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USING HYPERBOLA AND ONE-SHEETED HYPERBOLOID IN ENGINEERING PRACTICE

32.1 INTRODUCTION

Since the year 2012, we have begun to innovate our lectures within the course "Geometry on Computer". These innovations are supported by the FRVŠ grants. We use programs GeoGebra and OpenSCAD for teaching plane and 3D geometry. The GeoGebra allows us to concentrate on mathematical and geometrical principles of solving problems. OpenSCAD is basic programmable 3D CAD modeler, that produce STL files for 3D printing. The content of the course has been compiled with respect to requirements of students in their future studies and practical engineering use. In this paper, we present several application problems where the following principles of classic geometry are used: hyperbola, articulated antiparallelogram, parametrization of one-sheeted hyperboloid and their use in 3D modeling and solving navigation problems.

32.2 LORAN NAVIGATION

The LORAN (LOng RAnge Navigation) system enables the navigator of a ship or airplane to find its position without relying on visible landmarks. Radio stations located at the points F_1 and F_2 simultaneously broadcast signals. These signals are received by the ship located at the point P. The navigator measures the interval $\Delta t = t_2 - t_1$ between the time t_2 when he receives the signal sent by F_2, and the time t_1 when he receives the signal from F_1. The difference between the distances of the ship from F_1 and from F_2 is given by

$$|PF_2 - PF_1| = c \cdot \Delta t,$$

where c is the speed of light. The ship is located at some point P which lies on a hyperbola. If the navigator receives signals from three stations F_1, F_2 and F_3, the position of the ship will be uniquely determined.
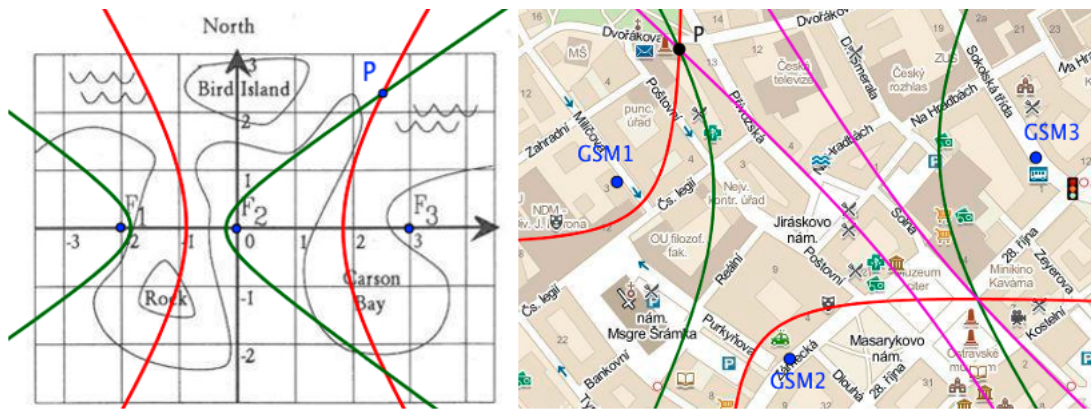
Fig. 32.1 LORAN position determination, GSM modification

Source: own elaboration

In the seminars the students become familiar with the use of the LORAN system which is already obsolete. We discuss with students the possibilities how to use these principles in the context of current technology. One possibility is to determine the position using ground stations GSM.

32.3 ARTICULATED ANTIPARALLELOGRAM

In this example, articulated antiparallelogram, we show the use of conics to students. This mechanism might seem a little bit too complicated is at the first sight, but it can be created quite simply.
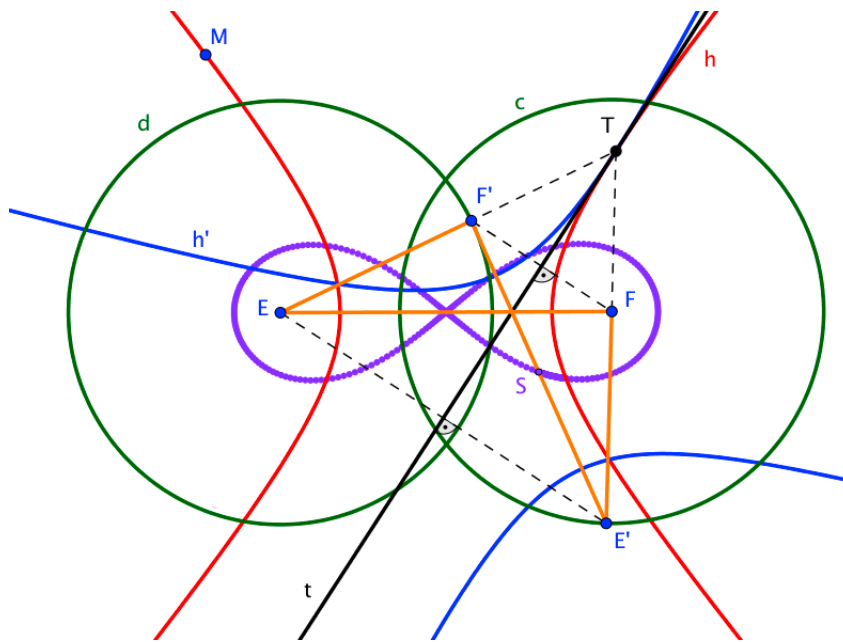
Fig. 32.2 Articulated antiparallelogram

Source: own elaboration

A hyperbola h is determined by two foci E, F and some point M. T is random point on hyperbola (moveable). At this point, we construct the tangent line t to hyperbola. We find images of foci E', F' in axisymmetric designed by tangent t. Points E', F' and original T determine hyperbola h' axisymmetric to original hyperbola h. If the point T moves along the hyperbola h, then the focal points E', F' of the hyperbola h' will move in circles, $E' \in c(F, r = |E'F|)$, $F' \in d(E, r' = |EF'|)$. Center S of the segment $E'F'$ will move along the curve called lemniscate.

One of the variants of transmission of longitudinal forces between a chassis frame and the main crossbeam wagon body is lemniscate mechanism (also called antiparallelogram). This mechanism is used in chassis of Czech locomotive Škoda 109E1.

32.4 ONE-SHEETED HYPERBOLOID

One-sheeted hyperboloid (students love to call it "spaghetti bundle") is unique surface of revolution because it integrates special properties of ruled and quadratic surfaces. Well known using of this surface is as cooling towers for powerplants, another posibility is to be used to build a transmission between skew axies or way to arrange blades on leather cutting machines.

```
1  a = 2;
2  v = 0.2;
3  jm = 100;
4
5  P_x = 20; P_y = 30; P_z = 0;
6  Q_x = -20; Q_y = 30; Q_z = 100+2*a;
7  v_x = P_x-Q_x; v_y = P_y-Q_y; v_z = P_z-Q_z;
8
9  delka_p = sqrt(pow(v_x,2)+pow(v_y,2));
10 uhel_p = atan(v_y/v_x);
11 delka_s = sqrt(pow(v_x,2)+pow(v_y,2)+pow(v_z,2));
12 uhel_s = 90-atan(Q_z/delka_p);
13
14 difference(){
15 for(i=[a:v:Q_z-a]){
16 translate([0,0,i])
17 cylinder(
18     h=v,
19     r=sqrt(pow(P_x-i*(v_x)/Q_z,2)+pow(P_y-i*(v_y)/Q_z,2))+a,
20     center=true, $fn=jm);
21 }//end for
22
23 for(i=[0:9*a:360]){
24 rotate([0,0,i])
25 translate([P_x,P_y+a/2,0]) rotate([0,-uhel_s,uhel_p])
26 cylinder(h=delka_s, r=1.6*a, center=false, $fn=jm);
27 }//end for
28
29 cylinder(h=4*Q_z, r=2*a, center=true, $fn=jm);
30 }//end diff
```

Fig. 32.3 One-sheeted hyperboloid as gear from transmission between skew axies
Source: own elaboration

This surface is very easy to program and prepare for 3D print. We have even two way to do it. As a ruled surface, we just rotate a line (cylinder with small radius) around axis. Line and axis are skew. There are two possible direction of rotation — left or

righ-hand rules. Or we use parametrization of making line given with point P and Q and for each point M from this line we calculate its distance from axis. Hyperboloid consists of cylinders with the radius equal to calculated distance and small height (about 0,2mm — height 1 layer of Prusa i3 MK2 3D printer) lying on one another.
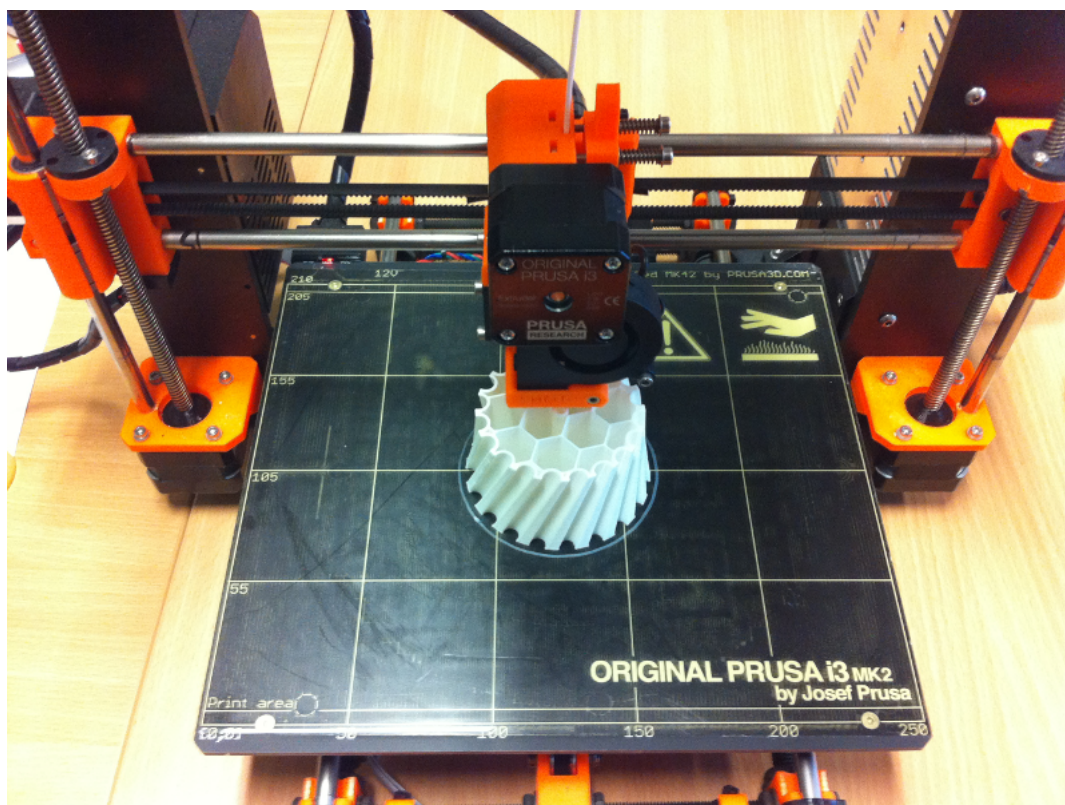
**Fig. 32.4 Prusa i3 MK2 3D printer**

Source: own elaboration

CONCLUSION

We have shown the students a connection between theoretical background and real problems. The response of students was very positive. So we created a textbook and we prepare two new practical lectures for Faculty of Mechanical Engineering and Faculty of Civil Engineering with focus on modeling gears, arcs and terrains and their possibilities for 3D print.

ACKNOWLEDGMENTS
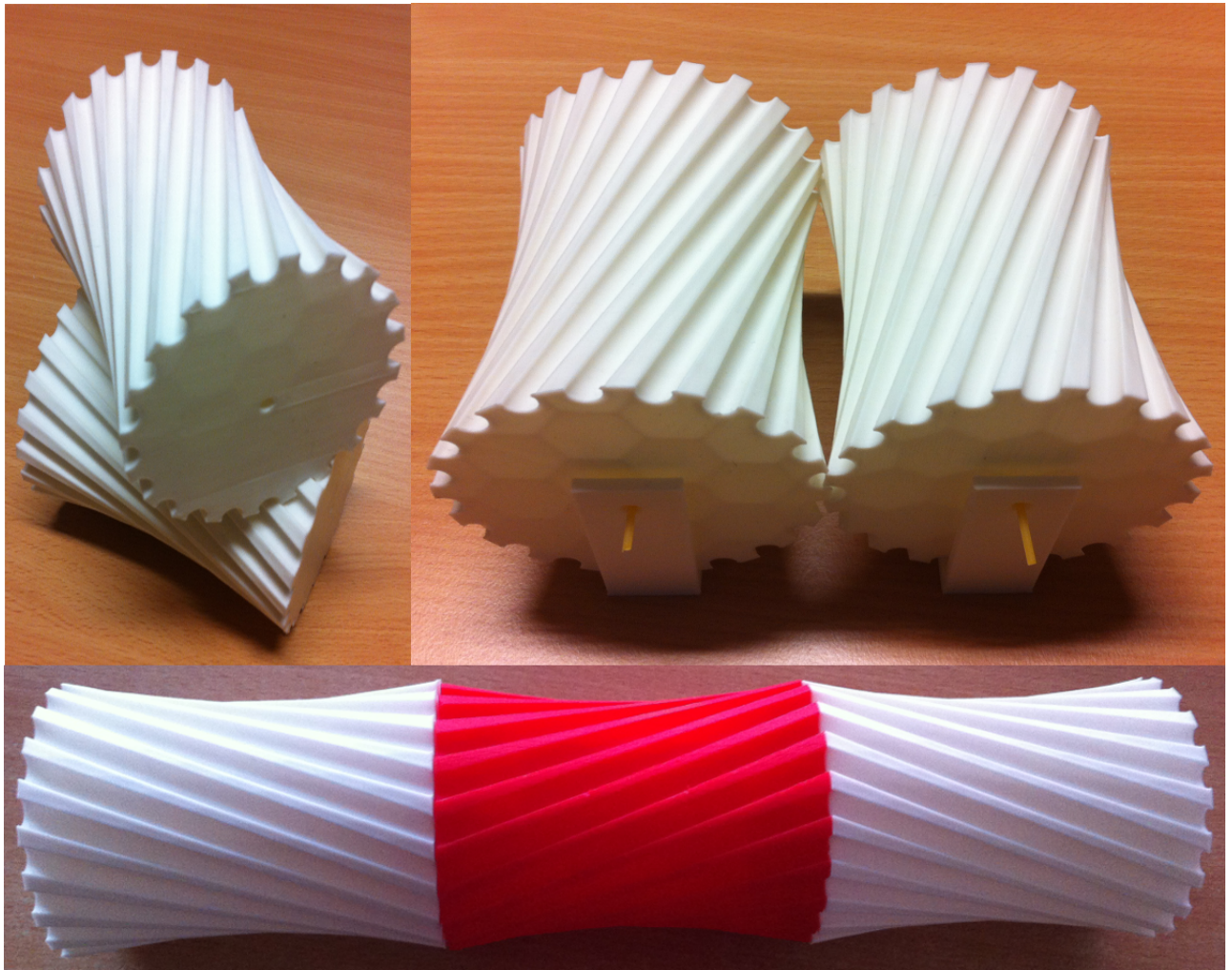
Fig. 32.5 One sheet hyperboloid 3D print

Source: own elaboration

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Abstract: *We have begun to innovate our lectures within the course "Geometry on Computer". These innovations are supported by the FRVS grants. We use programs GeoGebra and Open-SCAD for teaching plane and 3D geometry. The GeoGebra allows us to concentrate on mathematical and geometrical principles of solving problems. OpenSCAD is basic programmable 3D CAD modeler. The content of the course has been compiled with respect to requirements of students in their future studies and practical engineering use. In this paper we present application problems where following principals of classic geometry are used: articulated antiparallelogram, parametrization of functions and their use in 3D modeling and solving navigation problems.*

Keywords: *GeoGebra, OpenSCAD, 3D modelling, hyperbola, hyperboloid, antiparallelogram*

VYUŽITÍ HYPERBOLY A HYPERBOLOIDU V TECHNICKÉ PRAXI

Abstrakt: *Díky grantu FRVŠ jsme inovovali předmět "Geometrie na počítači". Pro názornou výuku rovinné a prostorové geometrie využíváme programy GeoGebra a OpenSCAD. Snadností použití nám GeoGebra umožňuje zaměřit se na matematické a geometrické problémy řešených úloh. OpenSCAD je jednoduchý programovatelný 3D CAD systém. Problémy řešené v tomto kurzu jsme se snažili vybírat s ohledem na jejich možné využití v následujícím studiu nebo praxi. V tomto článku ukážeme praktické využití několika úloh klasické geometrie: kloubový antiparalelogram, parametrizace křivky a její využití ve 3D modelování, řešení navigační úlohy.*

Klíčová slova: *GeoGebra, OpenSCAD, 3D modelování, hyperbola, hyporboloid, antiparalelogram*

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