 panels c1 and c2, i.e., $G_{kIi} = 0.9$ and $G_{kIj} = 0.26$ at unit cost $F_{ki} = F_{kj} = 100$ by government k against ideologue labours I_i and I_j, and $G_{kKi} = G_{kKj} = G_{kMi} = G_{kMj} = G_{kCi} = G_{kCj} = 0$, $i, j = 1, 2$, $i \neq j$. This eliminates both terrorist organisations quicker than in panel c1. Capital sponsoring in organisation i decreases to $K_i = 0$ when $t > 2.01$, contributing to ideologue labour reaching $I_i = 0$ when $t > 11.27$. Ideologue labour I_j in organisation j decreases to $I_j = 0$ when $t > 10.80$, which decreases capital sponsoring to $K_j = 0$ when $t > 10.87$. Mercenary labour M_i in organisation i experiences a high inverse U shape due to the inflow of mercenary labour M_j from organisation j, which helps sustain ideologue labour I_i in organisation i for a while. Mercenary labours and captive participants labours in both organisations eventually approach zero asymptotically, $\lim_{t\to\infty} M_i = \lim_{t\to\infty} M_j = \lim_{t\to\infty} C_i = \lim_{t\to\infty} C_j = 0.$ Government k's instantaneous utility U_k approaches $\lim_{t\to\infty} U_k = 0$ as the organisations approach extinction. Government k's accumulated utility U_{ka} approaches $\lim_{t\to\infty} U_{ka}/10 = -145.33,$ which government k prefers compared with panels a, b, and c.

6. Analysing government intervention in two terrorist organisations with emigration and competition as in Section 4

Government intervention with the same amounts as in Fig. 3, which assumes no competition between the terrorist organisations, is advantageous for government k since competition between the two terrorist organisations aids in their own destruction. This section considers alternative illuminative intervention strategies. Figure 4 panels a1 and a2 introduce into Fig. 2 panels a1 and a2 government intervention $G_{kIi} = G_{kIj} = 0.27$ at unit costs $F_{ki} = F_{kj} = 100$ by government k against ideologue labours I_i and I_j, and $G_{kKi} = G_{kKj} = G_{kMi} = G_{kMj} = G_{kCi} = G_{kCj} = 0$, $i, j = 1, 2$, $i \neq j$. Whereas $G_{kIi} = G_{kIj} = 0.4$ fails to terminate the terrorist organisations in Fig. 3 panels a1 and a2, in Fig. 4 panels a1 and a2 both terrorist organisations are terminated, i.e., $K_i = K_j = 0$ when $t > 29.78$, $I_i = I_j = 0$ when $t > 38.10$, $\lim_{t\to\infty} M_i = \lim_{t\to\infty} M_j = \lim_{t\to\infty} C_i = \lim_{t\to\infty} C_j = 0$, $\lim_{t\to\infty} U_k = 0$, $\lim_{t\to\infty} U_{ka} = -268.93$. The termination is similar to Fig. 3 panels a1 and a2, but less intervention is needed since competition $\alpha_{ij} = \alpha_{ji} = 0.1$ between the terrorist organisations aids in their demise. Lowering the intervention further to $G_{kIi} = G_{kIj} = 0.26$ fails to terminate them. Instead, in Fig. 4 panels a1 and a2, a steady state situation within bounds is obtained, i.e., $\lim_{t\to\infty} I_i = \lim_{t\to\infty} I_j = 2.53$, $\lim_{t\to\infty} K_i = \lim_{t\to\infty} K_j = 0.45$, $\lim_{t\to\infty} M_i = \lim_{t\to\infty} M_j = 2.08$, $\lim_{t\to\infty} C_i = \lim_{t\to\infty} C_j = 1.15$, $\lim_{t\to\infty} U_k = -97.60$, $\lim_{t\to\infty} U_{ka} = -\infty$. Although $G_{kIi} = G_{kIj} = 0.27$ is lower than $G_{kIi} = G_{kIj} = 0.41$ in Fig. 3, the termination takes more time, i.e., $t > 38.10$ rather than $t > 22.89$ to obtain $I_i = I_j = 0$. Government k may not have the capacity for higher intervention than $G_{kIi} = G_{kIj} = 0.27$. If government k focuses on its accumulated utility U_{ka}, it should intervene with more than $G_{kIi} = G_{kIj} = 0.27$ to ensure faster termination.

Figure 4 panels b1 and b2 introduce into Fig. 2 panels b1 and b2 (where ideologues compete and ideologue labour I_i in organisation j moves to organisation i) government intervention $G_{kIi} = 0.68$ and $G_{kIj} = 0$ when $0 < t < 13.05$ at unit costs $F_{ki} = F_{kj} = 100$ by government k against ideologue labour I_i in organisation i, and $G_{kKi} = G_{kKj} = G_{kMi} = G_{kMj} = G_{kCi} = G_{kCj} = 0$, $i, j = 1, 2$, $i \neq j$. Whereas intervening with $G_{kIi} = 0.40$ and $G_{kIj} = 0.24$ fails to eliminate organisation i in Fig. 3 panels b1 and b2, in Figure 4 panels b1 and b2 capital sponsoring decreases to $K_i = 0$ when $t > 10.06$, and ideologue labour decreases to $I_i = 0$ when $t > 13.04$, aided by competition $\alpha_{ij} = \alpha_{ji} = 0.1$ from increasing ideologue labour I_j in organisation j. Thereafter, $\lim_{t\to\infty} M_i = \lim_{t\to\infty} C_i = 0$. Lower intervention $G_{kIi} = 0.67$ and $G_{kIj} = 0$ fails to terminate organisation i. Figure 4 panels b1 and b2 assume government intervention $G_{kIi} = 0$ and $G_{kIj} = 0.5$ when $13.05 < t < 28.09$. That causes capital sponsoring to decrease to $K_j = 0$ when $t > 20.13$, ideologue labour to decrease to $I_j = 0$ when $t > 28.09$, and

$\lim_{t\to\infty} M_j = \lim_{t\to\infty} C_j = 0$. Lowering the latter intervention towards organisation j to $G_{klj} = 0.25$ fails to terminate it, but constrains it to $\lim_{t\to\infty} I_j = 2.33$, $\lim_{t\to\infty} K_j = 0.33$, $\lim_{t\to\infty} M_j = 2.01$, $\lim_{t\to\infty} C_j = 1.09$. Lowering the latter intervention towards organisation j to $G_{klj} = 0.3$ terminates it, but is considerably more time consuming than when $G_{klj} = 0.5$. For example, capital sponsoring decreases to $K_j = 0$ when $t > 75.54$. Government k's instantaneous utility U_k approaches $\lim_{t\to\infty} U_k = 0$. Government k's accumulated utility U_{ka} approaches $\lim_{t\to\infty} U_{ka}/10 = -212.46$. Also here, if government k focuses on its accumulated utility U_{ka}, and has sufficient budget, it should to some extent intervene more to ensure faster termination.

Figure 4 panels c1 and c2 introduce into Fig. 2 panels c1 and c2 (where ideologues compete and mercenary labour M_j in organisation j moves to organisation i) government intervention $G_{kIi} = 0.69$ and $G_{klj} = 0$ when $0 < t < 30.62$ at unit costs $F_{ki} = F_{kj} = 100$ by government k against ideologue labour I_i in organisation i, and $G_{kKi} = G_{kKj} = G_{kMi} = G_{kMj} = G_{kCi} = G_{kCj} = 0$, $i, j = 1, 2$, $i \neq j$. Intervening with $G_{kIi} = 0.68$ and $G_{klj} = 0$, as in Fig. 4 panels b1 and b2, is insufficient to eliminate organisation i. Whereas intervening with $G_{kIi} = 0.89$ and $G_{klj} = 0.26$ fails to eliminate organisation i in Fig. 4 panels b1 and b2, in Fig. 4 panels c1 and c2 capital sponsoring of organisation i decreases quickly to $K_i = 0$ when $t > 1.83$, mainly caused by the substantial immigration of mercenary labour M_j from organisation j. Supported by mercenary labour M_i, and immigration of ideologue labour I_j from organisation j, ideologue labour I_i in organisation i continues to grow despite the substantial government intervention $G_{kIi} = 0.69$. However, ideologue labour I_j in organisation j grows more due to no government intervention $G_{klj} = 0$. Mercenary labour M_j in organisation j experiences an inverse U shape, due to emigration to organisation i, and reaches $M_j = 0$ when $t > 22.86$. That causes mercenary labour M_i in organisation i to decrease substantially, $\lim_{t\to\infty} M_i = \lim_{t\to\infty} C_i = 0$. With support of neither capital sponsoring K_i nor mercenary labour M_i, substantial government intervention $G_{kIi} = 0.69$, and ideologue competition $\alpha_{ij} = \alpha_{ji} = 0.1$ with organisation j, ideologue labour in organisation i decreases to $I_i = 0$ when $t > 30.62$. Figure 4 panels c1 and c2 assume government intervention $G_{kIi} = 0$ and $G_{klj} = 3$ when $30.62 < t < 38.64$, and $\lim_{t\to\infty} M_i = \lim_{t\to\infty} C_i = 0$, which terminates organisation j. Government intervention $G_{kIi} = 0$ and $G_{klj} = 2.5$ causes $I_j = 0$ when $t > 42.78$. Lower government intervention $G_{klj} = 2$ causes unbounded growth of organisation j. Government k's instantaneous utility U_k approaches $\lim_{t\to\infty} U_k = 0$. Government k's accumulated utility U_{ka} approaches $\lim_{t\to\infty} U_{ka}/10 = -446.12$. If government k focuses on its accumulated utility U_{ka}, and has sufficient budget, it should to some extent

intervene more, and earlier, to ensure faster termination. In particular, eliminating organisation i earlier, and then intervening to eliminate organisation j, prevents organisation j to grow substantially before elimination.

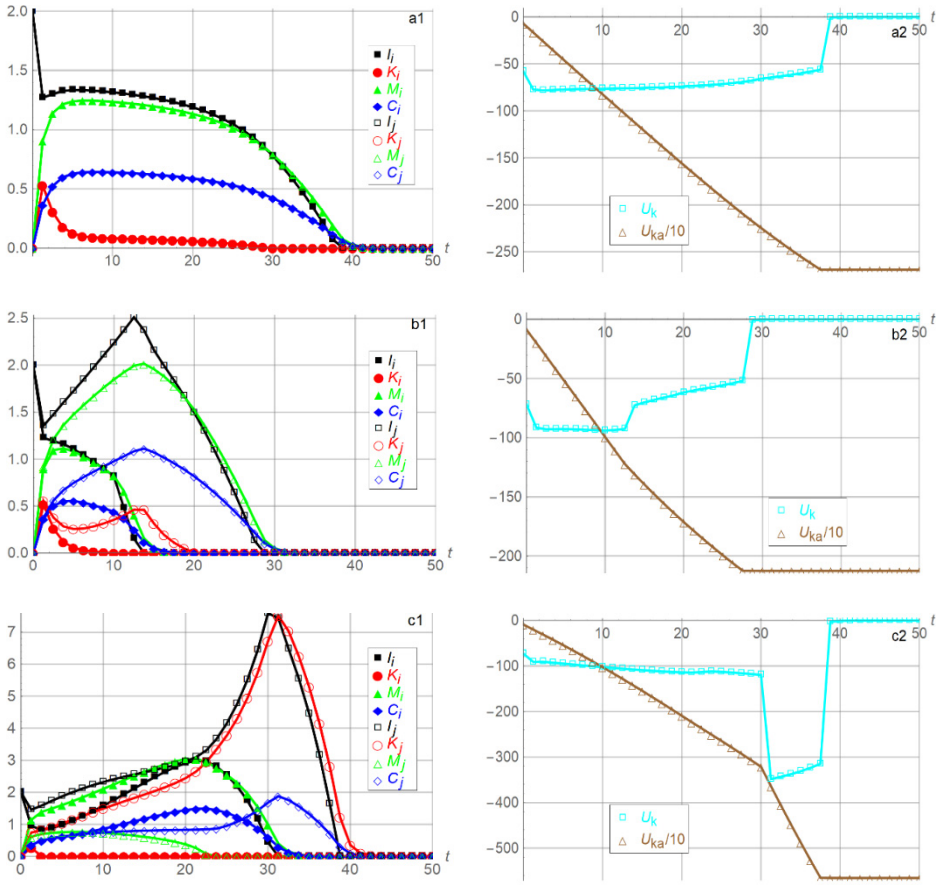


Fig. 4. Ideologue labours I_i and I_j, capital K_i and K_j, mercenary labours M_i and M_j, and captive participants labours C_i and C_j, in $N = 2$ organisations i and j, $i, j = 1, 2$, $i \neq j$, and government k's instantaneous utility U_k and accumulated utility U_{ka}, $k = 1, ..., Q$, as functions of time t with competition $\alpha_{ij} = \alpha_{ji} = 0.1$ and benchmark parameter values $a_i = a_j = c_i = c_j = d_i = d_j = e_i = e_j = f_i = f_j = h_i = h_j = m_i = m_j = r_i = r_j = s_i = s_j = u_i = u_j = v_i = v_j = w_i = w_j = \mu_{Iijt} = \mu_{Ijit} = \mu_{Mijt} = \mu_{Mjit} = 1$, $b_i = b_j = g_i = g_j = 0.8$, $\theta_i = \theta_j = 0.3$, $\varphi_i = \varphi_j = 0.6$, $n_i = n_j = o_i = o_j = p_i = p_j = q_i = q_j = 0.25$, $\beta_{ij} = \beta_{ji} = \gamma_{ij} = \gamma_{ji} = \lambda_{ij} = \lambda_{ji} = 0$, $I_i(0) = I_j(0) = 2$, $K_i(0) = K_j(0) = M_i(0) = M_j(0) = C_i(0) = C_j(0) = 0$, $A_{ki} = A_{kj} = B_{ki} = B_{kj} = D_{ki} = D_{kj} = E_{ki} = E_{kj} = \delta_k = 1$, $z_{ki} = z_{kj} = 0.5$, $G_{kIi} = G_{kIj} = G_{kKi} = G_{kKj} = G_{kMi} = G_{kMj} = G_{kCi} = G_{kCj} = 0$, $F_{ki} = F_{kj} = 100$. Division of U_{ka} with 10 is for scaling purposes. Panels a1 and a2: benchmark, $G_{kIi} = G_{kIj} = 0.27$, panels b1 and b2: $\gamma_{jit} = 0.125$, $G_{kIi} = 0.68$ and $G_{kIj} = 0$ when $0 < t < 13.05$, and $G_{kIi} = 0$ and $G_{kIj} = 0.5$ when $13.05 < t < 28.09$, panels c1 and c2: $\lambda_{jit} = 1$, $G_{kIi} = 0.69$ and $G_{kIj} = 0$ when $0 < t < 30.62$, and $G_{kIi} = 0$ and $G_{kIj} = 3$ when $30.62 < t < 38.64$

7. Conclusion

A model is developed to analyse migration and competition through time between terrorist organisations subject to government intervention. Each terrorist organisation is composed of ideologues, criminal mercenaries, and captive participants, and may be supported by sponsors. The evolution of the three labours and capital sponsoring is analysed with four coupled differential time equations. Ideologues and mercenaries may migrate between the terrorist organisations, which may compete with each other. Governments may intervene to regulate, control, prevent unbounded growth, or eradicate the terrorist organisations.

For two terrorist organisations, we first analyse a typical and illustrative benchmark of neither emigration, competition, nor government intervention. Ideologue labour grows supported by growing capital sponsoring and captive participants labour, while mercenary labour vanishes. Emigration of ideologues causes unbounded growth for the organisation receiving ideologues, while the organisation losing ideologues reaches a stationary state where ideologues are supported by both capital sponsoring and mercenaries which tolerate each other. Emigration of mercenaries causes the organisation losing mercenaries to experience growth. The organisation receiving mercenaries experiences a dip in capital sponsoring. If the dip is temporary and capital sponsoring rebounds, mercenaries vanish and the organisation grows. Emigration of both ideologues and mercenaries may eradicate an organisation.

Introducing competition between terrorist organisations may prevent their unbounded growth. Competition and emigration of ideologues or mercenaries may eliminate an organisation that would survive without competition.

Government intervention is shown to contain or eliminate terrorist organisations that would otherwise grow. Emigration of ideologues or mercenaries from one organisation to another requires more government intervention into the latter than the former to eliminate both organisations. Emigration of both ideologues and mercenaries from one organisation to another may enable quicker elimination since mercenaries more quickly go extinct in the former organisation. No immigration of mercenaries into the latter means less support for the ideologues which are then more easily terminated by government intervention.

Government intervention more easily extinguishes competing terrorist organisations since competition facilitates their extinction. Various alternative intervention strategies are considered where the most threatening organisation is eliminated first, aided by the competition from the least threatening organisation, after which the remaining organisation is eliminated. The government's instantaneous and accumulated utilities are analysed through time and compared depending on emigration, competition, and government intervention strategies.

The article provides tools to analyse government intervention through time into terrorist organisations interacting through migration and competition while accounting for how each terrorist organisation functions and evolves internally. Future research should gather empirical data to compare against the model and gain a richer understanding of how to fight terrorism. Future research may also analyse the phenomenon by applying other tools, e.g., a differential game approach.

Nomenclature

Q – number of governments, $Q \geq 1$

N – number of terrorist organisations, $N \geq 1$

t – time, $t \geq 0$

I_i – amount or stock of labour exerted by ideologues in terrorist organisation, $i = 1, ..., N$, $I_i \geq 0$

K_i – amount of capital provided by sponsors to terrorist organisation, $i = 1, ..., N$, $K_i \geq 0$

M_i – amount or stock of labour exerted by mercenaries in terrorist organisation $i = 1, ..., N$, $M_i \geq 0$

C_i – amount or stock of labour exerted by captive participants in terrorist organisation $i, ..., N$, $C_i \geq 0$

G_{kIi} – government k's labour effort at time t to combat ideologue labour I_i, $k = 1, ..., Q$, $-\infty < G_{kIi} < \infty$

G_{kKi} – government k's labour effort at time t to combat capital K_i, $k = 1, ..., Q$, $-\infty < G_{kKi} < \infty$

G_{kMi} – government k's labour effort at time t to combat mercenary labour M_i, $k = 1, ..., Q$, $-\infty < G_{kMi} < \infty$

G_{kCi} – government k's labour effort at time t to combat captive participants C_i, $k = 1, ..., Q$, $-\infty < G_{kCi} < \infty$

α_{ij} – loss rate for organisation i due to ideologue competition or warfare with organisation j, $i, j = 1, ..., N$, $i \neq j$, $\alpha_{ij} \geq 0$

β_{ij} – loss rate for organisation i due to mercenary competition or warfare with organisation j, $i, j = 1, ..., N$, $i \neq j$, $\beta_{ij} \geq 0$

x_{ij} – fraction of ideologue labour I_i fighting organisation j, $0 \leq x_{ij} \leq 1$, $\sum_{j=1,\, j\neq i}^{N} x_{ij} = 1$

y_{ij} – fraction of mercenary labour M_i fighting organisation j, $0 \leq y_{ij} \leq 1$, $\sum_{j=1,\, j\neq i}^{N} y_{ij} = 1$

$\gamma_{ijt} = -\gamma_{jit}$ – attractiveness of organisation j relative to organisation i for ideologue labour I_i at time t, $-\infty < \gamma_{ijt} < \infty$

$\lambda_{ijt} = -\lambda_{jit}$ – attractiveness of organisation j relative to organisation i for mercenary labour M_i at time t, $-\infty < \lambda_{ijt} < \infty$

μ_{Iijt} – terrorist organisation i's control parameter for the outflow of ideologue labour I_i from organisation i to organisation j at time t, $0 \leq \mu_{Iijt} \leq 1$

μ_{Mijt} – terrorist organisation i's control parameter for the outflow of mercenary labour M_i from organisation i to organisation j at time t, $0 \leq \mu_{Mijt} \leq 1$

a_i – growth rate for capital K_i impacting ideologue labour I_i, $a_i \geq 0$

c_i – growth rate for ideologue labour I_i impacting capital sponsoring K_i, $c_i \geq 0$

b_i – depreciation rate of ideologue labour I_i $b_i \geq 0$

d_i – depreciation rate of capital K_i, $d_i \geq 0$

e_i – growth rate for mercenary labour M_i impacting ideologue labour I_i, $e_i \geq 0$

f_i – depreciation rate of mercenary labour impacting M_i capital sponsoring K_i, $f_i \geq 0$

g_i – growth rate for ideologue labour I_i impacting mercenary labour M_i, $g_i \geq 0$
h_i – growth rate for capital K_i impacting mercenary labour M_i, $h_i \geq 0$
m_i – depreciation rate of mercenary labour M_i, $m_i \geq 0$
θ_i – depreciation rate of the product K_iM_i of capital K_i and mercenary labour M_i, $\theta_i \geq 0$
φ_i – depreciation rate of the product I_iK_i of ideologue labour I_i and capital K_i, $\varphi_i \geq 0$
n_i – growth rate for captive participants C_i impacting ideologue labour I_i, $n_i \geq 0$
o_i – growth rate for captive participants C_i impacting mercenary labour M_i, $o_i \geq 0$
p_i – growth rate for ideologue labour I_i impacting captive participants C_i, $p_i \geq 0$
q_i – growth rate for mercenary labour M_i impacting captive participants C_i, $q_i \geq 0$
r_i – depreciation rate of captive participants C_i, $r_i \geq 0$
s_i – unit effort costs of government labour to combat ideologue labour I_i, $s_i \geq 0$
u_i – unit effort costs of government labour to combat capital K_i, $u_i \geq 0$
v_i – unit effort costs of government labour to combat mercenary labour M_i, $v_i \geq 0$
w_i – unit effort costs of government labour to combat captive participants C_i, $w_i \geq 0$
U_{ki} – government k's instantaneous utility at time τ, $0 \leq \tau \leq t$, due to the presence of terrorist organisation $i = 1, ..., N$, $-\infty < U_{ki} < \infty$
U_{kai} – government k's accumulated utility due to terrorist organisation $i = 1, ..., N$ from time $\tau = 0$ to time $\tau = t$, $U_{kai} = \int_{\tau=0}^{\tau=t} \delta_k^{\tau} U_{ki} d\tau$, $-\infty < U_{kai} < \infty$
A_{ki} – weight parameter for I_i for government k for the constant elasticity of substitution utility, $A_{ki} \geq 0$
B_{ki} – weight parameter for K_i for government k for the constant elasticity of substitution utility, $B_{ki} \geq 0$
D_{ki} – weight parameter for M_i for government k for the constant elasticity of substitution utility, $D_{ki} \geq 0$
E_{ki} – weight parameter for C_i for government k for the constant elasticity of substitution utility, $E_{ki} \geq 0$
$1/(1 - z_{ki})$ – government k's elasticity of substitution as impacted by terrorist organisation $i = 1, ..., N$, $-\infty < z_{ki} \leq 1$
F_{ki} – government k's unit cost of exerting effort G_{kIi} at time t to combat ideologue labour I_i, $F_{ki} \geq 0$
H_{ki} – government k's unit cost of exerting effort G_{kKi} at time t to combat capital K_i, $H_i \geq 0$
J_{ki} – government k's unit cost of exerting effort G_{kMi} at time t to combat mercenary labour M_i, $J_i \geq 0$
L_{ki} – government k's unit cost of exerting effort G_{kCi} at time t to combat captive participants C_i, $L_i \geq 0$
δ_k – government k's time discount parameter, $0 \leq \delta_k \leq 1$

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