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Position Fixing and Uncertainty

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ABSTRACT: Taken random observations are usually accompanied by rectified knowledge regarding their behaviour. In modern computer applications, raw data sets are usually exploited at learning phase. At this stage, available data are explored in order to extract necessary parameters required within the inference scheme computations. Crude data processing enables conditional dependencies extraction. It starts with upgrading histograms and their uncertainty estimation. Exploiting principles of fuzzy systems one can obtain modified step-wise structure in the form of locally injective density functions. They can be perceived as conditional dependency diagrams with identified uncertainty that enables constructing basic probability assignments. Belief, uncertainty and plausibility measures are extracted from initial raw data sets. The paper undertakes problem of belief structures upgraded from uncertainty model in order to solve the position fixing problem. The author intention is presenting position fixing as an inference scheme. The scheme engages evidence, hypothesis and revokes concept of conditional relationships.

1 INTRODUCTION

In recent papers delivered by the author, fuzzy systems and Mathematical Theory of Evidence were used as a platform for processing uncertainty [4, 5, 6]. Models require methods to obtain objective evaluation of uncertainty. In nautical science, sets of random variables instances are exploited as main source of knowledge on observations. Usually they were perceived as governed by Gaussian dispersion patterns. Knowing the magnitude of standard deviations enables introduction of an observation rough assessment. This attitude is popular among navigators. Modern computer procedures accept uncertainty as an element of a processing scheme. Identification of doubtfulness was discussed in the author's publications [7, 8]. Proposed approach exploited evidence proximity exploration and engaged principles of fuzzy systems. Suggestion was that an application should include analyses of available raw recorded instances in order to extract required range of useful parameters. Introducing uncertainty model enriches the approach. This paper contains presentation of dealing with doubtfulness using its popular model. Association of uncertainty items are included and result with its increased informative context discussed. Further, histograms conversion is recalled and their embedded belief and uncertainty extracted. Concluded part concentrates on numerical example being an output of the application implementing presented approach. For those who want to get deeper insight into the terminology and the engaging scheme of reasoning the author recommends recent excellent book [1].

2 MODELLING UNCERTAINTY

Let us consider a problem of evaluation of a truth of a statement. It is popular that to some extent the proposition is considered true. There is also interval where it is treated as uncertain. Finally, range of false truth of the statement might exist. Figure 1 presents probability vs possibility diagram as uncertainty representation. The polyline is a membership function that specifies fuzzy probability set of the proposition truth. One can use interval [a, b] to define the function [10]. The abbreviation indicates three subsets: [0, a); [a, b); [b, 1] that feature the diagram. It should be noticed that a and b are equivalent to belief and plausibility measures adopted in Mathematical Theory of Evidence (MTE) [11]. One can upgrade models for popular statements such as "I am convinced that something is true, at the same time some doubtfulness exists and lack of acceptance might also be present". In formal way, the proposition can be written using Equation (1). Formula (1a) could be followed for mentioned uncertainty model [2].

$$\mathrm{m}(e)=\left\{\left(T,\, m(T)\right),\, \left(F,\, m(F)\right), \left(\{T,F\},\, m(\{T,F\})\right)\right\} \tag{1}$$

$$\mathrm{m}(e)=\left\{\left(T,\, a\right),\, \left(F,\, 1-b\right), \left(\{T,F\},\; b-a\right)\right\} \tag{1a}$$

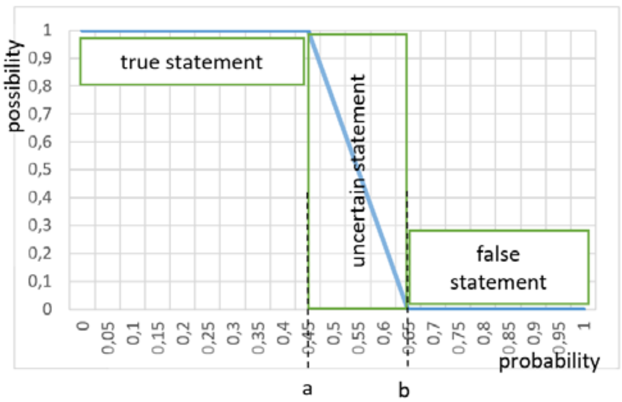


Figure 1. Diagram of uncertainty representation

Fragments of evidence and hypotheses exist in inference schemes. To some extent each piece of available data supports given hypothesis, uncertainty of the backing is unavoidable. Quite often lack of support also occurs. In nautical science, there are observations and sets of points considered as potential true locations of a vessel. Each measurement supports the true location to certain degree. Mentioned relations can be meant as conditional dependencies. Conditional relationships can be considered as a function that identifies belief and plausibility of support measures for hypothesis items embedded within each of the evidence fragments. Observations neighbourhood explorations were subject of the recent publication by the author [8].

3 COMBINING UNCERTAINTIES

Many persons or methods might evaluate the same statement. It is usual that extent the proposition is considered true varies. The same refers to uncertainty and false truth of the statement. Problem of combined assessment of a truth of the statement assessed by different experts or delivered from various sources appears practical. Figure 2 presents diagrams of uncertainty representations and result of their combination. Intervals [ai, bi] were used to define the respective membership functions. Note that assignment feature lower uncertainty contributes more decisively to the result of combination. It is in line with popular meaning of weighted contribution from various quality inputs. The idea is native for MTE's scheme of combination.

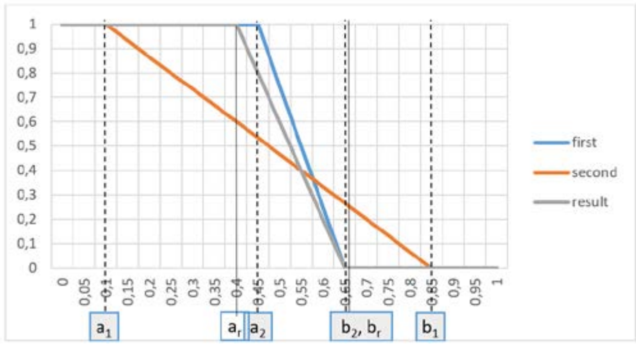


Figure 2. Two diagrams of uncertainty representations and result of their combination

In formal way, the two propositions can be written using Equation (2). Formula (2a) could be followed for mentioned uncertainty models.

$$\begin{aligned}\mathrm{m}(e_1)&=\left\{\left(T_1,\, m(T_1)\right),\, \left(F_1,\, m(F_1)\right), \left(\{T_1,F_1\},\, m(\{T_1,F_1\})\right)\right\}\\ \mathrm{m}(e_2)&=\left\{\left(T_2,\, m(T_2)\right),\, \left(F_2,\, m(F_2)\right), \left(\{T_2,F_2\},\, m(\{T_2,F_2\})\right)\right\}\end{aligned} \tag{2}$$

$$\begin{aligned}\mathrm{m}(e_1)&=\left\{\left(T_1, a_1\right),\, \left(F_1,\; 1-b_1\right), \left(\{T_1,\, F_1\},\, b_1-a_1\right)\right\}\\ \mathrm{m}(e_2)&=\left\{\left(T_2,\, a_2\right),\, \left(F_2, 1-b_2\right), \left(\{T_2,F_2\},\, b_2-a_2\right)\right\}\end{aligned} \tag{2a}$$

Combination scheme [3, 10] adopted for association of two structures being belief distributions and illustrated at Figure 2 is presented in Table 1. First structure is shown in shaded part of the first raw, second one presents first column. Each cell contains two elements: involved set and a mass attributed to the set. Note that for an assignment total mass is equal to 1. Single element sets identify range of true (T) and false statement (F). Sets consisting of two items represent uncertainty. Association partial results are included in other cells of the Table. Each result contains two elements: combined involved sets and product of their masses. Intersection of engaged sets are required while conjunctive structures association is carried out. In considered case two sets common part is empty when {T} and {F} are being combined. Instead of empty set ∅ uncertainty equivalent molecule {T, F} was used in respective cells. In this way non-null generating associations are obtained. It should be noted that the idea follows the Hau-Kashyap and Yager concepts [9, 12] of normalization. The one is recommended for discussed scope of applications.

Table 1. Combination scheme

set	$\{T_1\}$	$\{F_1\}$	$\{T_1, F_1\}$	Result
mass	0.100	0.150	0.750	
$\{T_2\}$	{T}	{T, F}	{T}	{Tr}
0.45	0.045	0.068	0.338	0.403
$\{F_2\}$	{T, F}	{F}	{F}	{Fr}
0.35	0.035	0.053	0.263	0.345
$\{T_2, F_2\}$	{T}	{F}	{T, F}	{Tr, Fr}
0.20	0.020	0.030	0.150	0.253

T_i – range of *i*-th true statement equals to a_i
F_i – range of *i*-th false statement equals to $1-b_i$
T_i, F_i – range of *i*-th uncertainty equals to $b_i - a_i$

One expert claims that he beliefs that given statement is true with probability at most 0.10. The upper limit of accepting the proposition as true is 0.85. Thus, the range of uncertainty is 0.75 and simple model takes the form [0.10, 0.85]. Other expert beliefs that the statement is true with probability up to 0.45. The upper limit of accepting the proposition as true is 0.65. This time the range of uncertainty is 0.20 and simple model takes the form [0.45, 0.65]. Obtaining overall opinion on the truth of the statement is the challenge. Adequate solution delivers combination of available expertise, obtained model is [0.403, 0.656] (see result diagram at Figure 2).

In nautical practice, there are randomly distorted indications. Referring to one of the observations given position represents the true observer location with probability at most a_1. Note that probability is meant as product of density and width of adjacent area. Hereto unitary range is assumed. The upper limit of accepting the representation as true is b_1. Thus, the range of uncertainty is $b_1 - a_1$ and simple model takes the form $[a_1, b_1]$. Other observation indicates that the statement is true with probability up to a_2. The upper limit of accepting the proposition as true is b_2. This time the range of uncertainty is $b_2 - a_2$ and simple model takes the form $[a_2, b_2]$. Association of available indications evaluates the truth regarding given location. Combination result diagram is very much like the one shown at Figure 2. Note that more assertive individual, this with lower uncertainty, dominates the final solution. Challenging are proposals of methods estimating uncertainty models elements $[a_i, b_i]$.

4 DISCOVERING DEPENDENCIES AND THEIR UNCERTAINTIES

Figure 3 illustrates the idea of proposed processing scheme aiming at dependencies extracting. Part a) displays a piece of evidence (labelled *o2*) with a set of instances showing its dispersion. The four hypothesis points labelled with *Hi* is also presented. The statement that a given location *Hi* is the true position gain some endorsement from this piece of available evidence. From the Figure, one can perceive that *H1* is the point with the highest support in this matter provided lack of a systematic deflection. For the case, conditional dependence $P(H \mid O)$ is to be considered as a function that identify measure of support for hypothesis item *Hi* embedded within an evidence fragment o_j. Thanks to the relationships appropriate supports can be obtained and evaluated. Respective metric is calculated and analysed. To obtain the support, x- and y-axis histograms are upgraded based on crude data, instances related to the indication. Then they are converted to gain stipulated continuous shape.

Part b) of Figure 1 shows the result of processing the initial instance set. At first raw data are converted to step-wise histograms. For these structures, evaluation of trustfulness of an observation at hand is to take place. Uncertainty of 0.37 is estimated for presented case. The value is obtained based on vertical and horizontal expansion of each histogram. Respective partial metricises are also presented.

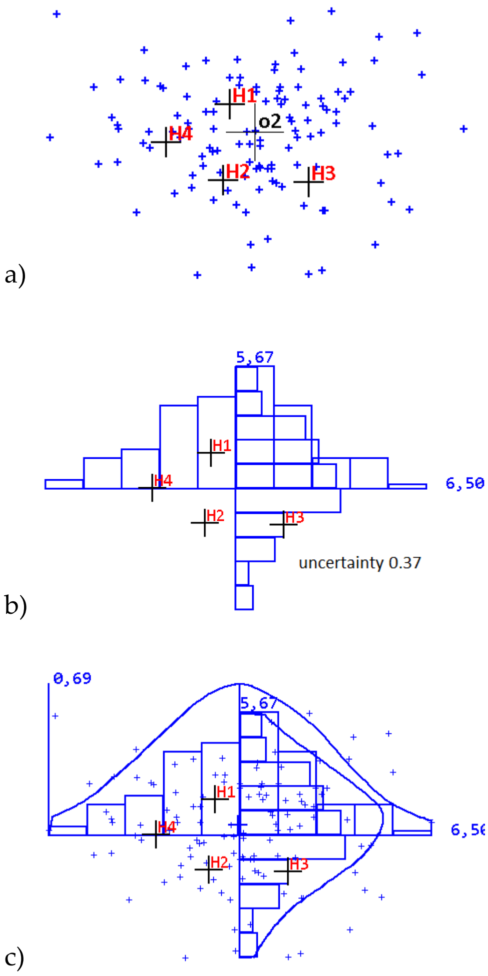


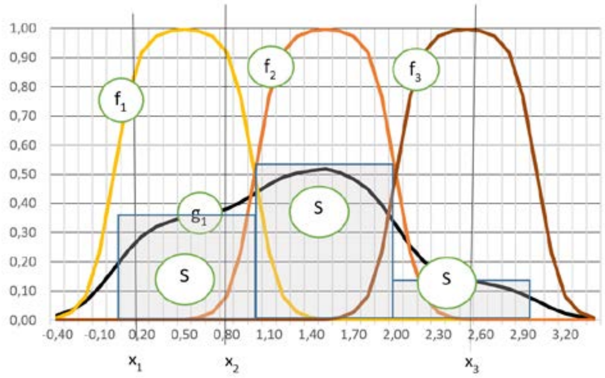
Figure 3. (a) Single observation with an example of its two-dimension dispersion set and four hypothesis locations; (b) horizontal and vertical axis histograms; (c) histograms and their converted to continuous function versions.

Based on obtained results, further on rectangular cells structures are transformed to injective diagrams of density functions shown at part c) of the Figure. Taking advantage of fuzzy sets and Bayesian conditional dependencies, an adequate converting method was implemented. Locally injective function is required in order to obtain solutions for many problems engaging random variables [7]. Figure 3 presents a basic scheme followed at the first stage of imprecise data handling. At the stage uncertainty embedded in initial data set are to be discovered and injective density functions identified. The function diagram shows support plausibility of density a position being located approximately particular point.

5 HISTOGRAMS EVALUATION AND CONVERSION

Randomly distorted evidence can be accompanied by sets of recorded instances, which are traditionally converted to a histogram. It displays a diagram of the distribution of observed data. A histogram consists of adjacent rectangles, primarily erected over non-overlapping intervals. The histogram is usually normalized and displays relative frequencies considered as empirical probability densities. It shows the proportion of cases that fall into each of bins. The intervals are usually chosen to be of the same width, percentage of the value limits the range very often. It is assumed that family of instances sets of relative frequencies are given as a result of a long term observations. Histograms quality should be evaluated in order to get knowledge and variables processed in many applications. Although histograms are popular and widely used, attempt of their quality assessment was proposed by the author [8].

Histograms should be subject to evaluation, intuitively and objectively. MTE's scheme of combination stipulates knowledge of uncertainty embedded within engaged structures. Histograms quality differs. Differentiations refer to their bin heights and ranges as well as to the whole structure expansion. Uncertainty refers to a certain feature within the discussed scope. The number of items with the same or almost the same value of the feature defines uncertainty. In this view, uncertainty might be related to distinguishability. One should note that points within single cell are not distinguishable. Following this way of reasoning one can conclude that the wider is histogram the higher is its uncertainty. For this reason, uncertainty of rectangular cells histogram is higher compare to continuous version of density distribution (see Figure 3).



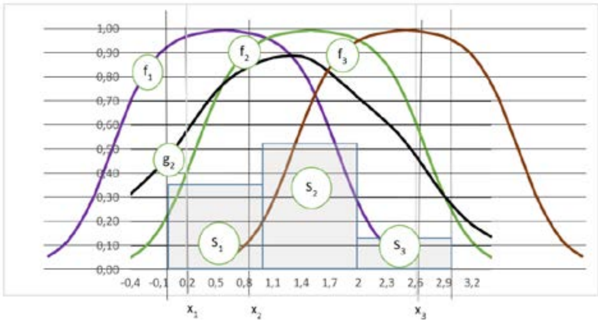
S_i i-th histogram cell, crisp valued limited area showing number of observations falling within the range.
f_i is membership function for i-th cell.
g_1 is a diagram of converted histogram.
Figure 4. Three bins histogram with membership functions for each cell for uncertainty level 0.05

The idea of bin-to-bin additive method is crucial for the histograms transformations concept [7]. Modern approach enables treating rectangular cells as fuzzy density sets. Limitations of such sets are established by membership functions which diagrams are uncertainty dependent. Figure 4 presents diagrams of membership functions for three cells histogram with rather low uncertainty level (assumed 0.05). Table 2 gathers data referring to items presented in the Figure. The Table contains membership grades for each of the marked points within consecutive cell. The assigned cell densities are included in the last raw. Result measures, highest densities approximately i-th point are included in the second last column. Included values are plausibility measures since way of their calculations refers to fuzzy systems well-known formula [3]. Note that density contributed by a cell is a product of the cell density and grade of belonging to the bin. One should perceive the value as bi, the title used in Figure 2. Thus to get belief (a_i) one should subtract uncertainty from plausibility value. The values are included in the last column.

Table 2. Grades of belonging to the histogram cells and result of bin to bin product summation for example set of points

point	$\mu_i(S_1)$	$\mu_i(S_2)$	$\mu_i(S_3)$	$pl(x_i)$	$bel(x_i)$
x_1	0.830	0.000	0.000	0.291	0.241
x_2	0.910	0.100	0.000	0.371	0.321
x_3	0.000	0.002	0.990	0.130	0.080
$m(S_i)$	0.350	0.520	0.130		

$\mu_i(S_k)$ grade of i-th point belonging to k-th bin
$pl(x_i)$ plausible, highest probable density measure in the vicinity of i-th point
$bel(x_i)$belief, highest certain density measure in the vicinity of i-th point



S_i i-th histogram cell, crisp valued limited area showing number of observations falling within the range.
f_i is membership function for i-th cell.
g_2 is converted histogram.
Figure 5. Three bins histogram with membership functions for consecutive cells for uncertainty level 0.44

Similar to Figure 4 is the next Figure 5, which presents diagrams of membership functions for three cells histogram with much higher uncertainty level assumed equal to 0.44. Due to higher uncertainty, ranges of membership functions expand compare to those presented at Figure 4. For discussion on relation between scope of range and doubtfulness, refer to [8]. Table 3 contains data referring to items presented in the Figure. The Table contains membership grades for each of marked points within consecutive cell. As before the assigned cell densities are included in the last raw. Result plausibility measures also referred to as b_i, highest possible densities approximately i-th point are included in the second last column. Result belief measures also referred to as a_i, lowest densities approximately i-th point are included in the last column. Grades of belonging to adjacent cells are much higher compare to those included in table 2. Figure 4 and 5 present three cells histogram and its two converted versions obtained for various uncertainty levels (see curves g_1 and g_2 at respective

Figure). Both were obtained using presented scheme of calculations regarding three example points. Obviously, number of locations involved was much greater.

Table 3. Example set of points, their belonging to the histogram cells and result plausibility

point	$\mu_i(S_1)$	$\mu_i(S_2)$	$\mu_i(S_3)$	$pl(x_i)$	$bel(x_i)$
x_1	0.960	0.410	0.005	0.550	0.110
x_2	0.980	0.910	0.120	0.832	0.392
x_3	0.040	0.590	0.980	0.448	0.008
$m(S_i)$	0.350	0.520	0.130		

$\mu_i(S_k)$ grade of i-th point belonging to k-th bin
$pl(x_i)$ plausible highest, probable density measure in the vicinity of i-th point
$bel(x_i)$ belief highest, certain density measure in the vicinity of i-th point

6 EVALUATING UNCERTAINTY

Random data evaluation aims at the discovery of certain patterns included within available sets of their instances. MTE combination scheme stipulates probability assignments, which include uncertainty. Differentiations refer to the histogram bin heights and ranges as well as to the structure expansion. Uncertainty refers to a certain feature within the discussed domain. In discussed field, uncertainty is related to discernibility. The more discernible abscissa points the less amount of uncertainty contains histogram. Thus, the diversity of bin heights can be proposed as a factor to measure distinguishability. The average of cases falling below and above the mean line of a histogram can be perceived as an objective measure of a bin heights diversity. Inserted numbers in Figure 2 are respective metrics for each the presented cases. The greater the numbers and smaller the horizontal extension the "better" is a histogram. Total of points that fall above and below presented horizontal line may be zero when uniform distribution of the feature is involved. Note that this is in line with the popular understanding of uncertainty. One can perceive doubtfulness as thinking of something that might be somewhere within a given scope but there is no hint as to where in particular. One can perceive histogram with the same cells heights as extremely unreliable. It should be noted that in such case total of instances below and above the central line is zero. As mentioned above, the number of items with the same value of the feature defines uncertainty. Thus, considering two histograms with the same numbers of bins and different bin widths, one can assume that wider structure embeds greater uncertainty.

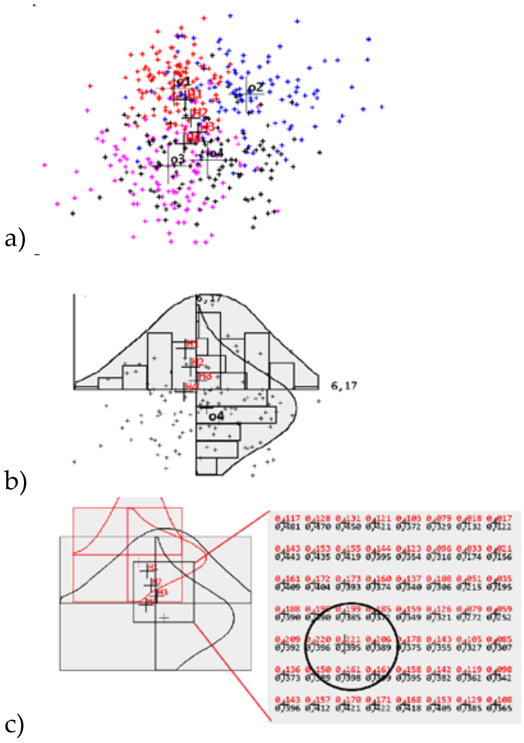


Figure 6. Results of case study engaging four observations, selected histograms converted versions and four hypotheses points. (a) four observations with their recorded dispersion sets, hypotheses points are also included (b) dispersion set for observation number 4, its histogram and converted diagram (c) converted histograms for observations 1 and 4 with exploded insertion containing belief and uncertainty measures for example mesh of points.

Results of a case study engaging four observations with their instances dispersions, selected sets converted versions; four hypotheses points histograms and result of exploration of included space of discernment are presented at Figure 6. Exploded insertion contains belief measures and uncertainties for selected mesh of points.

Set of hypothesis points from figure 6, their supporting by depicted observations and measures that they are the true location of the ship are gathered in the Table 4. Available indications feature included relative uncertainty. Given the input data one should consider point x_3 as the best approximation of the ship location. The point feature the highest belief and reasonable uncertainty.

6.1 *Normalizing a pool of data*

Position fixing problem exploits a pool of various quality observations. They need to be evaluated prior to the final usage. Preparing a pool of data requires their normalization in order to achieve uniform density distributions [8]. This enable construction of adequate conditional dependency functions. Also expected is relative uncertainties vector to enable definitions of basic probability assignments. Fixing aims at the selection of a point that represents the true location of a ship in best way. An evidence fragment supports the choice of each item from the considered set of hypothesis. Degrees of support are expressed as conditional dependability that rely on credibility attributed to the evidence items. Those with low uncertainty should contribute to the selection more decisively. To implement the idea, one has to create a hierarchy for available evidence.

It is quite often that relative rather than absolute uncertainty is of primary importance. Position fixing is an example problem where relativity of uncertainties really matters. The idea is establishing grades affecting the final solution by each piece of evidence compared to other ones. The relative weight of contributing to the final solution is an important issue. Given a set of histograms, their ranges and bin heights for each structure, adequate measures can be calculated [8].

Table 4. Example set of points, their supporting by observations at hand and measures that they are the true location of the ship

	plausibility of support from observation:				data on representing a fixed position		
point	$o_1/0.15$	$o_2/0.29$	$o_3/0.28$	$o_4/0.26$	$bel(x_i)$	$pl(x_i)$	$un(x_i)$
x_1	0.613	0.537	0.463	0.416	0,145	0.585	0,44
x_2	0.474	0.514	0.540	0.514	0,192	0.582	0,39
x_3	0.363	0.473	0.594	0.593	0,214	0.604	0,39
x_4	0.000	0.364	0.659	0.627	0,157	0.557	0,40

o_i/X *i*-th observation with its relative uncertainty
$bel(x_i)$ belief measure that x_i is the true location of the ship (refers to a_i in the uncertainty model)
$pl(x_i)$ plausibility measure that x_i is the true location of the ship (b_i in the uncertainty model)
$un(x_i)$ uncertainty that x_i represents true location of the ship (b_i - a_i) in the uncertainty model)

Vertical and horizontal layout uncertainty of a histogram are considered. Measures referred to variety of bin heights and to their widths enabled estimation of an overall uncertainty. Doubtfulness amount and shape of the membership functions are mutually dependent. Both are main factor that decide on shape of converted histograms, which can be seen as conditional dependencies adequate diagrams. Pool of data required for solving the problem engaging distorted data, needs additional normalization. Processing introduces comparable probability density distributions. For this purpose, expected is ranking regarding decisiveness on affecting the solution. Items with lower uncertainty are of greatest influence in this respect. Final ranking list regarding amounts of embedded uncertainty for the poll of four observations is {*o1/0.15, o4/0.26, o3/0.28, o2/0.29*}. Indication o1 mostly decide on the final selection. Figure 6 part c) includes example application screenshot with belief and uncertainty calculated for a rectangular frame of discernment. The distinguished fragment contains items with the highest beliefs along with rather low uncertainties.

7 CONCLUSIONS

In nautical practice, there are randomly distorted indications or observations. Referring to one of the available items one can discover support that given position represents the true observer location. Range the proposition is considered true varies from observation to observation. The same refers to uncertainty and false truth of the statement. Items such as belief, plausibility and uncertainty are included in uncertainty model that is presented at the beginning of the paper. Combining models, assessments of a truth of the statement extracted from various observations delivers fixed position. At the beginning, the paper contains combination of two structures being belief functions. It should be noted that result diagram is very much like less uncertain compound. More accurate observations dominate others while position fixing or other problems involving randomly distorted data. Individuals that are more assertive dominate final opinion.

Exploiting presented model one requires methods of extracting data from sets of recorded instances of given random variable. Proposal of exploration of the raw data deliver good estimates of uncertainty models. Partial results of processing are items of the popular uncertainty model architecture. For example, locally injective transformed histogram shows plausibility measures. It can upgraded with reference to given uncertainty. Overall evaluation of reasoning on the true location can be delivered by combination of the assignments created based on available indications. Calculated belief and uncertainty measures are helpful when the fixed position is selected. Solution is an item feature highest belief, in case of ambiguity one has to choose smallest uncertainty.

Uncertainty regarding vertical and horizontal layout of a histogram are considered. Measures referred to variety of bin heights and to their widths enabled estimation of an overall and relative uncertainty ranked among the available observations. Shape of the cell's membership functions depends on doubtfulness feature by given piece of evidence. Converted histograms can be seen as conditional dependencies diagrams [7]. Pool of data used for position fixing needs to be normalized. Additional processing introduces uniform, relatively balanced density distributions. Initial reference vector is required in order to obtain the hierarchy and subsequently adequate probability assignments.

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