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DESIGN RESISTANCE OF WELDED KNEES IN STEEL FRAMES

The paper presents issues related to calculations of welded knee joints, in which the interconnected load-bearing elements, beams and columns, can be made from plate girders with slender webs.
At the beginning, a typical knee joint of portal steel frame was characterized, along with presentation of calculations for the internal forces in characteristic zones of the knee, i.e. in tension, shear, and compression zone. Then, the checking procedures of resistance were presented in detail for each designated knee zone, paying particular attention to the influence of complex stresses state and loss of stability in the shear and the compression part of knee joint.
The work also presents a comprehensive calculation example, which illustrates the described method usage in practical design of welded knees of steel frames.

Keywords: steel frame, knee joint, transverse stiffeners, diagonal stiffeners, resistance

1. Introduction

Knee of steel frame, also called a knee joint (or an eaves joint), is a connection place of the load-bearing elements of the frame, i.e. the beam (rafter) and the column. Due to the kind of connected elements used, that joint can be classified as so-called, beam-to-column joints.

Designing this type of joints is quite difficult, mainly due to the complex geometry, which in turn causes a complex system of forces (stresses) acting in the knee. This mainly applies to design of bolted end-plate joints, where the need to transfer large internal forces and getting appropriate stiffness make calculations for this type of joints rather difficult [1–3].

Application of the solution in the form of welded knee joint often allows with less effort to design the joint with appropriate large resistance and stiffness, which often is also easier to manufacture. In that cases the bolted connection assembly can be located near the knee, e.g. in the beam, where there are generally smaller internal forces.

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Nowadays, the most commonly used method in design of steel frames joints is, a component method, recommended by Eurocode [4]. It is characterized by fairly high versatility, because the basic assumptions of that method allowed to generalize and adapt the computational algorithms so that it could be used to design welded and bolted internal joints, as well as column bases.

Despite this, application range of this method, included in the standard [4], has its own limitations. They result, among others, from the conditions of joint plates slenderness, limiting the use of Eurocode [4] to cases, where, e.g. sheared panel of column web is insensitive to instability.

This inconvenience makes it difficult to use the standard [4] when designing steel frames knees, in which the bearing elements (beams and columns) are designed from plate girders cross-sections, characterized by significant slenderness of the webs. Due to the fact that frames with plated structural elements are often used in steel construction, the procedures included in the standard [4] should be generalized in such a way that would allow to design the knee joints in that type of frames. This condition, according to the author, is met with the computational method of rectangular knees of steel frames discussed in this paper, which is partially based of information presented in [5], and also takes into account EC3 [4] provisions.

2. Analysis of forces acting in frame knee

Due to the complex geometry of a typical knee, the way of combining individual elements (type of welds used, technology used in performance of welded connections), as well as issues of plates instabilities cause that accurate static and strength analysis of the considered joint requires appropriate software usage (e.g. based on the finite elements method) and the use of very complex numerical models [6].

In practical design, generally, there is no need to conduct very detailed calculations of frames knees [7]. Considering the equilibrium conditions of the loaded model of knee joint (Fig. 1a), stress distribution and forces system acting on the individual components of the joint can be obtained at rough estimate (Fig. 1b and 1c). Thus, the values of the forces acting respectively in the tension zones (F_{bt} and F_{ct}) and the compression zones (F_{bc} and F_{cc}) of the joint can be approximately calculated from the equations:

$$F_{bt} = \frac{M_b}{b} - \frac{N_b}{2}, \quad F_{bc} = \frac{M_b}{b} + \frac{N_b}{2}, \quad F_{ct} = \frac{M_c}{c} - \frac{N_c}{2}, \quad F_{cc} = \frac{M_c}{c} + \frac{N_c}{2} \tag{1}$$

whereas, the shear forces values, corresponding to the tangential stresses in panel of the joint web can be determined with the use of expressions (Fig. 1d):

$$V_{sb} = F_{cc} - V_b = F_{ct}, \quad V_{sc} = F_{bc} - V_c = F_{bt} \tag{2}$$

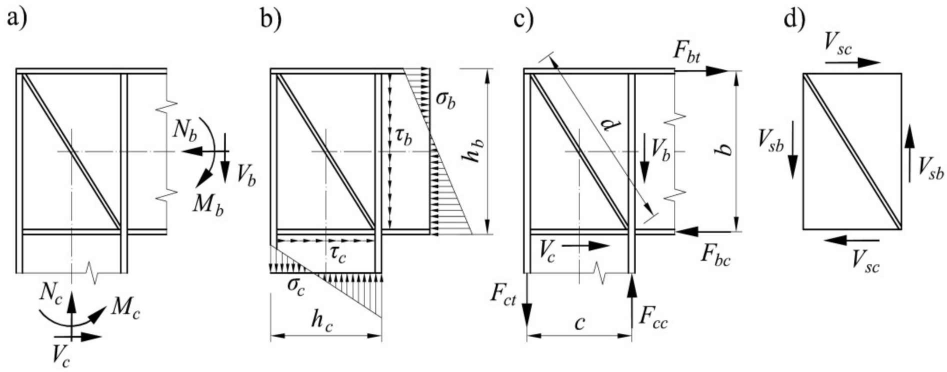

Fig. 1. Knee of frame analyzed: a) loading scheme of the knee, b) approx. stresses distribution in the knee, c) simplified system of the forces in the knee, d) scheme of the shear forces in the web panel

In case when the cross-sections of the column and the beam, as well as the welded connections have the required resistances, the calculation of the considered joint can be reduced to checking the resistance conditions in the tension, compression and shear zones.

3. Dimensioning of frame knee

Knowing the system and the type of forces acting in the separated zones of the joint, it is relatively easy to formulate the appropriate resistance conditions for each of mentioned zones. It gives the possibility to design the joint without having to run, often more complex calculations of resistance of the entire joint, e.g. due to bending.

3.1. Resistance of tension zone

Checking the resistance condition of the tension zone can be performed according to the expression:

$$\frac{F_{bt}}{F_{t.Rd}} \leq 1 \tag{3}$$

where $F_{t.Rd}$ is a resistance of tension zone of the welded joint, which can be obtained in accordance with point 6.2.6.3 of standards [4].

In case of knee joints of portal frames the top flange of beam is generally elongated in such a way that it could be directly connected with the exterior flange of column, thus covering the top edge of column (see Fig. 1a). In that case demonstration of the condition (3) can be presented in the form of resistance condition of the top flange cross-section of the beam, in which the tension resistance of the flange determines the equation:

$$F_{t.Rd} = \frac{b_{fb}\, t_{fb}\, f_y}{\gamma_{M0}} \tag{4}$$

where: b_{fb} and t_{fb} are respectively the beam flange width and thickness, f_y is the yield strength of steel, and γ_{M0} is the partial factor.

3.2. Resistance of shear zone

Checking of the shear zone resistance can be carried out on the basis of the condition in the form:

$$\frac{V_{sc}}{V_{wc.Rd}} \leq 1 \tag{5}$$

in which $V_{wc.Rd}$is the resistance of the shear zone in the form of the web panel.

The procedure of the resistance calculations should begin with sensitivity assessment of the column web panel due to the shear instability. In case, when the web panel considered is freely supported on all four edges (Fig. 2a), the sensitivity condition to instability of the web panel can be presented in the form [9]:

$$\lambda_w = \frac{c}{t_{wc}} \leq \frac{31}{\eta} \varepsilon \sqrt{k_\tau} \tag{6}$$

where: η = 1.2 if $f_y \leq$ 460 MPa, whereas k_τ is a coefficient taking into account boundary conditions and stresses distribution in the analyzed plate.

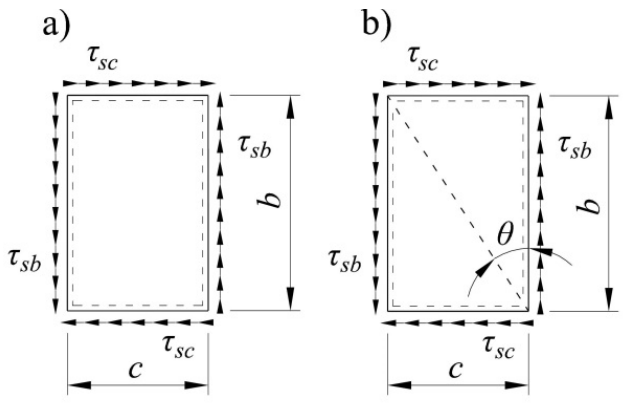

Fig. 2. Analyzed web panel of knee: a) single rectangular panel, b) system of two triangular panels

In the considered case the coefficient k_τ can be determined with the use of expressions [9]:

$$k_\tau(\alpha) = 4 + \frac{5.34}{\alpha^2}, \text{ if } \alpha \le 1$$
$$k_\tau(\alpha) = 5.34 + \frac{4}{\alpha^2}, \text{ if } \alpha > 1 \quad (7)$$

where $\alpha \approx b / c$ (see Fig. 2a).

Application of additional stiffening of the shear zone in the form of diagonal stiffeners leads to the change of supporting conditions in the web panel, converting one rectangular panel into two triangular panels (Fig. 2b). In such cases, the coefficient k_τ can be determined according to the relationships given in the works [10], [11]:

$$k_\tau(\xi) = 5.34\left(1+\xi^2\right)+19.3\xi, \text{ for compressed stiffeners}$$
$$k_\tau(\xi) = 5.34\left(1+\xi^2\right)+0.87\xi, \text{ for tensioned stiffeners} \quad (8)$$

which, as in case of the rectangular plate, were formulated on the assumption that the exterior edges of both panels are freely supported (Fig. 2). The graphical interpretation of coefficients value change k_τ, in the angle function θ of the stiffening ribs inclination, (see Fig. 2b) is shown in Figure 3.

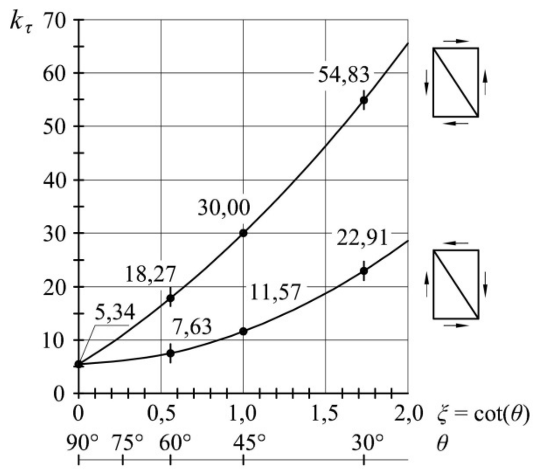

Fig. 3. Relationship of the coefficient k_τ from angle θ

It can be noticed that the presented graphs of the $k_\tau(\xi)$ function clearly indicate that supporting of the web panel with diagonal stiffeners reduces its sensitivity to loss of stability, especially when forces in the knee induce compression of the stiffeners.

Resistance determination of the web panel in case, when the condition (6) is met, can be determined according to the formula [12].

$$V_{wc.Rd} = \frac{A_{wc}\ f_y}{\sqrt{3}\ \gamma_{M0}} \sqrt{1 - \left(\frac{\sigma}{f_y}\right)^2} \tag{9}$$

where $A_{wc} = h_{wc}\, t_{wc}$ is a shear area, while σ is stresses value from axial force in the column. In case, where σ does not exceed 50% of yield strength f_y, the resistance of sheared plate can be determined according to the equation [4]:

$$V_{wc.Rd} = 0.9\ \frac{A_{wc}\ f_{y.wc}}{\sqrt{3}\ \gamma_{M0}} \tag{10}$$

If, however, the slenderness condition of stiffened web (6) is not met, then its resistance can be determined with the formula [9]:

$$V_{wc.Rd} = \chi_w \frac{A_{wc}\ f_y}{\sqrt{3}\ \gamma_{M0}} \tag{11}$$

in which χ_w is a shear buckling factor. Assuming, in accordance with the provisions of the standard [9], that the applied web stiffeners belong to, so-called, flexible stiffeners (non-rigid end post), that the parameter value χ_w can be determined based on the equation:

$$\chi_w = \min\left(0.9;\ \frac{0{,}83}{\bar{\lambda}_w}\right) \tag{12}$$

where $\bar{\lambda}_w$ is a shear panel slenderness:

$$\bar{\lambda}_w = \sqrt{\frac{f_y}{\sqrt{3}\ \tau_{cr}}} \tag{13}$$

Value of elastic shear buckling stresses τ_{cr} can be calculated on the basis of formula:

$$\tau_{cr} = \frac{\pi^2 E\, k_\tau}{12\left(1-\nu^2\right)\lambda_w^{\ 2}} \tag{14}$$

in which parameters E i ν are respectively the modulus of elasticity and the Poisson's ratio.

In case of the stiffened knee with diagonal stiffeners, special attention should be paid to the fact, that the components of the shear zone – web panel and the stiffeners can achieve resistance under the influence of significantly differing forces. In order to determine the appropriate load distribution for each of the listed components, a system of equations can be formulated in the form:

$$\begin{aligned} V_{sc} &= V_{wc} + V_{sd} \\ \delta &= \delta_{wc} = \delta_{sd} \end{aligned} \tag{15}$$

The first equation results from the equilibrium conditions of the forces in the shear zone, while the second one describes the conditions of displacements compatibility in this zone (Figure 4).

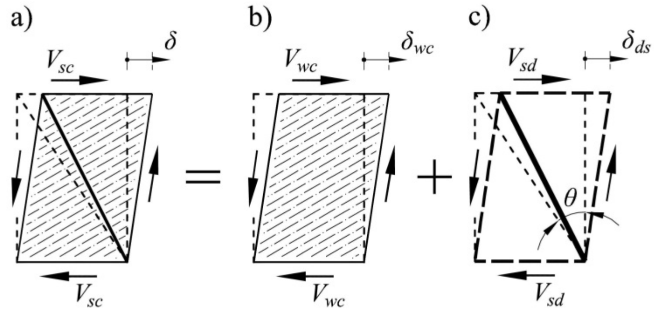

Fig. 4. Distribution of forces and deformation: a) whole shear area, b) web panel, c) stiffening ribs

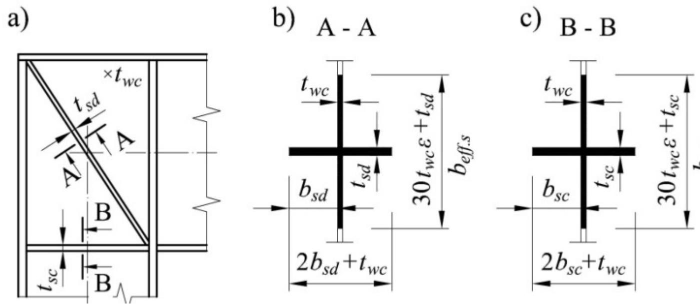

Fig. 5. Stiffeners of the knee joint: a) strengthening in the form of transverse and diagonal stiffeners, b) stiffened cross-section of the shear zone, c) stiffened cross-section of the compressed zone

Displacements caused by V_{wc} and V_{sd} forces acting on the relevant components are determined in the following equations:

$$\delta_{wc} = \frac{V_{wc}\, b}{A_{wc} G}, \quad \delta_{sd} = \frac{V_{sd}\, b}{E\, A_{sd} \sin^2(\theta)\cos(\theta)} \tag{16}$$

where: A_{sd} is the cross-section of the substitute compression element of the stiffened shear zone (Figure 5b). After some transformations, formulas on the values of the forces acting in each component of the shear zone can be determined:

$$V_{wc}=\left(\frac{A_{wc}}{A_{eq}+A_{wc}}\right)V_{sc},\ V_{ds}=\left(\frac{A_{eq}}{A_{eq}+A_{wc}}\right)V_{sc}, \tag{17}$$

where:

$$A_{eq}=A_{sd}\,\frac{E}{G}\sin^2(\theta)\,\cos(\theta) \tag{18}$$

Knowing these forces, the condition of the shear zone resistance can be formulated in the form of two inequalities:

$$V_{wc.Rd}=0.9\frac{A_{wc}\,f_y}{\sqrt{3}\,\gamma_{M0}}\le V_{wc},\ V_{sd.Rd}=\frac{A_{sd}\ f_y}{\gamma_{M0}}\sin(\theta)\le V_{sd}, \tag{19}$$

in case, when both components of the stiffened web are not sensitive to stability loss

Evaluation of the resistance of the stiffened shear zone, treated as a substitute diagonal member with a crossed cross-section (Fig 5b),which is sensitive to the loss of stability can be carried out in accordance with the algorithm used while checking resistance of compression elements [8]. For this purpose, the resistance condition of compression member, subject to the out-of-plane web buckling, should be determined using the formula:

$$V_{sd.Rd}=\chi\frac{A_{sd}\,f_y}{\gamma_{M1}}\sin(\theta)\le V_{sd} \tag{20}$$

where γ_{M1} is the partial factor, and χ is a reduction factor, calculated according to expression [8]:

$$\chi=\frac{1}{\phi+\sqrt{\phi^2-\overline{\lambda}^{\,2}}} \tag{21}$$

in which:

$$\phi=\frac{1}{2}\left[1+\alpha_0\left(\overline{\lambda}-0.2\right)+\overline{\lambda}^{\,2}\right] \tag{22}$$

The imperfection parameter α_0 can be assumed as value which equals 0.49, whereas a non-dimensional slenderness is calculated according to the formula:

$$\overline{\lambda}=\sqrt{\frac{A\ f_y}{N_{cr}}}=\frac{\lambda}{\lambda_1} \tag{23}$$

Slenderness of the substitutive compressed element λ due to the out-of-plane web buckling and a relative slenderness λ_1 are obtained with the use of equations:

$$\lambda = \frac{\mu\, l}{i} \tag{24}$$

$$\lambda_1 = \pi \sqrt{\frac{E}{f_y}} = 93.9\varepsilon \tag{25}$$

where: l is a length of substitutive member, μ is a buckling length coefficient, which can be assumed with a value of 0.75, if both ends of stiffeners are fixed (e.g. in flanges of column) [9], and a radius of gyration i should be determined for the cross-section of the compression member (Fig. 5b) with respect to the vertical symmetry axis.

In the compressive resistance calculations of the stiffened components of the joint one should also take into account their sensitivity to loss of stability due to the torsional buckling. The resistance to this type of instability can be tested using the condition [9]:

$$\frac{I_t}{I_p} \geq 5.3 \frac{f_y}{E} \tag{26}$$

where I_t is the St. Venant torsional constant of the stiffener alone, while I_p is the polar second moment of area of single stiffener alone around the edge fixed to the plate. The fulfillment of this condition means that the checked stiffener is resistant to the torsional form of stability loss.

3.3. Resistance of compression zone

Checking the resistance of compression zone of the beam-to-column joints can be carried out according to the resistance condition in the form:

$$\frac{F_{bc}}{F_{c.Rd}} \leq 1 \tag{27}$$

in which $F_{c.Rd}$ is the resistance of unstiffened compressed zone, which can be determined according with the standard [4] guidelines.

In case of occurrence of the diagonal stiffeners in the knee (see Fig. 5a) the resistance condition can be presented with the use of inequality:

$$\frac{F_{bc} - V_{sd}}{F_{c.Rd}} \leq 1 \tag{28}$$

However, when designing knee joints, in which the plate girders are connected together, the web panel of column is very often susceptible to stability loss, which does not guarantee the entire joint the adequate resistance. Strengthening the compressed zone with transverse stiffeners (flat ribs) greatly improves the resistance of the web to local buckling (Fig. 5a and 5c).

In case, when the value of a non-dimensional slenderness of the compression zone, treated as a substitute member with a crossed cross-section (Fig. 5c) is not larger than 0.2, and the stiffeners in the form of flat ribs are not sensitive to loss of the local stability, that resistance of that zone can be obtained from the equation:

$$F_{c.wc.Rd} = \frac{\omega k_{wc}\, b_{eff.c}\, t_{wc}\, f_y}{\gamma_{M0}} + \frac{2b_{sc} t_{sc} f_y}{\gamma_{M0}} \quad (29)$$

in which ω and k_{wc} are, calculated according to point 6.2.6.2. [4], coefficients taking into account the reduction of resistance due to the complex state of stresses in the web panel, whereas $b_{eff.c}$ is an effective width, equal $30t_{wc}\,\varepsilon + t_{sc}$ (Fig. 5c).

Evaluation of the resistance of the stiffened compression zone, which is susceptible to the loss of stability ($\bar{\lambda} > 0.2$) can be carried out in accordance with the algorithm used while checking resistance of compression elements [8]. For this purpose, the resistance of compression zone, sensitive to the out-of-plane web buckling, should be determined using the formula:

$$F_{c.Rd} = \chi\left(\frac{\omega k_{wc}\, b_{eff.c}\, t_{wc}\, f_y}{\gamma_{M1}} + \frac{2b_{sc} t_{sc} f_y}{\gamma_{M1}}\right) \quad (30)$$

Calculation of the coefficient χ in the formula (30) should be carried out analogously to resistance checking of the stiffened shear zone case, here taking into account the geometrical and material quantities of the transverse stiffeners of the web panel. Assessment of the sensitivity to loss of stability due to the torsional buckling of transverse stiffeners also should be checked.

4. Numerical example

On the basis of information presented in this paper, the knee of a certain steel frame, shown in Figure 6a), was calculated.

Using the results of static calculations in the form of forces acting on the frame knee (Fig. 6b), geometry of the cross-sections of the beam and the column

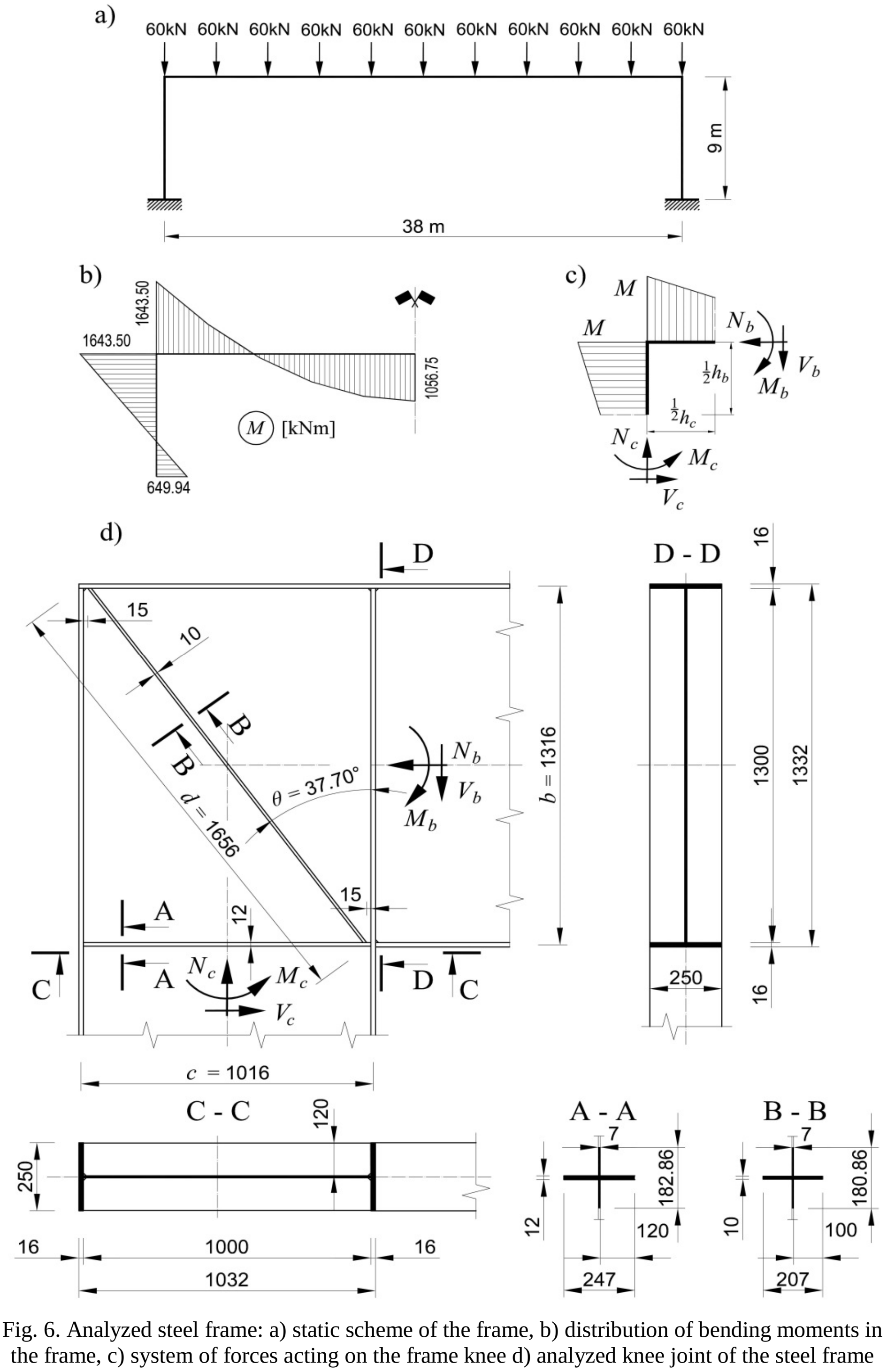

Fig. 6. Analyzed steel frame: a) static scheme of the frame, b) distribution of bending moments in the frame, c) system of forces acting on the frame knee d) analyzed knee joint of the steel frame

which roughly correspond to the proportions ofIKS girders, was assumed. Next, taking into account the geometry of the knee (Fig. 6d), and the local equilibrium conditions (Fig. 6c), the moments M_b and M_c acting on the calculated joint were determined.

Data: Steel S355: E = 210 GPa, G = 81 GPa, f_y = 355 MPa, ε = 0.814, η = 1.2; forces acting on the knee from the beam side (Fig. 6d; D – D cross-section): M_b = 1 473.22 kNm,N_b = 254.83 kN, V_b = 330 kN, forces acting on the knee from the column side (Fig. 6d;**C – C** cross-section): M_c = 1473.78 kNm, N_c = 330 kN, V_c = 254.83 kN.

Geometric properties of the column: area of the cross-section $A_c = 15\times10^3$ mm², second moment of the cross-section $I_c = 2.65\times10^9$ mm⁴.

Geometric properties of the tension zone: width of the beam flange b_{bf}= 250 mm, thickness of the beam flange t_{bf}= 16 mm.

Geometric properties of the shear zone: shear area: b_{sd} = 100 mm, t_{sd} = 10 mm, $A_{wc} = 7\times10^3$ mm², effective width of the web panel $b_{eff.s}$ = 180.86 mm, area of the cross-section $A_{sd} = 3.27\times10^3$ mm², $I_{sd} = 5.73\times10^6$ mm⁴, radius of the cross-section gyration i_{sd}= 41.90 mm.

Geometric properties of the compression zone: b_{sc} = 120 mm, t_{sc} = 12 mm, effective width of the web panel $b_{eff.c}$ = 182.86 mm, area of the cross-section $A_{sc} = 4.16\times10^3$ mm², second moment of the cross-section$I_{sc} = 11.62\times10^6$ mm⁴, radius of the cross-section gyration i_{sc} = 52.86 mm.

Resistance calculations of the knee were performed considering two cases:

- case I - the knee is not stiffened with diagonal stiffeners in the shear zone,
- case II - the knee is stiffened with diagonal stiffeners in the shear zone.

Determination of internal forces in the knee joint (acc. to 2)

$$F_{bt} = \frac{M_b}{b} - \frac{N_b}{2} = \frac{1\,473.22\,\text{kNm}}{1.316\,\text{m}} - \frac{254.83\,\text{kN}}{2} = 992.05\,\text{kN}$$

$$F_{bc} = \frac{M_b}{b} + \frac{N_b}{2} = \frac{1\,473.22\,\text{kNm}}{1.316\,\text{m}} + \frac{254.83\,\text{kN}}{2} = 1\,246.88\,\text{kN}$$

$$V_{sc} = F_{bt} = F_{bc} - V_c = 1\,246.88\,\text{kN} - 254.83\,\text{kN} = 992.05\,\text{kN}$$

Calculation of the tension zone (acc. to 3.1)

- Resistance of tension zone

$$F_{t.Rd} = \frac{b_{fb}\,t_{fb}\,f_y}{\gamma_{M0}} = \frac{250\cdot16\cdot355}{1.0} = 1\,420\,\text{kN}$$

- Resistance condition of the tension zone

$$\frac{F_{bt}}{F_{t.Rd}} \leq 1 \rightarrow \frac{992.05\,\text{kN}}{1\,420\,\text{kN}} = 0.70 \text{ - condition fulfilled}$$

Calculation of the shear zone – case I (acc. to 3.2)

- Sensitivity assessment of the column web panel due to the shear instability

$$\alpha \approx \frac{b}{c} = \frac{1316}{1016} = 1.30 \;\rightarrow\; k_\tau(\alpha) = 5.34 + \frac{4}{\alpha^2} = 5.34 + \frac{4}{1.30^2} = 7.72$$

$$\lambda_w = \frac{h_{wc}}{t_{wc}} \leq \frac{31}{\eta}\varepsilon\sqrt{k_\tau} \;\rightarrow\; \frac{1\,000}{7} \leq \frac{31}{1.2} 0.814\sqrt{7.72} \;\rightarrow\; 142.86 \geq 58.42$$

Elastic shear buckling stresses

$$\tau_{cr} = \frac{\pi^2 E\, k_\tau}{12\left(1-\nu^2\right)\lambda_w{}^2} = \frac{\pi^2 \cdot 210\times10^3 \cdot 7.72}{12\left(1-0.3^2\right)142.86^2} = 71.84 \text{ MPa}$$

Shear panel slenderness

$$\bar{\lambda}_w = \sqrt{\frac{f_y}{\sqrt{3}\,\tau_{cr}}} = \sqrt{\frac{355}{\sqrt{3}\cdot 71.84}} = 1.69$$

Shear buckling factor

$$\chi_w = \min\left(0.9; \frac{0.83}{\bar{\lambda}_w}\right) \rightarrow \chi_w = \min\left(0.9; \frac{0.83}{1.69}\right) = 0.49$$

- Resistance of the shear zone

$$V_{wc.Rd} = \chi_w \frac{A_{wc}\;f_y}{\sqrt{3}\;\gamma_{M1}} = 0.49 \cdot \frac{7\times10^3 \cdot 355}{\sqrt{3}\;\gamma_{M1}} = 704.98 \text{ kN}$$

- Resistance condition

$$\frac{V_{sc}}{V_{wc.Rd}} \leq 1 \rightarrow \frac{992.05\,\text{kN}}{704.98\,\text{kN}} = 1.41 \geq 1 \text{ - condition not met}$$

Calculation of the shear zone – case II (acc. to 3.2)

- Determine the load distribution for each of the shear components

$$\theta = 37.70°$$

$$A_{eq} = A_{sd}\,\frac{E}{G}\sin^2(\theta)\cos(\theta)$$

$$A_{eq} = 3.27\cdot 10^3\,\frac{210\times 10^6}{81\times 10^6}\sin^2(37.70)\cos(37.70) = 2.50\times 10^3\,\text{mm}^2$$

$$V_{wc} = \left(\frac{A_{wc}}{A_{eq} + A_{wc}}\right)V_{sc} = \left(\frac{7\cdot 10^3}{2.50\cdot 10^3 + 7\cdot 10^3}\right)992.05\,\text{kN} = 730.76\,\text{kN}$$

$$V_{sd} = \left(\frac{A_{eq}}{A_{eq} + A_{wc}}\right)V_{sc} = \left(\frac{2.50\cdot 10^3}{2.50\cdot 10^3 + 7\cdot 10^3}\right)992.05\,\text{kN} = 261.30\,\text{kN}$$

- Sensitivity assessment of the column web panel due to the shear instability

$$\theta = 37.70° \rightarrow \xi = \cot(\theta) = 1.30$$

$$k_\tau(\xi) = 5.34\left(1+\xi^2\right) + 19.3\xi = 5.34\left(1+1.30^2\right) + 19.3\cdot 1.30 = 39.48$$

$$\lambda_w = \frac{h_{wc}}{t_{wc}} \le \frac{31}{\eta}\varepsilon\sqrt{k_\tau} \;\rightarrow\; \frac{1\,000}{7} \le \frac{31}{1.2}0.814\sqrt{39.48} \;\rightarrow\; 142.86 \ge 132.07$$

Elastic shear buckling stresses

$$\tau_{cr} = \frac{\pi^2 E\,k_\tau}{12\left(1-\nu^2\right)\lambda_w^{\,2}} = \frac{\pi^2\cdot 210\times 10^3\cdot 39.48}{12\left(1-0.3^2\right)142.86^2} = 367.19\,\text{MPa}$$

Shear panel slenderness

$$\bar{\lambda}_w = \sqrt{\frac{f_y}{\sqrt{3}\,\tau_{cr}}} = \sqrt{\frac{355}{\sqrt{3}\cdot 367.19}} = 0.75$$

Shear buckling factor

$$\chi_w = \min\left(0.9;\,\frac{0{,}83}{\bar{\lambda}_w}\right) \rightarrow \chi_w = \min\left(0.9;\,\frac{0.83}{0.75}\right) = 0.90$$

- Resistance of the shear zone –web panel

$$V_{wc.Rd} = \chi_w \frac{A_{wc}\ f_y}{\sqrt{3}\ \gamma_{M0}} = 0.90\frac{7\times10^3\cdot355}{\sqrt{3}\cdot1.0} = 1\,291.24\text{kN}$$

- Resistance condition

$$\frac{V_{wc}}{V_{wc.Rd}} \leq 1 \rightarrow \frac{730.76\,\text{kN}}{1\,291.24\,\text{kN}} = 0.57 < 1 \text{ - condition fulfilled}$$

- Sensitivity assessment of the stiffened web due to the buckling instability

Slenderness of the substitutive compressed element

$$\lambda_{sd} = \frac{d\mu}{i_{sd}} = \frac{1\,663\cdot0.75}{41.90} = 29.76$$

Relative slenderness

$$\lambda_1 = 93.9\varepsilon = 76.41$$

Non-dimensional slenderness

$$\bar{\lambda}_{sd} = \frac{\lambda_{sd}}{\lambda_1} = \frac{29.76}{76.41} = 0.39$$

Reduction factor

$$\phi_{sd} = \frac{1}{2}\left(1+\alpha\left(\bar{\lambda}_{sd}-0.2\right)+\bar{\lambda}_{sd}^{\ 2}\right) = \frac{1}{2}\left(1+0.49(0.39-0.2)+0.39^2\right) = 0.62$$

$$\chi_{sd} = \frac{1}{\phi_{sd}+\sqrt{\phi_{sd}^{\ 2}-\bar{\lambda}_{sd}^{\ 2}}} = \frac{1}{0.62+\sqrt{0.62^2-0.39^2}} = 0.90$$

- Resistance of the stiffened web

$$V_{sd.Rd} = \chi_{sd}\frac{A_{sd}\ f_y}{\gamma_{M1}}\sin(\theta) = 0.90\frac{3.27\times10^3\cdot355}{1.0}\sin(37.70) = 639.73\,\text{kN}$$

- Resistance condition

$$\frac{V_{sd}}{V_{sd.Rd}} \leq 1 \rightarrow \frac{261.30\,\text{kN}}{639.73\,\text{kN}} = 0.41 < 1 \text{ - condition fulfilled}$$

- Sensitivity assessment of the web stiffeners due to the torsional instability

$$I_t = \frac{b_{sd} \cdot t_{sd}{}^3}{3} = \frac{100 \cdot 10^3}{3} = 33.33 \times 10^3 \,\text{mm}^4$$

$$I_p = \frac{b_{sd}{}^3 \cdot t_{sd}}{3} + \frac{b_{sd} \cdot t_{sd}{}^3}{12} = \frac{100^3 \cdot 10}{3} + \frac{100 \cdot 10^3}{12} = 3.34 \times 10^6 \,\text{mm}^4$$

$$\frac{I_t}{I_p} \geq 5.3 \frac{f_y}{E} \rightarrow \frac{33.33 \times 10^3}{3.34 \times 10^6} \geq 5.3 \frac{355}{210 \times 10^3} \rightarrow 0.010 \geq 0.009$$

Calculation of the compression zone - case I (acc. to 3.3)

- Coefficients taking into account the complex state of stresses in the web

$$\omega = \frac{1}{\sqrt{1 + 1.3\left(\frac{b_{eff.c}\, t_{wc}}{A_{wc}}\right)^2}} = \frac{1}{\sqrt{1 + 1.3\left(\frac{182.86 \cdot 7}{7 \cdot 10^3}\right)^2}} = 0.98$$

$$\sigma_c = \frac{N_c}{A_c} + \frac{M_c}{I_{yc}} z = \frac{330 \times 10^3}{15 \times 10^3} + \frac{1.474 \times 10^9}{2.65 \times 10^9} 500 = 300.28 \text{MPa}$$

$$k_{wc} = 1.7 - \frac{\sigma_c}{f_y} = 1.7 - \frac{300.28}{355} = 0.85$$

- Sensitivity assessment of the web stiffeners due to the buckling instability

Slenderness of the substitutive compressed element λ

$$\lambda_{sc} = \frac{c\mu}{i_{sc}} = \frac{1016 \cdot 0.75}{54.46} = 14.42$$

Non-dimensional slenderness

$$\bar{\lambda}_{sc} = \frac{\lambda_{sc}}{\lambda_1} = \frac{14.42}{76.41} = 0.19 \leq 0.20$$ - lateral buckling will not occur

- Resistance of the compression zone

$$F_{c.Rd} = \frac{\omega k_{wc}\, b_{eff.c}\, t_{wc}\, f_y}{\gamma_{M0}} + \frac{2 b_{sc} t_{sc} f_y}{\gamma_{M0}}$$

$$F_{c.Rd} = \frac{0.98 \cdot 0.85 \cdot 182.86 \cdot 7 \cdot 355}{1.0} + \frac{2 \cdot 120 \cdot 12 \cdot 355}{1.0} = 1402.36 \text{kN}$$

- Resistance condition

$$\frac{F_{bc}}{F_{c.Rd}} \leq 1 \rightarrow \frac{1\,246.88\,\text{kN}}{1\,402.36\,\text{kN}} = 0.89 < 1$$

- Sensitivity assessment of the web stiffeners due to the torsional instability

$$I_t = \frac{b_{sc} \cdot t_{sd}^{\ 3}}{3} = \frac{120 \cdot 12^3}{3} = 69.12 \times 10^3 \text{mm}^4$$

$$I_p = \frac{b_{sc}^{\ 3} \cdot t_{sc}}{3} + \frac{b_{sc} \cdot t_{sc}^{\ 3}}{12} = \frac{120^3 \cdot 10}{3} + \frac{120 \cdot 10^3}{12} = 6.93 \times 10^6 \text{mm}^4$$

$$\frac{I_t}{I_p} \geq 5{,}3 \frac{f_y}{E} \rightarrow \frac{69.29 \times 10^3}{6.93 \times 10^6} \geq 5{,}3 \frac{355}{210 \times 10^3} \rightarrow 0.010 \geq 0.009$$

Calculation of the compression zone - case II (acc. to 3.3)

- Resistance condition

$$\frac{F_{bc} - V_{sd}}{F_{c.Rd}} \leq 1 \rightarrow \frac{1\,246.88\,\text{kN} - 261.30\,\text{kN}}{1\,402.36\,\text{kN}} = 0.70 < 1$$

5. Summary

The paper presents the calculation problems of welded knees of steel frames, in which the main load-bearing elements of frames can be made of plate girders with slender webs.

The presented analysis method ofjoint loading state allows relatively easy to determine the forces acting in the three characteristic zones of the rectangle knee joint.

The algorithms for checking resistance have been developed in such a way as to take into account the complex state of stresses in the joint web, as well as the phenomenon of instability in the shear and the compression zone of the joint.

The proposed method of designing welded knee joints makes it possible to determine quite accurately the resistance of the individual zones of joint. This fact may in many practical cases favor more economical design of steel frames knees.

Furthermore, the method of internal forces determination and the dimensioning procedure presented in the paper were used in the calculations of certain knee joint of the portal steel frame.

Although the scope of the calculation method in this work relates directly to the design of welded knees of steel frames, the presented dimensioning algorithms can be relatively easy adapted to calculations of knees, in which end-plate bolted connections between beams and columns appear.

The issues presented in the paper are important from the practical point of view, and therefore, they should be taken into account in the design process of frames with plate girders' cross-sections.

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