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AN ANALYSIS OF HYDRODYNAMIC PRESSURE DISTRIBUTION AND LOAD CARRYING CAPACITY OF A CONICAL SLIDE BEARING LUBRICATED WITH NON-NEWTONIAN SECOND GRADE FLUI

ANALIZA ROZKŁADU CIŚNIENIA HYDRODYNAMICZNEGO I SIŁY NOŚNEJ W STOŻKOWYM ŁOŻYSKU ŚLIZGOWYM SMAROWANYM OLEJEM O WŁAŚCIWOŚCIACH NIENEWTONOWSKICH DRUGIEGO RZĘDU

Key words: small parameter method; Reynolds type equation, load carrying capacity, hydrodynamic pressure, Rivlin-Ericksen model, non-Newtonian oil, conical slide bearing.

Abstract In this paper, the authors present the equations of the hydrodynamic lubrication theory for conical slide bearings lubricated with the oil with properties described by the Rivlin-Ericksen model. It is assumed, that the considered lubricating oil shows non-Newtonian properties, i.e. it is an oil for which, apart from the classic dependence of oil viscosity on pressure, temperature and operating time, there is also a change in dynamic viscosity values caused by the changes of shear rate. The method of a small parameter was used to solve the conservation of momentum, stream continuity, and energy conservation equations.

The small parameter method consists in presenting the sought functions (pressure, temperature, components of the velocity vector) in the form of a uniformly convergent series expansion in powers of a constant small parameter. These functions are substituted into the system of basic equations, and then the series are multiplied by the Cauchy method. By a comparison of the coefficients with the same powers of a small parameter, we obtain systems of partial differential equations, from which the subsequent approximations of unknowns of the sought functions are determined. The small parameter method separates the non-linear system of partial differential equations and creates several linear systems of equations.

The aim of this work is to derive the equations describing and allowing the determination of the temperature distribution, hydrodynamic pressure distribution, velocity vector components, load carrying capacity, friction force and friction coefficient in the gap of conical slide bearing, lubricated with the oil of the properties described by the Rivlin-Ericksen model, taking into account its viscosity changes due to time of operation.

Słowa kluczowe: metoda małego parametru, równanie typu Reynoldsa, siła nośna, ciśnienie hydrodynamiczne, model Rivlina-Ericksena, olej o właściwościach nienewtonowskich, stożkowe łożysko ślizgowe.

Streszczenie W artykule autorzy przedstawiają równania hydrodynamicznej teorii smarowania olejem o modelu Rivlina-Ericksena stożkowego łożyska ślizgowego. Olej ten charakteryzuje się nienewtonowskimi właściwościami, czyli jest to olej, dla którego, oprócz klasycznych zależności lepkości oleju od ciśnienia, temperatury i czasu eksploatacji, występuje dodatkowo zmiana lepkości dynamicznej od szybkości ścinania. Do rozwiązania równań zachowania pędu, ciągłości strugi i zachowania energii wykorzystano metodę małego parametru.

Metoda ta polega na przedstawieniu poszukiwanych funkcji (ciśnienia, temperatury, składowych wektora prędkości) w formie jednostajnie zbieżnego szeregu potęgowego rozwiniętego względem stałego małego parametru. Funkcje te podstawia się do układu równań podstawowych, a następnie wymnaża te szeregi metodą Cauchy'ego. Porównując współczynniki przy jednakowych potęgach małego parametru, otrzymuje się układy równań różniczkowych cząstkowych, z których wyznacza się kolejne przybliżenia niewiadomych, poszukiwanych funkcji. Metoda małego parametru rozprzęga nieliniowy układ równań różniczkowych cząstkowych, tworząc kilka liniowych układów równań.

Celem niniejszej pracy jest wyprowadzenie równań umożliwiających wyznaczenie rozkładu temperatury, rozkładu ciśnienia hydrodynamicznego, składowych wektora prędkości, siły nośnej, siły tarcia i współczynnika tarcia w szczelinie poprzecznego łożyska ślizgowego smarowanego olejem o modelu Rivlina-Ericksena z uwzględnieniem zmian lepkości od czasu eksploatacji oleju.

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INTRODUCTION

The slide bearing load carrying capacity is a response to external bearing loads. It is calculated using the hydrodynamic pressure distribution in bearing gap. The hydrodynamic pressure for bearings of finite length is determined from the Reynolds type equation using various numerical methods **[L. 1–5]**. We have the possibility of using many computer programs that enable performing numerical calculations for the classical lubrication conditions of the slide bearings (ANSYS CFD, Numeca, Fluent, Autodesk Simulation CFD, etc.). The most popular methods of numerical calculations are based on the finite elements, finite volumes or boundary elements methods **[L. 1–2]**. The finite difference method can be also applied for such simulations. Each of these methods has its own advantages and disadvantages. The main disadvantage of CFD software programs is that usually there is no possibility to include unconventional elements into momentum equations (for non-Newtonian constitutive models **[L. 6–11]**) own extended models of dynamic viscosity changes. The use of appropriate methods for solving partial differential equations and our own calculation procedures allow eliiminating these inconveniences.

The value of hydrodynamic pressure is affected by the height of the lubrication gap, the bearing dimensions, lubricating oil dynamic viscosity, bearing operating conditions, including the angular velocity **[L. 9–20]** and others. Changes in the lubricant dynamic viscosity are influenced by pressure, temperature and shear rates as well as the time of operation of the lubricant **[L. 10–12**, **14–20]**. It is difficult to take into account the effect of viscosity changes on hydrodynamic pressure or temperature, because dynamic and temperature-dependent viscosity occurs in momentum or energy equations. The solution to this problem can be achieved by applying the small parameter method or the method of subsequent approximations. Both of these methods simplify the solution of the considered system of equations.

In the method of subsequent approximations, the hydrodynamic pressure distribution and temperature, where the dynamic viscosity depends only on time of operation, are determined in the first computational step. In subsequent calculation steps, changes in dynamic viscosity are assumed, depending on e.g. hydrodynamic pressure, temperature or shear rate, using the temperature and pressure values calculated in the previous step, saved in the form of a matrix. The calculation is repeated until the results are convergent.

In the method of the small parameter, the functions of hydrodynamic pressure, velocity components and temperature in momentum equations, the continuity of stream, and energy introduce uniformly convergent power series developed in relation to successive powers of small dimensionless parameters **[L. 11]**. The functions of dynamic viscosity of a lubricant, depending on pressure or temperature, are also developed into a power series with respect to dimensionless small parameters. The small parameter method separates the non-linear system of partial differential equations and creates several linear systems of equations. The first set of equations allows one to determine the flow parameters for classic, non-isothermal lubrication with Newtonian oil, without taking into account the influence of pressure and temperature on the change in viscosity. Subsequent systems of equations allow one to determine the velocity vector components, hydrodynamic pressure and temperature corrections, resulting from taking into account the influence of pressure or temperature on the dynamic viscosity of the lubricant. In addition, this method allows one to isolate and then analyse the influence of temperature, hydrodynamic pressure and non-Newtonian properties on the value of bearing operating parameters.

The aim of this work is to derive the equations describing and allowing one to determine the hydrodynamic pressure and temperature distribution, components of velocity vector, load carrying capacity, friction force and friction coefficient in the gap of conical slide bearing, lubricated with the oil of the properties described by the Rivlin-Ericksen model.

The geometry of the considered conical plain bearing is shown in **Fig. 1**.

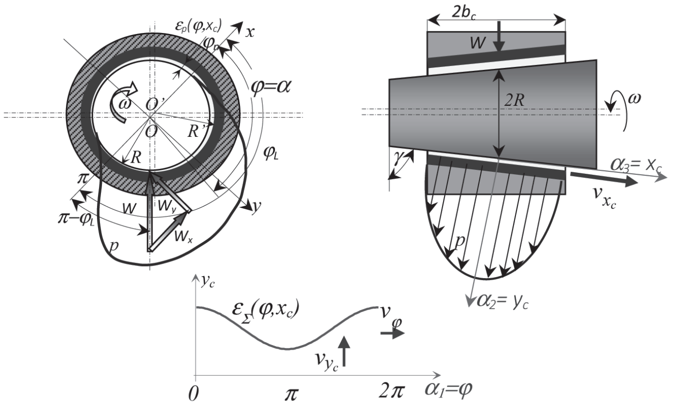

Fig. 1. The geometry of the conical surface of the sliding bearing

Rys. 1. Geometria powierzchni stożkowego łożyska ślizgowego

ANALYTICAL APPROACH

In order to determine the distribution of hydrodynamic pressure and then the load carrying capacity, frictional forces and friction coefficient, the equations for the conservation of momentum, stream continuity and energy conservation were estimated and have been presented in a dimensionless form **[L. 10–12**, **16–20]**. The calculation was made for stationary non-isothermal oil lubrication according to the Rivlin-Ericksen model. The dimensionless equations of momentum conservation, stream continuity, and energy conservation, as well as formulas describing viscosity changes and characteristic numbers, have the following form **[L. 10–11]**:

– The final form of the dimensionless equation of momentum conservation towards circumferential the direction φ:

$$\begin{gathered}
0=-\frac{1}{1+L_1x_{c1}\cos\gamma}\frac{\partial p_1}{\partial\varphi}+\frac{\partial}{\partial y_{c1}}\left(\eta_1\frac{\partial v_1}{\partial y_{c1}}\right)+De_\alpha\frac{1}{1+L_1x_{c1}\cos\gamma}\frac{\partial}{\partial\varphi}\left[\alpha_{01}\left(\frac{\partial v_1}{\partial y_{c1}}\right)^2\right]+\\
+De_\alpha\frac{1}{L_1^2}\frac{\partial}{\partial y_{c1}}\left[\alpha_{01}\frac{\partial v_3}{\partial y_{c1}}\left(\frac{1}{1+L_1x_{c1}\cos\gamma}\frac{\partial v_3}{\partial\varphi}-\frac{L_1v_1\cos\gamma}{1+L_1x_{c1}\cos\gamma}+\frac{\partial v_1}{\partial x_{c1}}\right)-2\alpha_{01}\frac{\partial v_1}{\partial y_{c1}}\frac{\partial v_3}{\partial x_{c1}}\right]+\\
+De_\alpha\left\{\frac{1}{L_1^2}\frac{\partial}{\partial x_{c1}}\left(\alpha_{01}\frac{\partial v_1}{\partial y_{c1}}\frac{\partial v_3}{\partial y_{c1}}\right)+\frac{\cos\gamma}{1+L_1x_{c1}\cos\gamma}\alpha_{01}\left[\left(\frac{\partial v_1}{\partial y_{c1}}\right)\left(\frac{1}{L_1}\frac{\partial v_3}{\partial y_{c1}}\right)\right]\right\}+\\
+De_\beta\frac{\partial}{\partial y_{c1}}\beta_{01}\left\{\frac{\partial}{\partial y_{c1}}\left(\frac{v_1}{1+L_1x_{c1}\cos\gamma}\frac{\partial v_1}{\partial\varphi}+v_2\frac{\partial v_1}{\partial y_{c1}}+\frac{1}{L_1^2}v_3\frac{\partial v_1}{\partial x_{c1}}+\frac{v_1v_3\cos\gamma}{L_1\left(1+L_1x_{c1}\cos\gamma\right)}\right)+\right.\\
\left.+\frac{2}{\left(1+L_1x_{c1}\cos\gamma\right)}\left[\frac{\partial v_1}{\partial y_{c1}}\left(\frac{\partial v_1}{\partial\varphi}+\frac{v_3\cos\gamma}{L_1}\right)+\frac{1}{L_1^2}\frac{\partial v_3}{\partial y_{c1}}\left(\frac{\partial v_3}{\partial\varphi}-L_1v_1\cos\gamma\right)\right]\right\}
\end{gathered}\tag{1}$$

where $0\leq\phi\leq2\pi$, $0\leq y_{c1}\leq\varepsilon_1$, $-1\leq x_{c1}\leq+1$.

– The final form of the dimensionless equation of conservation of momentum in the direction perpendicular to the cone generator line y_c:

$$\frac{\partial p}{\partial y_{c1}}=\frac{\partial}{\partial y_{c1}}\left\{\left(De_\alpha\alpha_{01}+2De_\beta\beta_{01}\right)\left[\left(\frac{\partial v_1}{\partial y_{c1}}\right)^2+\left(\frac{1}{L_1}\frac{\partial v_3}{\partial y_{c1}}\right)^2\right]\right\}\tag{2}$$

– The final form of the dimensionless equation of conservation of momentum in the direction of the cone generator line x_c:

$$\begin{gathered}
-\frac{L_1}{1+L_1x_{c1}\cos\gamma}v_1^2\cos\gamma=-\frac{\partial p_1}{\partial x_{c1}}+\frac{\partial}{\partial y_{c1}}\left(\eta_1\frac{\partial v_3}{\partial y_{c1}}\right)+De_\alpha\frac{1}{L_1^2}\frac{\partial}{\partial x_{c1}}\left[\alpha_{01}\left(\frac{\partial v_3}{\partial y_{c1}}\right)^2\right]+\\
+De_\alpha\left\{\frac{\partial}{\partial y_{c1}}\left[\alpha_{01}\frac{\partial v_1}{\partial y_{c1}}\left(\frac{1}{\left(1+L_1x_{c1}\cos\gamma\right)}\left(\frac{\partial v_3}{\partial\varphi}-L_1v_1\cos\gamma\right)+\frac{\partial v_1}{\partial x_{c1}}\right)+\right.\right.\\
\left.\left.-2\alpha_{01}\frac{\partial v_3}{\partial y_{c1}}\frac{1}{\left(1+L_1x_{c1}\cos\gamma\right)}\left(\frac{\partial v_1}{\partial\varphi}+\frac{v_3\cos\gamma}{L_1}\right)\right]+\frac{1}{\left(1+L_1x_{c1}\cos\gamma\right)}\frac{\partial}{\partial\varphi}\left(\alpha_{01}\frac{\partial v_1}{\partial y_{c1}}\frac{\partial v_3}{\partial y_{c1}}\right)\right\}+\\
+De_\alpha\left\{\frac{L_1\cos\gamma}{\left(1+L_1x_{c1}\cos\gamma\right)}\alpha_{01}\left[\left(\frac{1}{L_1}\frac{\partial v_3}{\partial y_{c1}}\right)^2-\left(\frac{\partial v_1}{\partial y_{c1}}\right)^2\right]\right\}+\\
+D_\beta\frac{\partial}{\partial y_{c1}}\beta_{01}\left[\frac{\partial}{\partial y_{c1}}\left(\frac{v_1}{\left(1+L_1x_{c1}\cos\gamma\right)}\left(\frac{\partial v_3}{\partial\varphi}-L_1v_1\cos\gamma\right)+v_2\frac{\partial v_3}{\partial y_{c1}}+\frac{1}{L_1^2}v_3\frac{\partial v_3}{\partial x_{c1}}\right)+\right.\\
\left.+2\left(\frac{\partial v_1}{\partial x_{c1}}\frac{\partial v_1}{\partial y_{c1}}+\frac{1}{L_1^2}\frac{\partial v_3}{\partial x_{c1}}\frac{\partial v_3}{\partial y_{c1}}\right)\right]
\end{gathered}\tag{3}$$

where $0\leq\phi\leq2\pi$, $0\leq y_{c1}\leq\varepsilon_1$, $-1\leq x_{c1}\leq+1$.

– The final form of the dimensionless stream continuity equation:

$$\frac{\partial(\rho_1 v_1)}{\partial\varphi}+(1+L_1 x_{c1}\cos\gamma)\frac{\partial(\rho_1 v_2)}{\partial y_{c1}}+\frac{1}{L_1^2}\frac{\partial\left[\rho_1 v_3(1+L_1 x_{c1}\cos\gamma)\right]}{\partial x_{c1}}=0 \tag{4}$$

– The final form of the dimensionless equation of energy conservation:

$$\begin{gathered}
\frac{\partial}{\partial y_{c1}}\left(\kappa_1\frac{\partial T_1}{\partial y_{c1}}\right)+\eta_1\left[\left(\frac{\partial v_1}{\partial y_{c1}}\right)^2+\left(\frac{1}{L_1}\frac{\partial v_3}{\partial y_{c1}}\right)^2\right]+\\
+De_\alpha\alpha_{01}\frac{3}{L_1^2}\left[\left(\frac{\partial v_1}{\partial y_{c1}}\right)\frac{\partial v_3}{\partial y_{c1}}\left(\frac{1}{(1+L_1 x_{c1}\cos\gamma)}\frac{\partial v_3}{\partial\varphi}-\frac{L_1 v_1\cos\gamma}{(1+L_1 x_{c1}\cos\gamma)}+\frac{\partial v_1}{\partial x_{c1}}\right)\right]+\\
-De_\alpha\alpha_{01}\frac{3}{L_1^2}\left[\left(\frac{\partial v_3}{\partial y_{c1}}\right)^2\left(\frac{v_3\cos\gamma}{L_1}+\frac{1}{(1+L_1 x_{c1}\cos\gamma)}\frac{\partial v_1}{\partial\varphi}\right)+\left(\frac{\partial v_1}{\partial y_{c1}}\right)^2\frac{\partial v_3}{\partial x_{c1}}\right]+\\
+2De_\beta\beta_{01}\left\{\left[\left(\frac{\partial v_1}{\partial y_{c1}}\right)^2+\frac{1}{L_1^2}\left(\frac{\partial v_3}{\partial y_{c1}}\right)^2\right]\frac{\partial v_2}{\partial y_{c1}}\right\}+\\
+De_\beta\beta_{01}\frac{\partial}{\partial y_{c1}}\left\{\left[v_1\left(\frac{1}{(1+L_1 x_{c1}\cos\gamma)}\frac{\partial v_1}{\partial\varphi}\right)+v_2\left(\frac{\partial v_1}{\partial y_{c1}}\right)+\frac{1}{L_1^2}v_3\left(\frac{\partial v_1}{\partial x_{c1}}\right)\right]+\frac{v_1 v_3}{L_1(1+L_1 x_{c1}\cos\gamma)}\cos\gamma\right\}\left[\frac{\partial v_1}{\partial y_{c1}}\right]+\\
+De_\beta\beta_{01}\frac{1}{L_1^2}\frac{\partial}{\partial y_{c1}}\left\{\left[\frac{v_1}{(1+L_1 x_{c1}\cos\gamma)}\left(\frac{\partial v_3}{\partial\varphi}-L_1 v_1\cos\gamma\right)+v_2\left(\frac{\partial v_3}{\partial y_{c1}}\right)+\frac{1}{L_1^2}v_3\left(\frac{\partial v_3}{\partial x_{c1}}\right)\right]\right\}\frac{\partial v_3}{\partial y_{c1}}+\\
+2De_\beta\beta_{01}\left\{\frac{1}{L_1^2}\frac{\partial v_3}{\partial y_{c1}}\left[\frac{\partial v_1}{\partial y_{c1}}\frac{\partial v_1}{\partial x_{c1}}+\frac{1}{L_1^2}\frac{\partial v_3}{\partial y_{c1}}\frac{\partial v_3}{\partial x_{c1}}\right]+\frac{1}{1+L_1 x_{c1}\cos\gamma}\frac{\partial v_1}{\partial y_{c1}}\left[\frac{\partial v_1}{\partial\varphi}\frac{\partial v_1}{\partial y_{c1}}+\frac{1}{L_1^2}\frac{\partial v_3}{\partial y_{c1}}\frac{\partial v_3}{\partial\varphi}\right]+\right.\\
\left.+\frac{\partial v_1}{\partial y_{c1}}\left[\frac{\partial v_1}{\partial y_{c1}}\frac{v_3\cos\gamma}{L_1(1+L_1 x_{c1}\cos\gamma)}-\frac{\partial v_3}{\partial y_{c1}}\frac{v_1\cos\gamma}{(1+L_1 x_{c1}\cos\gamma)}\right]\right\}
\end{gathered} \tag{5}$$

where $0\le\phi\le 2\pi$, $0\le y_{c1}\le\varepsilon_1$, $-1\le x_{c1}\le +1$.

$$\begin{gathered}
\eta_1\equiv\eta_{1T}(T_1)\eta_{1p}(p_1)\eta_{1\tau}(\tau_1),\quad \eta_{1p}(\varphi,z)=e^{\varsigma\cdot p_o\cdot p_1}=e^{\varsigma_p p_1},\quad \eta_{1T}(\varphi,z,r)\equiv e^{-\delta_T(T-T_o)}=e^{-Q_{Br}T_1},\\
\eta_{1\tau}(\varphi,z)=e^{\delta\cdot\tau_o\cdot\tau_1}=e^{\delta_\tau\tau_1},\ p_o\equiv\frac{RU\eta_o}{\varepsilon^2},\ \psi\equiv\frac{\varepsilon}{R}\cong 10^{-3},\ T=T_o+T_oBrT_1,\ Br\equiv\frac{U^2\eta_o}{\kappa_o T_o},\ Q_{Br}\equiv BrT_o\delta_T,\\
\varsigma_p=\varsigma\cdot p_o,\ p=p_o p_1,\ \delta_\tau=\delta\cdot\tau,\ \tau=\tau_o\tau_1,\ v_\varphi=Uv_1,\ v_r=U\psi v_2,\ v_z=\frac{U}{L_1}v_3,\ y_c=R(1+\Psi y_{c1}),\\
x_c=b_c x_{c1},\ L_1\equiv\frac{b_c}{R},\ 0\le y_{c1}<\varepsilon_1=(1+\lambda\cdot\cos\varphi)/\sin\gamma,\ 0\le\varphi<\varphi_k,\ -1\le x_{c1}<+1
\end{gathered} \tag{6}$$

where
Br – dimensionless Brinkman number,
L_1 – dimensionless length of bearing,
Q_{Br} – dimensionless coefficient of viscosity changes due to temperature,
R – bearing shaft radius at the origin of the coordinate system [m],
T_o – dimensional value of temperature, in [K],
T_1 – dimensionless temperature,
$U=\varpi\cdot R$ – peripheral speed in [m·s⁻¹],
$2b$ – bearing length in [m],
p_o – characteristic pressure value in [Pa],
p_1 – dimensionless value of hydrodynamic pressure,
τ_o – dimensional value of characteristic operating time in [km] or [mth],
τ_1 – dimensionless time of operation,
y_{c1} – dimensionless radial coordinate,
x_{c1} – dimensionless longitudinal coordinate,
γ – the angle of the cone generator,
δ – dimensional viscosity change coefficient dependent on the time of operation [km^{-1}] or [mth^{-1}],

δ_τ – dimensionless viscosity change coefficient dependent on the time of operation,
δ_T – dimensional coefficient of viscosity changes depending on temperature T [K^{-1}],
$\varepsilon = R' - R$ – bearing clearance [m],
ε_1 – dimensionless overall height of the lubrication gap,
ς – pressure dependent dimensional viscosity coefficient in [Pa^{-1}],
ς_p – pressure dependent dimensionless viscosity coefficient,
η_{1p} – dimensionless value of pressure-dependent dynamic viscosity,
η_{1T} – dimensionless value of temperature-dependent dynamic viscosity,
$\eta_{1\tau}$ – dimensionless value dynamic viscosity dependent on time of operation,
η_o – dimensional value of viscosity at T = T_o and p = p_{at}, in [Pa·s],
κ_o – dimensional value of the thermal conductivity coefficient of the lubricant, in [$W \cdot m^{-1} \cdot K^{-1}$],
κ_1 – dimensionless value of the thermal conductivity coefficient of the lubricant,
$\lambda = OO'/\varepsilon$ – relative eccentricity,
ρ_o – dimensional density of the lubricant [$kg \cdot m^{-3}$],
ρ_1 – dimensionless density of the lubricant,
ψ – dimensionless bearing clearance,
ϖ – angular velocity value of the bearing journal [s^{-1}].

In order to apply the small parameter method to the system of Equations (1) – (5) and the Relationship (6), the uniformly convergent power series, developed in relation to successive powers of small dimensionless parameters, should be introduced **[L. 10–11]**:

$$\begin{aligned}
v_i &= v_i^{(0)} + Q_{Br} v_{i0}^{(1)} + \ldots + Q_{Br}^j v_{i0}^{(j)} + \ldots + \varsigma_p v_{i1}^{(1)} + \ldots + \varsigma_p^j v_{i1}^{(j)} + \ldots \;, \\
p_1 &= p_1^{(0)} + Q_{Br} p_{10}^{(1)} + \ldots + Q_{Br}^j p_{10}^{(j)} + \ldots + \varsigma_p p_{11}^{(1)} + \ldots + \varsigma_p^j p_{11}^{(j)} + \ldots \;, \\
T_1 &= T_1^{(0)} + Q_{Br} T_{10}^{(1)} + \ldots + Q_{Br}^j T_{10}^{(j)} + \ldots + \varsigma_p T_{11}^{(1)} + \ldots + \varsigma_p^j T_{11}^{(j)} + \ldots \;, \\
\eta_{1p} &\equiv exp\left(\varsigma_p p_1\right) = 1 + \varsigma_p p_1 + \frac{1}{2!}\varsigma_p^2 p_1^2 + \ldots + \frac{1}{n!}\varsigma_p^n p_1^n + \ldots \;, \\
\eta_{1T} &\equiv exp\left(-Q_{Br} T_1\right) = 1 - Q_{Br} T_1 + \frac{1}{2} Q_{Br}^2 T_1^2 - \ldots - \frac{1}{n!} Q_{Br}^n T_1^n + \ldots
\end{aligned} \tag{7}$$

for i = 1, 2, 3, j = 1, 2... .

The above series are multiplied by the Cauchy method, and then equate the coefficients with the same powers of small parameters Q_{Br}, ς_p. The successive systems of partial differential equations are obtained. The unknown functions and their corrections can be determined with these systems of equations.

– The sought unknowns are as follows:
– The dimensional coordinates of the oil speed vector, pressure, and temperature $v_1^{(0)}, v_2^{(0)}, v_3^{(0)}, p_1^{(0)}, T_1^{(0)}$;
– The dimensionless corrections of the components of the oil velocity vector, pressure, and temperature caused by the change of the oil's dynamic viscosity with temperature $v_{10}^{(1)}, v_{20}^{(1)}, v_{30}^{(1)}, p_{10}^{(1)}, T_{10}^{(1)}$;
– The dimensionless corrections of the components of the oil velocity vector, pressure, and temperature caused by the change of the dynamic viscosity of the oil with pressure $v_{11}^{(1)}, v_{21}^{(1)}, v_{31}^{(1)}, p_{11}^{(1)}, T_{11}^{(1)}$; and,

The dimensionless corrections of oil velocity vector components, pressure, and temperature caused by the non-Newtonian, viscoelastic properties of the lubricant $v_1^{(1)}, v_2^{(1)}, v_3^{(1)}, p_1^{(1)}, T_1^{(1)}$.

In order to determine the unknown dimensionless corrections of velocity vector components, and hydrodynamic pressure and temperature, the previously determined functions $v_1^{(0)}, v_2^{(0)}, v_3^{(0)}, p_1^{(0)}, T_1^{(0)}$ have to be substituted into the corresponding systems of equations.

The functions of dimensionless values of the velocity vector components and their dimensionless corrections, resulting from taking into account non-Newtonian properties, as well as the influence of temperature and pressure on the viscosity change, will be determined by integrating the relevant equations and imposing the following boundary conditions:

$$\begin{gathered}
v_1^{(0)} = 0,\; v_2^{(0)} = 0,\; v_3^{(0)} = 0 \quad \text{at bearing sleeve } r_1 = \varepsilon_1,\; v_1^{(0)} = 1 + L_1 x_{c1} \cos\gamma, \\
v_2^{(0)} = 0,\; v_3^{(0)} = 0 \text{ at bearing shaft } r_1 = 0, \\
v_{i0}^{(1)} = 0 \quad \text{at bearing sleeve } r_1 = \varepsilon_1,\; v_{i0}^{(1)} = 0 \text{ at bearing shaft } r_1 = 0, \\
v_{i1}^{(1)} = 0 \quad \text{at bearing sleeve } r_1 = \varepsilon_1,\; v_{i1}^{(1)} = 0 \quad \text{at bearing shaft } r_1 = 0, \\
v_i^{(1)} = 0 \quad \text{at bearing sleeve } r_1 = \varepsilon_1, v_1^{(1)} = 0,\; v_2^{(1)} = 0,\; v_3^{(1)} = 0 \text{ at bearing shaft } r_1 = 0
\end{gathered} \tag{8}$$

where i = 1, 2, 3.

Reynolds type equations, on the basis of which the distributions of dimensionless hydrodynamic pressure and dimensionless hydrodynamic pressure corrections can be numerically determined, are obtained by integrating the stream continuity equations and applying boundary conditions (8) for the dimensionless radial velocity and its dimensionless corrections $v_{20}^{(1)}=0, v_{21}^{(1)}=0, v_{2}^{(1)}=0$ at the bearing sleeve $r_1=\varepsilon_1$.

In order to determine the dimensionless temperature distribution and its dimensionless corrections, the corresponding equations with the boundary imposed conditions are integrated **[L. 10–11, 20]**.

The functions of dimensionless components of velocity vector, hydrodynamic pressure and temperature, as well as their dimensionless corrections, were determined for the stationary and laminar flow in the lubrication gap of the conical non-porous bearing. It is also assumed, that there are no vibration and displacements of the bearing shaft and sleeve. These equations are, as follows:

– Equations of dimensionless components of velocity vector $v_1^{(0)}, v_2^{(0)}, v_3^{(0)}$ and their corrections $v_{10}^{(1)}, v_{11}^{(1)}, v_{1}^{(1)}$, $v_{20}^{(1)}, v_{21}^{(1)}, v_{2}^{(1)}$, $v_{30}^{(1)}, v_{31}^{(1)}, v_{3}^{(1)}$

$$v_1^{(0)}(\varphi, y_{c1}, x_{c1}) = \frac{1}{2\eta_{1\tau}}\left(\frac{1}{1+L_1 x_{c1}\cos\gamma}\frac{\partial p_1^{(0)}}{\partial\varphi}\right)\left(y_{c1}^2 - y_{c1}\varepsilon_1\right) + \left(1+L_1 x_{c1}\cos\gamma\right)\left(1-\frac{y_{c1}}{\varepsilon_1}\right) \tag{9}$$

$$v_3^{(0)}(\varphi, y_{c1}, x_{c1}) = \frac{1}{2\eta_{1\tau}}\left(\frac{\partial p_1^{(0)}}{\partial x_{c1}}\right)\left(y_{c1}^2 - y_{c1}\varepsilon_1\right) - \frac{1}{2\eta_{1\tau}}\frac{L_1\cos\gamma}{1+L_1 x_{c1}\cos\gamma}\left[\int_0^{y_{c1}}\int_0^{y_{c1}}\left(v_1^{(0)}\right)^2 dy_{c1}dy_{c1} - \frac{y_{c1}}{\varepsilon_1}\int_0^{\varepsilon_1}\int_0^{y_{c1}}\left(v_1^{(0)}\right)^2 dy_{c1}dy_{c1}\right] \tag{10}$$

$$\begin{aligned}
v_2^{(0)}(\varphi, y_{c1}, x_{c1}) = {} & \left\{\frac{1}{1+L_1 x_{c1}\cos\gamma}\frac{\partial}{\partial\varphi}\left[\frac{1}{6\eta_{1\tau}}\left(y_{c1}^2\varepsilon_1 - y_{c1}^3\right)\left(\frac{\partial p_1^{(0)}}{\partial\varphi}\right)\right] + \frac{1}{L_1^2}\frac{\partial}{\partial x_{c1}}\left[\frac{1}{6\eta_{1\tau}}\left(y_{c1}^2\varepsilon_1 - y_{c1}^3\right)\left(\frac{\partial p_1^{(0)}}{\partial x_{c1}}\right)\right]\right\} + \\
& + \frac{1}{L_1^2}\frac{\partial}{\partial x_{c1}}\left\{\frac{L_1\cos\gamma}{1+L_1 x_{c1}\cos\gamma}\left[\int_0^{y_{c1}}\int_0^{y_{c1}}\int_0^{y_{c1}}\left(v_1^{(0)}\right)^2 dy_{c1}dy_{c1}dy_{c1} - \int_0^{y_{c1}}\frac{y_{c1}}{\varepsilon_1}\int_0^{\varepsilon_1}\int_0^{y_{c1}}\left(v_1^{(0)}\right)^2 dy_{c1}dy_{c1}dy_{c1}\right]\right\} + \\
& - \int_0^{y_{c1}}\frac{\cos\gamma}{L_1\left(1+L_1 x_{c1}\cos\gamma\right)}\frac{1}{2\eta_{1\tau}}\left(\frac{\partial p_1^{(0)}}{\partial x_{c1}}\right)\left(y_{c1}^2 - y_{c1}\varepsilon_1\right)dy_{c1} - \frac{r_1^2}{2\varepsilon_1^2}\left(1+L_1 x_{c1}\cos\gamma\right)\frac{\partial\varepsilon_1}{\partial\varphi} + \\
& + \frac{L_1\cos\gamma}{1+L_1 x_{c1}\cos\gamma}\left[\int_0^{y_{c1}}\int_0^{y_{c1}}\int_0^{y_{c1}}\left(v_1^{(0)}\right)^2 dy_{c1}dy_{c1}dy_{c1} - \int_0^{y_{c1}}\frac{y_{c1}}{\varepsilon_1}\int_0^{\varepsilon_1}\int_0^{y_{c1}}\left(v_1^{(0)}\right)^2 dy_{c1}dy_{c1}dy_{c1}\right]
\end{aligned} \tag{11}$$

$$v_{10}^{(1)} = \frac{1}{2\eta_{1\tau}}\frac{1}{1+L_1 x_{c1}\cos\gamma}\frac{\partial p_{10}^{(1)}}{\partial\varphi}y_{c1}\left(y_{c1}-\varepsilon_1\right) + \int_0^{y_{c1}} T_1^{(0)}\frac{\partial v_1^{(0)}}{\partial y_{c1}}dy_{c1} - \frac{y_{c1}}{\varepsilon_1}\int_0^{\varepsilon_1} T_1^{(0)}\frac{\partial v_1^{(0)}}{\partial y_{c1}}dy_{c1} \tag{12}$$

$$\begin{aligned}
v_{30}^{(1)} = {} & \frac{1}{2\eta_{1\tau}}\frac{\partial p_{10}^{(1)}}{\partial x_{c1}}y_{c1}\left(y_{c1}-\varepsilon_1\right) + \int_0^{y_{c1}} T_1^{(0)}\frac{\partial v_3^{(0)}}{\partial y_{c1}}dy_{c1} - \frac{y_{c1}}{\varepsilon_1}\int_0^{\varepsilon_1} T_1^{(0)}\frac{\partial v_3^{(0)}}{\partial y_{c1}}dy_{c1} + \\
& - \frac{1}{2\eta_{1\tau}}\frac{L_1\cos\gamma}{1+L_1 x_{c1}\cos\gamma}\left[\int_0^{y_{c1}}\int_0^{y_{c1}}\left(v_{10}^{(1)}\right)^2 dy_{c1}dy_{c1} - \frac{y_{c1}}{\varepsilon_1}\int_0^{\varepsilon_1}\int_0^{y_{c1}}\left(v_{10}^{(1)}\right)^2 dy_{c1}dy_{c1}\right]
\end{aligned} \tag{13}$$

$$\begin{aligned}
v_{20}^{(1)} = {} & -\int_0^{y_{c1}}\frac{1}{\left(1+L_1 x_{c1}\cos\gamma\right)}\frac{\partial}{\partial\varphi}\left[\frac{1}{2\eta_{1\tau}}\frac{1}{1+L_1 x_{c1}\cos\gamma}\frac{\partial p_{10}^{(1)}}{\partial\varphi}y_{c1}\left(y_{c1}-\varepsilon_1\right) + \int_0^{y_{c1}} T_1^{(0)}\frac{\partial v_1^{(0)}}{\partial y_{c1}}dy_{c1} - \frac{y_{c1}}{\varepsilon_1}\int_0^{\varepsilon_1} T_1^{(0)}\frac{\partial v_1^{(0)}}{\partial y_{c1}}dy_{c1}\right]dy_{c1} + \\
& - \frac{1}{L_1^2}\int_0^{y_{c1}}\frac{\partial}{\partial x_{c1}}\left[\frac{1}{2\eta_{1\tau}}\frac{\partial p_{10}^{(1)}}{\partial x_{c1}}y_{c1}\left(y_{c1}-\varepsilon_1\right) + \int_0^{y_{c1}} T_1^{(0)}\frac{\partial v_3^{(0)}}{\partial y_{c1}}dy_{c1} - \frac{y_{c1}}{\varepsilon_1}\int_0^{\varepsilon_1} T_1^{(0)}\frac{\partial v_3^{(0)}}{\partial y_{c1}}dy_{c1}\right]dy_{c1} + \\
& + \frac{1}{L_1^2}\frac{\partial}{\partial x_{c1}}\left\{\frac{L_1\cos\gamma}{1+L_1 x_{c1}\cos\gamma}\left[\int_0^{y_{c1}}\int_0^{y_{c1}}\int_0^{y_{c1}}\left(v_{10}^{(0)}\right)^2 dy_{c1}dy_{c1}dy_{c1} - \int_0^{y_{c1}}\frac{y_{c1}}{\varepsilon_1}\int_0^{\varepsilon_1}\int_0^{y_{c1}}\left(v_{10}^{(0)}\right)^2 dy_{c1}dy_{c1}dy_{c1}\right]\right\} + \\
& - \int_0^{y_{c1}}\frac{\cos\gamma}{L_1\left(1+L_1 x_{c1}\cos\gamma\right)}\frac{1}{2\eta_{1\tau}}\left(\frac{\partial p_{10}^{(0)}}{\partial x_{c1}}\right)\left(y_{c1}^2 - y_{c1}\varepsilon_1\right)dy_{c1} + \\
& + \frac{L_1\cos\gamma}{1+L_1 x_{c1}\cos\gamma}\left[\int_0^{y_{c1}}\int_0^{y_{c1}}\int_0^{y_{c1}}\left(v_{10}^{(0)}\right)^2 dy_{c1}dy_{c1}dy_{c1} - \int_0^{y_{c1}}\frac{y_{c1}}{\varepsilon_1}\int_0^{\varepsilon_1}\int_0^{y_{c1}}\left(v_{10}^{(0)}\right)^2 dy_{c1}dy_{c1}dy_{c1}\right]
\end{aligned} \tag{14}$$

$$v_{11}^{(1)}=\frac{1}{2\eta_{1\tau}}\frac{1}{1+L_1x_{c1}\cos\gamma}\frac{\partial p_{11}^{(1)}}{\partial\varphi}y_{c1}\left(y_{c1}-\varepsilon_1\right)-\int_0^{y_{c1}}p_1^{(0)}\frac{\partial v_1^{(0)}}{\partial y_{c1}}dy_{c1}+\frac{y_{c1}}{\varepsilon_1}\int_0^{\varepsilon_1}p_1^{(0)}\frac{\partial v_1^{(0)}}{\partial y_{c1}}dy_{c1} \tag{15}$$

$$\begin{aligned}v_{31}^{(1)}=&\frac{1}{2\eta_{1\tau}}\frac{\partial p_{11}^{(1)}}{\partial x_{c1}}y_{c1}\left(y_{c1}-\varepsilon_1\right)-\int_0^{y_{c1}}p_1^{(0)}\frac{\partial v_3^{(0)}}{\partial y_{c1}}dy_{c1}+\frac{y_{c1}}{\varepsilon_1}\int_0^{\varepsilon_1}p_1^{(0)}\frac{\partial v_3^{(0)}}{\partial y_{c1}}dy_{c1}+\\&-\frac{1}{2\eta_{1\tau}}\frac{L_1\cos\gamma}{1+L_1x_{c1}\cos\gamma}\left[\int_0^{y_{c1}}\int_0^{y_{c1}}\left(v_{11}^{(1)}\right)^2dy_{c1}dy_{c1}-\frac{y_{c1}}{\varepsilon_1}\int_0^{\varepsilon_1}\int_0^{y_{c1}}\left(v_{11}^{(1)}\right)^2dy_{c1}dy_{c1}\right]\end{aligned} \tag{16}$$

$$\begin{aligned}v_{21}^{(1)}=&-\int_0^{r_1}\frac{1}{1+L_1x_{c1}\cos\gamma}\frac{\partial}{\partial\varphi}\left[\frac{1}{2\eta_{1\tau}}\frac{\partial p_{11}^{(1)}}{\partial\varphi}y_{c1}\left(y_{c1}-\varepsilon_1\right)-\int_0^{y_{c1}}p_1^{(0)}\frac{\partial v_1^{(0)}}{\partial y_{c1}}dy_{c1}+\frac{y_{c1}}{\varepsilon_1}\int_0^{\varepsilon_1}p_1^{(0)}\frac{\partial v_1^{(0)}}{\partial y_{c1}}dy_{c1}\right]dy_{c1}+\\&-\frac{1}{L_1^2}\int_0^{r_1}\frac{\partial}{\partial x_{c1}}\left[\frac{1}{2\eta_{1\tau}}\frac{\partial p_{11}^{(1)}}{\partial x_{c1}}y_{c1}\left(y_{c1}-\varepsilon_1\right)-\int_0^{y_{c1}}p_1^{(0)}\frac{\partial v_3^{(0)}}{\partial y_{c1}}dy_{c1}+\frac{y_{c1}}{\varepsilon_1}\int_0^{\varepsilon_1}p_1^{(0)}\frac{\partial v_3^{(0)}}{\partial y_{c1}}dy_{c1}\right]dy_{c1}\\&+\frac{1}{L_1^2}\frac{\partial}{\partial x_{c1}}\left\{\frac{L_1\cos\gamma}{1+L_1x_{c1}\cos\gamma}\left[\int_0^{y_{c1}}\int_0^{y_{c1}}\int_0^{y_{c1}}\left(v_{11}^{(0)}\right)^2dy_{c1}dy_{c1}-\int_0^{y_{c1}}\frac{y_{c1}}{\varepsilon_1}\int_0^{\varepsilon_1}\int_0^{y_{c1}}\left(v_{11}^{(0)}\right)^2dy_{c1}dy_{c1}dy_{c1}\right]\right\}+\\&-\int_0^{y_{c1}}\frac{\cos\gamma}{L_1\left(1+L_1x_{c1}\cos\gamma\right)}\frac{1}{2\eta_{1\tau}}\left(\frac{\partial p_{11}^{(0)}}{\partial x_{c1}}\right)\left(y_{c1}^2-y_{c1}\varepsilon_1\right)dy_{c1}+\\&+\frac{L_1\cos\gamma}{1+L_1x_{c1}\cos\gamma}\left[\int_0^{y_{c1}}\int_0^{y_{c1}}\int_0^{y_{c1}}\left(v_{11}^{(0)}\right)^2dy_{c1}dy_{c1}dy_{c1}-\int_0^{y_{c1}}\frac{y_{c1}}{\varepsilon_1}\int_0^{\varepsilon_1}\int_0^{y_{c1}}\left(v_{11}^{(0)}\right)^2dy_{c1}dy_{c1}dy_{c1}\right]\end{aligned} \tag{17}$$

$$v_1^{(1)}=\frac{1}{2\eta_{1\tau}}\frac{1}{1+L_1x_{c1}\cos\gamma}\frac{\partial p_{1p}^{(1)}}{\partial\varphi}\left(y_{c1}^2-\varepsilon_1y_{c1}\right)+\frac{1}{\eta_{1\tau}}\int_0^{y_{c1}}\int_0^{y_{c2}}F\left(\varphi,y_{c2},x_{c1}\right)dy_{c2}dy_{c1}-\frac{y_{c1}}{\varepsilon_1\eta_{1\tau}}\int_0^{\varepsilon_1}\int_0^{y_{c2}}F\left(\varphi,y_{c2},x_{c1}\right)dy_{c2}dy_{c1} \tag{18}$$

$$\begin{aligned}v_3^{(1)}=&\frac{1}{2\eta_{1\tau}}\frac{\partial p_{1p}^{(1)}}{\partial x_{c1}}\left(y_{c1}^2-\varepsilon_1y_{c1}\right)+\frac{1}{\eta_{1\tau}}\int_0^{y_{c1}}\int_0^{y_{c2}}G\left(\varphi,y_{c2},x_{c1}\right)dy_{c2}dy_{c1}-\frac{y_{c1}}{\varepsilon_1\eta_{1\tau}}\int_0^{\varepsilon_1}\int_0^{y_{c2}}G\left(\varphi,y_{c2},x_{c1}\right)dy_{c2}dy_{c1}+\\&-\frac{1}{2\eta_{1\tau}}\frac{L_1\cos\gamma}{1+L_1x_{c1}\cos\gamma}\left[\int_0^{y_{c1}}\int_0^{y_{c1}}\left(v_1^{(1)}\right)^2dy_{c1}dy_{c1}-\frac{y_{c1}}{\varepsilon_1}\int_0^{\varepsilon_1}\int_0^{y_{c1}}\left(v_1^{(1)}\right)^2dy_{c1}dy_{c1}\right]\end{aligned} \tag{19}$$

$$v_2^{(1)}=-\int_0^{y_{c1}}\frac{\partial v_1^{(1)}}{\partial\varphi}dy_{c1}-\frac{1}{L_1^2}\int_0^{y_{c1}}\frac{\partial v_3^{(1)}}{\partial x_{c1}}dy_{c1}-\frac{\cos\gamma}{L_1\left(1+L_1x_{c1}\cos\gamma\right)}\int_0^{y_{c1}}v_3^{(1)}dy_{c1} \tag{20}$$

where

$$\begin{aligned}F\left(\varphi,y_{c1},x_{c1}\right)\equiv&\left(1+2\frac{\beta_o}{\alpha_o}\right)\frac{1}{1+L_1x_{c1}\cos\gamma}\left(\frac{\partial X_1}{\partial\varphi}+\frac{1}{L_1^2}\frac{\partial Z_1}{\partial\varphi}\right)-\frac{1}{1+L_1x_{c1}\cos\gamma}\frac{\partial X_1}{\partial\varphi}+\\&-\frac{1}{L_1^2}\left(\frac{\partial X_2}{\partial y_{c1}}+\frac{\partial X_3}{\partial x_{c1}}+\frac{L_1\cos\gamma X_3}{1+L_1x_{c1}\cos\gamma}\right)-\frac{\beta_o}{\alpha_o}\left(\frac{\partial X_4}{\partial y_{c1}}+2\frac{\partial X_5}{\partial y_{c1}}\right),\\G\left(\varphi,y_{c1},x_{c1}\right)\equiv&\left(1+2\frac{\beta_o}{\alpha_o}\right)\left(\frac{\partial X_1}{\partial x_{c1}}+\frac{1}{L_1^2}\frac{\partial Z_1}{\partial x_{c1}}\right)-\frac{1}{L_1^2}\frac{\partial Z_1}{\partial x_{c1}}+\frac{L_1\cos\gamma}{1+L_1x_{c1}\cos\gamma}\left(X_1-\frac{1}{L_1^2}Z_1\right)+\\&-\left(\frac{\partial Z_2}{\partial y_{c1}}+\frac{1}{1+L_1x_{c1}\cos\gamma}\frac{\partial Z_3}{\partial\varphi}\right)-\frac{\beta_o}{\alpha_o}\left(\frac{\partial Z_4}{\partial y_{c1}}+2\frac{\partial Z_5}{\partial y_{c1}}\right),\end{aligned}$$

$$X_1 \equiv \left(\frac{\partial v_1^{(0)}}{\partial y_{c1}}\right)^2,\ X_2 \equiv \frac{\partial v_3^{(0)}}{\partial y_{c1}} Y_4 - 2\frac{\partial v_1^{(0)}}{\partial y_{c1}}\frac{\partial v_3^{(0)}}{\partial x_{c1}},\ X_3 \equiv \frac{\partial v_1^{(0)}}{\partial y_{c1}}\frac{\partial v_3^{(0)}}{\partial y_{c1}},$$

$$Y_1 \equiv \frac{1}{1+L_1 x_{c_1}\cos\gamma}\frac{\partial v_1^{(0)}}{\partial\varphi} + \frac{v_3^{(0)}\cos\gamma}{L_1},\ Y_2 \equiv \frac{\partial v_2^{(0)}}{\partial y_{c1}},\ Y_3 \equiv \frac{\partial v_3^{(0)}}{\partial x_{c1}},\ Y_4 \equiv \frac{1}{1+L_1 x_{c1}\cos\gamma}\frac{\partial v_3^{(0)}}{\partial\varphi} + \frac{\partial v_1^{(0)}}{\partial x_{c1}} - \frac{L_1 v_1^{(0)}\cos\gamma}{1+L_1 x_{c1}\cos\gamma},$$

$$Z_1 \equiv \left(\frac{\partial v_3^{(0)}}{\partial y_{c1}}\right)^2,\ Z_2 \equiv \frac{\partial v_1^{(0)}}{\partial y_{c1}} Y_4 - 2\frac{\partial v_3^{(0)}}{\partial y_{c1}}\frac{1}{1+L_1 x_{c1}\cos\gamma}\left(\frac{\partial v_1^{(0)}}{\partial\varphi} + \frac{v_3^{(0)}\cos\gamma}{L_1}\right),\quad Z_3 \equiv \frac{\partial v_1^{(0)}}{\partial y_{c1}}\frac{\partial v_3^{(0)}}{\partial y_{c1}},$$

$$X_4 \equiv \frac{\partial}{\partial y_{c1}}\left(\frac{v_1^{(0)}}{1+L_1 x_{c1}\cos\gamma}\frac{\partial v_1^{(0)}}{\partial\varphi} + v_2^{(0)}\frac{\partial v_1^{(0)}}{\partial y_{c1}} + \frac{1}{L_1^2}v_3^{(0)}\frac{\partial v_1^{(0)}}{\partial x_{c1}} + \frac{v_1^{(0)}v_3^{(0)}\cos\gamma}{L_1\left(1+L_1 x_{c1}\cos\gamma\right)}\right),$$

$$X_5 \equiv \frac{1}{1+x_{c1}L_1\cos\gamma}\left[\frac{\partial v_1^{(0)}}{\partial y_{c1}}\left(\frac{\partial v_1^{(0)}}{\partial\varphi} + \frac{v_3^{(0)}\cos\gamma}{L_1}\right) + \frac{1}{L_1^2}\frac{\partial v_3^{(0)}}{\partial y_{c1}}\left(\frac{\partial v_3^{(0)}}{\partial\varphi} - v_1^{(0)}\cos\gamma\right)\right],$$

$$Z_4 \equiv \frac{\partial}{\partial y_{c1}}\left(\frac{v_1^{(0)}}{1+L_1 x_{c1}\cos\gamma}\left(\frac{\partial v_3^{(0)}}{\partial\varphi} - v_1^{(0)}L_1\cos\gamma\right) + v_2^{(0)}\frac{\partial v_3^{(0)}}{\partial y_{c1}} + \frac{1}{L_1^2}v_3^{(0)}\frac{\partial v_3^{(0)}}{\partial x_{c1}}\right),$$

$$Z_5 \equiv \left(\frac{\partial v_1^{(0)}}{\partial y_{c1}}\frac{\partial v_1^{(0)}}{\partial x_{c1}} + \frac{1}{L_1^2}\frac{\partial v_3^{(0)}}{\partial x_{c1}}\frac{\partial v_3^{(0)}}{\partial y_{c1}}\right).$$

– Equations used to determine the dimensionless hydrodynamic pressure $p_1^{(0)}$ and its corrections $p_{10}^{(1)}, p_{11}^{(1)}, p_1^{(1)}$

$$\begin{aligned}
&\frac{1}{1+L_1 x_{c1}\cos\gamma}\frac{\partial}{\partial\varphi}\left[\frac{\varepsilon_1^3}{\eta_{1\tau}}\left(\frac{1}{1+L_1 x_{c1}\cos\gamma}\frac{\partial p_1^{(0)}}{\partial\varphi}\right)\right] + \frac{1}{L_1^2}\frac{\partial}{\partial x_{c1}}\left[\frac{\varepsilon_1^3}{\eta_{1\tau}}\left(\frac{\partial p_1^{(0)}}{\partial x_{c1}}\right)\right] + \\
&-\frac{12}{L_1^2}\frac{\partial}{\partial x_{c1}}\left\{\frac{L_1\cos\gamma}{1+L_1 x_{c1}\cos\gamma}\left[\int_0^{\varepsilon_1}\int_0^{y_{c1}}\int_0^{y_{c1}}\left(v_1^{(0)}\right)^2 dy_{c1}dy_{c1}dy_{c1} - \int_0^{\varepsilon_1}\frac{y_{c1}}{\varepsilon_1}\int_0^{\varepsilon_1}\int_0^{y_{c1}}\left(v_1^{(0)}\right)^2 dy_{c1}dy_{c1}dy_{c1}\right]\right\} + \\
&+12\int_0^{\varepsilon_1}\frac{\cos\gamma}{L_1\left(1+L_1 x_{c1}\cos\gamma\right)}\frac{1}{2\eta_{1\tau}}\left(\frac{\partial p_1^{(0)}}{\partial x_{c1}}\right)\left(y_{c1}^2 - y_{c1}\varepsilon_1\right)dy_{c1} + \\
&-\frac{12L_1\cos\gamma}{1+L_1 x_{c1}\cos\gamma}\left[\int_0^{\varepsilon_1}\int_0^{y_{c1}}\int_0^{y_{c1}}\left(v_1^{(0)}\right)^2 dy_{c1}dy_{c1}dy_{c1} - \int_0^{\varepsilon_1}\frac{y_{c1}}{\varepsilon_1}\int_0^{\varepsilon_1}\int_0^{y_{c1}}\left(v_1^{(0)}\right)^2 dy_{c1}dy_{c1}dy_{c1}\right] = 6\left(1+2L_1 x_{c1}\cos\gamma\right)\frac{\partial\varepsilon_1}{\partial\varphi}
\end{aligned} \tag{21}$$

$$\begin{aligned}
&\frac{1}{1+L_1 x_{c1}\cos\gamma}\frac{\partial}{\partial\varphi}\left[\frac{\varepsilon_1^3}{\eta_{1\tau}}\left(\frac{1}{1+L_1 x_{c1}\cos\gamma}\frac{\partial p_{10}^{(1)}}{\partial\varphi}\right)\right] + \frac{1}{L_1^2}\frac{\partial}{\partial x_{c1}}\left[\frac{\varepsilon_1^3}{\eta_{1\tau}}\left(\frac{\partial p_{10}^{(1)}}{\partial x_{c1}}\right)\right] + \\
&-\frac{12}{L_1^2}\frac{\partial}{\partial x_{c1}}\left\{\frac{L_1\cos\gamma}{1+L_1 x_{c1}\cos\gamma}\left[\int_0^{\varepsilon_1}\int_0^{y_{c1}}\int_0^{y_{c1}}\left(v_{10}^{(1)}\right)^2 dy_{c1}dy_{c1}dy_{c1} - \int_0^{\varepsilon_1}\frac{y_{c1}}{\varepsilon_1}\int_0^{\varepsilon_1}\int_0^{y_{c1}}\left(v_{10}^{(1)}\right)^2 dy_{c1}dy_{c1}dy_{c1}\right]\right\} + \\
&+12\int_0^{\varepsilon_1}\frac{\cos\gamma}{L_1\left(1+L_1 x_{c1}\cos\gamma\right)}\frac{1}{2\eta_{1\tau}}\left(\frac{\partial p_{10}^{(1)}}{\partial x_{c1}}\right)\left(y_{c1}^2 - y_{c1}\varepsilon_1\right)dy_{c1} + \\
&-\frac{12L_1\cos\gamma}{1+L_1 x_{c1}\cos\gamma}\left[\int_0^{\varepsilon_1}\int_0^{y_{c1}}\int_0^{y_{c1}}\left(v_{10}^{(1)}\right)^2 dy_{c1}dy_{c1}dy_{c1} - \int_0^{\varepsilon_1}\frac{y_{c1}}{\varepsilon_1}\int_0^{\varepsilon_1}\int_0^{y_{c1}}\left(v_{10}^{(1)}\right)^2 dy_{c1}dy_{c1}dy_{c1}\right] = \\
&= 12\left\{\frac{1}{1+L_1 x_{c1}\cos\gamma}\frac{\partial}{\partial\varphi}\left[\left(\int_0^{\varepsilon_1}\left(\int_0^{y_{c1}}T_1^{(0)}\frac{\partial v_1^{(0)}}{\partial y_{c1}}dy_{c1}\right)dy_{c1} - \int_0^{\varepsilon_1}\frac{y_{c1}}{\varepsilon_1}\left(\int_0^{\varepsilon_1}T_1^{(0)}\frac{\partial v_1^{(0)}}{\partial y_{c1}}dy_{c1}\right)dy_{c1}\right)\right] + \right. \\
&\left. + \frac{1}{L_1^2}\frac{\partial}{\partial x_{c1}}\left[\int_0^{\varepsilon_1}\left(\int_0^{y_{c1}}T_1^{(0)}\frac{\partial v_3^{(0)}}{\partial y_{c1}}dy_{c1}\right)dy_c - \int_0^{\varepsilon_1}\frac{y_{c1}}{\varepsilon_1}\left(\int_0^{\varepsilon_1}T_1^{(0)}\frac{\partial v_3^{(0)}}{\partial y_{c1}}dy_{c1}\right)dy_{c1}\right]\right\}
\end{aligned} \tag{22}$$

$$
\begin{aligned}
&\frac{1}{1+L_1x_{c1}\cos\gamma}\frac{\partial}{\partial\varphi}\left[\frac{\varepsilon_1^3}{\eta_{1\tau}}\left(\frac{1}{1+L_1x_{c1}\cos\gamma}\frac{\partial p_{11}^{(1)}}{\partial\varphi}\right)\right]+\frac{1}{L_1^2}\frac{\partial}{\partial x_{c1}}\left[\frac{\varepsilon_1^3}{\eta_{1\tau}}\left(\frac{\partial p_{11}^{(1)}}{\partial x_{c1}}\right)\right]+\\
&-\frac{12}{L_1^2}\frac{\partial}{\partial x_{c1}}\left\{\frac{L_1\cos\gamma}{1+L_1x_{c1}\cos\gamma}\left[\int_0^{\varepsilon_1}\int_0^{y_{c1}}\int_0^{y_{c1}}\left(v_{11}^{(1)}\right)^2dy_{c1}dy_{c1}dy_{c1}-\int_0^{\varepsilon_1}\frac{y_{c1}}{\varepsilon_1}\int_0^{\varepsilon_1}\int_0^{y_{c1}}\left(v_{11}^{(1)}\right)^2dy_{c1}dy_{c1}dy_{c1}\right]\right\}+\\
&+12\int_0^{\varepsilon_1}\frac{\cos\gamma}{L_1\left(1+L_1x_{c1}\cos\gamma\right)}\frac{1}{2\eta_{1\tau}}\left(\frac{\partial p_{11}^{(1)}}{\partial x_{c1}}\right)\left(y_{c1}^2-y_{c1}\varepsilon_1\right)dy_{c1}+\\
&-\frac{12L_1\cos\gamma}{1+L_1x_{c1}\cos\gamma}\left[\int_0^{\varepsilon_1}\int_0^{y_{c1}}\int_0^{y_{c1}}\left(v_{11}^{(1)}\right)^2dy_{c1}dy_{c1}dy_{c1}-\int_0^{\varepsilon_1}\frac{y_{c1}}{\varepsilon_1}\int_0^{\varepsilon_1}\int_0^{y_{c1}}\left(v_{11}^{(1)}\right)^2dy_{c1}dy_{c1}dy_{c1}\right]=\\
&=12\left\{\frac{1}{1+L_1x_{c1}\cos\gamma}\frac{\partial}{\partial\varphi}\left[\int_0^{\varepsilon_1}\frac{y_{c1}}{\varepsilon_1}\left(\int_0^{\varepsilon_1}p_1^{(0)}\frac{\partial v_1^{(0)}}{\partial y_{c1}}dy_{c1}\right)dy_{c1}-\int_0^{\varepsilon_1}\left(\int_0^{y_{c1}}p_1^{(0)}\frac{\partial v_1^{(0)}}{\partial y_{c1}}dy_{c1}\right)dy_{c1}\right]+\right.\\
&\left.+\frac{1}{L_1^2}\frac{\partial}{\partial x_{c1}}\left[\int_0^{\varepsilon_1}\frac{y_{c1}}{\varepsilon_1}\left(\int_0^{\varepsilon_1}p_1^{(0)}\frac{\partial v_3^{(0)}}{\partial y_{c1}}dy_{c1}\right)dy_{c1}-\int_0^{\varepsilon_1}\left(\int_0^{y_{c1}}p_1^{(0)}\frac{\partial v_3^{(0)}}{\partial y_{c1}}dy_{c1}\right)dy_{c1}\right]\right\}
\end{aligned}
\tag{23}
$$

$$
p_1^{(1)}\left(\varphi,y_{c1},x_{c1}\right)=\left(1+2\frac{\beta_0}{\alpha_0}\right)\left(X_1\left(\varphi,y_{c1},x_{c1}\right)+\frac{1}{L^2}Z_1\left(\varphi,y_{c1},x_{c1}\right)\right)+p_{1p}^{(1)}\left(\varphi,x_{c1}\right)
\tag{24}
$$

where
$p_{1p}^{(1)}(\varphi,\ x_{c1})$ – constant of integration with respect to y_{c1} coordinate, treated as an unknown auxiliary pressure correction function, depending on the position in relation to angle and longitudinal direction.

$$
\begin{aligned}
&\frac{1}{1+L_1x_{c1}\cos\gamma}\frac{\partial}{\partial\varphi}\left(\frac{\varepsilon_1^3}{\eta_{1\tau}}\frac{1}{1+L_1x_{c1}\cos\gamma}\frac{\partial p_{1p}^{(1)}}{\partial\varphi}\right)+\frac{1}{L_1^2}\frac{\partial}{\partial x_{c1}}\left(\frac{\varepsilon_1^3}{\eta_{1\tau}}\frac{\partial p_{1p}^{(1)}}{\partial x_{c1}}\right)+\\
&-\frac{12}{L_1^2}\frac{\partial}{\partial x_{c1}}\left\{\frac{L_1\cos\gamma}{1+L_1x_{c1}\cos\gamma}\left[\int_0^{\varepsilon_1}\int_0^{y_{c1}}\int_0^{y_{c1}}\left(v_1^{(1)}\right)^2dy_{c1}dy_{c1}dy_{c1}-\int_0^{\varepsilon_1}\frac{y_{c1}}{\varepsilon_1}\int_0^{\varepsilon_1}\int_0^{y_{c1}}\left(v_1^{(1)}\right)^2dy_{c1}dy_{c1}dy_{c1}\right]\right\}+\\
&+12\int_0^{\varepsilon_1}\frac{\cos\gamma}{L_1\left(1+L_1x_{c1}\cos\gamma\right)}\frac{1}{2\eta_{1\tau}}\left(\frac{\partial p_{1p}^{(1)}}{\partial x_{c1}}\right)\left(y_{c1}^2-y_{c1}\varepsilon_1\right)dy_{c1}+\\
&-\frac{12L_1\cos\gamma}{1+L_1x_{c1}\cos\gamma}\left[\int_0^{\varepsilon_1}\int_0^{y_{c1}}\int_0^{y_{c1}}\left(v_1^{(1)}\right)^2dy_{c1}dy_{c1}dy_{c1}-\int_0^{\varepsilon_1}\frac{y_{c1}}{\varepsilon_1}\int_0^{\varepsilon_1}\int_0^{y_{c1}}\left(v_1^{(1)}\right)^2dy_{c1}dy_{c1}dy_{c1}\right]=\\
&=12\left\{\frac{\partial}{\partial\varphi}\left(\frac{1}{\eta_{1\tau}}\int_0^{\varepsilon_1}\int_0^{y_{c1}}\int_0^{y_{c1}}F\left(\varphi,y_{c1},x_{c1}\right)dy_{c1}dy_{c1}dy_{c1}-\frac{\varepsilon_1}{2\eta_{1\tau}}\int_0^{\varepsilon_1}\int_0^{y_{c1}}F\left(\varphi,y_{c1},x_{c1}\right)dy_{c1}dy_{c1}\right)+\right.\\
&\left.+\frac{1}{L_1^2}\frac{\partial}{\partial x_{c1}}\left(\frac{1}{\eta_{1\tau}}\int_0^{\varepsilon_1}\int_0^{y_{c1}}\int_0^{y_{c1}}G\left(\varphi,y_{c1},x_{c1}\right)dy_{c1}dy_{c1}dy_{c1}-\frac{\varepsilon_1}{2\eta_{1\tau}}\int_0^{\varepsilon_1}\int_0^{y_{c1}}G\left(\varphi,y_{c1},x_{c1}\right)dy_{c1}dy_{c1}\right)\right\}
\end{aligned}
\tag{25}
$$

where $0\le y_{c1}\le y_{c2}\le y_{c3}<\varepsilon_1$, $0\le\phi<\phi_k$, $-1\le x_{c1}<+1$.

The differentia Equations (21)–(25) are solved numerically, with assumption of the Reynolds or Gümbel boundary condition **[L. 10–11]**. In order to obtain the final values of dimensionless pressure corrections, resulting from the viscoelastic properties of the lubricating oil, the values obtained from the numerical solution of Equation (21) are substituted into Equation (24). From Equation (24) the pressure corrections are determined and their average value is then calculated (in relation to the gap height direction).

– The equation of dimensionless temperature and its corrections

$$T_1^{(0)} = -\eta_{1\tau}\int_0^{y_{c1}}\int_0^{y_{c1}}\left[\left(\frac{\partial v_1^{(0)}}{\partial y_{c1}}\right)^2 + \frac{1}{L_1^2}\left(\frac{\partial v_3^{(0)}}{\partial y_{c1}}\right)^2\right]dy_{c1}dy_{c1} - q_{10c}^{(1)}r_1 + f_{1c}^{(0)} \tag{26}$$

$$T_{10}^{(1)} = \eta_{1\tau}\int_0^{y_{c1}}\int_0^{y_{c1}} X_{10}dy_{c1}dy_{c1} - 2\eta_{1\tau}\int_0^{y_{c1}}\int_0^{y_{c1}} Y_{10}dr_1dr_1 - q_{10c}^{(1)}r_1 + f_{10c}^{(1)} \tag{27}$$

where

$$X_{10} \equiv T_1^{(0)}\left[\left(\frac{\partial v_1^{(0)}}{\partial y_{c1}}\right)^2 + \frac{1}{L_1^2}\left(\frac{\partial v_3^{(0)}}{\partial y_{c1}}\right)^2\right],\ Y_{10} \equiv \left(\frac{\partial v_{10}^{(1)}}{\partial y_{c1}}\right)\left(\frac{\partial v_1^{(0)}}{\partial y_{c1}}\right) + \frac{1}{L_1^2}\left(\frac{\partial v_{30}^{(1)}}{\partial y_{c1}}\right)\left(\frac{\partial v_3^{(0)}}{\partial y_{c1}}\right)$$

$$T_{11}^{(1)} = -\eta_{1\tau}\int_0^{y_{c1}}\int_0^{y_{c1}} X_{11}dy_{c1}dy_{c1} - 2\eta_{1\tau}\int_0^{y_{c1}}\int_0^{y_{c1}} Y_{11}dy_{c1}dy_{c1} - q_{11}^{(1)}y_{c1} + f_{11c}^{(1)} \tag{28}$$

where

$$X_{11} \equiv p_1^{(0)}\left[\left(\frac{\partial v_1^{(0)}}{\partial y_{c1}}\right)^2 + \frac{1}{L_1^2}\left(\frac{\partial v_3^{(0)}}{\partial y_{c1}}\right)^2\right],\ Y_{11} \equiv \left(\frac{\partial v_{11}^{(1)}}{\partial y_{c1}}\right)\left(\frac{\partial v_1^{(0)}}{\partial y_{c1}}\right) + \frac{1}{L_1^2}\left(\frac{\partial v_{31}^{(1)}}{\partial y_{c1}}\right)\left(\frac{\partial v_3^{(0)}}{\partial y_{c1}}\right)$$

$$T_1^{(1)} = -2\eta_{1\tau}\int_0^{y_{c1}}\int_0^{y_{c1}} X_{22}dy_{c1}dy_{c1} - \frac{3}{L_1^2}\int_0^{y_{c1}}\int_0^{y_{c1}} Y_{22}dy_{c1}dy_{c1} - \frac{\beta_o}{\alpha_o}\int_0^{y_{c1}}\int_0^{y_{c1}} Z_{22}dy_{c1}dy_{c1} - q_1^{(1)}y_{c1} + f_{1c}^{(1)} \tag{29}$$

where

$$X_{22} \equiv \frac{\partial v_1^{(0)}}{\partial y_{c2}}\left(\frac{\partial v_1^{(1)}}{\partial y_{c2}}\right) + \frac{1}{L_1^2}\frac{\partial v_3^{(0)}}{\partial y_{c2}}\left(\frac{\partial v_3^{(1)}}{\partial y_{c2}}\right),\ Y_{22} \equiv X_3Y_4 - X_1Y_3 - Z_1Y_1,$$

$$Z_{22} \equiv 2\left(X_1 + \frac{1}{L_1^2}Z_1\right)Y_2 + X_4\sqrt{X_1} + \frac{1}{L_1^2}Z_4\sqrt{Z_1} + 2\left[X_5\sqrt{X_1} + \frac{1}{L_1^2}Z_5\sqrt{Z_1}\right]$$

where $0 \le y_{c1} \le \varepsilon_1$, $0 \le \phi \le 2\pi$, $-1 \le x_{c1} \le +1$ and

$f_{1c}^{(0)}$ – dimensionless temperature at bearing shaft,

$f_{10c}^{(1)}$ – dimensionless temperature correction at shaft surface, resulting from taking into account changes in temperature-dependent dynamic viscosity of oil,

$f_{11c}^{(1)}$ – dimensionless temperature correction at shaft surface, resulting from taking into account changes in the pressure-dependent dynamic viscosity of oil,

$f_{1c}^{(1)}$ – dimensionless temperature correction at shaft surface, resulting from the viscoelastic properties of the lubricating oil,

$q_{1c}^{(0)}$ – dimensionless heat flux density flowing from the shaft to the lubrication gap or from the gap to the shaft,

$q_{10c}^{(1)}$ – dimensionless heat flux density corrections, which flows from the shaft to the lubrication gap or from the gap to the shaft, (resulted from the temperature influence on viscosity of oil),

$q_{11c}^{(1)}$ – dimensionless heat flux density corrections, which flows from the shaft to the lubrication gap or from the gap to the shaft, (resulted from the pressure influence on viscosity of oil),

$q_{1c}^{(1)}$ – dimensionless heat flux density corrections, which flows from the shaft to the lubrication gap or from the gap to the shaft, (resulted from the influence of non-Newtonian properties of oil on its viscosity).

By imposing the condition $y_{c1} = \varepsilon_1$ on solutions (26)–(29), the temperature distribution on the sleeve $f_{1p}^{(0)}$ and its correction $f_{10p}^{(1)}, f_{11p}^{(1)}, f_{1p}^{(1)}$ are obtained. The total value of the dimensionless temperature on the surface of the bearing sleeve f_{1p} is determined from the following formula:

$$f_{1p} = f_{1p}^{(0)} + Q_{Br}f_{10p}^{(1)} + \varsigma_p f_{11p}^{(1)} + De_\alpha f_{1p}^{(1)} \tag{30}$$

For a slide bearing lubricated with non-Newtonian oil, the dimensional load carrying capacity C_Σ is determined by the following formula:

$$C_\Sigma = C_1 \cdot bR\eta_o\omega / \psi^2 \tag{31}$$

where

$$C_1 = C_1^{(0)} + Q_{Br}C_{10}^{(1)} + \varsigma_p C_{11}^{(1)} + De\, C_1^{(1)} \tag{32}$$

$$C_1^{(0)} = \sqrt{\left(\int_{-1}^{+1}\left(\int_0^{\varphi_k} p_1^{(0)}\left(1+L_1 x_{c1}\cos\gamma\right)\sin\varphi\, d\varphi\right)dx_{c1}\right)^2 + \left(\int_{-1}^{+1}\left(\int_0^{\varphi_k} p_1^{(0)}\left(1+L_1 x_{c1}\cos\gamma\right)\cos\varphi\, d\varphi\right)dx_{c1}\right)^2} \tag{33}$$

$$C_{10}^{(1)} = \sqrt{\left(\int_{-1}^{+1}\left(\int_0^{\varphi_k} p_{10}^{(1)}\left(1+L_1 x_{c1}\cos\gamma\right)\sin\varphi\, d\varphi\right)dx_{c1}\right)^2 + \left(\int_{-1}^{+1}\left(\int_0^{\varphi_k} p_{10}^{(1)}\left(1+L_1 x_{c1}\cos\gamma\right)\cos\varphi\, d\varphi\right)dx_{c1}\right)^2} \tag{34}$$

$$C_{11}^{(1)} = \sqrt{\left(\int_{-1}^{+1}\left(\int_0^{\varphi_k} p_{11}^{(1)}\left(1+L_1 x_{c1}\cos\gamma\right)\sin\varphi\, d\varphi\right)dx_{c1}\right)^2 + \left(\int_{-1}^{+1}\left(\int_0^{\varphi_k} p_{11}^{(1)}\left(1+L_1 x_{c1}\cos\gamma\right)\cos\varphi\, d\varphi\right)dx_{c1}\right)^2} \tag{35}$$

$$C_1^{(1)} = \sqrt{\left(\int_{-1}^{+1}\left(\int_0^{\varphi_k} p_1^{(1)}\left(1+L_1 x_{c1}\cos\gamma\right)\sin\varphi\, d\varphi\right)dx_{c1}\right)^2 + \left(\int_{-1}^{+1}\left(\int_0^{\varphi_k} p_1^{(1)}\left(1+L_1 x_{c1}\cos\gamma\right)\cos\varphi\, d\varphi\right)dx_{c1}\right)^2} \tag{36}$$

The total friction force value $F_{R\Sigma}$ or total dimensionless friction force F_{R1}, in the lubricating gap of the slide bearing, is calculated with the following formula:

$$Fr_\Sigma = Fr_1 \cdot bR\eta_o\omega/\psi = \left(Fr_1^{(0)} + Q_{Br}Fr_{10}^{(1)} + \varsigma_p Fr_{11}^{(1)} + De_\alpha Fr_1^{(1)}\right)\cdot bR\eta_o\omega/\psi \tag{37}$$

The dimensionless friction force for classic Newtonian oil and correction of the friction force resulting from the non-Newtonian viscoelastic properties of the lubricating oil, while taking into account the viscosity decrease with temperature increase and viscosity increase with pressure increase, are determined according to the following relations:

$$Fr_1^{(0)} = \int_{-1}^{+1}\left[\int_0^{\varphi}\left(\eta_{1\tau}\frac{\partial v_1^{(0)}}{\partial y_{c1}}\right)_{y_{c1}=\varepsilon_1}\left(R + x_c\cos\gamma\right)d\varphi\right]dx_{c1} \tag{38}$$

$$Fr_{10}^{(1)} = \int_{-1}^{+1}\left[\int_0^{\varphi}\left(\eta_{1\tau}\frac{\partial v_{10}^{(1)}}{\partial y_{c1}}\right)_{y_{c1}=\varepsilon_1}\left(R + x_c\cos\gamma\right)d\varphi\right]dx_{c1} \tag{39}$$

$$Fr_{11}^{(1)} = \int_{-1}^{+1}\left[\int_0^{\varphi}\left(\eta_{1\tau}\frac{\partial v_{11}^{(1)}}{\partial y_{c1}}\right)_{y_{c1}=\varepsilon_1}\left(R + x_c\cos\gamma\right)d\varphi\right]dx_{c1} \tag{40}$$

$$Fr_1^{(1)} = \int_{-1}^{+1}\left[\int_0^{\varphi_k}\eta_{1\tau}\left(\frac{\partial v_1^{(1)}}{\partial y_{c1}}\right)_{y_{c1}=\varepsilon_1}\left(R + x_c\cos\gamma\right)d\varphi\right]dx_{c1} \tag{41}$$

Formula (38) describes dimensionless values of friction force for a classical case, while Formula (39) presents dimensionless values of friction force corrections resulting from taking into account the change in dynamic viscosity of an oil under the influence of temperature. The dimensional values of friction force corrections resulting from changes in the dynamic viscosity of oil dependent on pressure are shown in Relation (40). The influence of viscoelastic properties of the lubricating oil on the correction of the friction force has been taken into account in Formula (41). The peripheral speed function (9) is inserted into Formula (38). The velocity function corrections, presented in the expressions (12), (15), and (18), are introduced into Formulas (39), (40), and (41). The peripheral speed function (9) consists of the velocity induced by the pressure gradient and the velocity caused by the peripheral movement of the shaft (shear flow).

In a similar way, the correction of peripheral speeds $v_{10}^{(1)}, v_{11}^{(1)}, v_1^{(1)}$ can also be divided into speed corrections caused by pressure and shear flow. In Formulas

(38–41), specifying the value of the friction force and its correction, the integration must be performed in the whole range of the wrap angle, i.e. from 0 to 2π, for the part describing the shear flow. The integral must always be positive in that case. However, the integration for the part depending on the pressure gradient is performed in the range of the angle of wrap from the line of centres, for $\varphi = 0$, to the angular coordinate value of the end of the oil film, for $\varphi = \varphi_k$. The result of this integration can take any positive or negative values in accordance with the situation.

The total contractual coefficient of friction is determined by the following formula:

$$\left(\frac{\mu}{\psi}\right)_{\Sigma} = \frac{Fr_{\Sigma}}{\psi C_{\Sigma}} = \frac{\left[Fr_1^{(0)} + Q_{Br}Fr_{10}^{(1)} + \varsigma_p Fr_{11}^{(1)} + De_\alpha Fr_1^{(1)} + O\left(Q_{Br}^2\right) + O\left(\varsigma_p^2\right) + O\left(De_\alpha^2\right)\right]\ bR\eta_o\omega/\psi}{\left[C_1^{(0)} + Q_{Br}C_{10}^{(1)} + \varsigma_p C_{11}^{(1)} + De_\alpha C_1^{(1)} + O\left(Q_{Br}^2\right) + O\left(\varsigma_p^2\right) + O\left(De_\alpha^2\right)\right]\ bR\eta_o\omega/\psi} \tag{42}$$

CONCLUSIONS AND OBSERVATIONS

The equations of the hydrodynamic theory lubrication with the non-Newtonian liquid of the second and higher orders are very complex. In order to obtain analytical and numerical solutions of these equations, it is necessary to simplify them. The small parameter method can be used for this purpose.

In the small parameter method, only the corrections multiplied by small parameters in the first power were taken into account, the remaining corrections were omitted. This is justified if the value of the small parameters is small, i.e. in the order of one-hundreds. If the small parameters are in the order of one-tenths (0.3–0.6), omitting the corrections multiplied by the small parameters in the second and higher power can cause large calculation errors.

The small parameter method, with only the first correction accepted in the calculation, linearizes the changes in viscosity from temperature and pressure.

The application of the Rivlin-Ericksen's constitutive equation causes, that the calculated pressure is not constant towards the height of the lubrication gap.

On the basis of these analytical considerations, in subsequent works, the authors will determine with numerical calculations, the total values of the bearing load carrying capacity, friction force, and coefficient of friction in the lubrication gap of the conical slide bearing.

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