Mirena TODOROVA
University of Transport "TodorKableshkov"
Geo Milev 158, Sofia, Bulgaria
Corresponding author. E-mail: mirena_todorova@abv.bg

GERT MODEL OF A TECHNICAL STATION FOR DETERMINING THE PASSENGER TRAINSET DELAY

Summary. The paper presents a technical station model using the graphical evaluation and review technique – GERT. The network model reflects the operation of trainset service in compliance with the technology and technical equipment as well as interaction between technological systems at the service stations that are different by track development but connected to big passenger stations, which gives a possibility of determining the service time of trainsets depending on the number of carriages and the specificity of operation conditions. This GERT model is applied to the case study of an individual service station within the railway network in Bulgaria. The regression functions of service time are worked out according to the type and the number of carriages taking into account the limitation parameters. The functions obtained could be used for trainset scheduling to optimize the operation of passenger carriage fleet.

1. INTRODUCTION

The complex of passenger and technical stations is one of the railway transport subsystems, which are most complex and difficult to describe and present. The passing and service of coach flows of different types (trains, trainsets, or coach sets subjected to repair or equipment) load railway subsystems in a different way. Due to the complex organization of coach flows maintenance, the time spent at technical stations is a considerable part of the total duration of stay. Waiting for service due to congestion of arrivals and/or lack of capacity results in increase in the duration of interoperation stays. The different options to organize coach flows at a technical station lead to:
- change in the duration of stay for equipment operation at a technical station;
- change in trainset scheduling, which is basic for scheduling plan and train traffic schedule in the railway network.

Hence, technical stations are complex technological sites where the processes of carriage handling and maintenance are discrete and stochastic, and the way of defining the duration of trainset stay should consider these peculiarities.

2. LITERATURE REVIEW

References [15, 23-25, 27] contain various descriptions of technical equipment and operation technology necessary at any complex of a passenger station and adjusting technical stations or areas.

The analysis of references has shown that different approaches and criteria are used to determine the duration of trainset stay at technical stations for technological operations. One of the suggestions to determine the duration of this stay is based on regulations where its determination depends on the train category [20]. However, the approach to determine the duration of stay for technological maintenance

according to the type (category) of train and traffic priority does not take into account the real time necessary for trainset service.

Another method used to determine the duration of trainset stay at a technical station is based on the regulations of carriage service and shunting and depends on the number of tracks and operation technology. The duration of technical service at a given yard is determined on the basis of the number of carriages in the set. Thus, the duration of service is determined on the basis of the above-mentioned regulations that vary from 3 to 5 hours per set, depending on the number of carriages included in each set [19, 20, 21, 25, 27]. The main disadvantage of this method is that the time of stay depending on the occupation of serving equipment and conflict train routes is not considered.

The determination of the duration of trainset stay for technological operations both in the passenger and technical stations can also be done using the network planning method [24]. It determines the time taken by events and activities, e.g., the time when the event started, the time when operation began and finished, etc. This method provides a possibility of determining the critical way, operational procedure, total duration, and resources needed for technological maintenance of trains. However, the drawback of this method is that it does not take into account the time consumed while waiting for service and other technological operations.

The total number of coaches in a trainset is determinative factor for both the nature of technological maintenance and the duration of stay necessary, as well as the labor costs of service provided during the determined stay of trainsets.

All three methods mentioned above are used to determine the time duration. The duration of service and interoperation stays (times of waiting) are of stochastic nature, and to determine them, we shall determine the duration of trainset stay at a technical station.

Another problem with coach set service in the complex of passenger and technical stations is shunting for translocation of sets [2]. Shunting occurs every time when the sets are not temporarily needed to for schedules or are translocated from one to another track. In the rush hour, railway operator's rolling stock is in operation according to the timetable. However, out of the rush hour, the operator usually has rolling stock redundancy. The redundancy has to be translocated on shunting tracks in order to entirely operate the main railway infrastructure. Especially at night, a lot of sets of passenger carriages have to be moved on shunting tracks because there are only a few passenger trains at that time. Some aspects of the problem are discussed in [2] and a number of mathematical models are suggested for its solution. Also, a technique for the effective and fast solution of problems using a column-generation algorithm is introduced. The computational results presented by authors are based on real examples taken from the Netherlands Railways. The rail complex of passenger and technical stations is examined in [2], as it is in most European countries, and operations for the stabling of passenger trains in the time between peak periods are daily performed, especially of suburban trains.

It is important to analyze the yard cost parameter, too. It is the time cost saved by each car while passing through a technical station without reclassification [8]. On the basis of the fact that trains are not technically inspected at every yard on their trip from origin to destination, one can establish a relation between train connect and total non-reclassification operating time cost because of technical inspection and change of locomotives. Therefore, it is suggested to replace the traditional yard cost parameter by non-resort operating cost and fix the delay suffered by cars in the yard due to operations: classification and makeup. Hence, the mathematical model of the train formation plan with non-resort operating time cost parameter has been developed with locomotive extended routing (NTFP) as a more suitable solution incorporating the characteristics of real-life applications.

To present the station and determine the length of the technological processing operations, it is possible to use "queuing theory" [6, 9, 7, 10, 14, 17, 26]. The paper [18] contains an imitation model of GPSS developed for the presentation of a technical station as a multichannel queuing system (GS). The duration of different operations is set according to the norms given in the station's technological process. The model is applied to a specific technical station, and the optimal number of shunting locomotives and the number of brigades for technical examination are determined depending on the duration of the stay.

3. METHODOLOGICAL APPROACH

Studying network algorithms and programs, it has turned out that the network models with a stochastic structure are more flexible and useful than the determined networks. Therefore, the method that should be used has to take into account these requirements, so we have chosen the graphical evaluation and review technique (GERT) networks and GERT systems [5, 11-13, 16, 27].

The stochastic GERT network is defined as a network that can be realized only on the basis of certain subsets and arcs. The realization of each branch/operation is selected according to probability distribution. For the realization of a node in stochastic models, it is not necessary to realize all the branches included in the node. Therefore, cycles and loops are allowed in these models.

Let us examine G = (N, A) network containing only GERT nodes, which form a set of N. Let the performance duration of operations (i, j) be random value Yij. A certain operation (i, j) can be performed only in case when node i is realized. In order to study the questions related to the realization of this operation, it is necessary to know the conditional probability (in a discrete case) or the distribution density (in a non-discrete case) of random quantity Yij under the condition that node i is realized. This allows us to carry out a study on the implementation of all networks. The aim of using a GERT network in the stochastic network analysis is to determine the mathematical expectation and dispersion of time to pass through the network (considered here as a general network parameter). Obviously, the coefficient of GERT branch traverse has a corresponding W-function.

The Laplace–Stilt transformation is used to determine the derivative function and transformations needed. The conditional derivative function of random quantity Yij moments is defined as follows:

$$M_{ij}(s) = E[e^{Y_{ij}}].\tag{1}$$

For non-discrete random variables:

$$M_{ij}(s) = \int_0^\infty e^{sY_{ij}} \cdot f\left(Y_{ij}\right) \cdot dY_{ij}.\tag{2}$$

For discrete randomvariables:

$$M_{ij}(s) = \sum_{Y_{ij}=0}^\infty f\left(Y_{ij}\right) e^{sY_{ij}}.\tag{3}$$

In particular, $M_{ij}(s) = E[e^{sa}] = e^{sa}$ with $Y_{ij} = a = const$.

If a=0, to $M_{ij}(s) = 1$.

Let Pij be the probability of the performance duration of operation (i, j) under the condition of i node realization. W-function for random variable Yij is determined as follows:

$$W_{ij}(s) = P_{ij}.M_{ij}(s).\tag{4}$$

Using transformations, it is always possible to define the G' network structure, which is identical to the G network structure, where there is only one parameter Wij instead of the two parameters of branches Pij and Yij. If the realization duration of operations in G network is represented as an independent random variable, then G' has a number of properties that are of certain interest from a computational point of view. To explore these properties, three particular cases are considered:

- G' consists of two or more consecutives: W-function of equivalent branch is equal to the product of W-functions of the consecutive branches.
- G' consists of two or more parallel branches: W-function of equivalent branch is equal to the sum of W-functions of the parallel branches.
- G' consists of a branch and a loop and is reduced to a single branch equivalent to it, for which W-function is equal to the following formula:

$$W_{ij}(s) = W_{ij}(s).[1 - W_{ii}(s)]^{-1} = \frac{W_{ij}(s)}{[1 - W_{ii}(s)]}.\tag{5}$$

If the GERT network consists of parallel and series chains and/or loops, it can be converted into an equivalent network consisting of only one branch. In fact, the result generalizes any GERT network, because the known basic transformations can be combined.

The main steps of using a GERT network are:
- to present the network as a stochastic network with GERT nodes;
- to calculate W-function for any other network; and
- to transform a network into an equivalent network consisting of only one branch.

To transform a network of any structure into an equivalent graph with a much simpler structure, most often Mason's topological equation is used having the following appearance:

$$H = 1 - \sum_{n=1}^{L_1} T(L_1) + \sum_{n=1}^{L_2} T(L_2) - \sum_{n=1}^{L_3} T(L_3) + ... + (-1)^m \sum_{n=1}^{Lm} T(L_m) = 0 \tag{6}$$

where $\sum_{n=1}^{Li} T(L_i)$ is the sum of the equivalent coefficients of traverse of all possible loops of i-order. In other words, using the topological equation, the following steps are necessary:

1. To determine the equivalent coefficient of traverse of all loops of m order.
2. To sum up the variables obtained on all loops of m order and to multiply the result by $(-1)^m$. For a loop of even order variable $(-1)^m$ is positive, and for a loop of odd order it is negative.
3. To add step 1 to the expression obtained from step 2 and equal the result to 0.

To apply (6) to an open network, it is necessary to introduce an additional branch with W-function $W_A(s)$, which joins chain end t with source s. Then, for the modified network, it is necessary to find all loops (to the maximum possible order). $W_A(s)$ function is needed in order to find the equivalent W-function for the output network. In Fig. 1, the output network is depicted as a "black box" with W-function $W_E(s)$.
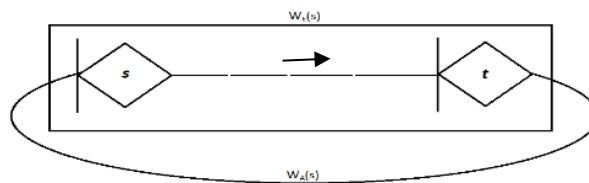

Fig. 1. Equivalent function

The given closed graphflow contains a loop of the first order with an equivalent coefficient equal to $W_A(s).W_E(s)$. Using Mason's rule, it is obtained that $1-W_A(s) W_E(s)=0$ or $W_A(s)=1/W_E(s)$.

It should be noted that the topological equation contains function $W_A(s)$, since it is an element of a loop of the first order. The significance of the result obtained with examining the above-mentioned example is due to the fact that if $W_A(s)$ in the topological equation is replaced with $1/W_E(s)$ and is solved in regard to $W_E(s)$, an equivalent W-function for the output stochastic network will be obtained.

To determine the equivalent W-function of an open stochastic network, the following steps are required:
- to close a given network with a branch coming out from the end node and entering the initial node;
- to find all loops of order m; and
- to apply topological equation (6) and obtain $W_E(s)$.

The expression for equivalent W-function of $W_E(s)$ network was obtained from topological equation (5). It should be reminded that $M_E(s)=1$ with s=0.

When $W_E(s)=P_E .M_E(s)$, $P_E= W_E(0)$, from where it follows that:

$$M_E(s) = \frac{W_E(s)}{P_E} = \frac{W_E(s)}{W_E(0)}. \tag{7}$$

It should be noted that $W_E(s)$ could be expressed by W-functions of all or some of the output network branches. It is not difficult to calculate the amount of $W_E(0)$; therefore, s=0 must be set in the expression for $W_E(s)$ derived from formula (7).

Computing the j-partial derivative by s-function ME(s) and laying s=0, jth first µjE of the random variable is found out, i.e.,

$$\mu_{jE} = \frac{d^j}{ds^j}\left(\frac{W_E(s)}{W_E(0)}\right)\bigg|_{s=0} = \frac{d^j}{ds^j}(M_E(s))\bigg|_{s=0}. \tag{8}$$

In particular, at s=0, the first moment μ_{1E} represents the mathematical expectation of network realization time where the dispersion of network realization time is equal to the difference between the second moment μ_{2E} and the square of mathematical expectation μ_{1E}, i.e.,

$$\sigma^2 = \mu_{2E} - (\mu_{1E})^2. \tag{9}$$

4. MODELING OF OPERATION OF TECHNICAL STATIONS

To determine the duration of trainset stay using GERT netsworks, an algorithm is developed following the steps given in Fig. 2:
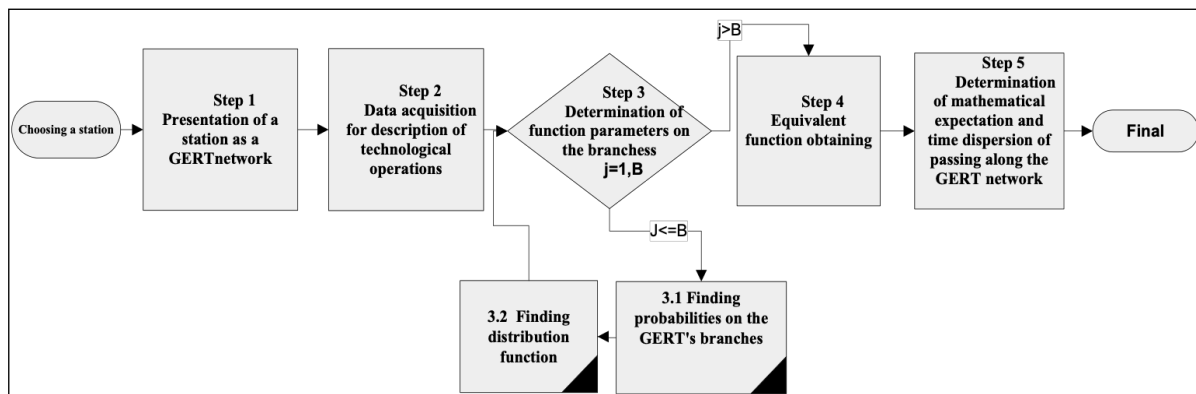

Fig. 2. Algorithm of determining the duration of trainset stay at a station

Step 1:Presentation of a station as a GERT network
The operation technology at the station serving trainsets is presented as a stochastic network with GERT nodes in regard to the track development and operations.

Step 2: Data acquisition for description of technological operations
The examined length of technological processes is a continuous random variable, which is set either with a distribution function (integral distribution law) or with the density of probabilities $f(x)$ (differential distribution law). To determine the distribution laws of technological times, it is necessary to make significant statistical observations on the duration of each technological operation.

Step3: Determination of equivalent function parameters
Determination of the probability and time featuress of «B» branches.
3.1. Finding the probabilities of GERT network arcs in the classical way.
3.2. Determining the type of law for distribution of service times for technological operations using software for statistical data processing or another selected method.

Step4: Equivalent function obtaining
The function of traverse of GERT network branches has a corresponding W-function, and it is determined depending on the composed stochastic network with GERT nodes presenting the station operation technology and then converted into an equivalent network consisting of only one branch.

Step 5: Determination of mathematical expectation and time dispersion of passing along the GERT network
The aim of using a GERT network is to determine the mathematical expectation and dispersion timing of the network. After determining the probabilities of realization of each arc and defining the law ofdistribution of service times for individual operations, the differential function of distribution f (x) is used with its parameters as given in Table 1.

Table 1

Parameters of distribution functions

Distribution law	Distribution density of probabilities $f(x)$	Function causing moments $M(x)$	First moment – mathematical expectation $E(x)$	Second moment – dispersion $D(x)$
Normal	$\dfrac{1}{b.\sqrt{2\pi}}.e^{-\frac{(x-a)^2}{2b^2}}$	$e^{(ax+\frac{b^2.x^2}{2})}$	a	b^2
Gamma	$\dfrac{x^{a-1}}{b.\Gamma(b)}.e^{-\frac{x}{b}}$	$\dfrac{1}{(b.s+1)^a}$	$a.b$	$a.b^2$

In order to determine the performance indicators of the station, using a GERT network, the following cases are considered: when there are two normal distributions; one normal and one gamma distributions, and two gamma distributions. Using the usual attributes of the exponent and the assumed substitutions, the equivalent function is deducted expressed by the first moment, mathematical expectation $E_{ij}(x)$, the second moment, dispersion $D_{ij}(x)$, and realization probability of the respective branch P_{ij}. It has been proved in [27] that examining different distributions, the result of service time dispersion is always one and the same depending on the initial scheme, which gives us the right to be interested only in the parameters of service time distribution law but not in the law itself. Therefore, to simplify differentiation for the GERT network of the station examined in this paper, it is assumed that all branches will be reduced to normal distribution with the distribution parameters that have already been found.

5. CASE STUDY OF TECHNICAL STATION NADEZHDA

The developed algorithm has been applied to Nadezhda technical station, which serves Sofia passenger station. The main task of Nadezhda–Sofia technical station, the largest one in Bulgaria serving 75% of all coaches, is to provide roadworthy, clean and equipped railway vehicles for safe and comfortable travel.

Step 1:

The main technological operations carried out at a technical station are [7, 21, 23, 25, 27] as follows.

o Technical inspection: it starts as soon as the shunting trainset passes through the switch crossing, where it meets the two inspectors standing on its both sides. The trainset is checked for sounds in order to detect faults on the bogie or in the automatic braking system. Any observed malfunctions, which can be removed without the detachment of the coach from the trainset, are taken care of in the workshop fully equipped for this purpose.

o The coaches with malfunctions are removed from operation, detached from the trainset, and lined on a track for repair. The coach taken out of the trainset is replaced with a repaired coach available. When necessary, a change of the trainset is made.

o Carriages are cleaned daily – internal cleaning of all types of carriages.

o Main internal and external cleaning of certain trainsets depends on the category of trains to be served in a certain time interval. External washing is done by pushing the trainset through a washing machine at a speed of 2 km/h.

o Technical maintenance and servicing of equipment of the trainsets are carried out in the workshop for the repair of coaches on one of the tracks for equipment.

When a set of coaches are ready for departure to the passenger station, a shunting locomotive is delivered to it in the equipping workshop or on the arrival–departure track where the set is stored after being prepared.

The track development of any technical station determines the technology of handling the trainsets. The work of the repair and equipping and the carriage and repair depots and inspection posts depends on the track development schemes and the number of tracks. The model of Nadezhda technical station has been developed as a stochastic network with GERT nodes, considering its track development and sequence of the operations performed and shown in Fig. 3:
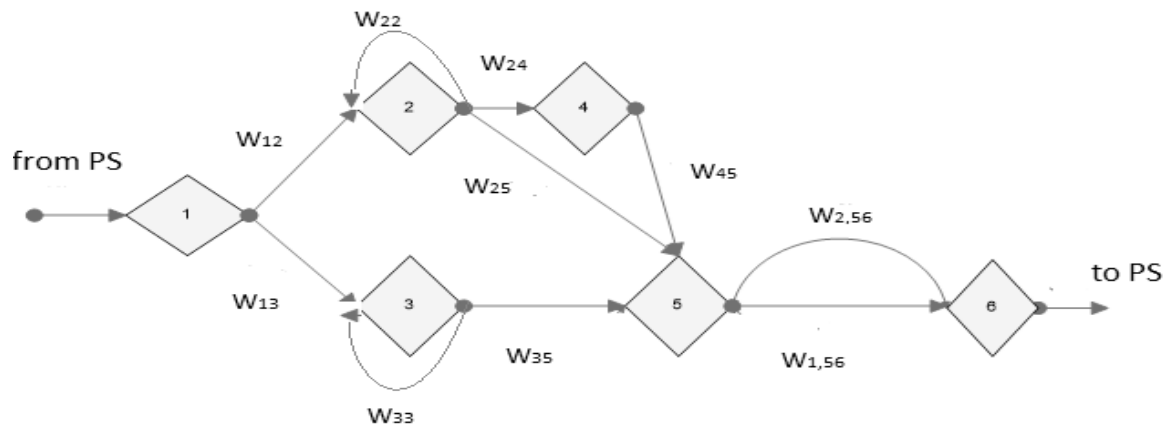

Fig. 3. The GERT model of a technical station

Node 1 – entry at the technical station (TS) and where the trainset is accepted, in a receiving park (RP) or directly on the equipment tracks
W_{12} – performing technical inspection, waiting and performance of internal cleaning in RP;
W_{13} – performing technical inspection and waiting and performing internal cleaning on the equipment tracks;
Node 2 – will the trainset be directly moved to the equipment tracks or placed for external washing
W_{24} – moving the trainset and passing through an external washing device;
W_{25} –technical maintenance and equipment on the equipment tracks;
W_{22} – due to the malfunction of any coach, it is replaced with an available one by shunting;
Node 3 –is the coach ready to be equipped?
W_{35} – technical maintenance and equipping on the equipment tracks;
W_{33} – due to malfunction of any coach, it is replaced with an available one by shunting;
Node 4 – is the trainset moved to the equipment track?
W_{45} – technical maintenance and equipping on the equipment track;
Node 5 – is the composition ready for moving to the arrival–departure track?
W_{56}^{1} – storage on the equipment tracks;
W_{56}^{2} – moving from the equipment tracks to the arrival–departure one;
Node 6 – shunting to leave the technical station and enter the passenger one.
Step 2: Due to the large number of equipped trainsets (37), they are divided into eight groups depending on the number of carriages and the approximate closeness in their service schedules for technological operations. Let us examine groups of 2, 5, 8, and 11 coaches. Traffic data on the technical station territory are collected from the Book of Chief Dispatcher, on 24-hour operation schedule of the tracks for equipping and the station manager on duty.
Step 3: The data collected are used to determine numerical characteristics and construct a histogram that allows us to propose the hypothesis about the type of distribution law. The verification of hypothesis proposed is based on Pearson criterion. The statistical processing has been performed using "STATGRAPHICS.Plus 5.1" software.
After determining the distribution laws of service times and numerical parameters, the probability of each arc is determined depending on the group under consideration and the collected statistical information. The results are shown in Table 2 giving the parameters of W_{ij} arcs described in Section 4, Step 1, as the results for each arc are given for selected groups under examination.

Step4:

Equivalent function W_{16} depending on the scheme in Fig. 3, formula 5, and the cases mentioned in the Methodological approach are obtained as follows:

$$W_{16} = \left[(W_{24}.W_{45} + W_{25}).\frac{W_{12}}{(1-W_{22})} + \frac{W_{13}.W_{35}}{(1-W_{33})} \right] (W_{56}{}^1 + W_{56}{}^2). \tag{10}$$

Formula 3.2.11 is used to close the chain:

$$W_{61}(s) = \frac{1}{W_{16}(0)} \quad, \text{ at } X=0 \text{ ; it is known that } e^0 = 1 \tag{11}$$

$$W_{16}(0) = \left[P_{12}.P_{24}.P_{45}.P_{56}^1 + P_{12}.P_{24}.P_{45}.P_{56}^2 + P_{12}.P_{25}.P_{56}^1 + P_{12}.P_{25}.P_{56}^2 \right](1-P_{22})^{-1} +$$
$$+ \left[P_{13}.P_{35}.P_{56}^1 + P_{13}.P_{35}.P_{56}^2 \right].(1-P_{33})^{-1}. \tag{12}$$

K1256 and K1356 are applied

$$V = \frac{1}{W_{16}(0)} \tag{13}$$

$$K_{1256} = \left[P_{12}.P_{24}.P_{45}.P_{56}^1 + P_{12}.P_{24}.P_{45}.P_{56}^2 + P_{12}.P_{25}.P_{56}^1 + P_{12}.P_{25}.P_{56}^2 \right] \tag{14}$$

$$K_{1356} = \left[P_{13}.P_{35}.P_{56}^1 + P_{13}.P_{35}.P_{56}^2 \right]. \tag{15}$$

The equivalent equation appears as follows:

$$M_{16}(s) = \frac{W_{16}(s)}{W_{16}(0)} = W_{16}(s).V \tag{16}$$

$$\mu_{1E} = \frac{d^1}{ds^1}(M_E(s))\Big|_{s=0} = \left[P_{12}.P_{24}.P_{45}.P_{56}^1(E_{12} + .E_{24} + E_{45} + E_{56}^1) + P_{12}.P_{24}.P_{45}.P_{56}^2(E_{12} + .E_{24} + E_{45} + E_{56}^1) + \right.$$
$$+ P_{12}.P_{25}.P_{56}^1(E_{12}. + E_{25}. + E_{56}^1) + P_{12}.P_{25}.P_{56}^2(E_{12}. + E_{25}. + E_{56}^2) \left.\right](1-P_{22})^{-1}.V + K_{1256}.V, P_{22}.E_{22} +$$
$$+ \left[P_{13}.P_{35}.P_{56}^1(E_{13}. + E_{35}. + E_{56}^1) + P_{13}.P_{35}.P_{56}^2(E_{13}. + E_{35}. + E_{56}^2) \right].(1-P_{33})^{-1}.V + K_{1356}.V, P_{33}.E_{33}. \tag{17}$$

In order to find the dispersion of operation time, it is necessary to find the second derivative of M16(s) in a similar way, and the dispersion itself at s=0 is equal to the difference between the values of the second derivative and the square of first derivative (formula 9).

$$\mu_{2E} = \frac{d^2}{ds^2}(M_E(s))\Big|_{s=0} = \left[P_{12}.P_{24}.P_{45}.P_{56}^1 \{(D_{12} + .D_{24} + D_{45} + D_{56}^1) + (E_{12} + .E_{24} + E_{45} + E_{56}^1)^2\} + \right. \tag{18}$$
$$P_{12}.P_{24}.P_{45}.P_{56}^2 \{(D_{12} + .D_{24} + D_{45} + D_{56}^1) + (E_{12} + .E_{24} + E_{45} + E_{56}^1)^2\} + P_{12}.P_{25}.P_{56}^1 \{(D_{12}. + D_{25}. + D_{56}^1) +$$
$$+ (E_{12}. + E_{25}. + E_{56}^1)^2\} + P_{12}.P_{25}.P_{56}^2 \{(D_{12}. + D_{25}. + D_{56}^2) + (E_{12}. + E_{25}. + E_{56}^2)^2\}\left](1-P_{22})^{-1}.V + \right.$$
$$K_{1256}.V, P_{22.}(D_{22} + E_{22}^2) + \left[P_{13}.P_{35}.P_{56}^1 \{(D_{13}. + D_{35}. + D_{56}^1) + (E_{13}. + E_{35}. + E_{56}^1)^2\} + \right.$$
$$+ P_{13}.P_{35}.P_{56}^2 \{(D_{13}. + D_{35}. + D_{56}^2)(E_{13}. + E_{35}. + E_{56}^2)^2\}\left](1-P_{33})^{-1}.V + K_{1356}.V, P_{33.}(D_{33} + E_{33}^2). \right.$$

To simplify the procedure for determining the mathematical expectation and dispersion of the duration of trainset stay at the technical station, an Excel computational file has been developed where the variables sought are determined by entering probability of the given arc and the two distribution moments.

Step 5:

Using the developed computational procedure, the features of selected groups for the GERT network are determined – the mathematical expectation and dispersion of the duration of trainset stay at the technical station given in Table 3.

Table 2

Parameters of arcs

Arcs	Probability P(x)	Distribution law	Mathematical expectation E(x)	Dispersion D(x)	Average Quadratic deviation	Variance coefficient-v(x)	P- Value
W12							
2 coach	0,683	Gamma	27,412	166,382	12,899	0,471	0,374
5 coach	0,227	Normal	50,985	221,733	14,891	0,292	0,219
8 coach	0,140	Normal	69,190	282,450	16,806	0,243	0,053
11 coach	0,383	Normal	25,906	158,404	12,586	0,486	0,261
W22							
2 coach	0,013						
5 coach	0,108	Normal	7,417	3,679	1,918	0,259	0,127
8 coach	0,178	Normal	9,331	3,952	1,988	0,213	0,465
11 coach	0,156	Normal	10,021	3,973	1,993	0,199	0,256
W24							
2 coach	0,518	Normal	82,547	346,400	18,612	0,225	0,147
5 coach	0,652	Normal	99,087	391,600	19,789	0,200	0,072
8 coach	0,744	Normal	126,964	452,440	21,271	0,168	0,690
11 coach	0,302	Normal	219,000	680,330	26,083	0,119	0,827
W45							
2 coach	1,000	Normal	7,500	3,364	1,834	0,245	0,223
5 coach	1,000	Normal	11,603	8,449	2,907	0,251	0,288
8 coach	1,000	Normal	12,903	9,024	3,004	0,233	0,187
11 coach	1,000	Normal	13,053	15,053	3,880	0,297	0,540
W25							
2 coach	0,469	Normal	90,417	422,992	20,567	0,227	0,513
5 coach	0,240	Normal	123,724	495,831	22,267	0,180	0,073
8 coach	0,078	Normal	110,625	489,583	22,127	0,200	0,432
11 coach	0,542	Normal	127,639	517,837	22,756	0,178	0,645
W13							
2 coach	0,317	Normal	8,632	25,023	5,002	0,580	0,090
5 coach	0,773	Normal	26,576	94,648	9,729	0,366	0,094
8 coach	0,860	Erlang	48,379	195,280	13,974	0,289	0,113
11 coach	0,617	Gamma	17,667	87,846	9,373	0,531	0,189
W33							
2 coach	0,009		5,200	2,100			
5 coach	0,043	Normal	6,842	4,268	2,066	0,302	0,143
8 coach	0,097	Normal	8,248	4,295	2,072	0,251	0,547
11 coach	0,117	Normal	9,321	6,531	2,556	0,274	0,326
W35							
2 coach	0,991	Normal	79,750	259,930	16,122	0,202	0,212
5 coach	0,957	Normal	81,346	387,115	19,675	0,242	0,541
8 coach	0,903	Normal	105,625	489,583	22,127	0,209	0,172
11 coach	0,883	Normal	138,929	506,217	22,499	0,162	0,099
W1,56							
2 coach	0,000		0,000	0,000			
5 coach	0,894	Normal	41,771	189,711	13,774	0,330	0,736
8 coach	0,147	Gamma	24,773	110,470	10,510	0,424	0,242
11 coach	0,058	Normal	36,714	135,569	11,643	0,317	малко данни
$W^{2,}56$							
2 coach	1,000	Erlang	26,189	124,694	11,167	0,426	0,494
5 coach	0,106	Normal	20,354	122,284	11,058	0,543	0,055
8 coach	0,853	Normal	19,163	126,044	11,227	0,586	0,081
11 coach	0,942	Gamma	32,706	124,972	11,179	0,342	0,259

Table 3

Features of selected groups of carriages

	2 coach	5 coach	8 coach	11 coach
Duration of the trainset average stay (minutes)	135	144	183	202
Dispersion of the trainset average stay	737	1150	1450	1611
Average square deviation (minutes)	27	34	38	40

Using the integral function of normal distribution, the cumulative function given in Fig. 4 is built using NORMDIST function (*x,mu,sigma,cumulative*) of Microsoft Excel, Microsoft Office (for the four groups under examination using the parameters determined in Table 3).
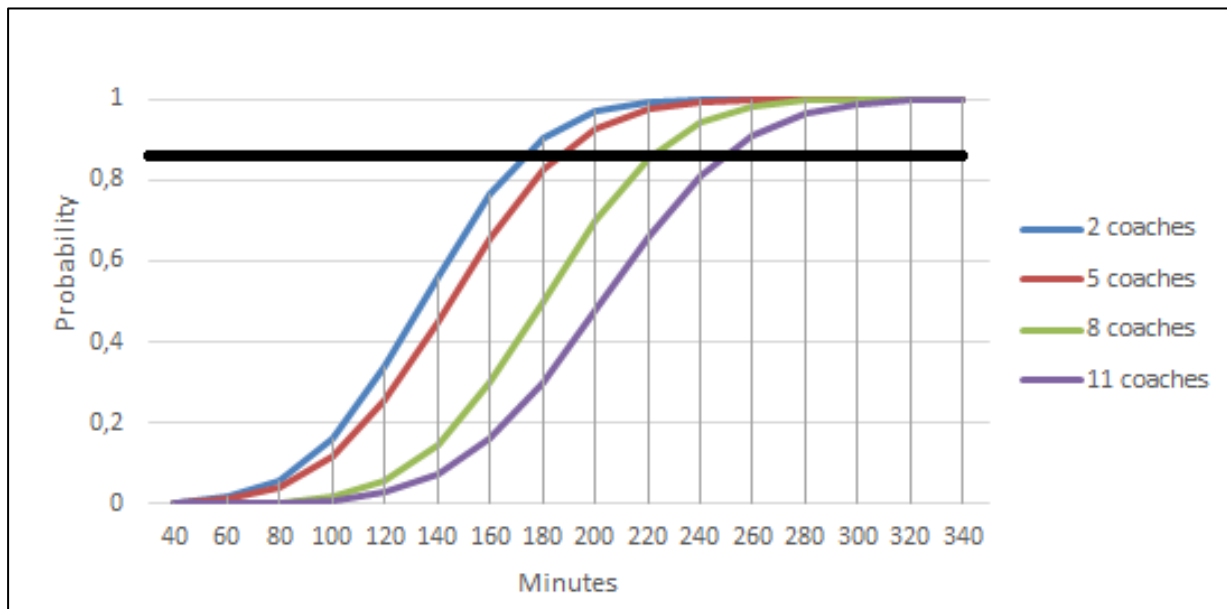

Fig. 4. Cumulative function of times for service of the selected groups

The average value plus the average square deviation involving 84% of the cases examined has been assumed as a standard of the duration of trainset stay. However, this would not reduce the reliability of the normal handling of trainsets because the rest of cases with longer stays are due to the inability to separate all trainset stays due to the set-up schedule. The standard of the duration of trainset stay Tdt (n) depends on the number of carriages in the trainsets to be served – n. An exponential function has been chosen to approximate the assumed standards, as approximation is practically implemented through add trendline of Microsoft Excel and Microsoft Office.

$$T_{dt}(n) = 0.1227.n^2 + 0.90874.n + 142.99 \quad \text{[minutes]}; \tag{19}$$

where n is the number of carriages in the set of coaches/ trainset; $R^2 = 0.98$ – coefficient of determination.

The rules used for determining the duration of trainset stay according to the number of coaches can be used to schedule trains in compliance with the passenger train traffic schedule.

Having defined the functions of different types of stations, their standardization could be included as regulations in references [19, 20, 21] that will decrease the downtime caused by the scheduling of passenger trains and increase their utilization because the functions of stations will be determined

according to their type, track development, and operatinal tecnology, but not as it is now, on average per carriage.

6. CONCLUSION

The analysis of the methods used to determine the duration of trainset stay at a technical station has led definition of problems, objectives and choice of an appropriate methodological approach to solve them. The algorithm developed for determining the duration of trainset stay using GERT networks is universal regardless of the type of station, which it is applied to.

The developed network model with a stochastic structure allows us to reflect the complex interaction of technological systems and to present stations arbitrary in schematic development. The model makes it possible to determine the duration of trainset stay according to their size and the technical equipment and operating technology at the stations under examination. The developed model (Fig. 3) and formulas 10-19 are original and reflect mathematical formalization of the accepted system approach of decomposition and synthesis.

In order to realize the model, it is necessary to provide a database of statistical information about the technological processes performed and determine the laws for the distribution of the service times and probabilities of the arc conditions. Appropriate regression functions of the duration of the service of coach/trainsets have been found depending on the type and number of carriages at technical stations taking into account technological features and technical constraints affecting their type (Figure 4, formula 19).

The general model of station as a network model is valuable at the level of object theoretical description. It makes possible to get specific results that can be used successfully for practical applications. The Nadezhda–Sofia station is the largest technical station in the country and the created model of stochastic network with GERT nodes as given in Figure 3, covering all possible technological operations and movements of the trains depending on its technical equipment. The obtained function for the technical station of Nadezhda–Sofia can be easily standardized and used in scheduling trainsets in compliance with the train formation plan and train traffic schedule in the railway network, which are changed in December every year.

This model can be used also for other technical areas by dropping some of its nodes corresponding to the state of the stochastic network taking into account the technical equipment. For example, the technical region of Plovdiv node 3 in the model should be dropped because there is no direct handling to the equipment rails, and the region of Varna node 2 should also be dropped because washing of the trains is carried out either in Nadezhda technical station or in the technical region of Plovdiv. The railway network in Bulgaria has many deviations and the nodal sectional stations, where the connection with main lines is made as well as the terminals of main lines are the end stations for passenger trains used to perform some of train servicing operations. The change of the stochastic network model with GERT-nodes for the two technical regions mentioned above or terminals leads to the simplification of equations for the first and second moments (formulas 17, 18) and, hence, finding out mathematical expectation and the dispersion of the network realization time (formulas 8, 9) is simplified too. Therefore, it can be said that the use of this model is universal in terms of determining the downtime of passenger trains at each station, which is the end destination of arriving trains and the place where these trains must be serviced. The model can also be applied to determine the minimum time of scheduling at each station regardless of the country where it is used due to the almost identical technological processes of passenger train servicing, as the stochastic network with GERT-nodes (Fig. 3) is reduced depending on the operations performed.

References

1. Kaakai, F. & Hayat, S. & El Moudni, A. A hybrid Petri nets-based simulation model for evaluating the design of railway transit stations. *Simulation Modelling Practice and Theory.* September. 2007. Vol. 15. No. 8. P. 935-969.
Available at: https://doi.org/10.1016/j.simpat.2007.05.003.
2. Freling, R. & Lentink, R. & Kroon, L. & Huisman, D. *Shunting of passenger train units in a railway station.* Rotterdam. Erasmus Research Institute of Management (ERIM).23 p. Available at: https://repub.eur.nl/pub/221/erimrs20020909125022.pdf.
3. Bekey, G.A. & Kogan, B.J. Modeling and Simulation: Theory and Practice. A Memorial Volume for Professor Walter J. Karplus (1927-2001). Springer Science & Business Media. 6.12.2012.
4. Reijers, H.A. *Design and Control of Work flow Processes.*Technical University of Eindhoven Department of Technology Management. Berlin, Heidelberg, New York: Springer-Verlag. ISSN 0302-9743. ISBN 3-540-01186-2
5. Ohta, H. GERT analysis of a single convey or system. *International Journal of Production Research.* 1979. Vol. 17. No. 4. P. 405-410.
6. Karagiozov, K. Aproximate method for modeling of the open general queueing networks In: *Internationale conference on operation research.* Operation research. 1990.
7. Kleinrock, L. *Queueing Systems.* Vol. I. – Theory. New York. Wiley Interscience. 1975. 417 p. ISBN 978-0471491101.
8. Lin, B. Non resort Operating Parameter Model for Technical Station's Train Formation Plan with Locomotive Extended Routing. *Journal of the China railway society.* 1999. Vol. 06.
9. Lipsky, L. *Queueing Theory. A Linear Algebraic Approach.* N.Y. Macmillan. 1992. 384 p. ISBN 0-02-370962-9.
10. Little, J. Proof of Queueing Formula. *Operations Research.* 1961. No. 9. P. 383-387.
11. Agarwal, M. & Mohan, P. GERT analysis of m-consecutive-k-out-of-n:F system with overlapping runs and (k-1)-step Markov dependence. *International Journal of Operational Research.* 2008. Vol. 3. No.1/2. P. 36-51.
12. Pritsker, A.A.B. & Whitehouse, G.E. GERT. Graphical Evaluation and Review Technique, Part I: Probabilistic and Engineering Applications. *Journal of Industrial Engineering.* 1966. Vol. 17(6). P. 293-303.
13. Pritsker, A.A.B. *GERT: Graphical Evaluation and Review Technique.* RM-4973-NASA April 1966. 152 p.
14. Saaty, T. *Elements of Queueing Theory with Applications.* N.Y. Dover. 1983.
15. Stoilova, Sv. & Nikolova, R. Classifying Railway Passenger Stations for use Transport Planning – Application to Bulgarian Railway Network. *Transport Problems.* 2016. Vol. 11. No. 2. P. 143-156.
16. Shankar, G. & Sahani, V. *GERT analysis of a two-unit cold stand by system with repair*, Pt. Ravishankar Shukla University. Raipur (M.P.). India. Accepted 6 May 1994. Available online 23 February 2000.
17. Taxa, H. *Operation research.* New Jersey. Prentice Hall, University of Arkansas. 910 p.
18. Todorova, M. GPSS Simulation of Coach Yard Operation and Application of the Model to Optimize Performance In: *22nd International Symposium EURO - Zel 2014 Recent Challenges for European Railways.* 03-04 June 2014. Žilina. P. 238-246.
19. *"БДЖ" ЕАД Поделение за почистване и екипиране, Технологията за ежедневно, основно вътрешно почистване и екипиране на пътническите вагони и ЕМВ.* София. 2015. [In Bulgarian: *BDZ EAD. Department for cleaning and equipping. Technology for daily, basic internal cleaning and equipping passenger coaches and EMU.* Sofia. 2015].
20. *"БДЖ" ЕАД Поделение за почистване и екипиране. Норми на време за ежедневно вътрешно почистване на вагоните и ЕМВ.* София. 2015. [In Bulgarian: *BDZ EAD. Department for cleaning and equipping. Standards of daily, basic internal cleaning and equipping passenger coaches and EMU.* Sofia. 2015].

21. *Министерство на транспорта и съобщенията, Нормативи за железопътния транспорт.* София. 2002. [In Bulgarian: *Ministry of Transport and Communications, Railway Transport Regulations*. Sofia. 2002].

22. *Министерство на транспорта и съобщенията, Закон за железопътния транспорт.* София. Двбр. 97/28.11.2000 [In Bulgarian: *Ministry of Transport and Communication. Railway Transport Act.* State Gazette. No. 97. Sofia. 28.11.2000].

23. *Организация процессов освоения дальних и пригородных пассажиропотоков.* Методические указания по выполнению курсового проекта. Чита. 2003. 44 с. [In Russian: *Organization of processes of using long distance and suburban passenger flows.* Methodological instructions for term project development.Chita. 2003. 44 p.].

24. Покацкая, Е.В. & Левченко, А.С. *Пассажирский железнодорожный комплекс - пассажирские станции.* Самарская государственная академия путей сообщения. 2007. [In Russian: Pokatskaya, E.V. & Levchenko, A.S. *Railway passenger complexes – passenger stations,* Samara State Academy of Communication Roads. 2007].

25. Правдин, Н.В. & Рябуха, Л.С. & Лукашев, В.И. *Технология работы вокзалов и пассажирских станций.* Москва: Транспорт. 1990. 320 p. [In Russian: Pravdin, N.V. & Ryabuha, L.S. & Lukashev, V.I. *Technology of operation of railway stations and passenger stations.* Moscow: Transport. 1990. 320 p.].

26. Тасев, Й. & Карагьозов, К. *Ръководство за курсово и дипломно проектиране по устройство, технология и проектиране на железопътни гари и възли.* ВМЕИ. София. 1983. 182 с. [In Bulgarian: Tasev, Y. & Karagyozov, K. *Handbook for development of term projects and diploma papers in designing railway stations and junctions.* HIMEE, Sofia. 1983. 182 p.].

27. Тодорова, М.Т. *Дисертационен труд "Оптимизация на използването на подвижния състав в пътническия железопътен транспорт".* ВТУ Т. Каблешков, София. 2013. [In Bulgarian: Todorova, M. *Optimization of rolling stock usingin passenger railway transport.* Todor Kableshkov University of Transport. PhD thesis. Sofia. 2013. 138 p.].