

5.1.3. ANALYSIS OF THE POSITION OF THE STATIONS FROM THE GPS NETWORK OF THE BALKAN PENINSULA

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5.1.3.1. General

A fundamental problem of the performed processings and analyses, interpretations and generalizations of periodic measurements for the purposes of geodynamics is to assume the initial (fixed) point. The word goes about the accepted starting points, for example of the IGS and EUREF systems, in the processing of periodic GNSS, respectively GPS campaigns in the region. The guarantee and the assessment of their position in the time are based on the permanent measurements and the time series analyses for the single stations. Moreover, their eventual displacement (velocity) is included in the processing of the periodic measurement campaigns. The accepted movement models and the respective software are used for this purpose. The experience and the results from the many years of measurement, processing and analyses prove that the accuracy, respectively – the determined vectors of displacement for the measured points in the region of study, depend substantially on the choice of fixed points or stations. This is the reason for certain discrepancies in the established vectors and velocities of displacements – magnitude and orientation, determined by different researchers. This is an extremely complicated problem and no exact solution has been found so far.

An attempt has been made here to find out a definite solution of above the problem, or more exactly – to verify to what extent the assumed prerequisites for the fixed point position correspond to the results obtained from the processing of a particular measurement campaign in the region of the Balkan Peninsula, with the aim of attaining more exact and reliable solution.

5.1.3.2. Adjustment of free GPS networks and S-transformation

This case is applied to establish the stable (fixed) points for the calculations of the movements of the geodynamic and the deformational networks. In principle this is an adjustment of a free spatial network.

The adjustment of the free spatial networks of measurements of a classical type is usually connected with the so called “seven-parameter transformation”, known also as the S-transformation, which is a conform transformation. This is a transformation, between two spatial coordinate systems $X(X,Y,Z)$ and $x(x,y,z)$. The seven transformation parameters in the general expression of the conform transformational model:

$$X = T + R.m.x \quad (1)$$

are: three translation components of the vector T , three rotational angles of the orthogonal rotational matrix R and scale factor m .

The translation components are the coordinates of the origin of the system x into the system X , and for small rotations R can be represented by a matrix like following:

$$R = \begin{pmatrix} 1 & -e_z & e_y \\ e_z & 1 & -e_x \\ -e_y & e_x & 1 \end{pmatrix} \quad (2)$$

The determination of these coefficients can be done on the basis of identical points, for which the coordinates in both systems are known. Since the parameters are 7, seven equations are needed or a minimum of two identical points with their three coordinates in both systems and another point with one coordinate in both systems. In all cases the number of the identical points is more than 3 and then the transformation parameters are processed according to the Method of Least Mean Squares (LMS). The general look of the equations of the corrections in this case is:

$$V = a.x_0 + b.y_0 + c.z_0 + d.m + e.ex + f.ey + g.ez + f \quad (3)$$

For each coordinate of every identical point one equation is composed, i.e. three equations for each point of the network.

$$\begin{aligned} V_x &= 1.x_0 + 0.y_0 + 0.z_0 + X.m + 0.ex + Z.ey - Y.ez + (x-X) \\ V_y &= 0.x_0 + 1.y_0 + 0.z_0 + Y.m - Z.ex + 0.ey + X.ez + (y-Y) \\ V_z &= 0.x_0 + 0.y_0 + 1.z_0 + Z.m + Y.ex - X.ey + 0.ez + (z-Z) \end{aligned} \quad (4)$$

The normal equations, corresponding to the above, written in the form

$$\begin{aligned} [a.V] &= [V_x] = 0, \\ [b.V] &= [V_y] = 0, \\ [c.V] &= [V_z] = 0, \\ [d.V] &= [0.V_x] - [Z.V_y] + [Y.V_z] = 0, \\ [e.V] &= [Z.V_x] + [0.V_y] - [Y.V_z] = 0, \\ [f.V] &= [-Y.V_x] + [X.V_y] + [0.V_z] = 0, \\ [g.V] &= [X.V_x] + [Y.V_y] + [Z.V_z] = 0, \end{aligned} \quad (5)$$

give the initial data as follows: the first three conditions are referring to the translations, the next three – the orientation according the axes and the last condition gives the scale.

If in the above conditions we substitute V_x, V_y, V_z with dx, dy, dz we'll get the necessary additional and obligatory conditions for the adjustment of free spatial networks.

With the adjustment of the network under the condition of LMS and the additional conditions (5) (for ensuring the regularity of the normal matrix) correlations are received, by means of which the transformational coefficients are also estimated.

When applying the S-transformation the coordinates of the points can be reduced by random quantities.

The mean errors of the transformed coordinates X, Y и Z after the adjustment can be calculated as follows

$$M_x^2 = me^2 \cdot Q_{xx} \quad (6)$$

where me is the mean error, calculated by the formula

$$me^2 = [V_x.V_x] / (n - 7) \quad (7)$$

and Q_{xx} are diagonal elements of the reverse (cofactor, variance-covariance) matrix Q . The correlative matrix K_{xx} of the adjusted coordinates X, Y, Z is calculated as follows

$$K_{xx} = me^2 \cdot Q_x \quad (8)$$

and of the corrections V_x, V_y, V_z , which are in fact corrections dX, dY, dZ

$$K_{vv} = (I - A \cdot Q_x \cdot A^*) \cdot me^2 = me^2 \cdot Q_v \quad (9)$$

There I is a single matrix. From the theory of the adjustment according to LMS follows that

$$Q_{I+v} = A^* \cdot Q_x \cdot A \quad (10)$$

When the matrixes Q_{I+v} and Q_v are symmetrical and idempotent.

In the adjustment of the free spatial networks besides the minimizing condition

$$V^* \cdot P \cdot V = [PVV] = \text{minimum} \quad (11)$$

the so called Helmert's condition is added:

$$X \cdot X^* = [dx \cdot dx + dy \cdot dy + dz \cdot dz] = \text{minimum} \quad (12)$$

From the Helmert requirement (12) it follows

$$\begin{aligned} [dx] &= 0, \\ [dy] &= 0, \\ [dz] &= 0, \\ [0 \cdot dx] - [Z \cdot dy] + [Y \cdot dz] &= 0, \\ [Z \cdot dx] + [0 \cdot dy] - [Y \cdot dz] &= 0, \\ [-Y \cdot dx] + [X \cdot dy] + [0 \cdot dz] &= 0, \\ [X \cdot dx] + [Y \cdot dy] + [Z \cdot dz] &= 0, \end{aligned} \quad (13)$$

which correspond to (5) and which we shall call Helmert's subconditions. Here, as well the first three conditions define the translations, the next three – the orientation in relation to the axes and the last condition gives the scale.

We emphasize, the subconditions must be applied for all the points, participated in the adjustment.

In the adjustment of free spatial networks we can choose different transformation models, from which different Helmert's subconditions follow from (5).

The first three subconditions (for the translations) are well defined, as well as this for the scale. If, however, distances are measured in the network, the subconditions for the scale will drop out.

As far as the orientation is concerned, the following calculations are not so simple because the orientation elements are given by means of specific measurements. When there are data from such measurements, the respective subconditions will drop out.

If in one of the points absolute definitions are carried out, all conditions will drop out. In the GPS networks with measured baselines, since the vectors are oriented and scaled quantities, only the first three subconditions should remain. In some special cases, however, it may be necessary to introduce also the subconditions for the scale.

5.1.3.3. Fixed points

The free network adjustment can be used for establishment of the stable (immobile) points in geodynamic networks, when periodic measurements are carried out. To define reliably the movements of the network points it is necessary some of the points to be accepted as "hard" (fixed). Usually the movements of the points are worked out in relation to the initial cycle of measurements, during the processing of which the choice of the initial data is not so significant. To the next cycles, however, the choice of the

initial data is extremely important. The initial data are connected with the coordinates of a certain number of stable points, which do not change their position during the measurements. These points are defined on the basis of geological, geophysical, tectonic and seismic investigations and analyses. But even if we know which are these points the question about their coordinated remains open. We don't have the right to accept as initial the coordinates of these points, received during the initial cycle, since they are loaded with inevitable errors as a result of the inevitable errors in the measurements during this cycle. That's why the coordinates of these points should undergo some changes within certain limits, which should not be interpreted as movements. The procedure is the following.

In the repeated measurements the geodynamic network is adjusted as a free one, and to the points that a priori are accepted as immobile (fixed) the three Helmert subcondition are applied.

$$\begin{aligned} [dx] &= 0, \\ [dy] &= 0, \\ [dz] &= 0, \end{aligned} \tag{14}$$

As a result of this adjustment these points will have new coordinates. From the differences in the coordinates in relation to the first cycle the following evaluations could be calculated

$$\begin{aligned} M_x &= \sqrt{\frac{[dx.dx]}{n}}; M_y = \sqrt{\frac{[dy.dy]}{n}}; M_z = \sqrt{\frac{[dz.dz]}{n}} \\ M_p &= \sqrt{M_x^2 + M_y^2 + M_z^2} \end{aligned}$$

If the differences dx, dy, dz in the coordinates of one of these points in absolute value exceeded a definite quantity (for example 3Mp), the conclusion has to be that either the point is not stable or its coordinates are not reliably defined. Besides, it is difficult to estimate in which cycle the coordinates are wrongly defined – the first, the second or both. Just in case this point is dropped out as an initial one and the adjustment is repeated without this point. If the impermissible differences appear for several points, the point with the biggest differences is removed first. In this way the adjustment is repeated several times until those points remain as fixed, for which the differences are in the accepted limits. As final coordinates of the fixed points we accept the coordinates received during this cycle. During the next cycle we use these coordinates as fixed, and we can also use the coordinates of some of the remaining points, received during the first cycle. The procedure is repeated several times until the fixed points for the respective cycle are established.

The Helmert's condition provides for configurations, where there is maximum vicinity among the chosen fixed points during the different cycles.

5.1.3.4. Practical application

The above approaches are applied for the investigation of the geodynamics of the Balkan Peninsula. The results of the GPS campaigns, carried out in 1997, 2003 and 2005 have been used. From the different daily solutions from the measurements the baselines and their weight matrixes have been calculated. With so prepared data an adjustment of a part of the European network has been carried out separately for the three campaigns. In the adjustment of the campaign the method of the fixed points is applied.

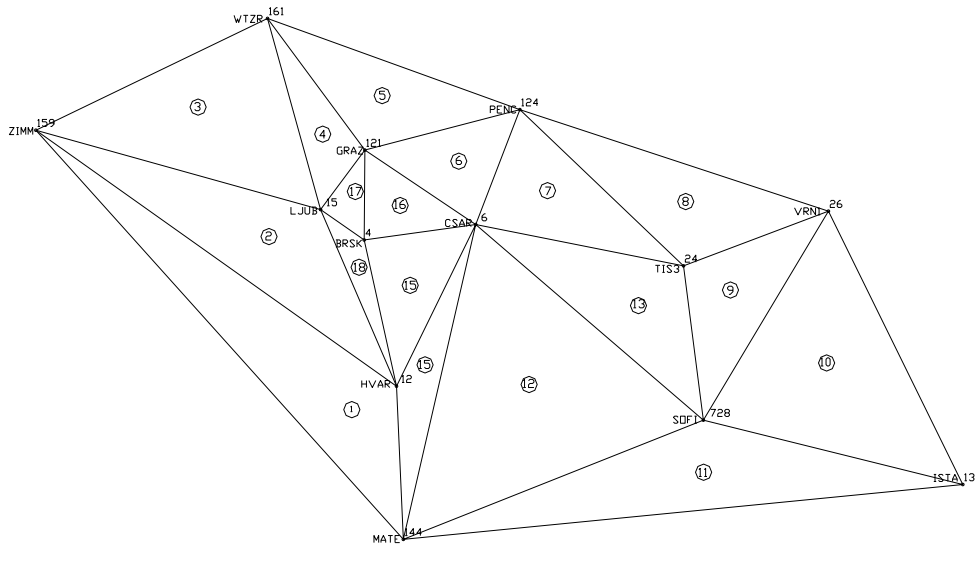


Fig. 5.1.3.1. Selected Network in M 1:10 000 000

The following six stations are introduced as reference points: 121(GRAZ), 124(PENC), 144(MATE), 159(ZIMM), 161(WTZR) и 728(SOFI). For these stations the received corrections to the initial coordinates have the order of the mean errors of the coordinates, while for the rest of the additional points these corrections exceed by a hundred times mean errors. It gives us ground to consider the received movements and annual velocities for real ones. The following results have been obtained.

LIST OF THE STATIONS

Station Number	Station Name	Location	Country
121	GRAZ	Graz	Austria
124	PENC	Penc	Hungary
144	MATE	Matera	Italy
159	ZIMM	Zimmerwald	Switzerland
161	WTZR	Wetzel	Germany
728	SOFI	Sofia	Bulgaria
4	BRSK	Brusnik	Croatia
6	CSAR	Csarnota	Hungary
12	HVAR	Hvar	Croatia
13	ISTA	Istambul	Turkey
15	LJUB	Ljubljana	Slovenia
24	TIS3	Tismania	Romania
26	VRN1	Vranca	Romania

BASELINES

Serial number	Baseline	From - to
1	ZIMM-MATE	159-144
2	CSAR-BRSK	6-4
3	CSAR-HVAR	6-12
4	CSAR-MATE	6-144
5	CSAR-PENC	6-124

6	CSAR-SOFI	6-728
7	CSAR-TIS3	6-24
8	GRAZ-BRSK	121-4
9	GRAZ-CSAR	121-6
10	GRAZ-LJUB	121-15
11	GRAZ-PENC	121-124
12	HVAR-BRSK	12-14
13	LJUB-BRSK	15-4
14	LJUB-HVAR	15-12
15	MATE-HVAR	144-12
16	MATE-SOFI	144-728
16A	MATE-ISTA	144-13
17	PENC-TIS3	124-24
18	PENC-VRN1	124-26
19	SOFI-TOS3	728-24
20	SOFI-VRN1	728-26
20A	SOFI-ISTA	728-13
21	TIS3-VRN1	24-26
22	ISTA-VRN1	13-26
22	WTZR-GRAZ	161-121
23	WTZR-LJUB	161-15
24	WTZR-PENC	161-124
25	WTZR-ZIMM	161-159
26	ZIMM-HVAR	159-12
27	ZIMM-LJUB	159-15
28	ISTA-VRN1	13-26

EXPERIMENTS – EPOCH 1997

No of exp.	1		2		3		4	
Me	4.9		4.9		4.9		4.9	
DX(121)	0.1	0.7	0.8	0.7	0.6	0.7	1.6	0.6
DY(121)	0.4	0.5	-0.2	0.5	1.6	0.4	1.2	0.4
DZ(121)	-1.4	0.7	-0.1	0.7	-1.7	0.7	-0.1	0.6
DX(124)	-3.4	0.9	-2.7	0.8	-2.9	0.9	-2.0	0.8
DY(124)	-4.5	0.5	-5.1	0.4	-3.3	0.5	-3.7	0.4
DZ(124)	2.6	0.7	3.9	0.7	2.3	0.7	3.9	0.7
DX(144)	0.4	0.9	1.1	0.9	0.9	0.9	1.9	0.9
DY(144)	0.6	0.5	-0.0	0.4	1.8	0.5	1.4	0.5
DZ(144)	-4.1	0.9	-2.8	0.9	-4.5	0.9	-2.8	0.9
DX(159)	3.3	1.0	3.9	1.2	3.8	0.9	4.7	1.2
DY(159)	-2.9	0.8	-3.5	0.9	-1.7	0.7	-2.1	0.9
DZ(159)	6.7	0.9	8.1	1.1	6.4	0.8	8.0	1.0
DX(161)	-2.9	0.8	-2.2	0.8	-2.4	0.7	-1.5	0.7
DY(161)	0.3	0.6	-0.3	0.6	1.5	0.5	1.1	0.5
DZ(161)	-2.2	0.8	-0.8	0.8	-2.5	0.7	-0.9	0.7
DX(728)	2.5	0.9	3.1	0.9	3.0	1.1	3.9	1.1
DY(728)	6.2	0.7	5.6	0.7	7.4	0.9	7.0	0.8
DZ(728)	-1.6	0.8	-0.2	0.8	-1.9	1.0	-0.3	1.0
Mx	2.5		2.2		2.4		1.7	

My	3.4		3.3		2.1		2.1	
Mz	3.6		2.2		3.9		2.4	
Mp	5.5		4.6		5.0		3.7	
Ms	1.4		1.4		1.5		1.5	
Mxy	0.8		0.8		0.9		0.9	
Mh	1.1		1.2		1.2		1.2	

EXPERIMENTS - EPOCH 2003

No of exp.	1		2		3		4		5	
Me	7.1		7.1		7.1		7.1		7.1	
DX(121)	3.5	1.3	5.5	1.3	1.8	1.3	3.9	1.2	4.7	1.1
DY(121)	-1.3	0.9	-1.0	0.9	-1.1	0.9	-0.7	0.9	1.2	0.7
DZ(121)	2.3	1.3	3.0	1.2	0.5	1.3	1.0	1.1	2.5	1.0
DX(124)	-4.7	1.5	-2.6	1.5	-6.3	1.4	-4.2	1.3	-3.5	1.3
DY(124)	-5.4	1.0	-5.1	1.0	-5.2	0.9	-4.8	0.9	-2.9	0.9
DZ(124)	-1.1	1.4	-0.4	1.3	-2.9	1.4	-2.4	1.2	-0.9	1.2
DX(144)	10.3	1.6	12.4	1.9	8.7	1.6	10.8	2.0	11.5	2.1
DY(144)	1.5	0.9	1.8	1.1	1.7	0.8	2.1	1.1	4.0	1.1
DZ(144)	3.8	1.5	4.5	1.8	2.0	1.5	2.5	1.8	4.0	2.2
DX(159)	-8.4	1.9	-6.3	1.9	-10.1	2.2	-7.9	2.4	-7.2	2.4
DY(159)	1.2	1.6	1.5	1.5	1.4	1.9	1.8	1.9	3.7	1.9
DZ(159)	-9.0	1.5	-8.2	1.5	-10.8	1.8	-10.3	1.9	-8.7	1.9
DX(161)	-2.4	1.4	-0.3	1.3	-4.1	1.4	-1.9	1.3	-1.2	1.1
DY(161)	-0.8	1.1	-0.5	1.1	-0.5	1.2	-0.1	1.1	1.7	0.9
DZ(161)	-2.0	1.4	-1.2	1.3	-3.7	1.4	-3.3	1.3	-1.7	1.1
DX(728)	1.6	1.7	3.7	1.7	-0.1	1.6	2.1	1.6	2.8	2.1
DY(728)	4.9	1.4	5.2	1.4	5.1	1.3	5.5	1.3	7.4	1.7
DZ(728)	6.0	1.6	6.8	1.7	4.2	1.5	4.7	1.6	6.3	2.2
Mx	6.05		4.26		5.19		3.20		3.42	
My	3.14		3.36		3.39		3.67		2.08	
Mz	4.85		4.98		2.99		3.15		1.83	
Mp	8.36		7.37		6.88		5.80		4.40	
Ms	2.8		2.8		2.8		2.9		3.0	
Mxy	1.7		1.8		1.7		1.8		1.9	
Mh	2.2		2.2		2.2		2.2		2.4	

EXPERIMENTS - EPOCH 2005

No of exp.	1		2		3		4		5	
Me	6.2		6.2		6.2		6.2		6.2	
DX(121)	12.1	1.0	14.5	1.3	16.0	1.3	12.9	1.3	14.4	1.3
DY(121)	6.9	0.7	8.3	0.8	9.2	0.9	7.8	0.8	8.9	0.9
DZ(121)	5.7	1.0	6.8	1.3	7.5	1.2	6.1	1.3	6.8	1.3

DX(124)	2.5	1.2	4.9	1.2	6.4	1.2	3.3	1.1	4.9	1.0
DY(124)	0.7	0.8	2.1	0.8	3.0	0.8	1.6	0.7	2.7	0.7
DZ(124)	-0.6	1.1	0.5	1.1	1.2	1.1	-0.2	1.1	0.5	1.0
DX(144)	3.6	1.1	6.1	1.1	7.6	1.3	4.5	1.1	6.0	1.5
DY(144)	2.5	0.6	3.8	0.6	4.8	0.8	3.3	0.6	4.4	0.8
DZ(144)	1.6	1.1	2.7	1.0	3.4	1.3	2.0	1.0	2.7	1.4
DX(159)	-8.5	1.3	-3.6	1.3	-4.6	1.3	-7.6	1.6	-6.1	1.8
DY(159)	-3.4	1.0	1.3	0.9	-1.1	1.0	-2.5	1.3	-1.4	1.3
DZ(159)	-3.9	1.2	-2.2	1.3	-2.1	1.1	-3.4	1.4	-2.7	1.5
DX(161)	-6.0	1.2	-6.1	1.3	-2.1	1.2	-5.1	1.3	-3.6	1.2
DY(161)	-0.1	0.9	-2.0	1.0	2.3	0.9	0.8	1.0	1.9	0.9
DZ(161)	-3.4	1.2	-2.7	1.1	-1.6	1.2	-2.9	1.3	-2.3	1.2
DX(728)	-3.7	1.3	-1.2	1.2	0.3	1.2	-2.8	1.1	-1.3	1.1
DY(728)	-6.6	1.0	-5.2	1.0	-4.2	1.0	-5.7	0.9	-4.6	0.8
DZ(728)	0.6	1.2	1.8	1.1	2.4	1.2	1.1	1.0	1.7	1.1
Mx	6.91		4.73		4.07		4.04		3.56	
My	4.26		3.22		2.89		3.41		3.26	
Mz	3.21		2.15		1.87		1.86		1.67	
Mp	8.73		6.11		5.33		5.61		5.10	
Ms	2.1		2.1		2.1		2.1		2.2	
Mxy	1.3		1.3		1.3		1.3		1.3	
Mh	1.6		1.7		1.7		1.7		1.7	

Remarks: In the tables above Me is the standard deviation after the adjustment (mean error for unit weight), DX, DY, DZ are corrections to coordinates of the fixed points, Mx,My, Mz and Mp are the mean errors, derived from the corrections, Ms – mean errors of the space position from the adjustment for the whole net, Mxy - mean errors of the horizontal position from the adjustment for the whole net, Mh - mean errors of the height position from the adjustment for the whole net. The eliminated points are signed by color. All dimensions the data are in millimeters.

5.1.3.5. Conclusions:

The proposed formulations, theoretical substantiations, algorithm, software and experimental investigations provide the grounds of drawing the following conclusions:

- The mean error Me remained approximately the same in the different variants.
- If the number of the fixed points increases, the mean errors of the whole network become smaller. The minimum occurs when all the points are considered as fixed ones.
- The mean errors of the whole network decrease when some of the fixed points with big coordinate corrections are eliminated.
- From the variants considered and for the number of points, respectively stations, investigated, the following fixed points could be accepted as most reliable: epoch 1997 -121,124,144 and 161, epoch 2003 – 121, 124 and 161, epoch 2005 – 124, 161 and 728.

- **The vectors of displacements, the stresses and the deformations in the region have to be further determined on the basis of the adjustment by these points as fixed ones.**

It has to be noted that the proposed solutions and results from the processing represent only the beginning, which could be developed and improved further on. This refers especially to networks of larger scope with greater number of studied points and stations.