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Heat transfer processes in installations with photovoltaic roof tiles

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The paper presents the classification of methods of heat transfer, and the procedure when determining criterion equations depending on the type of heat exchange. It describes the heat transfer coefficients and resistance of heat transfer by convection. It also indicates the workflow to determine the quantitative heat flow in the present system of photovoltaic roof tiles installed in the roof structure.

KEYWORDS: photovoltaic roof tile, convection, radiation, conduction, heat transfer

1. Introduction

Photovoltaic cells being under the constant influence of solar radiation, as well as generating the electric energy, get hot, which causes deterioration of their efficiency. In the photovoltaic installations built on the basis of solar roof tiles, other thermal processes (modes of heat transfer) may occur in comparison with the installations which consist of PV panels, placed on the supporting structure over the roof. PV roof tiles, built in the roof structure are not naturally cooled by wind and the air accumulated in the crevice between the roof tiles and the roof structure is subject to heating up from PV cells and transfers the accumulated heat with a varied degree, depending on the thermal insulation of the roof and the way of using the attic space of the building. In such cases, there is practically no forced movement of air in the roof gap, which could cool the underside of the photovoltaic roof tiles and mitigate the negative effect of temperature growth on the efficiency of the photovoltaic conversion.

Heat transfer between photovoltaic roof tiles, roof structure and air space between them, may occur in three physically different modes including [3, 4, 5]:

- conduction: transfer of kinetic energy of the microscopic motion of particles (atoms, ions, electrons) between the parts of one or different bodies being in direct contact with one another;
- convection: transfer of heat related to the macroscopic motion of particles. It may occur in restricted or open spaces and be caused by a difference in temperatures (and thus a difference in density and pressures – natural convection) or forced by the action of external forces (forced convection);

- radiation: transfer of energy between bodies or parts of the same body via temperature electromagnetic radiation. This phenomenon may be regarded as the generation of radial energy in the radiation source, as the response of the receiver which receives this energy or as the process of radiation propagation in the environment which separates the source from the receiver. Heat radiation emission by a body is accompanied by constant absorption of radiation which reaches it from outside. If there is a balance between these effects, the body remains in the state of thermodynamic balance. However, if the amount of energy radiated by the body or its part is different from the amount of absorbed energy, there is a transfer of radial energy between the bodies, and the resultant power will flow from the body with the higher temperature to the body with the lower temperature, which is called radiative heat transfer.

In very many cases, there is a heat transfer between different bodies with simultaneous occurrence of two or three modes of heat transfer to a different extent.

2. Heat transfer

Convection is defined as heat transfer occurring mainly in fluid bodies, in which particles are subject to macroscopic motions because of having great freedom of motion. At the same time, the conduction process is present; it consists in the transfer of energy through the chaotic motion of neighbouring particles. The value of the total heat flow density and the temperature field is significantly affected by convection and in the direction of the heat transfer determined by the temperature gradient, there are fluid velocity components different from zero. However, the heat transfer in this fluid may occur only by conduction (there are no components of convective motion in the direction determined by the temperature gradient), e.g. in the wall layer when such fluids flow around solid bodies. The fluid velocity is decreased as the distance to the solid body surface decreases, whereby the value which it reaches on that surface is equal to zero. If the surface is isothermal, the heat is transferred only by conduction. The existence of the wall layer means that the heat transfer between these bodies takes place by convection and conduction, which is called convective heat transfer. Additionally, when heat input or output takes place as a result of radiation, then the so called heat transfer occurs [3, 4, 5].

The issue of heat transfer is related to defined heat transfer coefficients occurring during the formulation of boundary conditions of the third kind referred to as Fourier conditions [3, 4]. If body *A* is in fluid *B*, then heat from the convection process reaches elementary surface *dF*. Also the heat transferred by radiation may reach that surface. Altogether, as a result of convection and radiation, *dF* is reached by the heat flow with the total density of $q_\alpha [W/m^2]$.

Assuming that the fluid which surrounds body *A* has temperature $t_{U,k} = t_U$ at a sufficiently great distance from surface *F*, e.g. at edge *U* of the thermokinetic system under consideration, then it can be assumed that heat flow density q_α is proportionate to temperature difference $t_F - t_U$, i.e.:

$$q_\alpha = \alpha(t_U - t_F) \quad (1)$$

where: α – heat transfer coefficient [W/m^{2}K].

Formula (1) constitutes a mathematical form of the boundary condition of the third kind. Coefficient α expresses the value of heat flow (heat output) equal to 1 W, inflowing (when $t_U > t_F$) or outflowing (when $t_U < t_F$) through the area unit equal to 1 m^2 at the temperature difference between *dF* and edge *U* equal to 1 K. With the simultaneous occurrence of radiation and convection, resultant heat flow density q_α is a sum of components of density of the radiation transferred by convection (q_k) and radiation (q_r):

$$q_\alpha = q_k + q_r \quad (2)$$

where: q_k – density of heat flow transferred to surface *dF* by convection, q_r – density of heat flow transferred to surface *dF* by radiation.

Heat may be transferred both form surface *U* to surface *F* and the other way round. Components of sum 2 are determined by the following relations:

$$q_k = \alpha_k(t_{U,k} - t_F) \quad (3)$$

$$q_r = \alpha_r(t_{U,r} - t_F) \quad (4)$$

The value of coefficient α in equation 1 takes into account the convective and radiative heat transfer and formula 1 assumes the following form:

$$q_\alpha = \alpha_k(t_{U,k} - t_F) + \alpha_r(t_{U,r} - t_F) \quad (5)$$

In the case of equation $t_{U,k} = t_{U,r} = t_U$, we obtain the following sum of proportionality factors: $\alpha_k + \alpha_r = \alpha$.

If the temperature field in the fluid body, at a sufficiently large distance from surface *F* is uniform, then heat flow P_k transferred by convection from body *A* to body *B* through surface *F* is equal to:

$$P_k = \int_F q_k dF \quad (6)$$

Heat flow density distribution q_k can be determined on the basis of the Newton's law (described by relation 3), and after comparing equations (3) and (6), relation (7), describing the heat transfer coefficient by convection is obtained:

$$\alpha_k = \frac{dP_k}{dF} \frac{1}{t_{U,k} - t_F} \quad (7)$$

If the components of velocity of the respective fluid streams directed towards different elements of surface F are not equal, it is necessary to determine the mean coefficient of heat transfer by convection:

$$\overline{\alpha}_k = \frac{1}{F}\int_F \alpha_k dF \tag{8}$$

and the total heat flow transferred by surface F is:

$$P_k = (t_{U,k} - t_F)\int_F \alpha_k dF = \overline{\alpha}_k F(t_{U,k} - t_F) \tag{9}$$

3. Resistance and coefficient of heat transfer by convection

Coefficient of heat transfer by convection α_k is a measure of heat flow (heat output) transferred by the unit element of the boundary surface between two bodies A and B, under the influence of the unit difference in temperature $t_F - t_{U,k}$, whereby t_F is the temperature of this element and $t_{U,k}$ is the temperature of fluid body A away from the boundary surface.

The heat movement by transfer is related to conduction heat resistance expressed by the quotient of difference in temperature and heat output. The occurrence of resistance on the heat transfer route in the one–dimensional temperature field causes a temperature decrease $\Delta t = t_{U,k} - t_F$, whereby a bigger decrease takes place at the wall layer $\Delta t' = t' - t_F$, where only heat conduction occurs. Heat resistance of this layer is [2, 4, 5]:

$$W' = \frac{\Delta l}{\lambda \Delta F} \qquad [\mathrm{K/W}] \tag{10}$$

where: λ – specific thermal conductivity for fluid in the wall layer with a thickness of Δl [W/mk], ΔF – finite element of the boundary surface to which heat is transferred [m^2].

Additionally, there is serial resistance W'', at which the temperature decrease is $\Delta t'' = t_{U,k} - t'$. The total heat transfer resistance is the sum of these two resistances, i.e.:

$$W_k = W' + W'' \tag{11}$$

By determining heat flow ΔP_k transferred to element of boundary surface ΔF and taking into account the Newton's law, the following formulas are obtained:

$$W_k = \frac{t_{U,k} - t_F}{\Delta P_k} = \frac{t_{U,k} - t_F}{q_k \Delta F} = \frac{1}{\alpha_k \Delta F} \tag{12}$$

or

$$\overline{W}_k = \frac{1}{\overline{\alpha}_k F} \tag{13}$$

where: $\overline{\alpha}_k$ – mean coefficient of heat transfer on entire boundary surface F.

Equation (13) implies that the heat transfer resistance is inversely proportional to heat transfer coefficient α, dependent, among other things, on specific heat of fluid c. The resistance increases with an increase in specific thermal conductivity λ of the wall layer and a decrease in its thickness. Specific thermal conductivity λ depends, among other things, on fluid density ρ while the wall layer thickness depends on the fluid velocity and its dynamic viscosity μ.

Knowing that the fluid layer near the boundary surface is not characterised by temperature uniformity t, and values such as λ, ρ, μ, c are functions of temperature, it is necessary to introduce the so called design temperature t_m, i.e. the temperature for which values of parameters that characterise liquid environment will be assumed in calculations of α_k. The design temperature value is determined on the basis of the following equation [3, 4, 6, 7]:

$$t_m = \frac{t_{U,k} + t_F}{2} \tag{14}$$

Value $t_{U,k}$ is the fluid temperature away from surface F, in the zone being outside the area of thermal interferences, often on the conventional boundary of the thermokinetic system in the fluid.

4. Types of heat transfer

The type of flow of the fluid participating in the heat transfer process allows three basic modes of heat transfer to be distinguished: at forced, free or mixed flows. The classification of types of heat transfer is presented in Figure 1 [4].

During the forced flow of the fluid, its particles are set in motion by external forces (e.g. fans, pumps), and in the case of the free flow, the particles move as a result of gravity. When the temperature of two particles is different, they will be subject to heat transfer, that is, their temperature will change and thus also their density. Owing to this, as a consequence of gravity, the particles will move. When both types occur at the same time, then heat transfer will take place at mixed flow. Frequently, especially at high fluid velocities, the percentage of heat transfer at free flow is hardly traceable in comparison with the forced flow.

If the nature of the flow is taken into consideration, it is possible to distinguish its three types: laminar flow, transition flow and turbulent flow, which is related to fluid viscosity. In the areas which are at a sufficient distance from the boundary surfaces, the impact of viscosity on the nature of the fluid flow is insignificant. However, it is of great importance in the areas adjacent to the boundary surfaces, where the fluid velocity w [m/s] goes down to zero at the contact point of the surfaces under consideration. Exceeding a certain critical fluid flow velocity w_{kr} changes the nature of the heat transfer in the system. Below the critical velocity, the fluid moves in an orderly, i.e. laminar manner (its streams are not subject to turbulences). Above this velocity, turbulences in the fluid's motion appear and its particles move in a chaotic manner. Such a flow is defined as turbulent flow. The value of heat transfer coefficient α_k at the turbulent flow is higher than at the laminar flow.

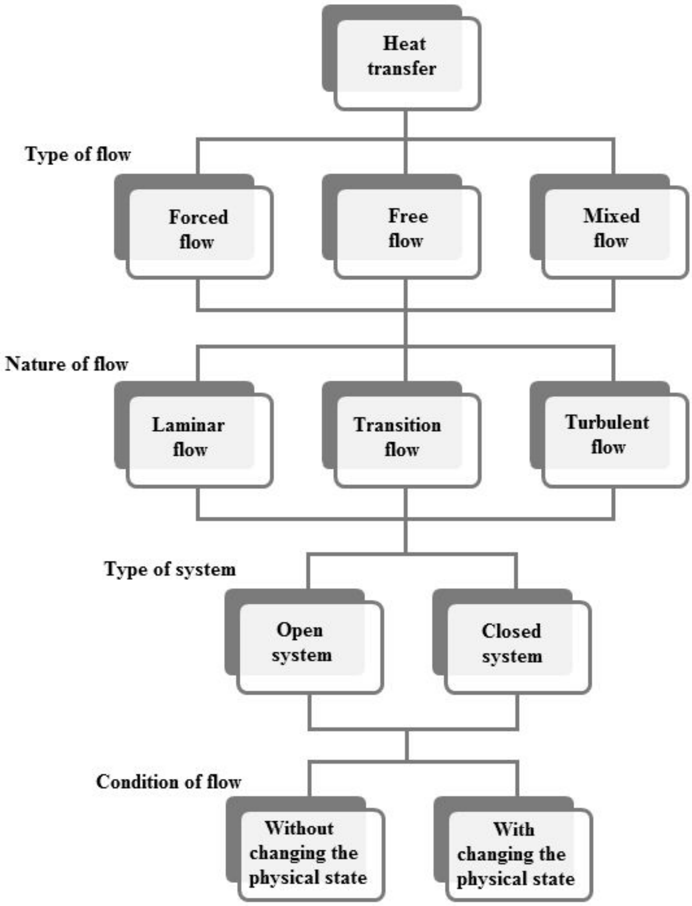

Fig. 1. Classification of types of heat transfer [4]

Kinetic velocity of fluid w_{kr} is directly proportional to its kinematic viscosity v and inversely proportional to characteristics dimension l of a body with which the fluid exchanges heat, which can be expressed by equation [4]:

$$w_{kr} = C\frac{v}{l} \tag{15}$$

where: v – kinematic viscosity of fluid [m^2/S], l – characteristic linear dimension [m], C – proportionality factor [–].

Factor C in equation (15) is equal to Reynolds' criterion *Re*. Below the critical value of the Reynold's criterion (just like in the case of critical velocity), the flow of fluid is laminar and its value depends on the type and geometry of the thermokinetic system. Because the value of Reynolds' criterion *Re* may be characterised by significant dispersion, the concept of the transition flow with the defined range of *Re* values was introduced.

The three distinguished flows mentioned above may also appear at free or mixed flow of fluid, however, the boundaries between them may not be as clear as in the case of the forced flow. Therefore, in the case of the free flow, the boundaries between the flows of different natures are not determined by means of the *Re* criterion, but by means of the Grashof's criterion – *Gr* or the product of the Grashof and Prandtl's *Pr* criteria.

While considering the type of the system, it is possible to distinguish open and closed systems. In the open systems, the surfaces that limit the system are at a far distance from the heat transfer location. On the other hand, in the closed systems, the thickness of the wall layer is comparable with the characteristic dimension of the system, and owing to this, the surfaces that limit the system exert a significant impact on the conditions of fluid motion.

Another criterion that classifies the type of heat transfer is the condition of flow when the physical state of fluid participating in the process is changed or not.

The specified criteria of division allow certain groups of processes of heat transfer, described by similar criteria equations, to be distinguished.

5. Criteria equations for the coefficient of heat transfer by convection

In order to determine the coefficient of heat transfer by convection, it is possible to use many analytical and empirical methods, based mainly on the Fourier–Kirchhoff equation, Navier–Stockes equation or continuity equation for incompressible fluid flow. Coefficient α_k depends on very many variables, which creates the necessity of using many simplifying assumptions while solving complex differential equations, and as a consequence of this, gives results characterised by little accuracy, incomparable with the experimentally determined values. Therefore, in order to analyse heat transfer, empirical studies combined with the theory of probability of physical phenomena turned out to be helpful. Instead of determination of α_k as a function of many variables, the process of heat transfer is described by means of characteristic numbers (criteria numbers, dimensionless modules) [3, 4].

Based on the theory of probability, the following criteria numbers may be determined for phenomena of heat transfer by convection [1, 3, 4, 5, 6, 7]:

- Nusselt number (it combines the heat transfer in the fluid flow with the penetration of heat into the wall):

$$Nu = \frac{\alpha_k l}{\lambda} \tag{16}$$

- Reynolds number (this number constitutes the ratio of inertia forces to fluid internal friction forces):

$$Re = \frac{wl}{\nu} \tag{17}$$

- Grashof number (this number determines the ratio of uplift forces to fluid internal friction forces):

$$Gr = \frac{\beta g l^3 \Delta t}{\nu^2} \tag{18}$$

where: β – volumetric expansion coefficient [1/K], determined as the inverse of design temperature from equation (14):

$$\beta = \frac{2}{t_{U,k} + t_F} \tag{19}$$

where: g – gravitational acceleration [m/s^2],

- Prandtl number (characterised by the similarity of fluid type):

$$Pr = \frac{c\mu}{\lambda} \tag{20}$$

where: c – specific heat [Ws/kgK], μ – dynamic viscosity [Ns/m^2].

In order to make the heat motion based on heat transfer by convection in geometrically similar systems similar (for steady states of heat and fluid flow), sought (non–determining) number *Nu* must be a function of the following determining numbers of heat and mechanical similarity [3, 4]:

- for free convection:

$$Nu \approx f(Gr, Pr) \tag{21}$$

- for forced convection:

$$Nu \approx f(Re, Pr) \tag{22}$$

If monoatomic gases are fluids, then $Pr = const$, therefore it is possible to exclude the *Pr* number, and value α_k can be determined after transforming equation (16) into the form of equation 23, where the Nusselt number is the function of only one criteria number: Reynolds or Grashof numbers, depending on the type of convection [4].

$$\alpha_k = \frac{Nu\lambda}{l} \tag{23}$$

Sometimes, it is necessary to supplement the determining characteristic numbers with additional, geometrical or physical dimensionless modules. The dimensional analysis and experimental studies show that behaviour of the functions mentioned above can be described by the following exponential behaviours [2, 3]:

$$Nu \approx C_1 (Gr\,Pr)^i \tag{24}$$

$$Nu \approx C_2\, Re^n\, Pr^m \tag{25}$$

Taking into consideration equations 24 and 25 as well as (16) ÷ (20), we get:

$$\frac{\alpha_k l}{\lambda} = C_1 \left(\frac{\beta g l^3 \Delta t}{\nu^2} \frac{c\mu}{\lambda} \right)^i \tag{26}$$

$$\frac{\alpha_k l}{\lambda} = C_2 \left(\frac{wl}{\nu} \right)^n \left(\frac{c\mu}{\lambda} \right)^m \tag{27}$$

The nature of the medium's flow determines the value of *GrPr* expressions for the free convection, or the *Re* expression for the forced convection. Small values of these expressions indicate the laminar nature of the flow, the average values – the transition flow – and the high values represent the turbulent flow. The experimentally determined behaviours of functions, which are described by equations (26) and (27) are provided by processes characterized by invariable values of coefficients: C_1, C_2, i, n, m, with regards to the variability of *GrPr* and *Re* expressions, corresponding to the nature of the flow. It is also possible to distinguish specific cases of free convection occurring in flat, cylindrical or spherical gaps.

In order to take advantage of the behaviours of functions described in literature, which deal with the phenomenon of convective heat transfer, it is necessary to [3]:

- select the behaviour of the function that corresponds to the studied phenomenon in physical and geometrical aspects,
- determine the physical values of fluid,
- calculate the values of expressions *GrPr* or *Re* and select the values of coefficients C_1, C_2, i, n, m, corresponding to them
- calculate the value of *Nu*,
- determine the value of α_k,
- find sought q_α based on formula 3.

Most frequently, for the heat transfer at free convection occurring in unrestricted spaces, the expressions proposed by Micheev are taken into account [3]:

$$\begin{gathered} Nu = 1{,}18(Gr\,Pr)^{0{,}125} \quad \text{dla } 10^{-3} < Gr\,Pr < 5 \cdot 10^{2} \\ Nu = 0{,}54(Gr\,Pr)^{0{,}25} \quad \text{dla } 5 \cdot 10^{-2} < Gr\,Pr < 2 \cdot 10^{7} \\ Nu = 0{,}135(Gr\,Pr)^{0{,}33} \quad \text{dla } 2 \cdot 10^{-7} < Gr\,Pr < 10^{13} \end{gathered} \tag{28}$$

The form of formulas (28) are sufficiently general to use them for the purpose of determination of value α_k in the case of planes located in parallel to the ground surface, assuming the plane width as characteristic dimension *l*. After determining value α'_k, based on the formulas given above, it is possible to calculate real value α_k for [3]:

- planes that transfer heat upwards:

$$\alpha_k = 1{,}3\alpha_k' \tag{29}$$

- planes that transfer heat downwards:

$$\alpha_k = 0{,}7\alpha_k' \tag{30}$$

In view of the specifics of assembly and operation of the photovoltaic installations integrated with a building, the convective heat transfer will occur at the free air flow.

6. Heat transfers at free flow

The free flow of fluid is possible as a result of uplift forces related to the change in the fluid density caused by changes in its temperature. In view of the fact that velocity of the fluid depends on the temperature (which is described in the *Gr* criterion), the free convection may be described by equation (21). However, in reality, the velocity field (except gravity) is also affected, though only slightly, by forces of inertia. Therefore the Grashof criterion occurs as a product together with the Prandtl criterion, which may be superseded by the Rayleigh criterion [4, 6, 7]:

$$Ra = Gr \cdot Pr = \frac{\beta g l^3 \Delta t}{a\nu} \tag{31}$$

where: a – thermal diffusivity [m^2/s], determined as:

$$a = \frac{\nu\lambda}{c\mu} \tag{32}$$

Thus, general relation (21) takes the form of (33):

$$Nu = f(Ra) \tag{33}$$

In general, with the free flow expressed according to the Rayleigh criterion, the laminar, transition and turbulent motions of the fluid may occur.

7. Heat transfers at forced flow

In the case of the roof installation consisting of photovoltaic tiles, the heat transfer will also occur at forced flow around the flat surface (as presented in Figure 2).

Velocity of inflowing air w_0 is equal to velocity of air beyond the area of the hydraulic wall layer, and in view of the oval shape of the roof tile front, velocity on its front (i.e. for y = 0) also amounts to w_0. A wall layer is formed and its thickness and nature depend on the distance of y from the beginning of the surface around which the air flows, and on the value of the *Re* criterion. In the first section (marked with number 1 in Figure 2 in the range of values of y between 0 and y_{kr}), the flow in the wall layer is always of laminar nature, and its thickness is determined by relation (34) [4]:

$$\Delta l_l = \frac{4.64y}{\sqrt{Re_{y,m}}} = 4.64\sqrt{\frac{y\nu_m}{w_o}} \tag{34}$$

where: ν_m – kinematic viscosity for temperature t_m determined on the basis of relation (14).

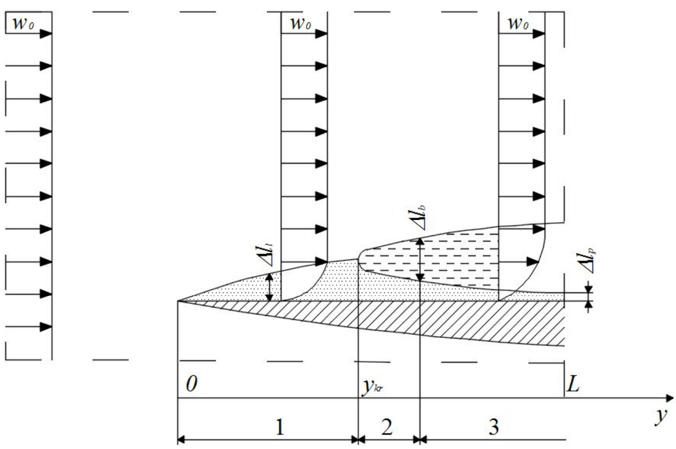

Fig. 2. Forced flow around the flat surface, where [4]: 1 – laminar wall layer, 2 – transition area, 3 – turbulent wall layer

Reynolds number present in equation 34 (described by relation (17)) takes the form of equation (35) now [4]:

$$Re_{y,m} = \frac{w_o y}{\nu_m} \tag{35}$$

For the values of the Re criteria higher than critical value $5\cdot10^5$, at a distance of y_{kr}, the airflow in the wall layer becomes turbulent. However, just at the surface around which the air flows, there still is a laminar layer with the thickness of Δl_p. Thickness of turbulent layer Δl_b can be determined using formula (36), length of section y_{kr} can be determined based on formula (37) and maximum thickness of laminar wall layer Δl_{kr} can be obtained from formula (38) [4]:

$$\Delta l_b = \frac{0.37y}{Re_{y,m}^{0.2}} = 0.37\sqrt[5]{\frac{y^4\nu_m}{w_o}} \tag{36}$$

$$y_{kr} = \frac{\nu_m Re_{kr,m}}{w_o} \tag{37}$$

$$\Delta l_{kr} = 3.28 \cdot 10^3 \frac{v_m}{w_o} \tag{38}$$

The local value of the Nusselt criterion at a location distant by y from the edge of the surface can be determined from formula (39) [4]:

$$Nu_{y,m} = 0.332\, Re_{y,m}^{1/2} \cdot Pr_m^{1/3} \quad \text{for} \quad Pr_m > 0.6 \tag{39}$$

For the critical value of the Reynolds criterion in the initial area of the flat surface with the length of y_{kr}, the fluid motion in the wall layer is laminar, further on, turbulent (Fig. 2), and the average value of the Nusselt number along entire length L can be determined by [4]:

$$Nu_{y,m} = 0.0366\, Pr_m^{1/3} (Re_{L,m}^{4/5} - 4200) \quad \text{for} \quad Re_{kr,m} = 1 \cdot 10^5 \tag{40}$$

$$Nu_m = 0.0366\, Pr_m^{1/3} (Re_{L,m}^{4/5} - 23100) \quad \text{for} \quad Re_{kr,m} = 5 \cdot 10^5 \tag{41}$$

Equations (40) and (41) are correct for $Pr_m > 0.6$, and the values of the correction coefficient for $Re_{kr,m}$ can be determined from among the limit values specified in formulas (40) and (41), on the basis of the linear interpolation, using limit values 4200 and 23100.

8. Conclusions

While considering the forms of heat transfer in the systems with photovoltaic roof tiles placed in the roof structure, attention must be paid to many factors having an impact on the distribution and mode of heat transfer. The presented equations allow the determination and assessment of relevant criteria equations in reference to heat transfer by free convection, which will be dominant in the case under consideration. The determination of the value of the respective criteria numbers will allow the type of convection to be specified, with the obtained type of fluid (air) motion, located in the space between the roof structure and the roof tiles. It is also necessary to determine the nature of the system (open or closed) in which the heat transfer occurs, and the shape of the gap and direction of heat transfer. In the majority of cases, the heat will be transferred from the underside of the PV roof tile, towards the roof structure, where these two planes are parallel to each other and inclined at a certain angle in reference to the substrate.

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