CZASOPISMO INŻYNIERII LĄDOWEJ, ŚRODOWISKA I ARCHITEKTURY
JOURNAL OF CIVIL ENGINEERING, ENVIRONMENT AND ARCHITECTURE
JCEEA, t. XXXV, z. 65 (4/18), październik-grudzień 2018, s. 21-29, DOI:10.7862/rb.2018.52

Oleksandr RIABENKO[1]
Volodymyr CHERNYUK[2]
Oksana KLYUHA[3]
Oksana HALYCH[4]
Dmytro POPLAVSKIY[5]

USING OF NEAR-CRITICAL FLOWS' THEORY IN PRACTICAL CALCULATIONS

The paper considers the problem of practical using of theory about near-critical flows. It describes the types of immovable and movable near-critical flow phenomena and cases of these phenomena formation during different hydrotechnical constructions operating. The paper gives generalized differential equation of free-surface profile of wavelike near-critical flows. The solution of mentioned generalized differential equation is given as well. The solution of generalized differential equation takes into account possible deviating from hydrostatic pressure in initial cross-section of considered flows. If the specifity of near-critical flows, especially wavelike free-surface profile and deviation of pressure distribution in initial section of considered flows, will not be taken into account, it can put to difference between designed and real hydraulic regimens. This factor can bring to miscalculation during designing, building and exploitation of hydrotechnical constructers. All that shows the issue urgency of near-critical flows characteristics determination and modelling for practical calculations. The equations for determination main depths (maximum and second conjugated) are given. Besides, the paper gives existence conditions of different types of near-critical flows. An objective of this work is to present the comparison between theoretical and experimental data of free-surface profile of cnoidal waves. The comparison shows good convergence of results.

Keywords: near-critical flows, non-hydrostatics, differential equations, laboratory researches

---

[1] Corresponding author: Oleksandr Riabenko, National University of Water and Environmental Engineering, Rivne, Ukraine, o.a.riabenko@nuwm.edu.ua

[2] Volodymyr Chernyuk, Lviv Polytechnic National University, Lviv, Ukraine, chernyuk@lp.edu.ua

[3] Oksana Klyuha, National University of Water and Environmental Engineering, Rivne, Ukraine, o.o.kluha@nuwm.edu.ua

[4] Oksana Halych, National University of Water and Environmental Engineering, Rivne, Ukraine, o.o.halych@nuwm.edu.ua

[5] Dmytro Poplavskiy, National University of Water and Environmental Engineering, Rivne, Ukraine, d.m.poplavskiy@nuwm.edu.ua

1. Introduction

1.1. General comments

Near-critical flow is called free surface water flow that is steady with rapidly varied movement and depths, which are close to critical value, and also unsteady flow (translational wave) with rapidly varied movement and velocities, which are close to critical value [1]. Critical depth and critical velocity for two-dimensional problem can be found by such well known formulas:

$$h_K = \sqrt[3]{\frac{\alpha q^2}{g}} \tag{1}$$

$$c_K = \sqrt{g h_1} \tag{2}$$

where: α – Coriolis coefficient, q – specific water discharge, g – acceleration of gravity, h_1 – depth of undisturbed flow.

Given definition can be expressed by Froude criterion which are close to unit for near-critical flows. This criterion appertains to initial cross-section with minimum depth h_1. For steady flow, it equals

$$Fr_1 = \frac{v_1^2}{gh_1} = \frac{q^2}{gh_1^3} \tag{3}$$

$$Fr_1 = \frac{c}{gh_1} \tag{4}$$

where $v=q/h_1$ – flow velocity in initial cross-section of steady phenomena, c – movement velocity of translational wave front.

Near-critical flows have a number of characteristic properties which distinguish appreciably these flows from usual subcritical and supercritical flows with smooth or slowly varied movement. Such properties include wavelike or roller nature of free-surface curve, availability of inclination and curvature, and also non-hydrostatic pressure distribution in depth mainly in vertical section of these phenomena [1].

Nonsufficient investigation of near-critical flow, accuracy's low level of calculation data cause that near-critical regimes during hydrotechnical structures' operation are not recommended, or excluded at all by normative documents [2]. The methods to avoid the near-critical regimes during hydrotechnical structures' operation are not always apposite, these methods require additional costs, but sometimes the formation of these regimes is impossible to avoid. In these cases, it is needed to apply expensive hydraulic modelling of hydrotechnical structures' operation to provide a reliable solution of complex technical problems.

2. Types of near-critical flows Cases of near-critical flows' formation. Actuality of issue

Based on conducted experimental investigations [1] and analysis of many other scientists' publications, considering flows on horizontal (or slightly inclined) plane bottom without rapids and steps, it is possible to detach several types of immovable and movable (translational waves) near-critical hydraulic phenomena, which are shown in figures 1 and 2 respectively. It is necessary to note such things during consideration suggested classification of near-critical flows.

Roller conjugating of ponds without jump, classic hydraulic jump, and solitary translational wave with surface roller, which are shown in figures 1a, 1j, 2f, belong to domain of near-critical flow not at all interval of characteristics of their existing, but only when peculiar depths and velocities are close to critical values.

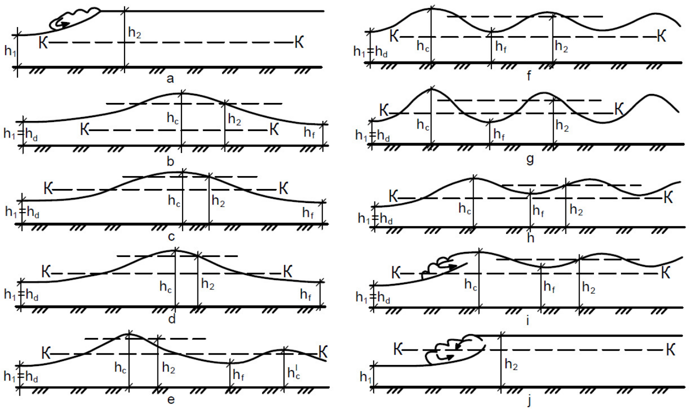

Fig. 1. Types of immovable near-critical phenomena: a – roller conjugating of ponds without jump, b – singular wave in subcritical, or critical flows, c – singular wave in supercritical flows, d – solitary wave, e – singular (solitary) wave with tail, f – cnoidal waves in subcritical, or critical flows, g – cnoidal waves in supercritical flows, h – undular jump with smooth surface, i – undular jump with surface roller on one or several wave crests, j – classic jump

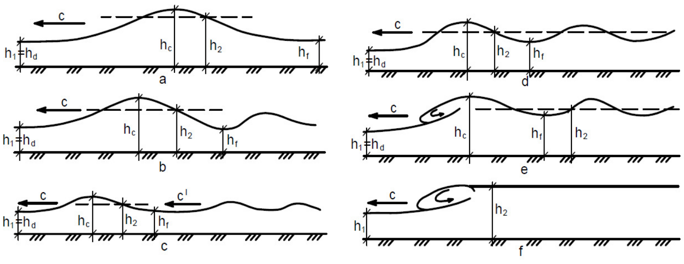

Fig. 2. Types of movable near-critical phenomena: a – solitary wave, b – solitary wave with tail, c – solitary wave with interrupted tail, d – grouped translational waves with smooth surface, e – grouped translational waves with surface roller on one or several wave crests, f – singular translational wave with surface roller (bore)

3. Cases of near-critical flows' formation. Actuality of issue

The near-critical flow may occur within different types of hydrotechnical structures: in tail water of water spillways, water outlets, hydropower plants, in channels, tunnels, passageways, pipes, during operating of geometrical shapes of flows, in the form of translational waves, etc. (fig. 3). Non-occurrence of general theory and reliable methods of near-critical flows' calculating, and indeterminateness of conditions of their existence are the reasons why near-critical flows sometimes aren't taken into account during designing. As a result many cases of damages and accidents of structures, which are operated in conditions of near-critical flows formation, happen. E.g. heavy damages of downstream floor were observed in water spillway Waco, shallow blankets were washed away of dams Krishna and Sardo, at a result, the dam Sardo was completely destroyed.

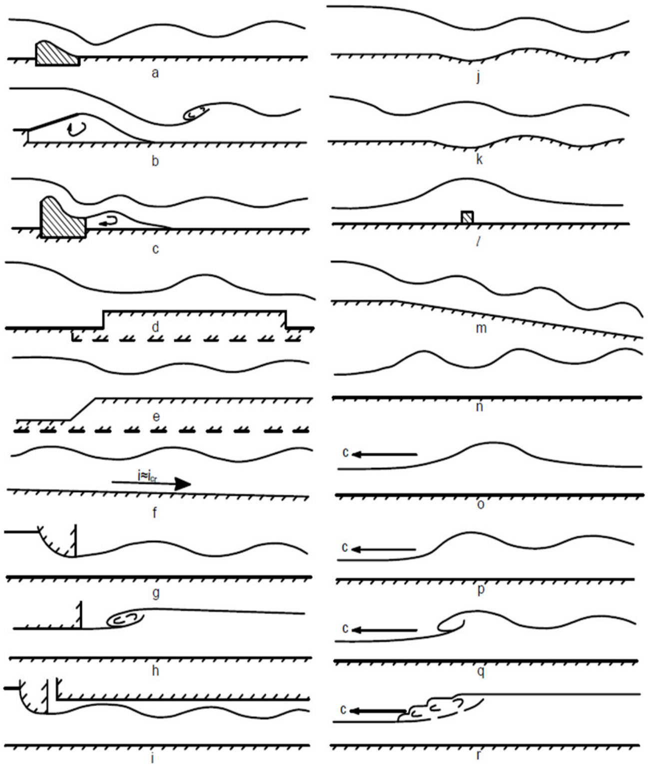

Fig. 3. Cases of near-critical flows' formation: a – after spillways, b – after regulator sluices and low pressure control structures, c – after spillways with drop wall and combine hydropower plants, d – on broad-crested weirs, e – at the inlet of channels or free-flow tunnels, f – in open channels, free-flow tunnels and pipes, g – during outflow from under gates with round or acicular configuration of bottom part, h – at the outlet of bottom discharges, i – in tunnels with boom, j, k – over undular bottom, l – during flow-around of bottom hindrance, m – at the inlet of inclined drops, n – during controlling of supercritical flows, o, p, q, r – in the form of translational waves

The damages of free-flow tunnels, that are Arpa–Sevan tunnel, Yalta tunnel, Spandaryan tunnel, Infiernillo tunnel, etc., were held fix.

For these cases of damages and accidents it is common that all of mentioned hydrotechnical structures were operated in domain of near-critical regimes' existence at certain stages. The analysis of operating conditions of free-flow diversion tunnels showed that Froude number in outlet section equalled Fr = 0.3 – 4.0. It shows that water flow was in domain of near-critical flows' existence. Obviously, noticed cases of damages and accidents of hydrotechnical structures happen as a result of large number of very different factors but these factors also include negative development of near-critical flow. Given information is declarative of actuality considered issue.

4. Generalized differential equation of the wavelike near-critical flow

Paper [1] shows generalized differential equation of the free-surface profile of near-critical flows with undular surface, which was developed as a result of investigation of near-critical flows' theory

$$h'^2 = \frac{3}{Fr_1}\left[-\eta^3 + (2\beta_1 + Fr_1)\eta^2 - (2\beta_1 - 1 + 2Fr_1)\eta + Fr_1\right] \tag{5}$$

where: $h' = dh/dx$ – first derivative of function h(x) at any point of free-surface curve, $\eta = h/h_1$ – dimensionless running ordinate of this curve, Fr_1 – Froude number in initial cross-section of considered phenomena which can be calculated by formula (3) or (4).

Integrating of this equation gives its general solution in the form of such system

$$\left.\begin{aligned}
&\eta = \frac{h}{h_1} = 1 + (\eta_c - 1)\operatorname{cn}^2\left(\frac{x}{\Delta}, k\right),\\
&\Delta = 2h_1\sqrt{\frac{\eta_c Fr_1}{3(\eta_c^2 - Fr_1)}},\\
&k = \sqrt{\frac{\eta_c(\eta_c - 1)}{\eta_c^2 - Fr_1}},\\
&\eta_c = \frac{1}{2}\left[t_1 + Fr_1 + \sqrt{(t_1 + Fr_1)^2 - 4Fr_1}\right].
\end{aligned}\right\} \tag{6}$$

where: $\eta_c = h_c/h_1$ – dimensionless depth under the first wave crest, Δ and k – parameters of cnoidal waves.

It is necessary to emphasize that expressions (5) and (6) take into account possible deviating from hydrostatics in initial cross-section of near-critical flows. Such accounting is made by related coefficients of non-hydrostatics s_1, hydrodynamic pressure t_1, and potential energy β_1 in considered cross-section.

From the practical point of view, the most important matters among various manifestations of near-critical flows are determination of maximum depth, the existence conditions of different types of near-critical phenomena and the calculations of free-surface profile.

5. Determination of maximum depth

The maximum depth h_c of wavelike near-critical flows is one of the most important of their characteristics because this depth defines the upper level of side dikes of open channels, bottom level of bridge girders, the height of free-flow tunnels, pipes and galleries. The maximum height of such waves can outdo their average height over 60-80% [3]. This problem is more investigated for undular jump and solitary wave. In case of phenomena with surface roller the maximum depth is second conjugated depth h_2 which can be calculated by known Bélanger's equation.

The existed equations, which find the maximum depth h_c of undular jump and solitary wave, and second conjugated depth h_2 of hydraulic jump, have some imperfections:

- the formulas cannot be used for all domain of near-critical flows,
- the formulas cannot be used when Froude number is less than unit $Fr_1 \leq 1$,
- the formulas do not take into account the possible deviating from hydrostatics in initial cross-section of considered phenomena.

The developed theory of near-critical flow [1] gives equations, which determine depths h_1 and h_2, and avoids the above-stated imperfections:

$$\eta_c = \frac{h_c}{h_1} = \frac{1}{2}\left[t_1 + Fr_1 + \sqrt{(t_1 + Fr_1)^2 - 4Fr_1}\right] \tag{7}$$

$$\eta_2 = \frac{h_2}{h_1} = \frac{2}{\sqrt{3}}\sqrt{k_1 + 2\alpha_{01}Fr_1 - T}\,\cos\left\{\frac{\pi}{3} - \frac{1}{3}\arccos\left[\frac{3\sqrt{3}\alpha_{02}Fr_1}{\sqrt{(k_1 + 2\alpha_{01}Fr_1 - T)^3}}\right]\right\} \tag{8}$$

where: α_{01} and α_{02} – coefficients of momentum in cross-sections with first and second conjugated depths respectively, T – dimensionless frictional force.

6. Existence conditions of different types of near-critical flows

From practical point of view the cognizance of reliable existence conditions of different types of near-critical flows is necessary to assign the favourable regimes of water movement through various constructions, to choose peculiar methodologies of calculations of considered phenomena's main characteristics, to determine the optimal size of structures, etc.

General imperfection of existed views on this matter is effort to find mentioned conditions only by one factor – Froude number Fr_1 in initial section of considered phenomena, and to except the influence of other factors. The disregard of possible deviating from hydrostatics in initial cross-section of considered flows can lead to absurdities and paradoxes [4].

The paper [1] showed that existence conditions of different types of near-critical flows should be characterized by not only one factor but two factors in their initial cross-section – by Froude number and one of the coefficients s_1, t_1, β_1, which take into account the possible deviating from hydrostatic pressure distribution in depth. This paper gave the existence conditions of different types of immovable near-critical phenomena by Froude number Fr_1 and coefficient of non-hydrostatics s_1.

7. Calculations of free-surface profiles

Practically the calculations of free-surface profile of wavelike near-critical flows are made for: determination of high-altitude size of constructions, determination of reach length of wave formation, operation of supercritical flows, checking of verity of developed theories of these flows.

Considered calculations are based on known differential equations of Korteweg-de Vries, Serre, Selezov, and others and use solutions of these equations in forms of solitary wave and cnoidal waves. In this connection undular jump and grouped translational waves are often considered as superimposition of mentioned waves' solutions. In this paper, the calculations were based on differential equation (5) and its general solution (6), which in explicit form takes into account possible deviating from hydrostatics in initial cross-section of considered flows.

8. Experimental validation of theoretical equations (5) and (6)

The complex laboratory investigations of near-critical flows were made in National University of Water and Environmental Engineering at four setups. During researches all types of immovable phenomena, which are shown in figure 1, were investigated. Considered phenomena occurred in two-dimensional conditions during outflow from under gates [1, 5], and also in three-dimensional conditions of single and double span regulator sluices with pivot-leaf gates [6, 7]. Each setup was equipped by system of bottom piezometers (all setups included about 346 piezometers). Existence of free-surface curve and piezometric line allowed to determinate the positions of sections with first and second conjugated depths, and also to calculate the coefficient non-hydrostatics in initial and other sections. Experimental setups and the investigated methodology were described in [1, 5-8].

Figure 4 shows collating of theoretical free-surface profiles of cnoidal waves, which were calculated by system (6), with experimental data. Adduced results and similar comparisons, which were made for different types of near-critical flows [1, 5-8], show their good convergence. It shows the principle verity idea about necessity of taking into account possible deviating from hydrostatics in initial cross-section of near-critical flows.

9. Conclusions

1. It is necessary to realize the calculation of free-surface profile, determination of the main characteristics (maximum h_c and second conjugated h_2 depths), and existence conditions of different types of near-critical flows with possible deviating from hydrostatics in initial cross-section of these flows.
2. It is recommended to take into account the results of developed theory of near-critical flow of this paper during making practical calculations of hydrotechnical structures.

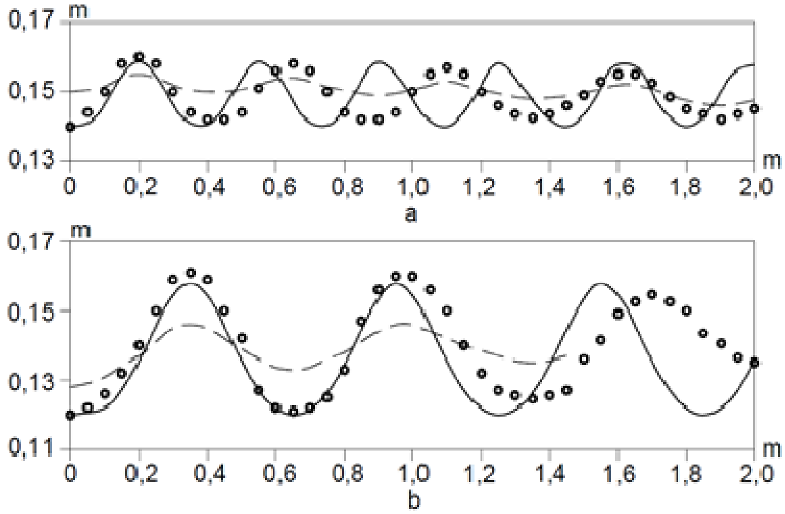

Fig. 4. The free-surface profile of cnoidal waves: a – q=0,094 m^3/s, h_1=0,14 m, Fr_1=0,322, s_1=1,05; b – q=0,111 m^3/s, h_1=0,12 m, Fr_1=0,722, s_1=1,06; 1 – piezometric line; 2 – fee-surface profile which was calculated by system (6); ○ – experimental data of free-surface

References

[1] Riabenko O.A. (2003). Theoretical elements and methods of calculations of near-critical fluid flows with a free surface. Thesis for academic degree of doctor of technical sciences: 05.23.16. Rivne. 393 ps. (in Ukrainian).

[2] State building rulers V.2.4-1-99. (1999) Meliorative systems and structures. Derzhbud of Ukraine. Kyiv. 112 ps. (in Ukraine).

[3] Tursunov A.A. (1969). Near-critical state of gravity water flows, Izvestiya VNIIG. Vol. 90. p. 201–224 (in Russian).

[4] Riabenko O.A. (2011). Problems and paradoxes of near-critical fluid flow. Applied Fluid Mechanics, Vol. 13(85), №4. Kyiv. p. 37–51 (in Ukrainian).

[5] Riabenko O.A., Halych O.O., Poplavskiy D.M. (2013) Using of differential equations to calculate the free-surface profile of undular jump. Georgian Technical University, Hydroengineering, Vol. 1-2(15-16). Tbilisi. p. 36–44 (in Russian).

[6] Klyuha O.O. (2006). Flow hydraulics after single-span regulator sluice with pivot-leaf gates with accounting properties of near-critical flow. Synopsis of a thesis. Rivne, p. 19 (in Ukrainian).

[7] Chernobyl O.Y. (2010). The improvement's methods of hydraulic calculations and designs of double-span regulator sluice with accounting properties of near-critical flow. Synopsis of a thesis. Rivne, p. 20 (in Ukrainian).

[8] Riabenko O.A., Cherniuk V., Klyuha O.A., Halych O.A., Poplavskiy D.M. Mathematical and hydraulic modeling. XV International Scientific Conference. Current issues of civil and environmental engineering and architecture. Book of abstracts. Rzeszów – Lviv – Košice. Rzeszów. Poland. 2015, p. 132–133.

Przesłano do redakcji: 24.03.2017 r.
Przyjęto do druku: 28.09.2018 r.