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Mathematical process quality models based on the example of the baking industry

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ABSTRACT:

This article sets out a new approach to quantitative, dimensionless assessment of the efficiency and competitiveness of production processes. New concepts of process quality and relative product quality have been introduced. The process quality is expressed by means of vectors and scalars. The process quality $\overrightarrow{\mathrm{JP}}$ vector was expressed by the product of reliability by the vector from the sum of three components taking into account the composition of the repetition of the basis weight and the minimum costs. Two new scalar process quality measures as a module of the quality vector and the mixed product of three process base vectors are proposed. A new concept of relative product quality (WJP) is also highlighted. Separability and relationship of both concepts are presented. The assessment method is illustrated by calculations on the example of baking processes carried out at the bakery in the Pomerania Province. The usefulness of robotization leading to a significant increase of process quality indicators has been demonstrated. New quantitative and vector process quality measures developed give the opportunity to build technological, economic and organisational algorithms that lead to innovation and increase the efficiency of production processes.

Modele matematyczne jakości procesów na przykładzie branży piekarniczej

Słowa kluczowe: modele matematyczne, wektor jakości procesów, względna jakość produktu, jakość procesu

STRESZCZENIE:

W niniejszym artykule przedstawiono nowe podejście do ilościowej, bezwymiarowej oceny efektywności i konkurencyjności procesów produkcyjnych. Wprowadzono nowe pojęcia jakości procesu i względnej jakości produktu. Jakość procesu wyrażono wektorowo i skalarnie. Wektor jakości procesu $\overrightarrow{\mathrm{JP}}$ wyrażono iloczynem niezawodności przez wektor z sumy trzech składowych uwzględniającej skład powtarzalności gramatury i minimum kosztów. Zaproponowano dwie nowe skalarne miary jakości procesu jako modułu wektora jakości i iloczynu mieszanego trzech wektorów bazy procesu. Wyróżniono też nowe pojęcie względnej jakości produktu (WJP). Przedstawiono separowalność i związek obydwu pojęć. Metodę oceny zilustrowano obliczeniami na przykładzie procesów piekarniczych realizowanych w piekarni w województwie pomorskim. Wykazano przydatność robotyzacji prowadzącej do znacznego podwyższenia wskaźników jakości procesu. Opracowane nowe ilościowe i wektorowe miary jakości procesu dają możliwości budowy algorytmów technologicznych, ekonomicznych i organizacyjnych prowadzących do innowacyjności i wzrostu efektywności procesów produkcyjnych.

1. INTRODUCTION

Currently, the methods of analyzing the quality of the product are being considered. This pertains in particular to the food and health industry [4, 5].
Relevant factors having a dominant influence on the product quality include:
a) quality parameters and costs of raw materials;
b) precision and monitoring of ingredient dosing;
c) formulation of technology and composition of substitutes;
d) repeatability of the designed product characteristics in the manufacturing process;
e) high reliability value of process systems operation.
An important issue in assessing the quality of a product is the organisation of the structure of this concept and its metrizability.
Here we encounter fundamental difficulties.
On the basis of the above factors strongly related to product quality, an organic chain of the following links can be seen: technical, technological, economic and a number of other ones relating to different fields of science and organisation.
Therefore, there is an urgent need to formulate a new approach to the assessment of product quality in the context of its relationship with the entire production process and consumer needs.

The purpose of the study is to propose to formulate a new concept of "process quality", hereinafter referred to as the (JP) symbol.
Process quality (JP) combines relative product quality (WJP) with system conditions of manufacturing processes and economics. It expresses the efficiency of the process and the directions of innovation. A vector and scalar measure of process quality (JP) and relative product quality (WJP) is also proposed.
The metrizability of the terms (JP) and (WJP) are the starting points for the development of modern manufacturing technologies. The problem is illustrated by an example of baking processes.

2. STRUCTURE AND METRIZABILITY OF PROCESS QUALITY (JP) AND RELATIVE PRODUCT QUALITY (WJP) IN THE BAKING INDUSTRY

2.1 Initial remarks

The model of process quality (JP) in the baking process should take into account the following main features:
1. Permissible ingredient-related standards for the bread to ensure palatability (S).
2. Repeatability and stability of the grammage of the structural composition of the product (P).
3. Minimisation of manufacturing costs (K).

4. Monitoring of process reliability and performance (N).
The model of relative product quality (WJP) will meet the following characteristics:

1(S), 2(P) and 3(K).

Vector and scalar quality measures of (JP) and (WJP) are presented below. Scalar values for both measures are between zero and unity.

2.2 Vector representations of (JP) and (WJP)

We consider a three-dimensional orthogonal coordinate system with versors $\vec{e}_1, \vec{e}_2, \vec{e}_3$.
The vector of the permitted compositional standards is expressed as:

$$\vec{S} = s \times \vec{e}_1, \tag{1}$$

where:

$$s = \frac{\sum_{i=1}^{k} \times \mathrm{Si(r)}}{\sum_{i=1}^{j} \times \mathrm{Si(n)}}. \tag{2}$$

Si(r) values are the values of the subsequent “ith” actual ingredients of the product structure in the process.
Si(n) values are the values of the successive “ith” ingredients of the structure determined by the standard.
The quantity of standard ingredients “j” should be equal to the number of actual ingredients “k”.
Thus, in a properly conducted baking process

$$j = k. \tag{3}$$

The value of the vector module $\vec{s}$ in a properly executed process meets the relation:

$$0 \le S \le 1. \tag{4}$$

The vector of repetition and stability of the grammage $\vec{P}$ is expressed as:

$$\vec{P} = p \times \vec{e}_2, \tag{5}$$

where:

$$p = \frac{\sum_{i=1}^{k} \times \frac{\mathrm{Si(r)}}{\Delta \mathrm{ti(r)}}}{\sum_{i=1}^{j} \times \frac{\mathrm{Si(n)}}{\Delta \mathrm{ti(n)}}}. \tag{6}$$

The value p is the quotient of the actual and standard ingredient values by the actual times Δti(r) and normative times Δti(n) determined by the process grammage.
The value of the vector module $\vec{P}$ in a properly executed process meets the relation:

$$0 \le P \le 1. \tag{7}$$

The subsequent vector responsible for the minimum manufacturing cost resulting from the consumption of raw materials and energy $\vec{K}$ is expressed as:

$$\vec{K} = k \times \vec{e}_3\,. \tag{8}$$

The value of the vector module $\vec{K}$ in a properly executed process is expressed as:

$$K = \frac{k_r}{k_n}. \tag{9}$$

The value of k_r is the total cost of actual contribution of all components of the product. It is expressed as:

$$k_r = \sum_{i=1}^{k} \times \mathrm{Si(r)} \times \mathrm{ki(r)}. \tag{10}$$

The value of K is the total standard cost of the share of all components of the product. It is expressed as:

$$k_n = \sum_{i=1}^{j} \times \mathrm{Si(n)} \times \mathrm{ki(n)}. \tag{11}$$

K_r(r) and K_i(n) values mean, respectively, the appropriately modified unit cost of the share of components in the actual process and the unit cost determined by the standard.
The value of the vector module $\vec{K}$ in a properly executed process meets the relation:

$$0 \le K \le 1. \tag{12}$$

The vectors $\vec{S}, \vec{P}, \vec{K}$ shown above and the scalar reliability N are further the basis for the determination of vector and scalar measures of process quality (JP) and relative product quality (WJP).
The values: s, p, k in formulas (2), (6), (9) are dimensionless.

3. PROCESS QUALITY (JP) METRIZABILITY

It is proposed to express metrizability of (JP) by two values, namely a vector and a scalar.
The process quality vector is in the form of $\overrightarrow{JP}$

$$\overrightarrow{JP} = \frac{N_{\overrightarrow{JP}}}{\sqrt{3}}, \tag{13}$$

where:

$$\overrightarrow{jp} = \frac{N}{\sqrt{3}}\vec{S} + \vec{P} + \vec{K}. \tag{14}$$

Taking into account the dependencies (2), (6), (9), we have:

$$\overrightarrow{JP} = \frac{N}{\sqrt{3}}(\overrightarrow{s_{e_1}} + \overrightarrow{p_{e_2}} + \overrightarrow{k_{e_3}}). \tag{15}$$

The value of N means the reliability and the controllability of the process.
The N value is determined empirically in a specific production process using the stream and system method or on a probabilistic basis. These issues are discussed in the works of authors [1-3].
With output base of vectors $\vec{S}, \vec{P}, \vec{K}$ and N values, we may propose two scalars of JP.

The first scalar is the module of vector $\overrightarrow{JP}$.
It is designated as jP_m.
The value of the jP_m module meets the relation:

$$jP_m = \frac{N}{\sqrt{3}}\sqrt{s^2 + p^2 + k^2}. \quad (16)$$

The values of jP_m are included in the inequality:

$$0 \leq jP_m \leq 1. \quad (17)$$

The second measure of process quality is the product of reliability N and the values of mixed product of vectors $\vec{S}, \vec{P}, \vec{K}$.
It is expressed as:

$$jP_0 = N(\vec{S} \times \vec{P}) \times \vec{K} =$$

$$N \begin{vmatrix} \vec{e}_1 & \vec{e}_2 & \vec{e}_3 \\ s & 0 & 0 \\ 0 & p & 0 \\ 0 & 0 & k \end{vmatrix} = \quad (18)$$

$$= N \times s \times p \times k.$$

The scalar $\overrightarrow{JP}_0$ is interpreted as the volume determined by the product of mixed vectors $\vec{S}, \vec{P}, \vec{K}$ multiplied by the reliability value. The value $\overrightarrow{JP}_0$ always meets the inequality:

$$0 \leq \overrightarrow{JP}_0 \leq 1. \quad (19)$$

The process quality vector $\overrightarrow{JP}$ is illustrated in Figure 1.

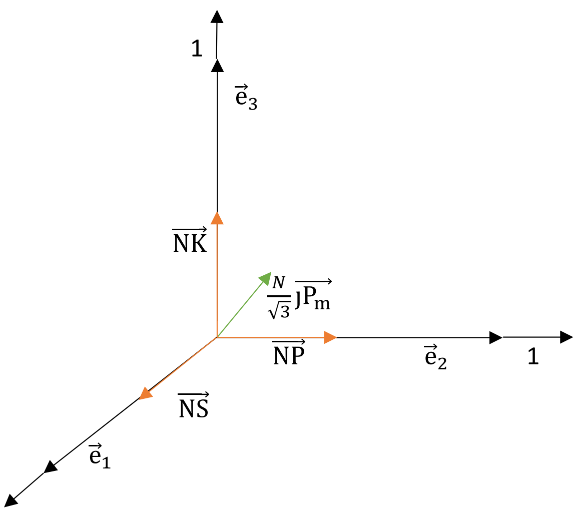

Figure 1 Orientation of the vector $\overrightarrow{JP}$ relative to the axis (process base) gives the possibility of quick evaluation in relation to individual components of the base of value $\overrightarrow{JP}$. Source: Own study based on research performed in the bakery X

4. RELATIVE PRODUCT QUALITY (JWP) METRIZABILITY

This term applies only to the fulfilment of the first three characteristics of the product listed in section 2.
Therefore, the product quality vector $\overrightarrow{JWP}$ is in the form of:

$$\overrightarrow{JWP} = \frac{1}{\sqrt{3}}(\vec{S} + \vec{P} + \vec{K}). \quad (20)$$

The scalar measure of the vector $\overrightarrow{JWP}$ is its module jwp_m:

$$jwp_m = \frac{1}{\sqrt{3}}\sqrt{s^2 + p^2 + k^2}. \quad (21)$$

The second scalar measure is the mixed product of the vector $\vec{s}, \vec{p}$ and $\vec{k}$ given as:

$$jwp_0 = \frac{1}{\sqrt{3}}(\vec{s} \times \vec{p}) \times \vec{k} = \frac{1}{\sqrt{3}} spk. \quad (22)$$

Designations in formulas (20), (21) and (22) are the same as for the process quality vector.
The values jwp_m and jwp_0 meet the following inequalities:

$$0 \leq jwp_m \leq 1, \quad (23)$$

$$0 \leq jwp_0 \leq 1. \quad (24)$$

5. EXAMPLES OF CALCULATIONS FOR BAKERIES IN THE POMERANIA PROVINCE

The process quality and relative product quality measures presented above were used to assess the efficiency in the actual baking processes for baking bread in a bakery in the Pomerania Province.
Details of the process are included in the works [1, 2, 3].
After taking into account the tables, standards, composition of bread ensuring palatability and process measurements in the procedures of the subsequent process stages, as well as the reliability calculations, the following process quality values were obtained:
jp_m = 0.69 (non-robotized bakery);
jp_m = 0.89 (robotized bakery);
jp_m = 0.71 (non-robotized bakery);
jp_m = 0.92 (robotized bakery).

6. CONCLUSIONS

The demonstrated vector and scalar measures of process quality and relative product quality provide new opportunities to comprehensively eva-

luate process performance for obtaining the best products, including the minimization of costs, materials and energy. They also indicate ways of quality improvement.
The quantitative, dimensionless process quality assessments also showed a clear profit from the robotization in the bakery X.

Quantitative, dimensionless assessments of process quality give the possibility of economical prediction of production efficiency, reasonable economy and accurate identification of competition.

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