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DETERMINATION OF THE STATIONARY THERMAL STATE OF SIMPLE GEOMETRY LAYERED STRUCTURES WITH THEMPERATURE DEPENDENT HEAT CONDUCTIVITY FACTORS

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Abstract. An analytical-numerical method to determine the one-dimensional stationary thermal state of simple geometry multilayer structures for arbitrary dependences of heat-conductivity factors on temperature is proposed (the multilayer bodies of thermosensitive materials, referred to one of the classical orthogonal coordinate systems (α,β,γ) are considered, the thermal state caused by thermal load is characterized by a one-dimensional stationary temperature field $t(\alpha)$).

The method is based on:

- utilization of elements of generalized functions algebra;
- approximation of temperature dependences of heat-conductivity factors of materials by piecewise constant temperature functions –

$$\lambda_t^{(i)}(t) \approx \Lambda^{(i)}(t) = \Lambda_1^{(i)} + \sum_{j=1}^{m}\left(\Lambda_{j+1}^{(i)} - \Lambda_j^{(i)}\right) S_+\left(t - t_i\right),$$

  where $t_0 = 0 < t_1 < t_2 < ... < t_m < t_k = t_{m+1}$; $\Lambda_j^{(i)}$ – with the given accuracy corresponds to the value of the heat-conductivity factor of the corresponding layer in the interval $t_{j-1} < t < t_j$, $S_+(\alpha) = \{0,\ \alpha \le 0;\ 1,\ \alpha > 0\}$;
- introduction into consideration the function of Kirchhoff function type –

$$\vartheta(t) = \int_0^t \sum_{i=1}^{n} \Lambda^{(i)}(\xi) N_i(\alpha)\, d\xi,$$

  where $N_i(\alpha) = S_+(\alpha - \alpha_{i-1}) - S_+(\alpha - \alpha_i)$.

Therefore, the temperature field is determined by the relation

$$t = \left[\vartheta + \sum_{i=1}^{n} F_i(\vartheta) N_i(\alpha)\right] \Big/ \left[\sum_{i=1}^{n} \Lambda^{(i)}(\vartheta) N_i(\alpha)\right],$$

where $\vartheta = C_1 f_k(\alpha) + C_2 + \sum_{i=1}^{n-1}\left(K_i\, \vartheta\big|_{\alpha_i} + Q_i\right) S_+(\alpha - \alpha_i)$

is the solution of the partially degenerate equation derived from the heat equation in accordance with generalized functions algebra, taking into account the perfect thermal contact of the layers; C_1, C_2 are the constants of integration, in the general case determined from the system of two nonlinear algebraic equations obtained from the boundary conditions; $f_k(\alpha)$, K_i, Q_i are the functions and constants, determined by the recurrence relations obtained in the work.

Approbation of the methodology by studying the stationary thermal state of a two-layer cylinder is realized. The cases of existence of a closed-form analytic solutions for the nonlinear heat conduction problem are considered.

Key words: multilayer structures, solids of simple geometry, steady thermal state, temperature-dependent heat conduction factors, generalized functions.

INTRODUCTION

A considerable part of the methods of prediction the parameters of the technological processes of manufacturing and operation of structure elements at high-temperature heating is based on the concept of an effective heat source [1], whose parameters are thermophysical characteristics of materials, the characteristics of the technological process of manufacturing or operation a structural element and its geometric structure and, of course, heating.

In these methods it is assumed that the temperature field is a single independent characteristic, in terms of which all other are determined. Investigation in accordance with these methods is carried out in two steps: at the first step, the heat problem is formulated and solved, that is, the temperature field is determined, and at the second one – the temperature field is assumed to be known and the fast-moving processes that do not affect the temperature distribution are calculated. For example, the study of static or quasistatic thermoelastic behavior of solids is preceded by determination of their thermal state (temperature field) [2-9].

ANALYSIS OF RECENT INVESTIGATIONS AND PUBLICATIONS

Adequate determination of the thermal state of structural elements, including the layered ones, for high temperature heating is based on the model of thermosensitive body (the model takes into account the temperature dependence of the physical and mechanical characteristics of materials) [2, 3]. According to this model, the thermal state of the body is determined by solution of the nonlinear heat conduction problem.

The papers [10-12] proposed the methods to determine the stationary temperature fields in layered bodies for linear, quadratic and cubic dependence of heat-conductivity factors of materials of layers on temperature.

For an arbitrary number of layers these methods by Kirchhoff transformation [1] reduce the problem to the solution of one or a system of two nonlinear algebraic equations (according to the conditions of heat exchange), solution of which is recommended to be searched by numerical methods, by numerical methods, of successive approximations in particular.

In this case, the choice of initial approximation and clearning out the existence and uniqueness of the solution requires additional investigation. An overview of works related to this theme can be found in the works [2-15]

OBJECTIVES

The objective of this work is the development of a method of construction the analytical and numerical solutions for one-dimensional steady heat conduction problems of multilayer heat-sensitive bodies of simple geometry at high temperature heating.

The method is based on the use of the apparatus of generalized functions and allows us to investigate the thermal state of the layered bodies, irrespective of the temperature dependences type of the thermophysical characteristics of the material of the layers. Its use also makes it possible to find out the existence and uniqueness of the solution for the nonlinear heat conduction problem.

STATEMENT OF THE PROBLEM AND INITIAL EXPRESSIONS

Consider the multilayer bodies of thermosensitive materials assigned to one of the classical orthogonal coordinate systems (α,β,γ) (Cartesian – x,y,z (Fig. 1 a); cylindrical – r,φ,z (Fig. 1 b); spherical – r,φ,θ (Fig. 1 c)).

The boundary surfaces of the bodies coincide with the coordinate surfaces $(\alpha,\beta,\gamma)\ni\eta_i=const$ $(i=0,n)$ (multilayer structures of simple geometry [2]), and the surfaces of conjugation of materials – with the coordinate surfaces $\alpha=\alpha_i=const$ $(i=\overline{1,n-1})$ on which the conditions of the perfect contact are satisfied. We believe that the thermal state due to the thermal load is characterized by a one-dimensional stationary temperature field $t(\alpha)$.

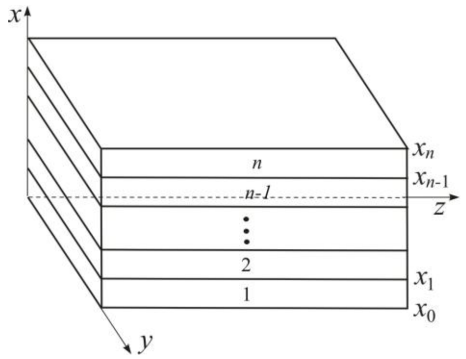

Fig. 1a. Fragment of a multilayer plate

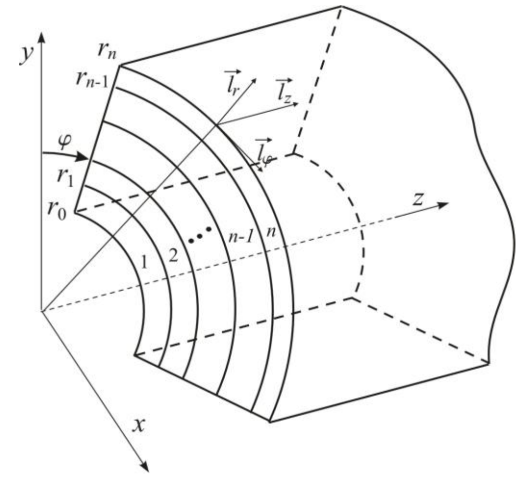

Fig. 1b Fragment of a multilayer cylinder

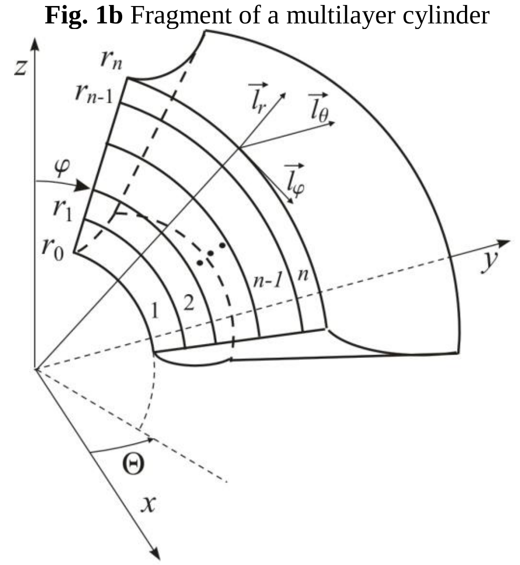

Fig. 1c Fragment of a multilayer ball

Fig.1 Multilayer structures of simple geometry

A mathematical model of thermal behavior of such bodies, according to the theory of an inhomogeneous body [16, 17], is the nonlinear boundary-value problem of the steady heat conductivity, which consists in determining the temperature function t by solving the heat equation

$$\left(\alpha^k\right)^{-1}\frac{\partial}{\partial\alpha}\left(\alpha^k\lambda_t(t,\alpha)\frac{\partial t}{\partial\alpha}\right)=-w_t(\alpha), \qquad (1)$$

which satisfies the conditions of perfect thermal contact on the surfaces of conjugation

$$\left.\begin{aligned} t\big|_{\alpha_i-0}&=t\big|_{\alpha_i+0}, \\ \left.\left(\lambda_t(t,\alpha)\frac{dt}{d\alpha}\right)\right|_{\alpha_i-0}&=\left.\left(\lambda_t(t,\alpha)\frac{dt}{d\alpha}\right)\right|_{\alpha_i+0}, \end{aligned}\right\} \qquad (2)$$

and the boundary conditions that describe the external thermal load

$$\left.\left(a_i(t)\frac{dt}{d\alpha}+b_i(t)\right)\right|_{\alpha_i}=0,\ \left(i=0,n\right). \qquad (3)$$

Here, for the case of: Cartesian coordinate system $k=0,\alpha=x$, cylindrical – $k=1$, $\alpha=r$, spherical – $k=2,\alpha=r$, the functions $a_i(t),b_i(t)$ are chosen according to the method of heating, $w_t(\alpha)$ is the power density of the internal heat sources and sinks, all derivatives are taken in the classical sense, and the temperature-coordinate dependence of the heat conductivity factor $\lambda_t(t,\alpha)$ has the form [18-20]

$$\lambda_t(t,\alpha)=\lambda_t^{(1)}(t)+\sum_{i=1}^{n-1}\left(\lambda_t^{(i+1)}(t)-\lambda_t^{(i)}(t)\right)S_+(\alpha-\alpha_i), \quad (4)$$

where $\lambda_t^{(i)}(t)$ is the temperature dependence of heat conductivity factor of the material of the i-th layer.

MAIN RESULTS OF THE INVESTIGATIONS

The basis of the method of analytical-numerical solution of the boundary-value problem (1) – (3) makes the approximation of the temperature dependences of the heat conductivity factors $\lambda_t^{(i)}(t)$ of the material of the layers by piecewise constant functions of the temperature in the form [13, 18, 20],

$$\lambda_t^{(i)}(t)=\Lambda^{(i)}(t)\approx\Lambda_1^{(i)}+\sum_{j=1}^{m}\left(\Lambda_{j+1}^{(i)}-\Lambda_j^{(i)}\right)S_+(t-t_i), \quad (5)$$
$$t_0=t_p<t_1<t_2<\ldots<t_m<t_k=t_{m+1}$$

and introduction into consideration the function of the Kirchhoff function type

$$\vartheta(t)=\int_0^t\sum_{i=1}^{n}\Lambda^{(i)}(\xi)N_i(\alpha)d\xi, \quad (6)$$

where $N_i(\xi)=S_+(\alpha-\alpha_{i-1})-S_+(\alpha-\alpha_i)$, $\alpha_0<\alpha_1<\ldots<\alpha_{n-1}<\alpha_n$, $[t_p;t_k]$ is the common temperature interval of determination $\lambda_t^{(i)}(t)$, ($i=\overline{1,n}$), the value of $\Lambda_j^{(i)}$ with the given accuracy corresponds to the value $\lambda_t^{(i)}(t)$ in the temperature range $t_{j-1}<t<t_j$, α_0, α_n are the coordinates of the structure's boundary surfaces; α_i $(i=\overline{1,n-1})$ are the conjugation (contact) coordinate surfaces of "i-th" and "i+1-th" layer, $S_+(\varsigma-\varsigma_i)=\{1,\varsigma>\varsigma_i;0,\varsigma\le\varsigma_i\}$.

As a result, the problem of determining the stationary thermal state of a multilayer structure is reduced to the solution of the relation

$$\vartheta(t)=t\sum_{i=1}^{n}\Lambda^{(i)}(t)N_i(\alpha)-\sum_{i=1}^{n}\left[\sum_{j=1}^{m}\left(\Lambda_{j+1}^{(i)}-\Lambda_j^{(i)}\right)t_jS_+(t-t_j)\right]N_i(\alpha), \quad (7)$$

by the solution of a partially degenerate equation

$$\alpha^{-k}\frac{d}{d\alpha}\left\{\alpha^k\frac{d\vartheta}{d\alpha}-\sum_{i=1}^{n-1}\left(\vartheta|_{\alpha_i+0}-\vartheta|_{\alpha_i-0}\right)\alpha_i^k\delta_+(\alpha-\alpha_i)\right\}=\sum_{i=1}^{n}\left(\left.\frac{\tilde d\vartheta}{\tilde d\alpha}\right|_{\alpha_i+0}-\left.\frac{\tilde d\vartheta}{\tilde d\alpha}\right|_{\alpha_i-0}\right)\delta_+(\alpha-\alpha_i)-w_t(\alpha), \quad (8)$$

obtained from the heat equation (1) with the help of the generalized functions apparatus [16], where $d/d\alpha$ is the generalized derivative, $\tilde d/\tilde d\alpha$ is the classical derivative.

Between ϑ and t according to the relation (6) there is a one-to-one correspondence, therefore

$$S_+(t-t_i)=S_+(\vartheta-\vartheta_i), \quad (9)$$

and as a result $\Lambda^{(i)}(t)=\Lambda^{(i)}(\vartheta)$, and

$$\vartheta(t)=t\sum_{i=1}^{n}\Lambda^{(i)}(\vartheta)N_i(\alpha)-\sum_{i=1}^{n}\left[\sum_{j=1}^{m}\left(\Lambda_{j+1}^{(i)}-\Lambda_j^{(i)}\right)t_jS_+(\vartheta-\vartheta_j)\right]N_i(\alpha). \quad (10)$$

Where from we obtain

$$t=\left(\vartheta+\sum_{i=1}^{n}F_i(\vartheta)N_i(\alpha)\right)\left(\sum_{i=1}^{n}\Lambda^{(i)}(\vartheta)N_i(\alpha)\right)^{-1}. \quad (11)$$

Here

$$F_i(\vartheta)=\sum_{j=1}^{m}\left(\Lambda_{j+1}^{(i)}-\Lambda_j^{(i)}\right)t_jS_+(\vartheta-\vartheta_j), \qquad \vartheta_l=\vartheta_l(\alpha),$$

$$\vartheta_l(\alpha)=\sum_{i=1}^{n}\left[t_l\Lambda^{(i)}(t_l)-\sum_{j=1}^{m}\left(\Lambda_{j+1}^{(i)}-\Lambda_j^{(i)}\right)t_jS_+(t_l-t_j)\right]N_i(\alpha).$$

On the surfaces of conjugation α_i, the perfect thermal contact conditions (2) are satisfied, therefore having taken into account (9), (10) we can assert the validity of relations

$$t|_{\alpha_i-0}=t|_{\alpha_i+0}=t|_{\alpha_i}, \quad \vartheta|_{\alpha_i-0}=\vartheta|_{\alpha_i},$$
$$S_+(t-t_i)|_{\alpha_i}=S_+(\vartheta-\vartheta_i)|_{\alpha_i}=S_+(\vartheta-\vartheta_i)|_{\alpha_i+0}, \quad (12)$$

$$\left.\begin{aligned}&\vartheta|_{\alpha_i-0}=\left(\Lambda^{(i)}(\vartheta)t-F_i(\vartheta)\right)\Big|_{\alpha_i},\\&\vartheta|_{\alpha_i+0}=\left(\Lambda^{(i+1)}(\vartheta)t-F_{i+1}(\vartheta)\right)\Big|_{\alpha_i},\\&\left.\frac{\tilde d\vartheta}{\tilde d\alpha}\right|_{\alpha_i-0}=\left.\left(\Lambda^{(i)}(t)\frac{\tilde dt}{\tilde d\alpha}\right)\right|_{\alpha_i-0},\\&\left.\frac{\tilde d\vartheta}{\tilde d\alpha}\right|_{\alpha_i+0}=\left.\left(\Lambda^{(i)}(t)\frac{\tilde dt}{\tilde d\alpha}\right)\right|_{\alpha_i+0},\end{aligned}\right\} \quad (13)$$

basing ourselves on which we obtain

$$\left.\begin{aligned}&\vartheta|_{\alpha_i+0}-\vartheta|_{\alpha_i-0}=K_i\,\vartheta|_{\alpha_i}+Q_i,\\&\left.\frac{\tilde d\vartheta}{\tilde d\alpha}\right|_{\alpha_i+0}-\left.\frac{\tilde d\vartheta}{\tilde d\alpha}\right|_{\alpha_i-0}=0\end{aligned}\right\}, \quad (14)$$

where $K_l=\left.\left(\frac{\Lambda^{(l+1)}(\vartheta)}{\Lambda^{(l)}(\vartheta)}-1\right)\right|_{\alpha_l}$, $Q_l=\left.\left(\frac{\Lambda^{(l+1)}(\vartheta)}{\Lambda^{(l)}(\vartheta)}F_l-F_{l+1}(\vartheta)\right)\right|_{\alpha_l}$. (15)

As a result, the solution of equation (8) reads:

$$\vartheta=C_1f_k(\alpha)+C_2+\sum_{i=1}^{n}\left(K_i\,\vartheta|_{\alpha_i}+Q_i\right)S_+(\alpha-\alpha_i)-W_t. \quad (16)$$

Here $f_1(\alpha)=\alpha-\alpha_0$ for the plate;

$f_2(\alpha)=\ln\frac{\alpha}{\alpha_0}$ for the cylinder;

$f_3(\alpha)=\frac{\alpha_i-\alpha}{\alpha_i\alpha}$ for the ball,

$$W_t(\alpha)=\int_0^{\alpha}\eta^{-k}\int_0^{\eta}\zeta^kw_t(\zeta)d\zeta d\eta.$$

Presenting $\vartheta|_{\alpha_i}$ in the form $\vartheta|_{\alpha_i}=\tilde K_1^{(i)}C_1+\tilde K_2^{(i)}C_2+\tilde K_3^{(i)}$, from (16) the recurrence relations to determine $\tilde K_j^{(i)},(j=1,2,3)$ are obtained

$$\tilde{K}_1^{(i)} = f_k(\alpha_i) + \sum_{j=1}^{i-1} K_j \tilde{K}_1^{(j)},$$
$$\tilde{K}_2^{(i)} = 1 + \sum_{j=1}^{i-1} K_j \tilde{K}_2^{(j)}, \qquad (17)$$
$$\tilde{K}_3^{(i)} = \sum_{j=1}^{i-1} \left(K_j \tilde{K}_3^{(i)} + Q_j \right) - W(\alpha_i),$$

as a result, the expression (16) for ϑ takes the form

$$\vartheta = C_1 \left[f_k(\alpha) + \sum_{j=1}^{n-1} K_j \tilde{K}_1^{(j)} S_+(\alpha - \alpha_j) \right] + C_2 \left[1 + \sum_{j=1}^{n-1} K_j \tilde{K}_2^{(j)} S_+(\alpha - \alpha_j) \right] - \left[W_t(\alpha) - \sum_{j=1}^{n-1} \left(K_j \tilde{K}_3^{(i)} + Q_j \right) S_+(\alpha - \alpha_j) \right]. \qquad (18)$$

In the general case, the values of constants of integration C_1, C_2 are determined from the system of two nonlinear algebraic equations obtained by substituting expression (11) into boundary conditions (3), which describe the heat exchange of the structure with the environment (external thermal load). Note that in the case of specifying on one of the boundary surfaces ($\alpha = \alpha_i$, $i = 0, n$) the heat exchange conditions of the 1-st or 2-nd type the finding of C_1, C_2, reduces to the solution of one nonlinear algebraic equation. When on the one of the boundary surfaces the heat exchange condition of the I-t type ($t|_{\alpha_0} = t_{0,n}^*$), and on another – the II-nd type ($\left(\lambda_t(t) \frac{\partial t}{\partial \alpha}\right)\Big|_{\alpha_{0,n}} = q$) are given, we obtain a closed analytical solution.

APPROBATION OF RESULTS

Approbation of the proposed analytical-numerical approach was carried out on the example of numerical study of the hypothetical two-layer cylinder's stationary thermal state, the internal surface of which $\alpha_0 = r_0$ is under the action of a steady temperature t_0^*, and the external $\alpha_2 = r_2$ – the heat flow q. At the same time, it was believed that on the surface of the conjugation of the layers $\alpha_1 = r_1$ the conditions of perfect thermal contact are satisfied.

The temperature field of such layered structure according to the relations (11), (15), (16), (17), (18) is determined by the formula

$$t = \frac{\vartheta + F_1(\vartheta) S_-(\alpha_1 - \alpha) + F_2(\vartheta) S_+(\alpha - \alpha_1)}{\Lambda^{(1)}(\vartheta) S_-(\alpha_1 - \alpha) + \Lambda^{(2)}(\vartheta) S_+(\alpha - \alpha_1)},$$

where $\vartheta = C_1 \left[f_2(\alpha) + K_1 f_2(\alpha_1) S_+(\alpha - \alpha_1) \right] + C_2 \left[1 + K_1 S_+(\alpha - \alpha_1) \right] + Q_1 S_+(\alpha - \alpha_1),$

$$K_1 = \left(\frac{\Lambda^{(2)}(\vartheta)}{\Lambda^{(1)}(\vartheta)} - 1 \right)\Bigg|_{\alpha_1}, \quad Q_1 = \left(\frac{\Lambda^{(2)}(\vartheta)}{\Lambda^{(1)}(\vartheta)} F_1 - F_2(\vartheta) \right)\Bigg|_{\alpha_1},$$
$$S_-(\alpha_1 - \alpha) = 1 - S_+(\alpha - \alpha_1)$$
$$F_i(\vartheta) = \sum_{j=1}^{m} \left(\Lambda_{j+1}^{(i)} - \Lambda_j^{(i)} \right) t_j S_+(\vartheta - \vartheta_j),$$
$$\vartheta_l = \sum_{i=1}^{n} \left[t_l \Lambda^{(i)}(t_l) - \sum_{j=1}^{m} \left(\Lambda_{j+1}^{(i)} - \Lambda_j^{(i)} \right) t_j S_+(t_l - t_j) \right] N_i(\alpha).$$

The constants of integration C_1, C_2 are determined from the boundary conditions by relations

$$C_1 = q\alpha_2, \; C_2 = \vartheta(\alpha_0).$$

NUMERICAL STUDIES AND THEIR ANALYSIS

Numerical studies were carried out for the cylinders, whose layers are made of molybdenum, tungsten, aluminum or steel.

Expressions, describing the temperature dependence of heat-conductivity factors were obtained by tabular data's approximation by polynomial functions using the least squares method in the form [2, 11, 12]:

- for molybdenum $t \in [273\,K;\; 1800\,K]$:

$$\lambda_{M0}(t) = 110.987 [W/mK];$$
$$\lambda_{M1}(t) = 137.5 - 214.653 \cdot 10^{-4} (t - 273) [W/mK];$$
$$\lambda_{M2}(t) = 146.146 - 425.178 \cdot 10^{-4} (t - 273) + 850.591 \cdot 10^{-8} (t - 273)^2 [W/mK];$$
$$\lambda_{M3}(t) = 151.73 - 702.736 \cdot 10^{-4} (t - 273) + 366.838 \cdot 10^{-7} (t - 273)^2 - 7.59 \cdot 10^{-9} (t - 273)^3 [W/mK];$$

- for tungsten $t \in [273\,K;\; 1800\,K]$

$$\lambda_{W0}(t) = 124.335 [W/mK];$$
$$\lambda_{W1}(t) = 114.819 - 768.94210^{-5} (t - 273) [W/mK];$$
$$\lambda_{W2}(t) = 163.269 - 110.96210^{-3} (t - 273) + 479.40410^{-7} (t - 273)^2 [W/mK];$$

- for steel $t \in [273\,K;\; 1000\,K]$:

$$\lambda_S(t) = 45{,}04 \left(1 - 0{,}5 \frac{t - 373}{873} \right) [W/mK];$$

- for aluminum $t \in [273\,K;\; 1000\,K]$:

$$\lambda_A(t) = 247 \left[1 - 0{,}493 \cdot 10^{-3} (t - 273) + 0{,}49 \cdot 10^{-6} (t - 273)^2 \right] [W/mK],$$

During studies, the corresponding temperature dependence of the layer's material heat-conductivity factor $\lambda_t^{(i)}(t)$ in the temperature range of their determination $t \in [t_p \,;\, t_k]$ was approximated by expression (5), in which the approximation coefficients $\Lambda_j^{(i)}$ and the approximation nodes t_j were given in this way

$$\Lambda_j^{(i)} = \lambda_t^{(i)}(t_j), \quad t_j = t_p + j(t_k - t_p) m^{-1}, \; j = \overline{1, m},$$

where m is the number of approximation nodes.

Typical results of numerical studies in graphical form are shown in Figs. 2-6

The behavior of the temperature dependence $\lambda_{M3}(t)$ and its approximation $\Lambda^{(M3)}(t)$ at $m = 30$ are given in Fig. 2. The variation of the relative error of approximation

$\varepsilon = \left| \dfrac{\lambda_{M3}(t) - \Lambda^{(M3)}(t)}{\lambda_{M3}(t)} \right| 100\%$ is illustrated in Fig. 3

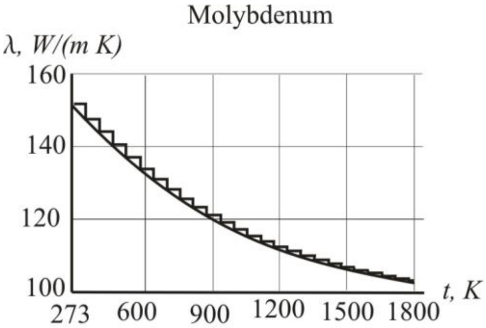

Fig. 2 Temperature dependence $\lambda_{M3}(t)$ and its piecewise constant approximation $\Lambda^{(M3)}(t)$ at $m = 30$.

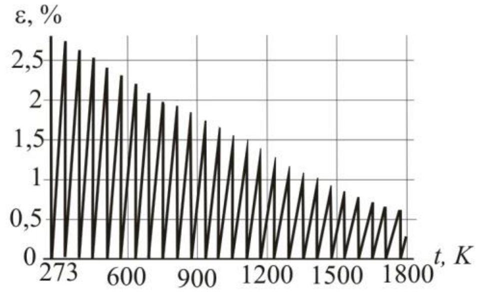

Fig. 3 The changing of the relative error for approximation $\lambda_{M3}(t)$ by a piecewise-continuous function $\Lambda^{(M3)}(t)$ at $m = 30$

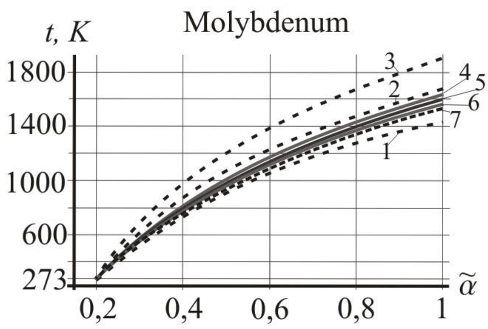

Fig. 4 Distribution of temperature along the radial coordinate $\tilde{\alpha} = \alpha/\alpha_2$ in a homogeneous cylinder of molybdenum for different accuracy of temperature dependence approximation $\lambda_{M1}(t)$.

Fig. 4 shows the distribution of temperature along the radial coordinate $\tilde{\alpha} = \alpha/\alpha_2$ in a homogeneous molybdenum cylinder for different accuracy of approximation of temperature dependence $\lambda_{M1}(t)$ at $t_0^* = 273K$, $q = 100\,\kappa W$:

- dash line 1, 2, 3 correspond to the temperature distribution calculated for the approximation of the heat-conductivity factor's dependence on the temperature $\lambda_{M1}(t)$ by its values at the beginning, middle and at the end of the temperature interval of its determination;
- solid lines 4, 5, 6 – for the approximation of the themperature dependence $\lambda_{M1}(t)$ according to the representation (5) for the number of nodes of approximation $m = 5, 10, 30$, respectively;
- dash line 7 corresponds to the temperature distribution calculated for the dependence $\lambda_{M1}(t)$.

The temperature distribution along the coordinate $\tilde{\alpha}$ in a two-layer thermosensitive cylinder (steel-molybdenum $\lambda_t^{(1)}(t) = \lambda_S(t)$, $\lambda_t^{(2)}(t) = \lambda_{M1}(t)$), for the piecewise constant approximation of the temperature dependence of the heat-conductivity factor (dash curves: 1 – for $\lambda_{M1}(t_k)$; 2 – $\lambda_{M1}\left[0,5\left(t_p - t_k\right) + 273\right]$; 3 – $\lambda_{M1}(t_p)$; solid lines 4,5,6 – $\lambda_{S,M1}(t) \approx \Lambda^{(S,M1)}(t)$ for $m = 5, 10, 30$, respectively), is shown in Fig. 5.

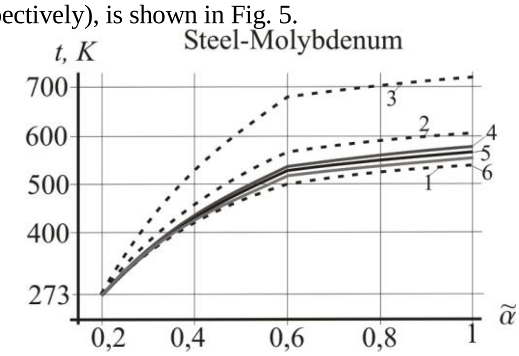

Fig. 5 Distribution of temperature along the coordinate $\tilde{\alpha}$ in a two-layer thermosensitive cylinder (steel-molybdenum) for different piecewise-constant approximation accuracy of dependences of heat-conductivity factors of its components.

The temperature variation along the coordinate $\tilde{\alpha}$ in a two-layer thermosensitive cylinder (aluminum-tungsten) at $t_0^* = 273K$, $q = 100\,\kappa W$, for a piecewise constant approximation (5) at $m = 30$ nodes of approximation (solid curves 1,2,3 for different dependences of the tungsten heat-conductivity factor $\lambda_{W0,W1,W2}(t) \approx \Lambda^{(W0,W1,W2)}(t)$, respectively; dash line – for values of dependencies $\lambda_A(t) \approx 247[W/mK]$, and $\lambda_{W0}(t) = 124.335[W/mK]$) is illustrated in Fig. 6.

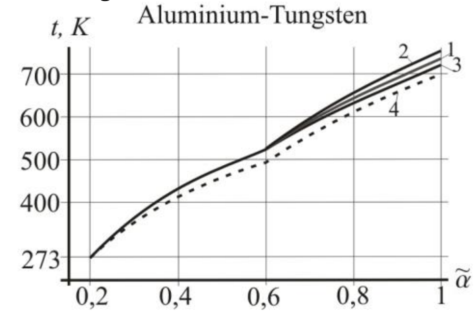

Fig. 6 Distribution of temperature along the coordinate $\tilde{\alpha}$ in a two-layer thermosensitive cylinder (aluminum-tungsten) for different piecewise-constant approximation accuracy of dependences of heat-conductivity factors of its components.

The presented results testify that:

– the use of piecewise constant approximation (5) makes it possible to determine the temperature state with arbitrary accuracy Fig. 3;

– the use of approximation of the temperature dependence of the heat- conductivity factor by constant value most likely results in significant errors (Fig. 4-6);

– neglecting the type of the temperature dependence of the heat-conductivity factor of the layers material can causes the inadequate estimation of the thermal state of the object (Fig. 6);

– the proposed procedure ensures rapid convergence of the process of temperature numerical determination (increase in the number of approximation nodes $m > 30$ in the investigated cases practically did not affect the numerical value of temperature Fig. 4).

CONCLUSION

The analytical-numerical approach to solving one-dimensional problems of steady heat conductivity of layered bodies of simple geometry, the materials of which have thermal nonlinearity, is proposed. This approach makes it possible to investigate the thermal state of layered bodies, regardless of the type of the temperature dependences of the thermophysical characteristics. Its use also avoids the need to find out the existence and uniqueness of solution to the nonlinear heat conduction problem. The use of such an approach is useful in determining the thermo-stressed state of typical parts and structure elements of composite structure, in in which the heat conductivity factors of components materials have different nature of temperature dependence.

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