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A new approach to maximize the overall return on investment with price and stock dependent demand under the nonlinear holding cost*

by

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Abstract: This study investigates an inventory model for deteriorating products with a price and stock-dependent demand pattern where the holding cost is a non-linear function of both time and stock level. Moreover, a decreased price and a higher stock level lead to a higher rate of demand. Consequently, in this article, we present a new approach, aiming at maximization of the return on investment by maximizing the profit/cost ratio. If an inventory manager has the potential to invest in a variety of projects, but disposes of only limited resources, it is logical to strategically plan towards a better return on investment. As a result, the manager's objective will be to develop an inventory policy with a possibly high return on investment. Therefore, a new strategy is considered in this article to optimize the profitability ratio in terms of replenishment time and selling price, which is determined as the proportion between the profit and the overall cost of the inventory scheme. This research demonstrates that optimizing the profitability ratio is equivalent to decreasing the average inventory cost of a product per unit. Also, the optimality is graphically checked and one numerical illustration is discussed to explain the result of the proposed model. Finally,

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sensitivity analysis of key parameters is performed to show the applicability of the proposed model. The profit/cost ratio is more sensitive to price elasticity markup or purchasing cost compared to the other parameters used. Also, for decision-makers, several helpful management insights are derived.

Keywords: deterioration, nonlinear holding cost, price and stock dependent demand, profit/cost ratio

1. Introduction

1.1. The outline of the problem

This article suggests an inventory model, in which the demand rate is determined by the price and inventory level, and the holding cost is also determined by the inventory level. Many models that adopt one of these two assumptions have been developed in the inventory literature since the 1980s. Most of them consider the objective of decreasing inventory cost per unit time, while others consider the objective of maximizing profit per unit time. When working with these models, we determined that the most effective policy for the maximum profit per unit time resulted in a high inventory cost per unit time, whereas the optimal policy for least inventory cost per unit time results in a low profit per unit time. However, in many real situations, the inventory manager may favor a balance of both policies with a high profit per unit time and a low inventory cost. That is, the objective should be the maximization of the ratio between the profit and the total cost of the inventory system.

In this situation, the model should be required to optimize the profit/cost ratio. In addition, during storage, most of the objects will deteriorate and their original value will diminish or be lost. During the recent period, the matter of deterioration and imperfect quality has been paid increasing attention from the side of numerous researches around the world. The first applications of mathematical models for deteriorating products were encouraged by the case of the center for blood bank, and were followed by some extended models related to, for example, organic products, vegetables, and flowers. As far as modelling of deterioration is concerned, the temporal behavior of the perishable things may be quite complex, but two main groups can generally be distinguished:

(1) the instantaneously deteriorating items; for a specific time the nature of this category remains unchanged, i.e., there is a date of expiry, after which the product loses the entire cost promptly; subsequently, such products would be rejected; blood units and dairy products are examples of this kind;

(2) non-instantaneously deteriorating things; this category is highly sensitive to the conditions of keeping; some products, such as, for instance, vegetables, ice-cream, dairy products in general, but also some electronic products, do deteriorate during storage; certain deteriorating products have a fresh-product period, during which they maintain their original condition and value, before

starting to deteriorate; afterwards, a part of or all products begin to deteriorate; this category of products may also have an expiry date, like those in the first category.

In addition, customers are less interested in buying those items that have either deteriorated or lost their originality. Therefore, the retailer applies different techniques to maintain the deteriorating product's quality. Furthermore, the demand for some items may be influenced by the stock level. Indeed, large piles of goods displayed in a supermarket sometimes lead customers to buy more. This issue called into question two common rules for inventory managers: always keep low stock levels to minimize the costs of the inventory management, and match a new order just when the stock is depleted. The reason for this lies in the fact that the high stock levels and the removal of stock-out increase the sales of the item and the profit of the inventory system per unit time, although the inventory costs are also increased. Then, it might be profitable to raise the order level in each cycle and request a new order before the stock runs out.

The buying behavior of consumers demonstrates, of course, that the price of goods is another important factor that affects the demand pattern. Consequently, the price should not be considered in the analysis of the market in a separate manner. The price of the goods, the optimization of inventories, and the integration of the supply chain are the subjects of vital decisions for the profit from the entire system. It is definitely realistic to suppose that demand is affected by both the stock level and the selling price at the same time. Another important factor results from the fact that the holding cost in classic inventory models is mostly assumed to be known and constant. However, the cost of holding may not always be constant. This is especially true in the storage of deteriorating and perishable items, such as food, dairy product, fruits, vegetables, and meat, whose freshness deteriorates with each passing day, requiring increased holding costs to retain the freshness of the products and prevent deterioration. Therefore, the holding cost per unit time changes with time.

1.2. Aim of the study

According to the above explanations, the objective of the research reported in this paper is to address the following research questions:

- What would be the optimum sales price of the product which maximizes the profit/cost ratio of the retailer?
- Since a larger display of goods stimulates higher sales, what is the influence of stock efficiency of demand on the retailer's optimal order quantity?
- What would be the preferred behaviour of the retailer and how do the key parameters of the model affect this preference?
- What would be the managerial implications of implementing this model for the use in business reality?

In order to effectively respond to the above questions, we aim to calculate the profit/cost ratio, and for this purpose we build an appropriate inventory model for deteriorating products.

1.3. Structure of the paper

In Section 2 the existing literature of the subject is reviewed. In Section 3, the notation and assumptions are introduced. Then, Section 4 provides the formulation of the mathematical model. Section 5 presents and discusses the numerical illustration, sensitivity analysis of main parameters, and managerial implications. In addition, the conclusions and future possibilities are deliberated in Section 6.

2. Literature review

2.1. The review

The rate of deterioration is a key measurement tool for describing the essential characteristics of perishable goods. Several kinds of products have deteriorating nature over time, such as fresh fruits and vegetables, milk, juice, any dairy product, meat, and so on. Stock models considering deterioration have been widely analysed in recent years under different hypotheses. Thus, first, this article reviews the literature on the deteriorating inventory models.

So, Liao and Huang (2010) proposed a stock model to optimize the cycle time and trade credit, this problem being addressed under deteriorating inventory. Kumar et al. (2012) studied a deterministic inventory model for time-varying deteriorating products with price-dependent demand under parabolic time-dependent holding charge. Sarkar et al. (2015) proposed a stock control policy with a maximum fixed lifetime of the product. Shaikh et al. (2018) established an economic production quantity (EPQ) model for an exponentially deteriorating product under allowable delay in payment. Further, Panda et al. (2019) analyzed an inventory model for deteriorating product under price, stock and advertisement–dependent demand. Later, Shah et al. (2020) studied and assessed an EOQ model, in which the demand is depending on the time-credit cycle, and also considered the constant deterioration rate. An inventory model for time-dependent decaying items was explored by Jani et al. (2020). Furthermore, according to the product life cycle hypothesis, Mashud et al. (2021) investigated an inventory model for deteriorating items with a discount facility. Rana et al. (2021) derived an inventory model for livestock farming products under a permissible delay in payment. In this general direction, several researchers, like, for instance, Halim et al. (2021), Duary et al. (2022), and Shah et al. (2022) further studied the inventory models for deteriorating items.

In addition to the rate and proportion of deterioration, another essential factor that practitioners have widely acknowledged as having a significant effect on managerial decision-making is market demand. In supermarkets, it has been found that if the on-hand stock level rises or decreases, the demand rate reacts by going up and down in response. The respective condition usually emerges from the form of inventory of consumer products. Sarkar et al. (2011) calculated an economic manufacturing quantity (EMQ) with time-variant demand and also examined the model for the time value of money. Pando et al. (2012) implemented the inventory model of stock-based demand and proposed a strategy to increase benefit.

It is obvious that customers try to satisfy their demand for any product at a reasonable price, and thus market demand for a specific item is mainly dependent on the price. In this direction, Soni (2013) investigated a model of inventory, in which demand depends on price and on the stock level. Shah et al. (2018) proposed an economic order quantity model focused on the quadratic demand with three different model variants, in which the time-dependent holding cost was considered. Bhattacharyya and Sana (2019) proposed a mathematical model of a green product manufacturing inventory system, in which they defined a profit function under the service level and a random variable dependant demand. Sana (2020) developed a newsvendor inventory model with sales price, carbon emission, and corporate social responsibility index-based demand. Further, Shah and Naik (2020) built a multi-purpose model for inventory policies with the price-stock reliability-based rate of demand. Pando et al. (2020) proposed an inventory model, in which they assumed that the rate of demand is a concave power function on the stock level. Further, Barman et al. (2021) proposed a green supply chain model that includes government subsidies for green products and tax implementation for non-green products under price and greening level dependant demand. Saha et al. (2021) also investigated the effect of a subsidy and horizontal strategic collaboration on a green supply chain under price and green quality level dependent demand.

Another development of the basic inventory models, which is frequently undertaken by researchers is the relaxation of the assumption that the holding cost per unit time is constant. Consequently, several researchers hypothesized that the inventory system's holding cost may be a power function of the stock level, the duration of the cycle time, and sometimes both. Taking into consideration the nonlinear holding cost, Cardenas-Barron et al. (2021) established the policy under the power demand function. In this direction, different inventory models with non-linear holding costs have been developed in the literature, including the articles by San-Jose et al. (2017) and Macias-Lopez et al. (2021).

Some articles in the inventory model literature are focused on investigating inventory systems with the aim of optimizing return on investment. Thus, Chen (2001) adopted the mathematical model meant to maximize return on investment (ROI). Also, Li et al. (2008) investigated a model aimed at maximizing ROI in an inventory with capital investments in setup and quality operations. Other articles, focusing on optimizing return on investment are those by Chen and Liao (2014) and Ishfaq and Bajwa (2019). Most recently, Pando et al. (2021) also considered the ROI with the stock-dependent demand. Further, Jani et al. (2021), Sana (2021), and Khanna and Jaggi (2021) investigated also several effective ways of respective mathematical modelling. Having reviewed some of the most important relevant positions of literature of the subject, we now show in Table 1 those that inspired our work, classifying them according to a number of aspects, accounted for in the respective models.

Table 1. Contribution of related research works

Source	**Demand type**	**Non-linear holding cost**	**Deterioration**	**Minimum cost**	**Maximum profit**	**Profit/cost ratio**
Panda et al. (2019)	Advertisement, price, and stock dependent	No	Yes	Yes	No	No
Pando et al. (2020)	Stock dependent	Yes	No	Yes	Yes	Yes
Halim et al. (2021)	Price and stock dependent	No	Yes	Yes	No	No
Macias-Lopez et al. (2021)	Price, stock and time dependent	Yes	Yes	No	Yes	No
This article	Price and stock dependent	Yes	Yes	Yes	Yes	Yes

2.2. Research gaps and contributions

In many real-life conditions, the inventory manager may prefer a solution that generates a high profit per unit time without significantly increasing the total cost of inventory. In such a case, business would be more concerned with optimizing the return on investment as defined by the profit/cost ratio. Hence also the corresponding model developed should focus on maximizing the profit/cost ratio. We could not find any research articles on inventory models for deteriorating items with price and stock-dependent demand rates that include considering the goal of optimizing the return on inventory management expenditure. This is the gap in the published literature that this research is intended to effectively fill.

Table 1 combines the most significant articles previously referred to, categorizing them according to the different types of demand, holding cost, deterioration, and the aim considered (minimum cost, maximum profit, or profit/cost ratio), in order to compare the contribution of the listed references to the inventory theory. The most similar work to this article is the one by Pando et al. (2020). In two aspects, the present research differs from the work of Pando et al. (2020).

First, the model examined in this study is the only one that analyses the issues of maximum profit/cost ratio using price and stock-dependent demand rate. Generally, demand is affected by the price and stock, and so this is a real-life scenario. Second, in the currently considered scenario, it is assumed that product deterioration plays an important role for any organization, and therefore the research here reported considers product deterioration, which is a novelty of this research.

So, altogether, the present article investigates the issues mentioned above together and formulates a comprehensive model to determine the optimal setting with respect to the profit/cost ratio, which is measured as the proportion between the profit and the overall cost per unit time. Furthermore, the model developed incorporates a novel managerial concept that assists inventory managers in optimizing the profit/cost ratio.

3. Notation and assumptions

In formulating a mathematical model of the problem considered, this paper uses the following notation and assumptions.

3.1. Notation

Parameters	
A_0	Ordering cost; (in \$/order)
P_c	Purchasing cost; (in \$/unit)
λ_0	Scale demand; $\lambda_0 > 0$
β_0	Inventory level elasticity of demand rate; $0 \leq \beta_0 < 1$
η_0	Price elasticity mark-up; $\eta_0 > 1$
h_c	Scale parameter of the holding cost (\$/unit/unit time)
γ_0	Holding cost elasticity parameter; $\gamma_0 > 1$
θ	Constant deterioration rate; $0 < \theta \leq 1$
Q_0	Order quantity; $Q_0 > 0$ (units)
Decision variables	
T	Cycle time (in years)
p	The sales price of the product (in \$/unit)

Functions	
$I_r(t)$	The level of inventory at a time t (in units)
$TC(p,T)$	Overall cost (in \$/unit time)
$\pi_r(p,T)$	Profit (in \$/unit time)
$r_0(p,T)$	Average inventory cost (in \$/unit time)
$Z_r(p,T)$	Profit/cost ratio (in %)

3.2. Assumptions

The following assumptions were adopted in the development of the here presented model:

- The inventory structure works with only a single product.
- The unit ordering cost and purchasing cost per order are known and stable.
- The demand rate $D_r(p,t) = \lambda_0 p^{-\eta_0}\left[I_r(t)\right]^{\beta_0}$, $\lambda_0 > 0$ and $0 \le \beta_0 < 1$, is a concave power function of the stock level, where p is selling price per unit, $\eta_0 > 1$ is a mark-up of the selling price that depends on the level of inventory, $I_r(t)$.
- In terms of the number of storage items, the holding charge $HC(t) = h_c\left[I_r(t)\right]^{\gamma_0}$ (as in Pando et al., 2020) is nonlinear when $\gamma_0 > 1$.
- The lead time is zero.
- The planning horizon is infinite.
- The shortages are not permitted.

4. Mathematical model

In this section, according to the above assumptions, we focus on how to optimize the profit/cost ratio in the situation, in which depletion of the inventory occurs due to the combined effects of demand and deterioration. The inventory level curve $I_r(t)$ is obtained by solving the equation

$$\frac{dI_r(t)}{dt} + \theta I_r(t) = -D_r(p,t), \qquad 0 \le \text{t ¡T} \tag{1}$$

with boundary condition $I_r(T) = 0$. Now, using this condition, the solution of equation (1) is given by

$$I_r(t) = \left[\frac{-\lambda_0 p^{-\eta_0}}{\theta} + \frac{\lambda_0 p^{-\eta_0}}{\theta} e^{(1-\beta_0)\theta(T-t)}\right]^{\frac{1}{1-\beta_o}}. \tag{2}$$

The holding charge per unit time in an inventory cycle can be calculated as follows

$$HC = \int_0^T h_c \left[I_r\left(t\right)\right]^{\gamma_0} dt = K_1 \left[\left(t-T\right)^{1-\frac{\gamma_0}{\beta_0-1}}\right] \tag{3}$$

with

$$K_1 = \frac{h_c \left[\lambda_0 p^{-\eta_0}\left(\beta_0-1\right)\right]^{\frac{\gamma_0}{1-\beta_0}}}{1+\frac{\gamma_0}{1-\beta_0}} \quad . \tag{4}$$

As

$$I_r\left(0\right) = \left(\frac{\lambda_0 p^{-\eta_0}}{\theta}\right)^{\frac{1}{1-\beta_o}} \left[-1+e^{(1-\beta_0)\theta(T)}\right]^{\frac{1}{1-\beta_o}} \text{ and } I_r\left(T\right)=0,$$

the order quantity is

$$Q_0 = I_r\left(0\right) - I_r\left(T\right) = K_2\left(-1+e^{(1-\beta_0)\theta T}\right)^{\frac{1}{1-\beta_0}} \tag{5}$$

with

$$K_2 = \left(\frac{\lambda_0 p^{-\eta_0}}{\theta}\right)^{\frac{1}{1-\beta_0}} . \tag{6}$$

Now, the retailer's sales revenue per cycle time T is given by

$$SR = \left(\frac{pQ_0}{T}\right) \tag{7}$$

Next, consider that the retailer overall cost components per cycle period T are composed of:

- Ordering cost: $\quad \mathrm{OC} = A_0$
- Purchasing cost: $\quad \mathrm{PC} = P_c Q_0$.

As a result, the overall cost per unit time during the cycle time T for the retailer is given by

$$TC(p,T) = \frac{1}{T}\left(OC+PC+HC\right). \tag{8}$$

The profit per unit time, $\pi_r\left(p,T\right)$, is generated as

$$\pi_r\left(p,T\right) = \frac{1}{T}\left(SR-OC-PC-HC\right), \tag{9}$$

and for the inventory model, the profit/cost ratio $Z_r\left(p,T\right)$ is calculated by

$$Z_r\left(p,T\right)=\frac{\pi_r\left(p,T\right)}{TC\left(p,T\right)}=\frac{p}{P_c+r_0\left(p,T\right)}-1 \tag{10}$$

where

$$r_0\left(p,T\right)=\frac{OC+HC}{Q_0}=\frac{A_0+K_1\left[\left(t-T\right)^{1-\frac{\gamma_0}{\beta_0-1}}\right]}{K_2\left(-1+e^{(1-\beta_0)\theta T}\right)^{\frac{1}{1-\beta_0}}}. \tag{11}$$

If we describe the cost of inventory as the cost of holding and ordering, then the function $r_0(p,T)$ estimates the average cost of inventory per unit of an item. Furthermore, as $r_0(p,T)>0$, it is clear that $1<Z_r(p,T)<p/P_c-1$. The purpose of the model is to optimize with respect to the profit/cost ratio $Z_r(p,T)$. Consequently, the mathematical problem is

$$\max_{T>0} Z_r\left(p,T\right) \tag{12}$$

which is equivalent to

$$\min_{T>0} r_0\left(p,T\right). \tag{13}$$

The essence of the model is based on observing that while working towards the revenue one focuses, as a rule, on the maximum profit problem, while one is faced, at the same time, with the lowest cost dilemma, complicating the revenue maximisation issue. On the other hand, if the aim is to optimize with respect to the profit/cost ratio, then we should also consider that the rate of demand depends on the inventory level when $\beta_0>0$. This is the reason why this article is meant to offer an optimal approach to the problem of achieving the highest profit/cost ratio.

5. Numerical illustration and sensitivity analysis

5.1. Numerical illustration

In this part of the paper, a numerical illustration is used to determine a concrete form of the proposed model, to show the solution technique, and perform sensitivity analysis. The hypothetical exemplary data for the numerical illustration are given as follows:

$A_0=\$\ 25$ per order, $h_c=\$\ 0.5$ per unit per year, $P_c=\$\ 15$ per unit, $\gamma_0=2$, $\beta_0=0.5$, $\lambda_0=15$, $\eta_0=1$, $\theta=0.1$.

The corresponding optimal solution is shown in Table 2.

Using the software Maple $XVIII$ we plot the concavity of average inventory cost per unit of an item vs. cycle length T and selling price p as shown in Fig. 1, and the convexity of the total profit/cost ratio, as shown in Fig. 2.

Table 2. Exemplary optimal solution

$\pi_r\,(p,T)$ (\$/unit time)	$TC\,(p,T)$ (\$/unit time)	$r_0\,(p,T)$ (\$/unit time)	$Z_r\,(p,T)$ (in %)	T (years)	p (\$/unit)	Q_0 (\$/unit)
8.71	5.38	5.95	46	9.284	30.57	8.404

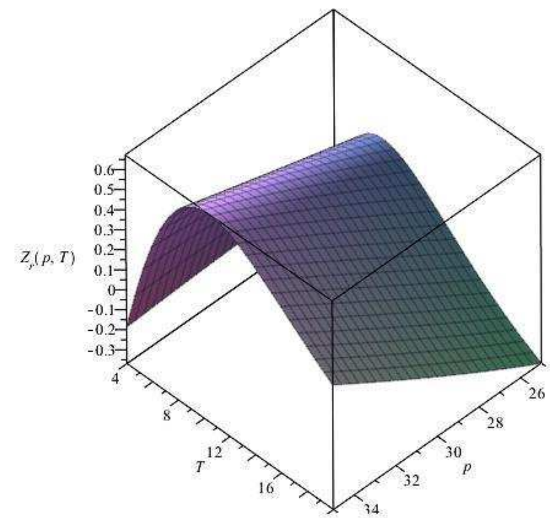

Figure 1. Optimality of average inventory cost per unit of an item against cycle time T and selling price p

5.2. Sensitivity analysis

In this section, for the varying parameter values, sensitivity analysis is performed on the numerical example considered. The results are provided in Table 3 for the values of the individual parameters changed with respect to the original ones, previously given, by, respectively -20%, -10%, 0%, 10% and 20%.

The following conclusions can be drawn from the results, given in Table 3: 63 1140 2004 0000 3702 6807 8853 z opisem wycieczka zakładowa

- The profit/cost ratio, $Z_r(p,T)$, is highly sensitive with regard to the holding cost, h_c, and the scale of demand, λ_0. On the other hand, the profit/cost ratio, $Z_r(p,T)$ is moderately sensitive with respect to the ordering cost, A_0, the deterioration rate, θ, the price elasticity mark-up, η_0, and the purchasing cost, P_c.
- The profit, $\pi_r(p,T)$, is highly sensitive with regard to the scale of demand, λ_0, while being moderately sensitive with respect to holding cost, h_c, the

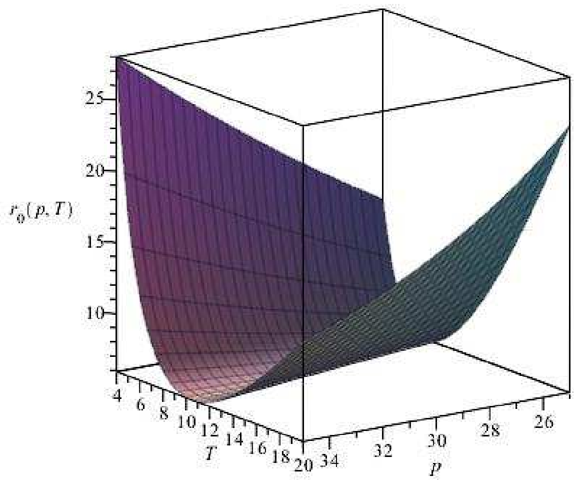

Figure 2. Optimality of total profit/cost ratio against selling price p and cycle time T

ordering cost, A_0, the deterioration rate, θ, the price elasticity mark-up, η_0, and the purchasing cost, P_c.

- On the one hand, the overall cost, $TC(p,T)$, is highly sensitive with respect to the ordering cost, A_0, and the deterioration rate, θ. On the other hand, it is insensitive with respect to parameters h_c, η_0, P_c and the scale of demand λ_0.
- The order quantity, Q_0, is highly sensitive to the ordering cost, A_0, and the deterioration rate, θ, and less sensitive with respect to holding cost, h_c. It is, however, insensitive with respect to the parameters η_0, P_c, and the scale of demand, λ_0.
- It is clear that the selling price, p, is highly sensitive with regard to the parameters of the scale of demand, λ_0, and holding cost, h_c, while being moderately sensitive to A_0, θ, and η_0, and insensitive with respect to purchasing cost, P_c.

5.3. Managerial implication

The proposed model might bring benefits to industry managers and others through the maintenance of appropriate inventory in a competitive marketplace to stimulate demand for a product and maximize the profit/cost ratio.

- In inventory models, the maximum profit/cost ratio solution does not

Table 3. Sensitivity analysis

Parameters	Values	Solution					
		T	p	$\pi_r(p,T)$	$TC(p,T)$	$Z_r(p,T)$	Q_0
		years	\$	\$/time	\$/time	%	\$/unit
A_0	20	9.284	32.32	9.72	4.31	59	7.52
	22.5	9.284	31.38	9.22	4.85	52	7.97
	25	9.284	30.57	8.71	5.38	46	8.40
	27.5	9.284	29.85	8.17	5.92	41	8.81
	30	9.284	29.21	7.62	6.46	36	9.21
h_c	0.4	-	-	-	-	-	-
	0.45	9.284	29.77	8.71	5.38	44	8.86
	0.5	9.284	30.57	8.70	5.38	46	8.40
	0.55	9.284	31.30	8.69	5.38	47	8.01
	0.6	9.284	31.99	8.66	5.38	49	7.67
θ	0.08	11.605	40.40	12.14	4.31	86	7.51
	0.09	10.316	34.87	10.51	4.85	64	7.97
	0.1	9.284	30.57	8.70	5.38	46	8.40
	0.11	8.440	27.13	6.75	5.92	31	8.81
	0.12	7.737	24.36	4.65	6.46	19	9.21
η_0	0.8	9.284	71.87	46.10	5.38	243	8.40
	0.9	9.284	44.70	21.50	5.38	113	8.40
	1	9.284	30.56	18.71	5.38	46	8.40
	1.1	9.284	22.39	11.31	5.38	6	8.40
	1.2	-	-	-	-	-	-
λ_0	12	9.284	24.45	3.17	5.38	16	8.40
	13.5	9.284	27.51	5.93	5.38	31	8.40
	15	9.284	30.57	8.71	5.38	46	8.40
	16.5	9.284	33.62	11.47	5.38	60	8.40
	18	9.284	36.68	14.24	5.38	75	8.40
P_c	12	9.284	30.57	11.42	5.38	70	8.40
	13.5	9.284	30.57	10.06	5.38	57	8.40
	15	9.284	30.57	8.71	5.38	46	8.40
	16.5	9.284	30.57	7.35	5.38	36	8.40
	18	9.284	30.57	5.99	5.38	28	8.40

equal the maximum profit per unit time solution. Hence, the managers must first choose the appropriate inventory system objective.

- The solution of our model allows us to conclude that, since demand is affected by price and inventory level, the ideal length of the cycle time is longer for maximizing the profit/cost ratio than for lowering the inventory cost per unit time. As a result, if prices change, the inventory manager must modify the warehouse order policy. However, this is not true if the aim is to maximize profit per unit time.
- The maximum profit/cost ratio is more sensitive to the relative changes in the unit purchasing cost or the deterioration rate than to the relative changes in the ordering cost or the demand rate and holding cost scale factors. Therefore, the inventory manager should pay adequate attention to the parameters P_c, θ, η_0, h_c, and A_0 while holding goods for future commercial purposes.
- Table 3 shows that the total order quantity increases as ordering cost increases, but it decreases as the scale parameter of holding cost increases.
- The optimal lot size to obtain the maximum profitability is greater than the optimal lot size to obtain the minimum inventory cost per unit time.

6. Conclusions

This article analyses an inventory system with the aim to estimate the profit/cost ratio for deteriorating products, under the assumption that the rate of demand is a concave power function, depending upon the price and stock levels. In addition, the holding charge per unit time is a concave power function of the number of products in stock, which allows for a wide scope of the actual realistic situations to be represented. In the previous literature of the subject, many researchers have been focusing on either maximization of the total profit of the system or on minimizing the total cost, but this article discusses a new concept, consisting in the maximization of profit/cost ratio with regard to the decision variables of selling price and total cycle time.

Moreover, the mathematical model developed for the problem makes it possible to check that the maximization of the profit/cost ratio is equivalent to the minimization of the average inventory cost per unit of an item.

In this article we illustrate the optimality graphically and resolve a numerical example in order to show the properties of the proposed model. Finally, we perform the sensitivity analysis with respect to the key parameters of the model.

Since demand is price and stock dependent and the selling price is also a decision variable, the solution with the maximum ROI and the highest profit per unit time could be significantly different. According to the numerical simulations, concerning the solution of maximum profit per unit time, the maximum profit/cost ratio requires a longer cycle time, higher selling price, smaller lot size, higher inventory cost, and a lower overall system expenditure per unit time. Then, the inventory manager could invest the resources in other profitable businesses to obtain a higher profit per unit time. In economic theory, when monetary resources are limited and different investment alternatives are possible, the best option is usually the solution with the maximum profitability. If the final objective is the sale of the item, the inventory manager will always be interested in obtaining the maximum return on the money.

The limitation of the model presented is that this study considers price and stock dependent demand only without accounting for other aspects, like, e.g., the carbon emission effect. There are, therefore, some possible extensions of the model that can be implemented, namely, e.g.: (i) to take into account other functions for the demand rate, for example stock level and advertisement dependent demand, as shown in Mandal et al. (2021); (ii) to include discounts in the unit purchase price, as shown in Duary et al. (2022); (iii) to investigate the case of the non-instantaneously deteriorating items, as shown in Das et al. (2021), or (iv) to add carbon tax strategies, as shown in Shi et al. (2020). This would allow for the development of yet more realistic models.

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