A STUDY ON MITIGATION OF THE DISTANCE-DEPENDENT BIASES IN THE NETWORK RTK TECHNIQUE

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1. INTRODUCTION

Today the Network – based Real Time Kinematic (Network RTK) technique is the most accurate relative GNSS kinematic positioning in real-time. Also in Poland the multiple reference stations network ASG-EUPOS which is a part of European Position Determination System (Bosy et al., 2008) provides the real-time data to use this technique. The basic idea of Network RTK is to use the network of reference stations to determinate the distance-dependent errors: ionospheric and geometric (tropospheric, orbital) biases. The biases can be used to generate the spatial correction terms and reduce the distance-dependent errors between the reference station and user receiver. This allows, within the coverage of the reference stations network, to fix carrier phase ambiguities and achieve cm-level positioning accuracy for up to 50km baseline (Rizos, 2002).

One of the most important issues for Network RTK techniques is how to estimate the ionospheric and geometric biases for the user's location. In this paper, several interpolation methods are reviewed in details. The formulas of each of these methods as well as the numerical results for the test network (part of the ASG-EUPOS network) are also presented.

2. METHODOLOGY FOR SPATIAL CORRECTION TERMS GENERATION

The main steps of spatial correction terms algorithm can be describe as follows:

- **to fix the double-differenced carrier phase ambiguities between the reference stations (which coordinates are well known);**
- **to determinate the distance-dependent biases (dispersive/ionospheric and non-dispersive/geometric biases between reference stations;**
- **to interpolate the distance-dependent biases for the user's receiver location.**

2.1 Ambiguity resolution between reference stations

Double-differenced carrier phase ambiguity resolution between reference stations is the first step to determinate spatial correlated errors. The length of baseline between reference stations on test area is up to 50km so ambiguity determination follows a wide lane/narrow lane (L_5/L_3) approach (Mervart, 1995; Chen et al., 2000). In these approach double-differenced wide lane (L_5) ambiguity was resolved at first using

“Melbourne-Wübbena” phase-code combination (Melbourne, 1985; Wübbena, 1985; Hofmann-Wellenhof et al., 2008):

$$\phi_{M+W} = \frac{c}{f_1 - f_2}(\phi_1 - \phi_2) - \frac{1}{f_1 + f_2}(P_1 f_1 + P_2 f_2) \quad (1)$$

$$\nabla\Delta N_{L5} = \frac{f_1 - f_2}{c}\nabla\Delta\phi_{M+W} \quad (2)$$

where Φ_k is the carrier phase observable in cycles; P_k is code pseudorange observables; f_k is the frequency of the carrier wave; $\nabla\Delta\phi_k$ is the double-difference carrier phase observable in cycles; $\nabla\Delta N_k$ is the double-difference integer ambiguity.

The Melbourne-Wübbena double-difference phase-code combination cancels out all bias terms in the observation equations (ionospheric and tropospheric delay, ephemeris errors) except integer ambiguity parameter. It allows to resolve wide lane integer ambiguity without a'priori models of biases.

In the second step the ionosphere-free phase combination:

$$\phi_3 = \phi_1 - \frac{f_2}{f_1}\phi_2 \quad (3)$$

is using to determinate L_1 integer ambiguity $(\nabla\Delta N_1)$ by expression:

$$\nabla\Delta N_1 = \frac{f_1}{f_1 - f_2}\nabla\Delta\phi_3 - \frac{f_1 + f_2}{c}(\nabla\Delta\rho + \nabla\Delta T) - \frac{f_2}{f_1 - f_2}\nabla\Delta N_5 \quad (4)$$

where $\nabla\Delta\rho$ is the double-difference geometric satellite-to-receiver distance and $\nabla\Delta T$ is the double-difference tropospheric delay (computed from tropospheric model). The effective wavelength of L_1 in equation (4) is 10.7cm like in narrow lane combination. In the end the L_2 integer ambiguity can be derived as:

$$\nabla\Delta N_2 = \nabla\Delta N_1 - \nabla\Delta N_5 \quad (5)$$

In order to estimate float ambiguity $(\nabla\Delta\breve{N})$ as well as covariance matrix $(Q_{\nabla\Delta\breve{N}})$, the Kalman filter was used. The Least-squares AMBiguity Decorrelation Adjustment (LAMBDA) method (Teunissen, 1995) and Modified LAMBDA method (Chang et al., 2005) were applied to fix ambiguity with ratio test (more than 3.0) as a threshold of validation. Cycle slips were detected and repaired using discontinuities in the double-differenced ionospheric residuals.

2.2 Distance-dependent biases between reference stations

After solving out the double-differenced integer ambiguity for L_1 and L_2 frequencies between reference stations, the double-differenced carrier phase residuals $(\nabla\Delta\bar{\phi}_k)$ can be obtained, as follows:

$$\nabla\Delta\bar{\phi}_k = \nabla\Delta\phi_k - \frac{f_k}{c}(\nabla\Delta\rho + \nabla\Delta T) - \nabla\Delta N_k \quad (6)$$

The double-differenced tropospheric delay was computed using Saastamoinen model (Saastamoinen, 1973) with a standard atmosphere parameters and Niell mapping functions for wet and dry components (Niell, 1996). Final precise ephemerides from International GNSS Service (IGS) and relative antenna calibrations from National Geodetic Survey (NGS) (Mader, 1999) also were applied to compute the double-difference geometric satellite-to-receiver distance.

The dispersive (double-difference ionospheric delay for L_1 in meters - $\nabla\Delta I_1$) and non-dispersive (double-difference geometric delay reflecting residual tropospheric delay, orbit errors, reference stations coordinates errors and etc., in meters - $\nabla\Delta G$) parts of the carrier phase residuals can by separated using the formulas:

$$\nabla\Delta I_1 = \frac{f_2^2}{f_1^2 - f_2^2}\left(\frac{c}{f_1}\nabla\Delta\bar{\phi}_1 - \frac{c}{f_2}\nabla\Delta\bar{\phi}_2\right) \quad (7)$$

$$\nabla\Delta G = \frac{f_1^2}{f_1^2 - f_2^2}\left(\frac{c}{f_1}\nabla\Delta\bar{\phi}_1 - \frac{cf_2}{f_1^2}\nabla\Delta\bar{\phi}_2\right) \quad (8)$$

2.3 Models of interpolation the distance-dependent biases

On the basis of computed distance-dependent error between reference stations and precise known reference stations coordinates, the spatial correction terms (separately for dispersive and non-dispersive parts) can by generated using one of the existing interpolations methods.

2.3.1 Linear Interpolation Algorithm (LIA)

LIA algorithm, also called Distance-based linear Interpolation Method (DIM) (Gao et al., 1997), allows to derive ionospheric delay (and geometric bias) for the master station - user station baseline ($\nabla\Delta I_{m,u}$), as follows:

$$\nabla\Delta I_{m,u} = \sum_{r=1}^{n-1}\frac{s_{r,u}}{s}\nabla\Delta I_{m,r} \quad (9)$$

$$s_{r,u} = \frac{1}{d_{r,u}} \quad (10)$$

$$s = \sum_{r=1}^{n-1} s_{r,u} \quad (11)$$

where n is the number of reference stations, $d_{r,u}$ is the distance between reference stations (r) and user stations (u) (the approximate coordinates of the user station must be known). $\nabla\Delta I_{m,r}$ is the double-differenced ionospheric delay for pair: master reference station (m) – r^{th} reference station. LIA needs at least three reference stations.

2.3.2 Linear Interpolation Method (LIM)

LIM method proposed by Wanniger (Wanniger, 1995) and extended by Wübbena et al. (Wübbena et al., 1996) describes distance-dependent biases as a two parameters plane model, where variables a and b, so-called network coefficients, estimates for north and east gradient. The correction from the master to user can be describe as:

$$\nabla\Delta I_{m,u} = [\Delta x_{m,u} \quad \Delta y_{m,u}]\begin{bmatrix} a \\ b \end{bmatrix} \quad (12)$$

where $\Delta x_{m,u}$ and $\Delta y_{m,u}$ are the plane coordinate differences that refer to master reference station (m) and user station (u). At least three reference stations are needed to compute the network coefficients; if the number of reference stations is bigger than three, the coefficients can be estimate by a least-square adjustment:

$$\begin{bmatrix} a \\ b \end{bmatrix} = \left(A^T A\right)^{-1} A^T V, \quad A = \begin{bmatrix} \Delta x_{m,1} & \Delta y_{m,1} \\ \Delta x_{m,2} & \Delta y_{m,2} \\ \vdots & \vdots \\ \Delta x_{m,n-1} & \Delta y_{m,n-1} \end{bmatrix}, \quad V = \begin{bmatrix} \nabla\Delta I_{m,1} \\ \nabla\Delta I_{m,2} \\ \vdots \\ \nabla\Delta I_{m,n-1} \end{bmatrix} \tag{13}$$

where subscript 1,2,..,*n* denotes number of reference stations.

Some variety of LIM is Weighted Linear Interpolation Method (WLIM), which is a standard interpolation method of Trimble software (Chen et al., 2003). In this method the distance dependent biases weighted by the distance between reference stations and user station are used to calculate correction terms. Also an additional parameter c, estimates for constant part that represents the station-specific error, are used as opposed to LIM, which makes at least four reference stations are needed. The correction term of master-user baseline can be describe as:

$$\nabla\Delta I_{m,u} = \begin{bmatrix} \Delta x_{m,u} & \Delta y_{m,u} & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} \tag{14}$$

2.3.3 Linear Combination Model (LCM)

LCM method, proposed by Han and Rizos (Han and Rizos, 1996), describe spatial correlated errors between master reference station (*m*) and user stations (*u*) as linear combination of double-differenced biases between reference stations (*r*) and master reference station:

$$\nabla\Delta I_{m,u} = \alpha_1 \nabla\Delta I_{m,1} + \alpha_2 \nabla\Delta I_{m,2} + \cdots + \alpha_{n-1} \nabla\Delta I_{m,n-1} \tag{15}$$

where α is the set of determined parameters and n is the number of reference stations. The parameters α are resolve with the following conditions:

$$\sum_{r=1}^{n} \alpha_r = 1, \quad \sum_{r=1}^{n} \alpha_r \left(\vec{X}_u - \vec{X}_r\right) = 0, \quad \sum_{r=1}^{n} \alpha_r^2 = min \tag{16}$$

where $\vec{X}_u$ and $\vec{X}_r$ are horizontal coordinates vector for the user and reference stations respectively. The parameters of linear combination are determined using equations:

$$\vec{\alpha} = B^T \left(BB^T\right)^{-1} C \tag{17}$$

$$\vec{\alpha} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 1 & \cdots & 1 & 1 \\ \Delta x_{m,1} & \Delta x_{m,2} & \cdots & \Delta x_{m,n-1} & 0 \\ \Delta y_{m,1} & \Delta y_{m,2} & \cdots & \Delta y_{m,n-1} & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 \\ \Delta x_{m,u} \\ \Delta y_{m,u} \end{bmatrix} \tag{18}$$

LCM needs at least three reference stations and the parameters depend on the position of the user station and should be recalculated on epoch-by-epoch in kinematic applications.

2.3.4 Low-order Surface Model (LSM)

LSM method uses a low-order surface to fit the spatial correlated biases in network of reference stations. As a variables are used 2D (horizontal) or 3D spatial (horizontal and altitude) coordinates. First or second order fitting function is used. The number of reference stations (*n*) needed to resolve depends on the number of coefficients (*l*) of LSM: *n=l+1*. Some function are (Fotopoulos and Cannon, 2000):

$$f(\Delta x_{m,u}, \Delta y_{m,u}) = a\Delta x_{m,u} + b\Delta y_{m,u} + c \quad (19)$$

$$f(\Delta x_{m,u}, \Delta y_{m,u}) = a\Delta x_{m,u} + b\Delta y_{m,u} + c\Delta x_{m,u}\Delta y_{m,u} + d \quad (20)$$

$$f(\Delta x_{m,u}, \Delta y_{m,u}) = a\Delta x_{m,u} + b\Delta y_{m,u} + c\Delta x_{m,u}\Delta y_{m,u} + d\Delta x_{m,u}^2 + e \quad (21)$$

$$f(\Delta x_{m,u}, \Delta y_{m,u}) = a\Delta x_{m,u} + b\Delta y_{m,u} + c\Delta x_{m,u}\Delta y_{m,u} + d\Delta x_{m,u}^2 + e\Delta y_{m,u}^2 + f \quad (22)$$

2.3.5 Partial Derivative Algorithm (PDA)

The concept of PDA uses a first and second order partial derivative of GPS measurement error function to model spatially correlated errors (Varner and Cannon, 1997; Fotopoulos and Cannon, 2001), as follow:

$$g(P) = g_0 + \frac{\partial g}{\partial x}\Delta x + \frac{\partial g}{\partial y}\Delta y + \frac{\partial g}{\partial h}\Delta h + \frac{\partial^2 g}{2\partial h^2}\Delta h^2 + v \quad (23)$$

where g is GPS measurement error function expanded in Taylor series. In practice, based on distance-dependent error between reference stations, the coefficients equal first-order partial derivative with respect to the horizontal coordinates (β, γ) and altitude (δ) are computed. Also second-order partial derivative with respect to altitude (ε), which takes into consideration the nonlinear effects in the vertical direction due to ionosphere and troposphere is used. The constant coefficient (α) describe the station-specific error at master station. The model for the dispersive bias describe the expression:

$$\nabla\Delta I_{m,u} = \alpha + \beta\Delta x_{m,u} + \gamma\Delta x_{m,u} + \delta\Delta h_{m,u} + \epsilon\Delta z_{m,u}^2 \quad (24)$$

in which nonlinear effects in the horizontal directions are omitted.

2.3.6 Least-Squares Collocation (LSC)

LSC method is used to interpolate some value at any given location using known covariance between interpolated and measurement values. The interpolation equation can be written (Dai et al., 2001):

$$\hat{U} = C_{vu} \cdot C_v^{-1} \cdot V \quad (25)$$

where $\hat{U}$ denotes interpolated vector, C_v is the covariance matrix of the measurement vector V, and C_{uv} is the cross-covariance matrix between interpolated and measurement vector.
To obtain the optimal estimator of interpolated values, the measurement (distance-dependent biases) must satisfy the condition of normal distribution and zero mean (Raquet and Lachapell, 2001). In practice, the interpolation equations for ionospheric biases is presented (Odijk et al., 2000):

$$\nabla\Delta\hat{I}_{1,u}^{k,l} = \begin{bmatrix} C_{u,1}^l & C_{u,2}^l & \cdots & C_{u,n}^l \end{bmatrix} \begin{bmatrix} C_0 & C_{1,2}^l & \cdots & C_{1,n}^l \\ C_{2,1}^l & C_0 & & \vdots \\ \vdots & & \ddots & C_{n-1,n}^l \\ C_{n,1}^l & \cdots & C_{n,n-1}^l & C_0 \end{bmatrix}^{-1} \begin{bmatrix} \nabla\Delta I_{1,1}^{k,l} = 0 \\ \nabla\Delta I_{1,2}^{k,l} \\ \vdots \\ \nabla\Delta I_{1,n}^{k,l} \end{bmatrix} \quad (26)$$

where: subscript r=1, 2, … n denotes reference stations (r=1 is the master station) and u is the user station; superscript s=k,l denotes satellites (k is the reference satellite). The covariance function $C^j{}_{a,b}$ is linearly dependent on the ionospheric pierce points:

(27)

where $l^j_{a,b}$ is the distance between the ionospheric points (where the vectors satellite-reference stations pierce an assumed infinitely thin ionospheric layer at 350km height above the Earth surface, see Fig. 1) for stations *a* and *b* with respect to satellite *j*. l_{max} is the distance larger than the longest distance between ionospheric pierce points: $l_{max} > l^j_{a,b}$. The presented algorithm assumes dependence of the covariance function on distance between reference stations only and it is approximate approach.

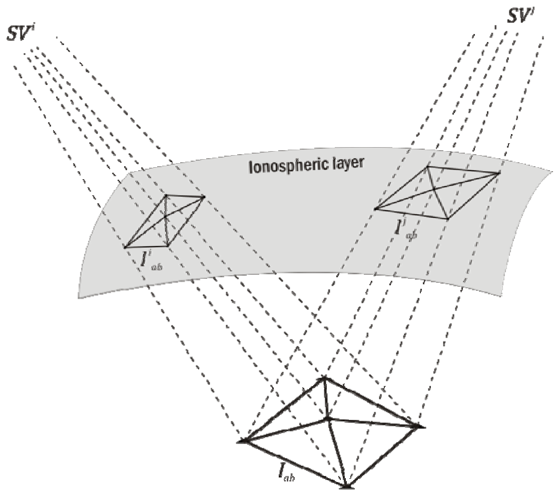

Fig. 1. Infinitely thin ionospheric layer model.

3. TESTING THE PERFORMANCE OF SPATIAL CORRECTION TERMS GENERATION

To test how well the interpolation algorithms can approximate the real distance-dependent biases the test network consisting of the nine ASG-EUPOS reference stations was used (Fig. 2). The reference stations BYDG located in the middle of the test network was used as a master reference station. The distance between master and other reference stations is ca 70km. The station TORU was selected as the user station. The ionospheric and geometric biases for the baseline BYDG-TORU (ca 45km) were not included in the interpolation algorithms and were used as a *true* values to verify the correction terms performance. All calculation were performed by using the MATLAB scripts developed by author.

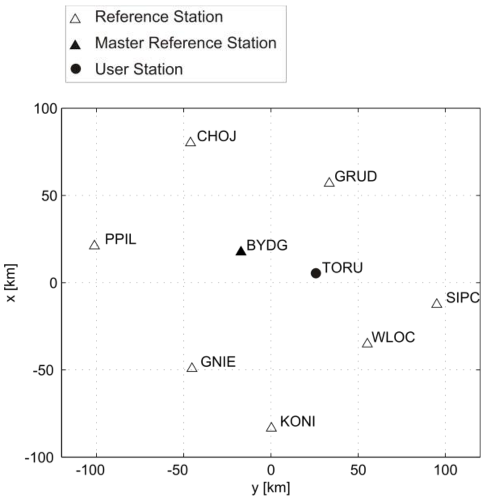

Fig. 1. Reference station test network.

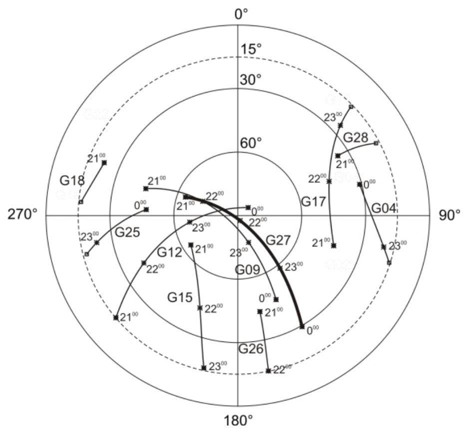

Fig. 2. Sky plot for the test network, 14 February 2001, 21.30 – 23.30 UT.

The two hours (21.30 – 23.30 UT) of GPS dual frequency phase and code data was collected on 14 February 2011 (during that time the ionosphere was active with Kp index of 4). A 10 seconds sampling rate and 15° elevation cut-off was used in experiment. Figure 3 shows the sky plot during the test period (bold line marks reference satellite).

Figures 4 and 5 shows the *true* value of double-difference ionospheric (for L_1 in meters) and geometric (in meters) biases for all satellites (related to reference satellite) for BYDG-TORU baseline. Using the interpolation algorithms described by formulas: (9), (12), (14), (15), (21), (24), (26) the estimate ionospheric and geometric biases (correction terms) were computed. Table 1 shows the results (RMS, mean and range values) for the different interpolation methods for ionospheric and geometric biases. The first column (*raw*) shows the *true* biases and the next columns the biases after the correction terms have been applied. The LSC method was use only to interpolate the ionospheric delay.

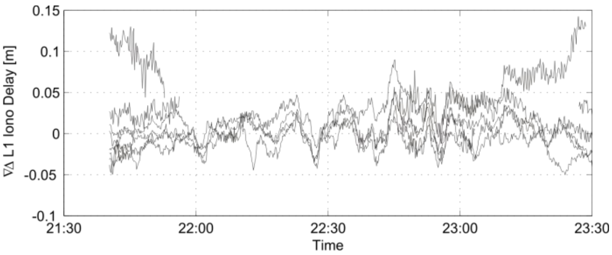

Fig. 3. DD ionospheric delay time series for BYDG-TORU – true value.

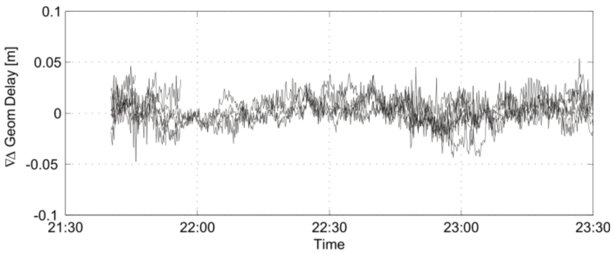

Fig. 4. DD geometric bias time series for BYDG-TORU – true value.

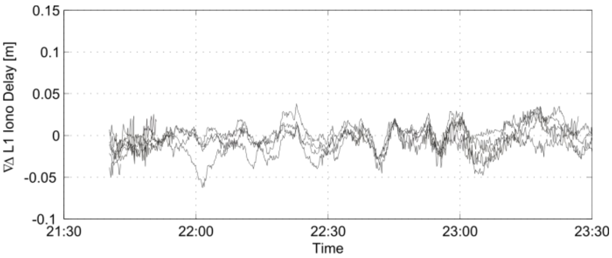

Fig. 5. DD ionospheric delay time series for BYDG-TORU – with the LCM.

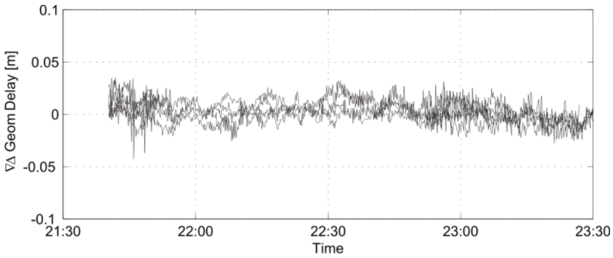

Fig. 6. DD geometric bias time series for BYDG-TORU – with the LCM.

Table 1. Statistic for ionospheric and geometric biases before and after the different correction terms were applied

		RAW	LIA	LIM	WLIM	LCM	LSM	PDA	LSC
DD L1 Iono Delay	RMS [mm]	0.030	0.020	0.015	0.016	0.015	0.017	0.016	0.015
	Mean [mm]	0.010	-0.009	0.000	-0.005	-0.004	-0.004	-0.005	-0.005
	Range [mm]	0.193	0.131	0.112	0.105	0.100	0.108	0.102	0.108
DD Geom Delay	RMS [mm]	0.013	0.011	0.011	0.010	0.010	0.012	0.010	-
	Mean [mm]	0.002	0.004	0.000	0.002	0.002	0.003	0.002	-
	Range [mm]	0.101	0.079	0.093	0.078	0.077	0.083	0.078	-

It can be seen from Table 1 that all the interpolation algorithms significantly mitigate the ionospheric delay (by about 50%). Only the LIA method gives slightly worse results (by about 30%). The geometric biases did not exceed (after applying a'priori troposphere model) ±5cm (see Fig. 5) and can be considered that they reflect the measurement noise and residual multipath delay. Therefore after applying the correction terms the biases (and RMS) are slightly reduced. Figures 6 and 7 show the distance-dependent errors for BYGD-TORU baseline after applying the LCM (the smallest RMS values).

The dispersive and non-dispersive correction terms were also used to mitigate the distance-dependent biases in the double-differenced carrier phase residuals. Table 2 shows reduction of RMS value (in percentage terms) for carrier phase residuals after using tested interpolation algorithms. The LSC method was used only to model the ionospheric delay thus the geometric biases computed by LIM was used to determinate spatial correction terms for carrier phase residuals. It can be seen that reduction of RMS values for all using algorithms, except LIA method, are very close. In the Figures 8 and 9 the L_1 double-differenced carrier phase residuals before and after applying the LIM correction terms are presented.

Table 2. The reduction of RMS value for L1 and L2 DD residuals

	DD L1	DD L2
LIA	14.2%	22.1%
LIM	53.8%	55.1%
WLIM	33.8%	39.8%
LCM	38.0%	43.8%
LSM	33.9%	39.8%
PDA	36.4%	42.1%
LSC+LIM	46.1%	50.4%

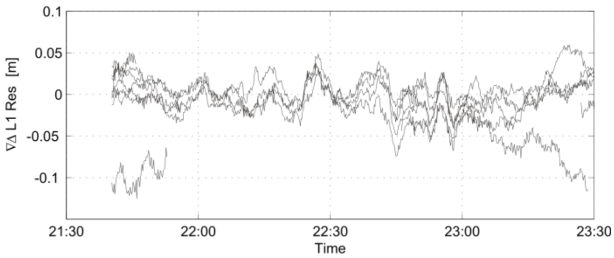

Fig. 7. DD L1 residuals time series for BYDG-TORU – true value.

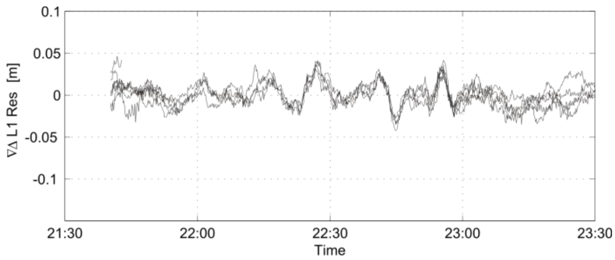

Fig. 8. DD L1 residuals time series for BYDG-TORU – with the LIM.

An important threshold of remaining ionospheric biases after correction terms applied is 8cm. If the biases are larger than these threshold the ambiguity determination will take significantly longer (Landau et al., 2003). For the test baseline 96.9% of ionospheric biases were less than the threshold before and 100% after the correction terms applied for all tested models.

4. CONCLUSIONS

In this paper the algorithm of generation the spatial correction terms - especially the methodology of interpolation of the distance-dependent biases have been reviewed in detail. The performance of described algorithms were tested using the reference station test network consisting of nine GPS stations. The numerical results shows that all tested methods significantly mitigate ionospheric and geometric biases for master station – user station baseline. The reduction of RMS value for L1 and L2 double-differenced carrier phase residuals for all algorithms are very close, except LIA method which gives slightly worse results. Also each of tested models reduced all the ionospheric biases to the threshold of 8cm.

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