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## Abstract

The article analyses the effects of paper moisture content on energy absorbed during its tensile testing in the range of moisture equilibrium obtained during paper air conditioning at a temperature of 23 °C. To predict changes in breaking energy caused by changes in moisture content, two calculation methods were proposed. The methods were verified on the basis of laboratory tests and calculation results, and their practical usefulness was confirmed.

**Key words:** breaking energy, paper, humidity, breaking strength, tensile strength.

## Introduction

Breaking energy is of great importance for many practical applications of paper and paper products. It is work done by external forces in a one direction tensile test of paper until the moment of its breaking. The energy depends on many factors, and on a tension graph it is illustrated by the surface area under a curve depicting force changes in the elongation function (**Figure 1**).

Breaking energy is proportional to the area of the stretched sample. To avoid relying on this parameter, tensile energy absorption (TEA) was introduced. It is expressed by the following formula:

$$TEA = \frac{W_z}{s \cdot l} \quad (1)$$

where,  $W_z$  – breaking energy,  $l$  – initial length of test piece,  $s$  – width of sample.

Another parameter affecting the value of breaking energy is paper grammage. In order to eliminate its effect, the tensile energy absorption index (TEAI) was introduced. It is expressed by the following formula:

$$TEAI = \frac{TEA}{g} \quad (2)$$

where,  $g$  – paper grammage.

As is known [1-7], paper mechanical properties depend on its moisture content. Higher moisture content reduces paper strength and increases tensile stretch. Also breaking energy changes along with variations in the paper moisture content. Paper is usually used in conditions with no direct contact with water; however, its moisture content may change as a result of the impact of surrounding air humidity, having an effect on the breaking energy. Such cases are analysed in this article. Despite the fact that breaking energy de-

pends on paper moisture content, in practice, it is easier to use the relative humidity of air at which paper reaches moisture equilibrium, especially in tests carried out in laboratory conditions at constant temperature and different air humidity values. An example is the TAPPI T 502 om-89 standard, which recommends determination of the equilibrium relative humidity of paper in stacks as moisture equilibrium with air in the stack. For this reason, in this article, changes in breaking energy and other paper properties are analysed depending on the relative humidity of air in which the paper is conditioned.

## Calculation of the theoretical value of breaking energy

Breaking energy  $W_z$  can be determined theoretically from the following formula:

$$W_z = \int_0^{\Delta l_z} F(\Delta l) d\Delta l \quad (3)$$

where,  $F(\Delta l)$  – function depicting relationship between elongation  $\Delta l$  and force  $F$  in one direction tensile test,  $\Delta l_z$  – elongation at moment of breaking.

In the first approximation it can be assumed that paper will behave as an elastic body (broken line in **Figure 1**). In such a case, the relationship between elongation and force is expressed by the following formula:

$$F = \frac{F_z}{\Delta l_z} \cdot \Delta l \quad (4)$$

After substituting (4) into (3), the following formula is obtained:

$$W_z = \frac{1}{2} F_z \cdot \Delta l_z \quad (5)$$

where,  $F_z$  – breaking force.

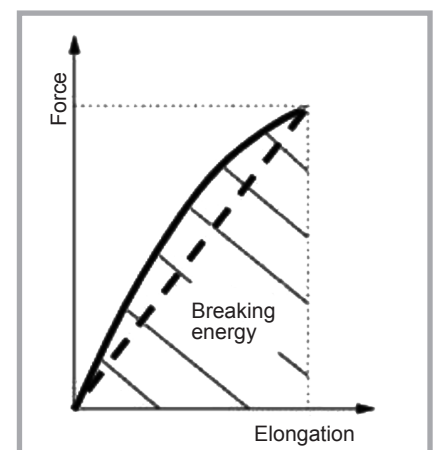
The assumption that paper will behave as an elastic body throughout the whole tensile test until rupture, contains a significant error. As **Figure 1** shows, such an assumption lowers the calculated value of breaking energy in relation to the real one. If in both cases  $\Delta l_z$  and  $F_z$  are the same, the ratio of the breaking energy value  $W_z$  and elastic body  $W_{z_s}$  can be expressed by coefficient  $h$

$$h = \frac{W_z}{W_{z_s}} \quad (6)$$

Knowing the value of coefficient  $h$ , the breaking force and strain at the moment of paper rupture, the breaking energy can be calculated on the basis of the following relationship:

$$W_z = h \cdot \frac{1}{2} F_z \cdot \Delta l_z \quad (7)$$

The calculations of breaking energy can also be carried out knowing the function depicting the tension curve  $F(\Delta l)$ , but for a different moisture content of fibrous material, the relationship between the force and strain is very different (**Figure 2**).



**Figure 1.** Breaking energy on tension graph.

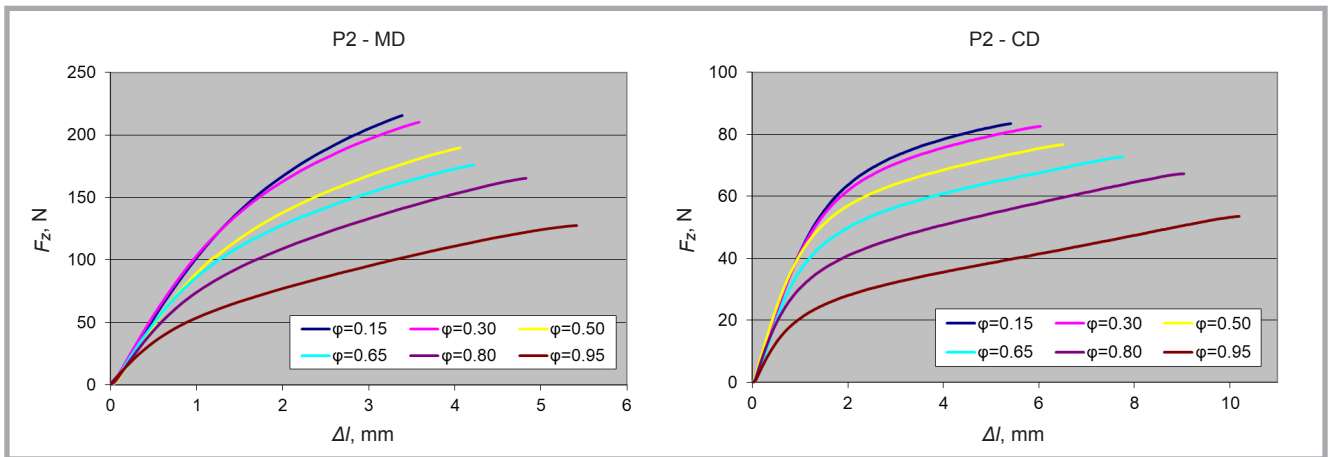


Figure 2. Tension graphs in MD and CD for paper P2.

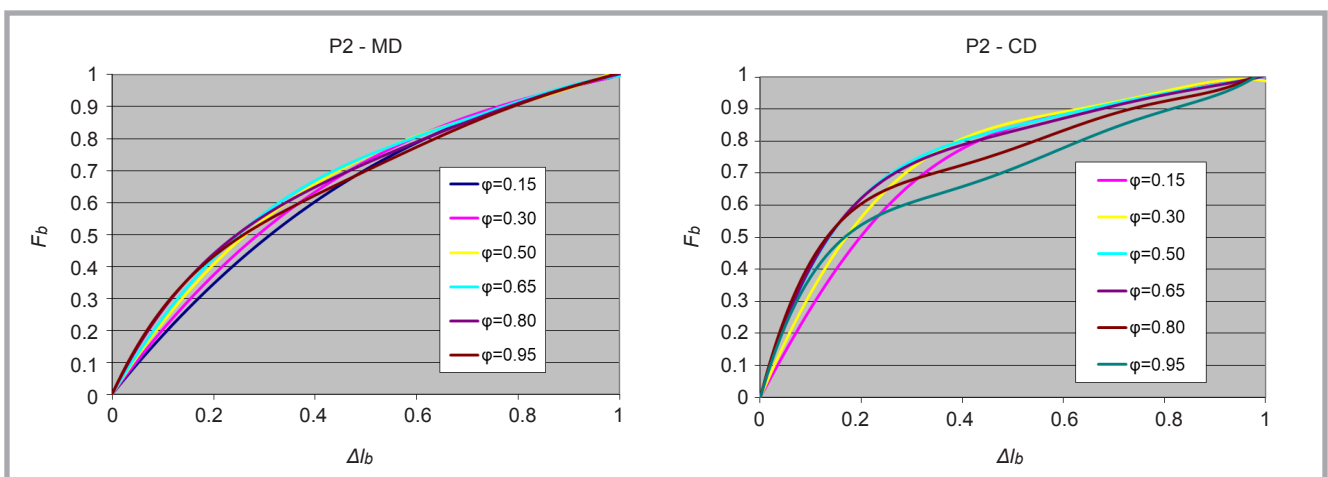


Figure 3. Tension graphs in the system of relative coordinates for paper P2.

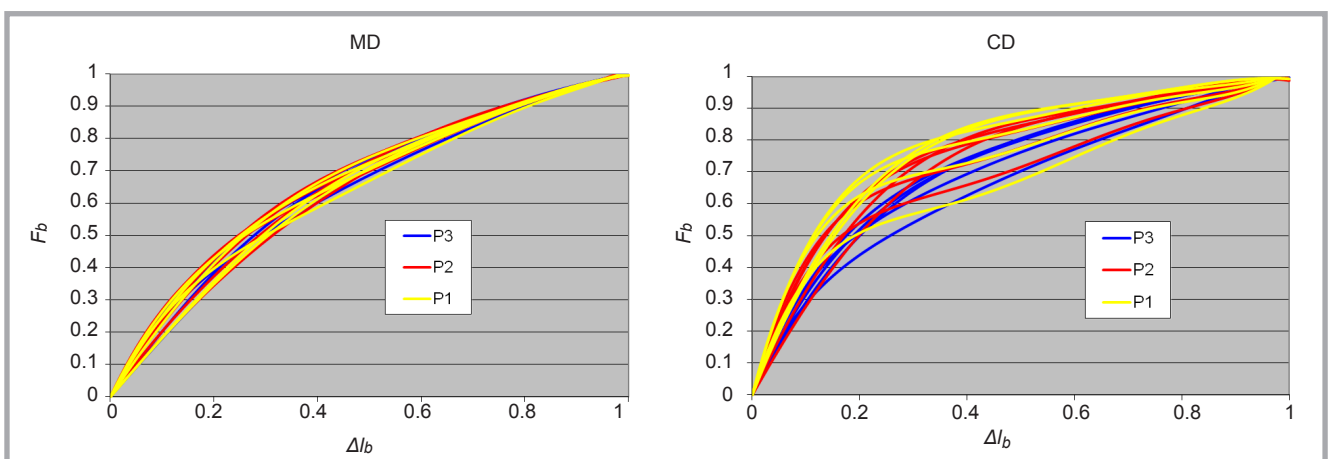


Figure 4. Tension curves of paper grades: P1, P2 & P3 after conditioning at different humidity.

The force  $F$  and elongation  $\Delta l$  can be presented as dimensionless values  $F_b$ ,  $\Delta l_b$ , expressed by the following relationships:

$$F_b = \frac{F}{F_z} \quad (8)$$

$$\Delta l_b = \frac{\Delta l}{\Delta l_z} \quad (9)$$

It allows to present all the tension curves in a single coordinate system ( $\Delta l_b$ ,  $F_b$ ). In the further part of the article it is called a relative coordinate system, where the curves have the same initial (0,0) and final (1,1) points, as illustrated in **Figure 3**.

Both a single and averaged tension curve (representing the dependence of the force from the strain for various tension curves in a dimensionless coordinate system) can be drawn using a polynomial. On the basis of analysis of the measurement results of all the papers tested, it was concluded that sufficient mapping accuracy

of the tension curves can be obtained using a polynomial of degree 6,

$$F_b = \sum_{i=1}^6 a_i \cdot \Delta l_b^i \quad (10)$$

where,  $a_i$  – constant determined on the basis of tensile test results.

In the non-relative co-ordinate system, the force has the following form:

$$F = F_z \cdot \sum_{i=1}^6 a_i \cdot \left( \frac{\Delta l}{\Delta l_z} \right)^i \quad (11)$$

After substituting **Equation (11)** into **(3)**, the relationship allowing to calculate breaking energy is obtained:

$$W_z = F_z \cdot \Delta l_z \cdot \sum_{i=1}^6 \frac{a_i}{i+1} \quad (12)$$

In order to calculate the breaking energy from relationships (5), (7) and (12), it is necessary to know the coordinates of the breaking point.

Analysing isothermal changes in strain at the moment of breaking and the breaking force as a function of equilibrium air humidity  $\varphi$  (**Figure 5**), it can be seen that (making a minor error) they can be described using a linear relationship.

**Table 1.** Characteristics of papers used in the tests.

Designation of paper	Grade of paper	Grammage, g/m <sup>2</sup>	Thickness, mm
P1	Fluting WB	175	0,27
P2	Testliner	200	0,32
P3	Printing paper	80	0,13
P4	Fluting WB	90	0,17
P5	Fluting WB	140	0,23
P6	Fluting SC	150	0,24
P7	Fluting SC	105	0,18
P8	Fluting SC	200	0,3
P9	Krafliner	90	0,11

With this assumption, coordinates of the paper breaking point ( $F_{z\varphi}$ ,  $\Delta l_{z\varphi}$ ) after conditioning in air of any given humidity  $\varphi$  can be calculated knowing the coordinates of breaking points of the paper after conditioning it in air of two different relative humidities  $\varphi_1$  and  $\varphi_2$ , for which breaking and tensile forces will marked- $F_{z1}$ ,  $F_{z2}$  and  $\Delta l_1$ ,  $\Delta l_2$ , respectively, from the following relationships:

$$F_{z\varphi} = \frac{F_{z1} - F_{z2}}{\varphi_{z1} - \varphi_{z2}} (\varphi - \varphi_1) + F_{z1} \quad (13)$$

$$\Delta l_{z\varphi} = \frac{\Delta l_{z1} - \Delta l_{z2}}{\varphi_{z1} - \varphi_{z2}} (\varphi - \varphi_1) + \Delta l_{z1} \quad (14)$$

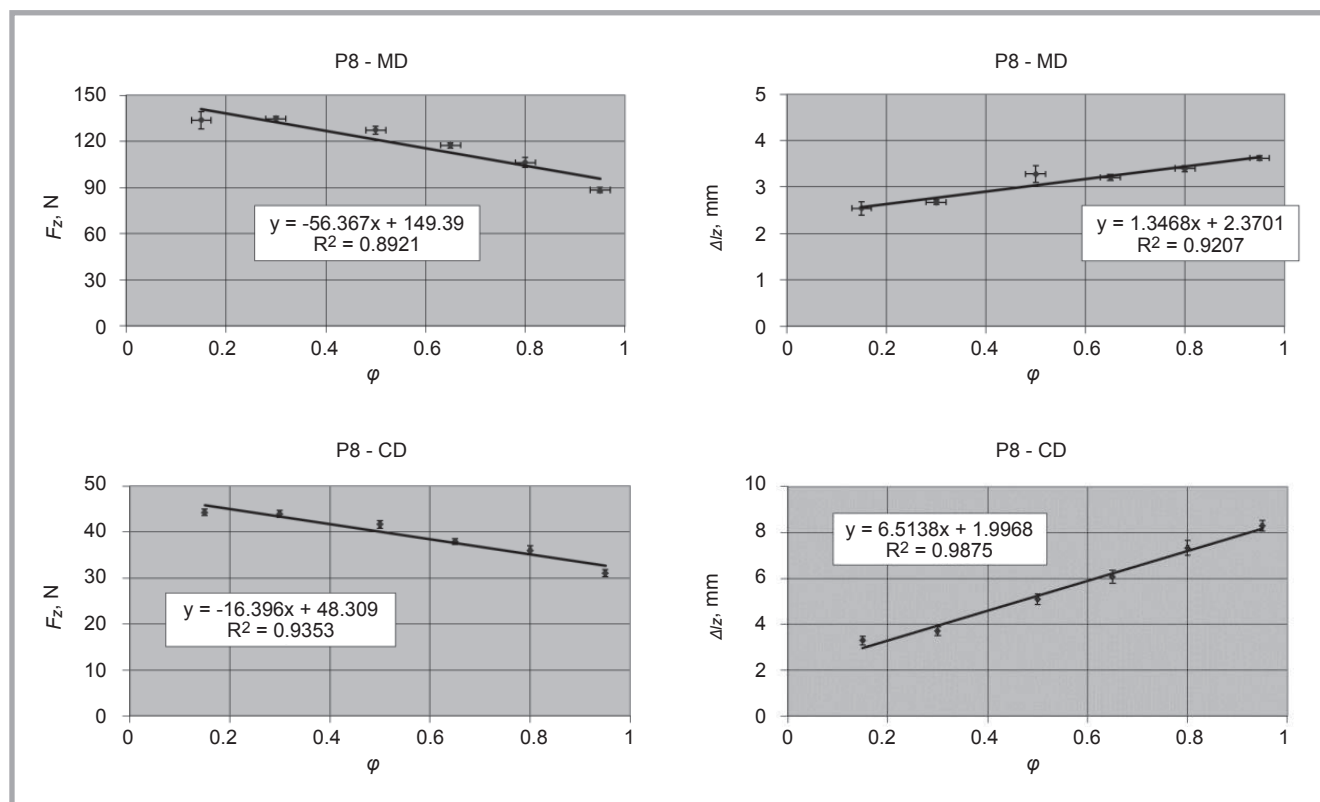
For all the papers, the breaking energy and tensile energy absorption index in MD and CD were measured after having conditioned them at a temperature of 23 °C and relative humidity of 15, 30, 50, 65, 80 and 95%. Conditioning was carried out in a way that moisture equilibrium was obtained in the adsorption process.

Tensile tests were carried out with 15 mm wide paper strips, whose initial clamping length was 180 mm.

Paper tensile test results after conditioning at different relative humidities  $\varphi$  are presented in **Figure 2**. The same results in the system of relative coordinates are shown in **Figure 3**.

## Measurement results

The paper samples listed in **Table 1** were used in the tests.



**Figure 5.** Relation of the breaking force and strain at the moment of breaking to humidity  $\varphi$ , for paper P8.

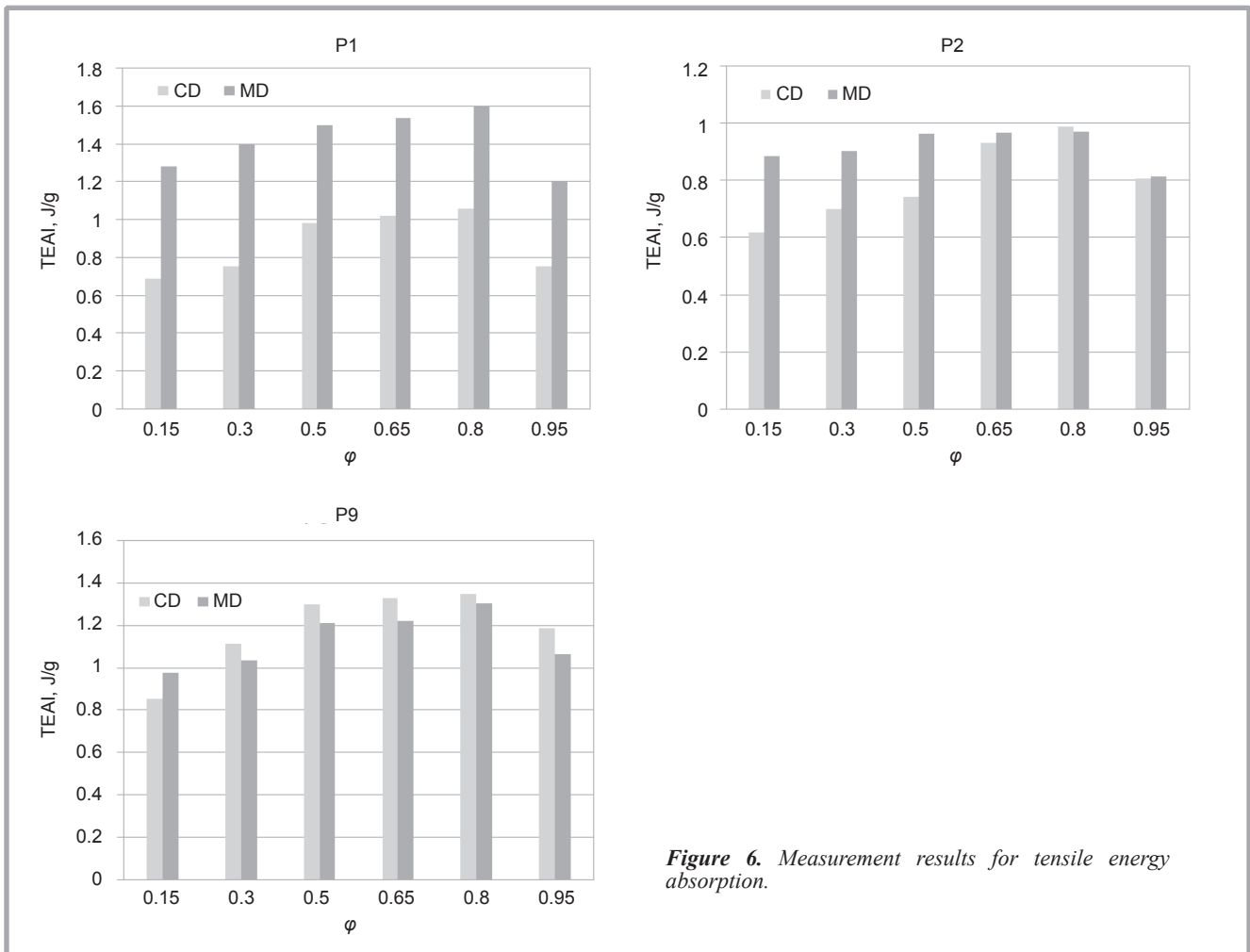


Figure 6. Measurement results for tensile energy absorption.

Figure 2 shows that in the range of paper moisture content at equilibrium relative humidity obtained after paper conditioning at a temperature of 23 °C and relative humidity from 15 to 95%, changes in the moisture content cause significant variations in the shape of tension curves.

After presenting paper tension in the system of relative coordinates (Figure 3), discrepancy in the shapes of curves diminishes significantly. It helps to express all the curves for a given tension direction with one relationship making a minor error, particularly for the machine direction (MD).

Figure 4 shows the tension curves for three different paper grades in the system of relative coordinates: testliner, fluting and writing paper, after conditioning in different conditions.

Despite the fact that the graphs show measurement results for papers of different material composition and structure, the differences in shapes of the curves are not significantly bigger than those in the

shape of the curves obtained during the test carried out for one paper (Figure 3).

With a probability of 95%, the value of breaking force  $F_b$ , for any of the papers tested in the range of relative humidity  $\varphi$  used is contained for a given value of strain  $\Delta l_b$  in the range  $\pm 0.018$  for MD and  $\pm 0.037$  for CD.

It allows to assume that in some cases, particularly for papers of similar structure and material composition, in the system of relative coordinates, it is possible to express their tension curves with a single relationship, with an error from ten to twenty percent of the value measured in relation to the real values.

Figure 5 shows graphs illustrating changes in breaking forces and strain at the moment of breaking as a function of  $\varphi$ .

At high relative humidity  $\varphi$  (over 0.8), graphs of the breaking force (Figure 5) show high differences between force values determined from a trend line and measured ones. Graphs illustrating

changes in the tensile energy absorption index of three selected papers in dependence on the relative humidity of air in which they were conditioned are shown in Figure 6.

In all cases tested, essential changes in  $TEAI$  as a function of the relative air humidity  $\varphi$  were found. Despite the fact that the distribution of  $TEAI$  for each sample tested and each direction are different, in all cases, along with an increase in humidity, its value first increases and then decreases; however, for each sample tested and direction, maximal values appear at different humidity.

## Calculation results

Using Equation (5), on the basis of measured values of coordinates of breaking points, approximate values of breaking energy were calculated, and then a relation of measured values of breaking energy to calculated values was determined. Figure 7 illustrates changes in coefficient  $h$  as a function  $\varphi$  for all

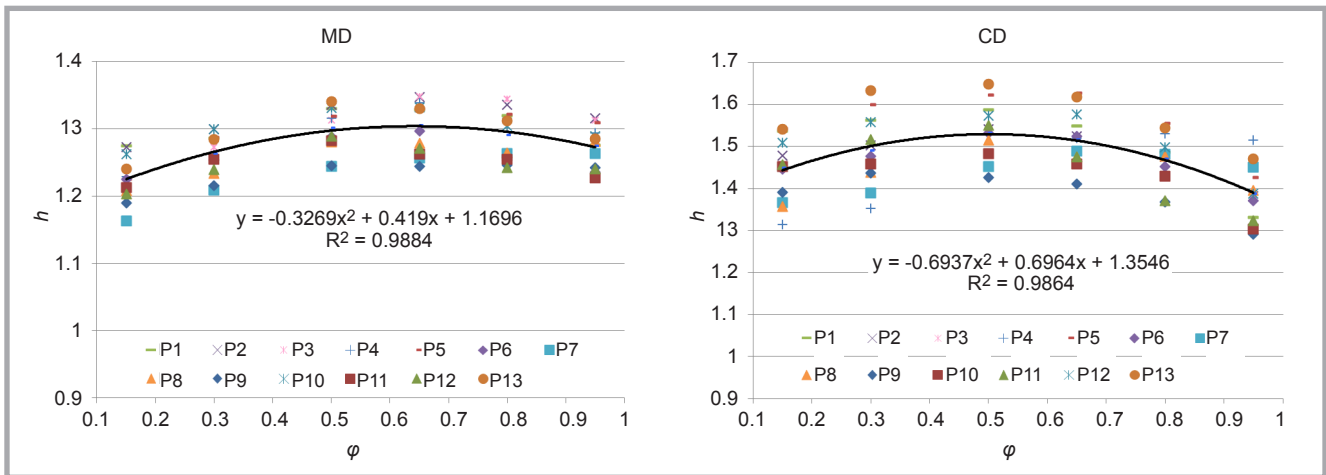


Figure 7. Changes in coefficient  $h$  in humidity function  $\varphi$ .

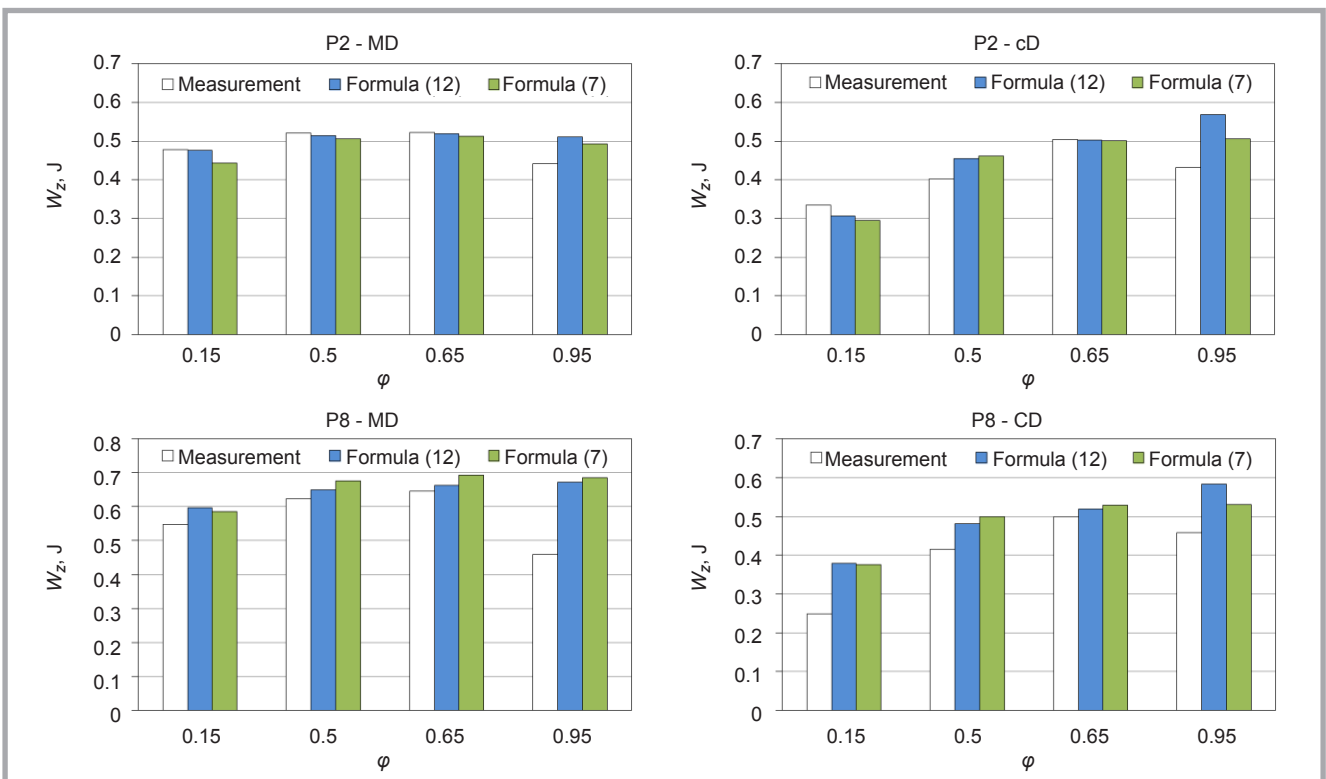


Figure 8. Comparison of measurement and calculation results of paper breaking energy.

the samples tested in both the machine and cross directions.

Changes in the average value of coefficient  $h$  for all papers tested are marked with a solid line. For each humidity  $\varphi$ , the differences between the average value of coefficient  $h$  and its extreme values in the machine direction are in the range of  $\pm 5\%$ , whereas for the cross direction they are in the range of  $\pm 10\%$  of the average value. On the basis of the results of the tensile test of papers after their conditioning at relative humidities of 30% and 80%, using *Equations (13) and (14)*, values of breaking forces and strain at the moment

of breaking for relative humidities of 15, 50, 65 and 95% were determined theoretically. Later on, for the humidity values above, breaking energy was calculated on the basis of *Equations (7) and (12)*. To determine constants  $a_i$  appearing in *Equation (12)*, the tensile curves obtained for papers conditioned at relative humidities of 30% and 80% were used.

### Comparison of measurement and calculation results

Figure 8 compares the values calculated theoretically with *Equations (7) and (12)*

to the values measured for two selected papers.

Comparison of the measured and calculated values of breaking energy shows that the calculation methods proposed give good results in a relative air humidity  $\varphi$  range from 30% up to 80%. Discrepancies between the measurement and calculation results occurring for low and high humidity are caused by inaccurate calculation of breaking point coordinates  $(F_{z\varphi}, \Delta l_{z\varphi})$ , from *Equations (13) and (14)*.



## ■ Conclusions

Changes in humidity of the test papers stored in different conditions have a significant impact on the value of breaking energy, both in the machine and cross direction. Both methods proposed for breaking energy calculation for papers conditioned at a given temperature and different humidity can be useful in certain practical applications. *Equation (7)* allows to calculate paper breaking energy on the basis of coordinates of two breaking points determined after paper conditioning at a given temperature and two different relative humidity values. However, first the method requires determining coefficient  $h$  for a given paper or a group of papers of the same type. It is possible to use *Equation (12)* when tensile curves of paper are known after its conditioning at a defined temperature and two different relative humidity values. Apart from that, this relationship allows to calculate the energy at any moment of the tensile test and not only at the moment of breaking. Determination of the values of breaking forces and strain at the moment of breaking with *Equations (13)* and *(14)* in the range of high humidity, above 80%, can involve serious error.

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■ Received 17.11.2017      Reviewed 25.01.2018



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- Phthalates
- Polychloro-Biphenyls (PCB)
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- Glyoxal
- Glycols
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