Archives of Hydro-Engineering and Environmental Mechanics
Vol. 66 (2019), No. 3–4, pp. 101–130
DOI: 10.1515/heem-2019-0007
ISSN 1231–3726

Flow-induced Vibrations of a Horizontal Elastic Band Plate Submerged in Fluid of Finite Depth

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(Received December 10, 2019; revised February 28, 2020)

Abstract

The paper deals with forced vibrations of a horizontal thin elastic plate submerged in a semi-infinite layer of fluid of constant depth. The pressure load on this plate is induced by water waves arriving at the plate. This load is accompanied by pressure resulting from the motion of the plate. The plate and fluid motions depend on boundary conditions, and, in particular, the pressure load depends on the width of the gap between the plate and the bottom. In theoretical description of the phenomenon, we deal with a coupled problem of hydrodynamics in which the plate and fluid motions are coupled through boundary conditions at the plate surfaces. The main attention is focused on transient solutions of the problem, which correspond to fluid (and plate) motion starting from rest. In formulation of this problem, a linear theory of small deflections of the plate is employed. In order to calculate the fluid pressure, a solution of Laplace's equation is constructed in a doubly connected fluid domain. With respect to the initial value problem considered, we confine our attention to a finite fluid domain. For a finite elapse of time, measured from the starting point, the solution in the finite fluid area mimics a solution within an infinite domain, inherent for wave propagation problems. Because of the complicated structure of boundary conditions of the coupled problem considered, the fluid domain is divided into sub-domains of simple geometry, and the solutions of the problem equations are constructed separately in each of these domains. Numerical experiments have been conducted to illustrate the formulation developed in this paper.

Key words: elastic plate vibrations, surface waves, initial value problem, coupled problem

1. Introduction

The problem considered in this paper is relevant to offshore engineering, where structures installed in sea coastal zones are loaded with water wave forces. These hydrodynamic forces depend on fluid flows in the vicinity of the structure, as well as on its size, shape, rigidity, and foundation. Usually, such a structure consists of elements of simple geometry such as bars, pipes, and plates, and therefore, in the theoretical description of the structure dynamics, it is reasonable to investigate dynamic characteristics of these individual elements. Among them, of primary importance are elastic plates submerged in fluid and loaded with forces induced by gravitational waves. These hydrodynamic forces depend not only on waves themselves, but also on the foundation of the plate and its orientation to the direction of wave propagation. For instance, the wave forces on a plate perpendicular to the wave propagation direction are different from those that act on a plate whose surface is parallel to the wave direction. In general, these forces may also depend on the distance between the plate and boundaries of the fluid domain. In cases of horizontal plates placed at a small distance from the sea bottom, one may expect certain changes (amplifications) in hydrodynamic forces loading these plates. This phenomenon is related to changes in flow velocities at the upper and bottom surfaces of the plates. In general, vibrations of the plate submerged in fluid lead to the so-called co-vibrating mass of fluid, which significantly changes the eigenfrequencies of this plate. Thus, with respect to the above, we focus our investigations on the coupled hydrodynamic problem of the flow-induced motion of a horizontal plate submerged in fluid of constant depth. In order to simplify our discussion, we confine our attention to a simply supported elastic band plate, which makes it possible to reduce the description of the physical three-dimensional problem to a two-dimensional one. In a formal way, the two-dimensional description model corresponds directly to a simply supported horizontal beam submerged in fluid of constant depth. An additional simplification is introduced into the problem description that plate deflections are assumed infinitesimally small. In theoretical investigations, we resort to approximate modeling that can describe the main features of this phenomenon.

In describing vibrations of plates submerged in fluid, we frequently deal with the problem of an infinite fluid body with a free surface, and thus with the problem of surface water waves (gravitational waves) propagating in an infinite domain. In a general case, vibrations of plates submerged in fluid are accompanied by the transmission of energy from the plate to the fluid. In a compressible fluid, this transmission is due to dilatational waves. In cases of fluids with a free surface, transmission also occurs through the gravitational waves. For small vibrations with small frequencies, one may ignore the transmission through the dilatational waves. On the other hand, for higher frequencies of vibrations, the transmission of the energy by gravitational waves may be neglected.

The literature on the subject is considerable. The vibration of plates submerged in fluid has been discussed in many papers over the past decades. In most of the papers, steady-state harmonic problems are considered in which the time factor may be eliminated from equations describing the problem given. In many papers, the fluid body is infinite, and thus, together with the plate motion, the problem of waves propagating in fluid emerges. There are also problems of practical importance in which the fluid body is finite – for example, vibrations of tanks filled with fluid. In some cases, the interaction of plates with compressible fluids is discussed, where propagation of acoustic waves is of primary importance. Some of the investigations relevant to the current work are shortly described below; many others are cited within these references.

As regards vibrations of elastic plates in contact with fluid, Solecki (1966) discussed the problem of an infinite elastic plate floating on a water half-space. A similar problem of the deformation of floating ice plates was investigated by Kerr and Palmer (1972). A detailed discussion on the dynamics of an elastic band plate floating on a tank with a rectangular cross section is given in Sawicki (1976). More recently, Martin and Farina (1997) studied the small-amplitude, time-harmonic problem of the radiation of water waves by a heaving submerged horizontal disc. This disc (a thin rigid plate) was submerged beneath the free surface of deep water. The boundary-value problem for the potential velocity function describing the fluid motion was reduced to a boundary integral equation over the surface of the body immersed in fluid. The surprising result of the analysis is that the added-mass coefficient becomes negative for a certain range of frequencies when the disc is sufficiently close to the free surface. Leniowska (1999) discussed the time-dependent problem of forced vibrations of a circular elastic plate interacting with an infinite compressible fluid. The damping of the plate motion is partially due to the plate material (the Kelvin-Voigt model of damping) and to the radiation of mechanical energy by acoustic waves. Vibrations of rectangular plates coupled with fluid were investigated by Kerboua et al (2008) and Hosseini-Hashemi et al (2012). In the first of these papers, the fluid finite element method was applied to the hydro-elastic vibration analysis of a plate-fluid system. Completely submerged or floating on the free fluid surface, horizontal plates coupled with a finite fluid body were considered. The accuracy of the discrete model developed in that paper was assessed by comparing its results with data obtained in laboratory experiments with plates placed in a rectangular fluid reservoir. The second paper discussed the problem of vibrations of moderately thick rectangular plates coupled with stationary fluid. The main attention of the study was focused on natural frequencies of plates vibrating in contact with fluid for a combination of six different boundary conditions of these plates. The results obtained were compared with those known from the literature on the subject, obtained by the finite element method and by experiments, to show the applicability of the proposed formulation. Similar to the above, the problem of free vibrations of a cantilever plate partially submerged in fluid is discussed by Kwak and Yang (2013). In their paper, the rectangular plate is placed vertically. In order to calculate the effect of the fluid on the natural vibration characteristics of the plate, elliptical coordinates were introduced in the description of the coupled problem. In this way, elements of the virtual mass matrix were expressed in an analytical form in terms of the Mathieu functions.

A transient problem of vibrations of a plate coupled with a finite fluid body is considered in Nandu and Jayaraj (2014). The authors resort to a finite element formulation to describe the transient motion of a plate-fluid system consisting of a rectangular finite fluid domain with an elastic plate forming one of the fluid boundaries.

The natural frequencies of the system are calculated, which depend on the density of the fluid filling the cavity. The initial plate motion induced by an assumed impact pressure load of the plate is also calculated. Jafari and Rahmanei (2016) investigated vibrations of a rectangular composite plate floating on a rectangular fluid tank. They considered free and forced vibrations of this plate. The main attention of the study was focused on the influence of the number of lamina on natural frequencies of the plate. In discussing the free vibrations of plates in contact with a finite fluid body, one should be aware that the reduction in the values of the plate eigenfrequencies depends on the fluid body motion, so the reduction will be different for different amounts of fluid.

With regard to water wave problems involving the scattering of water waves by submerged horizontal plates and radiation of water waves by forced motion of these plates, an important contribution was made by Porter (2014). The main attention of his research was focused on the interaction of surface waves with thin rigid plates, including both a two-dimensional problem of infinitely-long plates and a three-dimensional problem of circular discs. The discussion includes the scattering of oblique waves by an infinite plate of constant width, a radiation problem for forced motion of the plate, and the scattering of waves by a circular disc. Only time-harmonic rigid-body motion of these plates is considered. Analytic solutions presented in that paper are developed on the basis of either Fourier or Hankel transforms to formulate integral equations for functions related to the unknown jump in pressure across the thin plate.

With respect to the papers shortly described above, the problem discussed in the present paper partially corresponds to that investigated by Sawicki (1976), Martin and Farina (1997) and Porter (2014). Our main goal, however, is different from those discussed in their papers. In our formulation, a thin horizontal elastic plate placed at a small distance from the sea bottom is considered. For such a case, the width of the gap between the plate and the sea bottom is essential for hydrodynamic forces loading the plate. The motion of the plate is induced by water gravitational waves arriving at the plate. The solution obtained corresponds to the initial problem of a plate-fluid system that starts to move at a specific moment in time. In a limit, this solution may be applied to a steady harmonic problem of a plate loaded with steady harmonic water waves arriving at the plate from infinity.

2. Problem Formulation

Let us consider a two-dimensional problem of a simply supported thin elastic band plate submerged in fluid, as shown schematically in Fig. 1.

The motion of the plate and fluid system is induced by a piston-type generator placed at the beginning of the layer. In order to describe the evolution of the plate-fluid system in time, an initial value problem is considered in which the generator, initially at rest, starts to move at a certain moment in time. The generator motion $x_g(t)$ is assumed as a known function of time. Formally, one may consider an arbitrary displace-

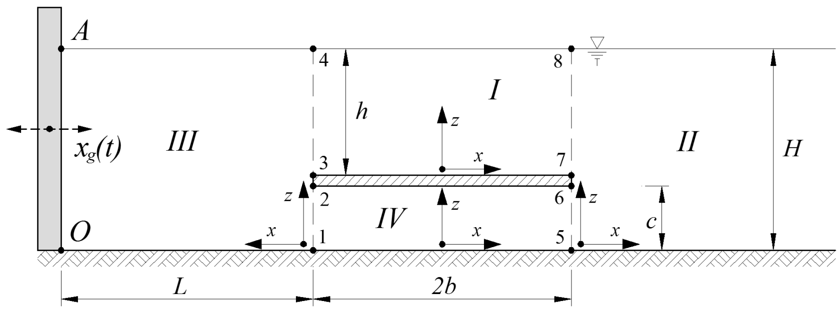

Fig. 1. Elastic plate submerged in a semi-infinite layer of fluid

ment of this generator, but, bearing in mind that under natural conditions sea waves are periodic in time, it is desirable to develop a solution corresponding to a steady harmonic generation of the waves. Such generation, however, requires additional explanations. Since we are dealing with motion that starts from rest, a harmonic generation may only be obtained as a limiting case of generation described, for instance, by the following formula (Wilde and Wilde 2001):

$$x_g(t) = C\left[A(\tau)\cos\omega t + D(\tau)\sin\omega t\right], \tag{1}$$

where $x_g(t)$ describes the generator horizontal displacement, t is the time measured from the starting point, C is a constant, $\tau = \mu t$ is a non-dimensional time factor, μ is a memory parameter responsible for an increase in wave generation in time, and

$$\begin{aligned} A(\tau) &= \frac{\tau^3}{3!}\exp(-\tau), \\ D(\tau) &= 1 - \left(1 + \tau + \frac{\tau^2}{2!} + \frac{\tau^3}{3!}\right)\exp(-\tau), \quad \tau = \mu t. \end{aligned} \tag{2}$$

One can check that, at the starting point $t = 0$, the displacement, velocity, and acceleration of the generator plate are all equal to zero. Moreover, with increasing time, the generator goes asymptotically to the harmonic motion with a constant amplitude (C in equation (1)). In practice, for a time exceeding one or two periods of wave generation ($t \gg T = 2\pi/\omega$), it can be assumed that we are dealing with the problem of harmonic generation with a constant amplitude. The motion of the plate is induced by water waves arriving at the plate from the left side of the layer (subdomain *III* in Fig. 1). Both plate and fluid motions are coupled through the boundary conditions at the plate-fluid boundaries (at the upper and bottom surfaces of the plate). These boundary conditions mean that the normal components of the fluid and plate velocities should be equal to each other. With respect to small deflections $w(x, t)$ of this plate, its motion is governed by the equation (e.g. Nowacki 1972)

$$m_{pl.}\frac{\partial^2 w}{\partial t^2} + D^*\frac{\partial^4 w}{\partial x^4} = p_{low.} - p_{upp.}, \tag{3}$$

where $m_{pl.}$ is the mass per unit width and length of the plate, $D^* = E\delta^3/12(1-\nu^2)$ is the flexural rigidity of the plate (δ is the plate thickness, and ν is Poisson's ratio), $p_{low.}$ and $p_{upp.}$ denote the fluid pressure at the lower and upper surfaces of the plate. It should be stressed that the "density" of the plate is defined here as $m_{pl.} = (\rho_{pl.} - \rho)\delta$, where $\rho_{pl.}$ is the density of the plate material, and ρ is the fluid density. Assuming the potential velocity of the fluid motion, the associated fluid pressure is described by the formula

$$p = -\rho\frac{\partial\Phi}{\partial t} + \rho g(H - z), \tag{4}$$

where g is the gravitational acceleration, and $\Phi(x, z, t)$ is the velocity potential satisfying Laplace's equation

$$\frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial z^2} = 0 \tag{5}$$

within the fluid domain and appropriate boundary and initial conditions.

In order to describe the evolution of the plate-fluid system in time, it is assumed that, at the starting point (at $t = 0$), the deflection of the plate, the dynamic part of pressure, and the fluid velocity are all equal to zero:

$$w(x, t = 0) = 0, \quad \text{and} \quad \frac{\partial\Phi}{\partial t} = \frac{\partial\Phi}{\partial x} = \frac{\partial\Phi}{\partial z} = 0 \quad \text{for} \quad t = 0. \tag{6}$$

At the same time, for the fluid domain, the following boundary conditions hold:

$$\begin{aligned}
&\left.\frac{\partial\Phi}{\partial x}\right|_{gener.} = \frac{\partial x_g(t)}{\partial t} = \dot{x}_g(t), \quad \left.\frac{\partial\Phi}{\partial x}\right|_{x\to\infty} = 0,\\
&\left.\frac{\partial\Phi}{\partial z}\right|_{z=0} = 0, \quad \left.\frac{\partial\Phi}{\partial z}\right|_{z=H} = \frac{\partial\eta(x,t)}{\partial t} = \dot{\eta}(x,t),\\
&\frac{\partial\Phi}{\partial n} \simeq \left.\frac{\partial\Phi}{\partial z}\right|_{plate} = \frac{\partial w(x,t)}{\partial t} = \dot{w}(x,t),
\end{aligned} \tag{7}$$

where $\partial x_g/\partial t$ denotes the horizontal velocity of the generator face (the rigid wall OA in Figure 1), and $\partial w/\partial t$ describes the transverse velocity of the plate (the velocity of its central plane).
Relations (7) are supplemented by the Bernoulli equation written for all points of the free surface:

$$\left.\frac{\partial\Phi}{\partial t} + gz\right|_{z=\eta(x,t)} = 0, \tag{8}$$

where $\eta(x, t)$ denotes the free surface elevation.

The kinematic and dynamic boundary conditions for the free fluid surface (described by the fourth equation in (7) and equation (8)) are usually reduced to one equation for the velocity potential:

$$\left.\frac{\partial^2 \Phi}{\partial t^2} + g\frac{\partial \Phi}{\partial z}\right|_{z=H} = 0. \tag{9}$$

In order to find a solution of equation (5), satisfying the above initial and boundary conditions, it is convenient to express the plate deflection $w(x,t)$ in terms of eigenfunctions of this plate vibrating in air. Thus, in the first step, free vibrations of the plate in air are considered. For this case, equation (3) reduces to the following one:

$$m_{pl.}\frac{\partial^2 w}{\partial t^2} + D^*\frac{\partial^4 w}{\partial x^4} = 0, \tag{10}$$

where $m_{pl.} = \rho_{pl.}\delta$.
For harmonic vibrations, the standard method of separation of variables is applied:

$$w(x,t) = W(x)\exp(i\omega t), \tag{11}$$

where ω is the vibration frequency, and $W(x)$ is the deflection amplitude. Substitution of this relation into equation (10) gives

$$-\omega^2 m_{pl.}W + D^*\frac{\partial^4 W}{\partial x^4} = 0. \tag{12}$$

A general solution of this equation, which satisfies boundary conditions at $x = \pm b$, reads

$$W(x) = \sum_n A_n \sin r_n(x+b), \quad r_n = \frac{n\pi}{2b}, \quad n = 1,2,..., \tag{13}$$

where A_n are constants, and $\sin r_n(x+b)$ are orthogonal eigenfunctions of the plate.

At the same time, equations (12) and (13) yield the set of eigenfrequencies of the plate vibrating in air:

$$\omega_n = (r_n)^2\sqrt{\frac{D^*}{m_{pl.}}} = \left(\frac{n\pi}{2b}\right)^2\sqrt{\frac{D^*}{m_{pl.}}}, \quad n = 1,2,\ldots. \tag{14}$$

In the further investigations, eigenfunctions (13) of the plate vibrating in air are employed in a description of the forced vibrations of the plate submerged in fluid. The latter are induced by time-dependent surface gravitational waves. To this end, generation of fluid motion, shown in Fig. 1, is considered in which the plate-fluid system starts to move at a specific moment in time (at $t = 0^+$). The problem is assumed linear, and therefore in its formulation it is convenient to divide the potential function into two parts:

$$\Phi^*(x,z,t) = \phi(x,z,t) + \Phi(x,z,t), \tag{15}$$

where Φ^* is a new notation for the potential Φ considered above (equations 4–9). The first part in this representation corresponds to vibrations of the plate, and the second part is associated with wave generation. Each of these parts satisfies Laplace's equation within the fluid domain and appropriate initial and boundary conditions. In the first step, the solution for the "external" fluid domain (i.e. the fluid domain except for the rectangular domain IV in Fig. 1) is considered. Thus, with respect to the notations and coordinate system shown in Fig. 1, the following boundary and initial conditions hold:

$$
\begin{aligned}
&\frac{\partial^2\Phi}{\partial t^2} + g\frac{\partial\Phi}{\partial z} + g\frac{\partial\phi}{\partial z}\bigg|_{z=H} = 0,\\
&\frac{\partial\Phi}{\partial x}\bigg|_{x=0} = \dot{x}_g(t), \quad \frac{\partial\phi}{\partial x}\bigg|_{x=0} = 0, \quad \frac{\partial\Phi}{\partial x}\bigg|_{x\to\infty} = 0, \quad \frac{\partial\phi}{\partial x}\bigg|_{x\to\infty} = 0,\\
&\frac{\partial\Phi}{\partial t}\bigg|_{x\to\infty} = 0, \qquad \frac{\partial\phi}{\partial t}\bigg|_{x\to\infty} = 0,\\
&\frac{\partial\Phi}{\partial z}\bigg|_{x=O-1 \text{ and } x>5,\ z=0} = 0, \quad \frac{\partial\Phi}{\partial z}\bigg|_{x=3-7, z=c} = 0, \quad \frac{\partial\Phi}{\partial x}\bigg|_{z=1-2} = \frac{\partial\Phi}{\partial x}\bigg|_{z=5-6} = 0, \qquad (16)\\
&\frac{\partial\phi}{\partial z}\bigg|_{x=O-1 \text{ and } x>5,\ z=0} = 0, \quad \frac{\partial\phi}{\partial z}\bigg|_{plate} = \frac{\partial w(x,t)}{\partial t}, \quad \phi|_{z=H} = 0,
\end{aligned}
$$

and

$$
\frac{\partial\Phi}{\partial t} = \frac{\partial\Phi}{\partial x} = \frac{\partial\Phi}{\partial z} = \frac{\partial\phi}{\partial t} = \frac{\partial\phi}{\partial x} = \frac{\partial\phi}{\partial z} = 0 \quad \text{for} \quad t = 0.
$$

The solution for the "external" domain (regions III, I, II in Fig. 1) should match a solution for the rectangular fluid domain (area IV in Fig. 1) . This means that the solution beneath the plate must be correlated with the potentials Φ and ϕ. Due to conditions (16), however, the potential for the rectangular fluid domain, say $\varphi(x, z, t)$, may by obtained independently of the potential Φ, and therefore it should be matched only with the potential ϕ. The matching with the potential Φ at common boundaries may be done in the second step by introducing an additional potential $\varphi_{add.}$ beneath the plate. Accordingly, in constructing a solution for the potential $\Phi(x, z, t)$, it is sufficient to confine our attention to the potentials ϕ and φ. Such a solution simplifies the solution of the whole problem considered.

A comment is needed here. In addition to the boundary conditions given above, it is necessary to investigate the potential behaviour at the plate end points. If the cross section of the plate is rectangular with two right angles at the corner points of the plate, the fluid velocity field is singular at these points. It may be shown, however, that this fluid velocity field is an integrable function along an arbitrary path in the vicinity of these end points, and thus, the potential function is a rational function within the fluid domain. Accordingly, with respect to the boundary conditions written above, one may construct separate solutions for the potential functions ϕ and φ. Due to the boundary condition at the free fluid surface ($\phi|_{z=H} = 0$), this potential describes a standing wave. Therefore, in the next section, the standing wave solution is considered.

3. Standing Wave Solution

With respect to the previous section, let us now consider solutions of placeLaplace's equation for the potentials $\phi(x, z, t)$ and $\varphi(x, z, t)$. It follows from the associated boundary conditions that these potential functions depend only on the plate motion. The problem for these potentials is formulated in the doubly connected fluid domain, and therefore, in constructing its solution, it is convenient to divide this domain into four parts: I, II, II and IV (see Fig. 1) and look for solutions in each of these parts separately. In descriptions of these potential functions within sub-domains, it is convenient to introduce local Cartesian coordinate systems. Thus, with respect to these coordinate systems, shown in Fig. 1, the general solutions of Laplace's equation are as follows:

Subdomain *III* ($0 \le x \le L, \quad 0 \le z \le H$):

$$\begin{gathered}\phi(x, z, t) = -\sum_{j=1} C_j(t)\frac{1}{k_j}\left(e^{-k_j x} + e^{-k_j(2L-x)}\right)\cos k_j z,\\ k_j = \frac{2j-1}{2H}\pi, \quad j = 1, 2, \ldots .\end{gathered} \tag{17}$$

Subdomain *II* ($0 \le x < \infty, \quad 0 \le z \le H$):

$$\phi(x, z, t) = -\sum_{j=1} B_j(t)\frac{1}{k_j}e^{-k_j x}\cos k_j z, \quad k_j = \frac{2j-1}{2H}\pi, \quad j = 1, 2, \ldots \tag{18}$$

Subdomain *I* ($-b \le x < +b, \quad 0 \le z \le h$):

$$\begin{gathered}\phi(x, z, t) = -\sum_{n=1} \dot{A}_n(t)\frac{1}{r_n}\frac{1}{\upsilon_n}\left(e^{-r_n z} - e^{r_n(z-2h)}\right)\sin r_n(x+b) +\\ + \sum_{m=1}\left[D_m^1(t)\frac{\cosh(k_m^* x)}{\cosh(k_m^* b)} + D_m^2(t)\frac{\sinh(k_m^* x)}{\sinh(k_m^* b)}\right]\cos k_m^* z,\\ r_n = \frac{n\pi}{2b},\ \upsilon_n = 1 + e^{-2r_n h},\ k_m^* = \frac{2m-1}{2h}\pi,\ m = 1, 2, \ldots,\ n = 1, 2, \ldots,\end{gathered} \tag{19}$$

where $\dot{A}_n(t) = dA_n(t)/dt$.

Subdomain IV $(-b \le x \le +b,\ \ 0 \le z \le d)$:

$$\varphi(x,z,t) = \sum_{n=1} \dot{A}_n(t)\frac{1}{r_n}\frac{\cosh r_n z}{\sinh r_n d}\sin r_n(x+b) +$$

$$+ E_0(t) + \sum_{m=1}\left[E_m^1(t)\frac{\cosh k_m x}{\cosh k_m b} + E_m^2(t)\frac{\sinh k_m x}{\sinh k_m b}\right]\cos k_m z, \tag{20}$$

$$r_n = \frac{n\pi}{2b}, \quad k_m = \frac{m\pi}{d}, \quad m = 1,2,\ldots, \quad n = 1,2,\ldots \,.$$

It is worth adding here that the potential $\phi(x,z,t)$, defined by equations (17) and (18), quickly decays with growing distance from the plate. Thus, for a sufficient distance, say $L >> 0$, the second term in the square brackets in equation (17) may be neglected in practical calculations. To save space, hereinafter we omit the time character t in description of the functions $A(t), \cdots, E(t)$, i.e. all the functions are named constants. In accordance with the linear problem considered, all constants: $B_j, C_j,\ \ j = 1,2,\ldots$, $D_m^1, D_m^2,\ \ m = 1,2,\ldots$, and E^0, and $E_m^1, E_m^2, m = 1,2,\ldots$, may be expressed in terms of the constants $\dot{A}_n = dA_n/dt,\ \ n = 1,2,\ldots$. It means that, for an arbitrary deflection of the plate, it is possible to find appropriate solutions within the corresponding fluid domains. To this end, we match solutions at the common boundaries of the subdomains. Thus, let us assume in advance that the solutions corresponding to B_j and C_j $(j = 1,2,\ldots)$ are known. The potential $\varphi(x = b, z)$ below the plate should be equal to that of the right-hand side domain, i.e. to $\phi(x = 0, z)$ at the common boundary. This condition gives

$$E_0 + \sum_{m=1}\left(E_m^1 + E_m^2\right)\cos k_m z = -\sum_{j=1} B_j \frac{1}{k_j}\cos k_j z. \tag{21}$$

In a similar way, at the boundary $(x = -b, z)$, we have

$$E_0 + \sum_{m=1}\left(E_m^1 - E_m^2\right)\cos k_m z = -\sum_{j=1} C_j \frac{1}{k_j}\left(1 + e^{-2k_j L}\right)\cos k_j z. \tag{22}$$

Multiplication of equation (21) in succession by $\cos k_m z$ $(m = 0,1,2,\ldots)$ and then integration in the range $(0 \le z \le d)$ leads to the following formulae:

$$E_0 = -\frac{1}{d}\sum_{j=1} B_j \frac{1}{(k_j)^2}\sin k_j d,$$

$$E_m^1 + E_m^2 = -\frac{2}{d}\sum_{j=1} B_j \left[\begin{array}{ll} \dfrac{(-1)^m \sin k_j d}{(k_j)^2 - (k_m)^2} & \text{for} \quad k_j \ne k_m, \\[2ex] \dfrac{d}{2k_j} & \text{for} \quad k_j = k_m, \end{array}\right. \qquad m = 1,2,\ldots \,. \tag{23}$$

Similar results hold for the left boundary:

$$
\begin{aligned}
E_0 &= -\frac{1}{d}\sum_{j=1} C_j\left(1+e^{-2k_jL}\right)\frac{1}{(k_j)^2}\sin k_j d,\\
E_m^1 - E_m^2 &= -\frac{2}{d}\sum_{j=1} C_j\left(1+e^{-2k_jL}\right)\times\\
&\times\left[\begin{array}{ll} \dfrac{(-1)^m \sin k_j d}{(k_j)^2-(k_m)^2} \quad \text{for} \quad k_j \neq k_m, \\ \dfrac{d}{2k_j} \quad \text{for} \quad k_j = k_m, \end{array}\right. \qquad m = 1,2,\ldots .
\end{aligned}
\tag{24}
$$

A similar procedure is employed for the upper fluid domains. Comparisons of the potential functions at the common boundaries of these domains give

$$
\begin{aligned}
\sum_{m=1}\left(D_m^1 + D_m^2\right)\cos k_m^* z &= -\sum_j B_j \frac{1}{k_j}\cos k_j(z+c),\\
\sum_{m=1}\left(D_m^1 - D_m^2\right)\cos k_m^* z &= -\sum_j C_j \frac{1}{k_j}\left(1+e^{-2k_jL}\right)\cos k_j(z+c).
\end{aligned}
\tag{25}
$$

From multiplication of these equations in succession by $\cos k_m^* z$ $(m = 1, 2, ...)$, and then integration in the range $(0 \le z \le h)$, the following is obtained:

$$
\begin{aligned}
D_m^1 + D_m^2 = &-\frac{2}{h}\sum_{j=1} B_j \times\\
\times&\left[\begin{array}{ll} \dfrac{\sin k_j c}{(k_m^*)^2-(k_j)^2} \quad \text{for} \quad k_j \neq k_m^*, \\ \dfrac{1}{2(k_j)^2}\left(k_j h \cdot \cos k_j c - \sin k_j c\right) \quad \text{for} \quad k_j = k_m^*, \end{array}\right. \qquad m = 1,2,\ldots,
\end{aligned}
\tag{26}
$$

and

$$
\begin{aligned}
D_m^1 - D_m^2 = &-\frac{2}{h}\sum_{j=1} C_j\left(1+e^{-2k_jL}\right)\times\\
\times&\left[\begin{array}{ll} \dfrac{\sin k_j c}{(k_m^*)^2-(k_j)^2} \quad \text{for} \quad k_j \neq k_m^*, \\ \dfrac{1}{2(k_j)^2}\left(k_j h \cdot \cos k_j c - \sin k_j c\right) \quad \text{for} \quad k_j = k_m^*, \end{array}\right. \qquad m = 1,2,\ldots
\end{aligned}
\tag{27}
$$

Equations (23–27) allow us to eliminate two sets of constants (E_m and D_n with $m = 0, 1, 2 \ldots$, $n = 1, 2, \ldots$) from the equations of the problem considered. In order

to eliminate the sets of constants B_j and C_j ($j = 1, 2, \ldots$), we employ the remaining boundary conditions that the velocity components normal to the fluid boundaries between the associated fluid domains (finite and infinite) must be the same. To write these conditions, it is necessary to calculate the normal fluid velocities. Following the potential functions described by equations (17–20), the normal fluid velocities are as follows:
The right fluid domain (at $x = +b$ of the plate):

$$\begin{aligned} & 0 \le z \le d \\ & u = \frac{\partial \varphi}{\partial x}\bigg|_{x=b} = \sum_{n=1} \dot{A}_n (-1)^n \frac{\cosh r_n z}{\sinh r_n d} + \\ & \quad + \sum_{m=1} k_m \left(E_m^1 \tanh k_m b + E_m^2 \frac{1}{\tanh k_m b} \right) \cos k_m z = \\ & = \sum_j B_j \cos k_j z \ , \end{aligned} \tag{28}$$

and

$$\begin{aligned} & c \le z \le H \\ & u = \frac{\partial \phi}{\partial x}\bigg|_{x=b} = -\sum_{n=1} \dot{A}_n (-1)^n \frac{1}{v_n} \left(e^{-r_n(z-c)} - e^{r_n(z-c-2h)} \right) + \\ & \quad + \sum_{m=1} k_m^* \left(D_m^1 \tanh k_m^* b + D_m^2 \frac{1}{\tanh k_m^* b} \right) \cos k_m^* (z-c) = \\ & = \sum_j B_j \cos k_j z \ . \end{aligned} \tag{29}$$

The left fluid domain (at $x = -b$ of the plate):

$$\begin{aligned} & 0 \le z \le d \\ & u = \frac{\partial \varphi}{\partial x}\bigg|_{x=-b} = \sum_{n=1} \dot{A}_n \frac{\cosh r_n z}{\sinh r_n d} + \\ & \quad + \sum_{m=1} k_m \left(-E_m^1 \tanh k_m b + E_m^2 \frac{1}{\tanh k_m b} \right) \cos k_m z = \\ & = -\sum_j C_j \left(1 - e^{-2k_j L} \right) \cos k_j z \end{aligned} \tag{30}$$

and

$$
\begin{aligned}
&c \le z \le H \\
&u = \left.\frac{\partial \phi}{\partial x}\right|_{x=-b} = -\sum_{n=1} \dot{A}_n \frac{1}{v_n}\left(e^{-r_n(z-c)} - e^{r_n(z-c-2h)}\right) + \\
&\quad + \sum_{m=1} k_m^* \left(-D_m^1 \tanh k_m^* b + D_m^2 \frac{1}{\tanh k_m^* b}\right)\cos k_m^*(z-c) = \\
&= -\sum_j C_j\left(1 - e^{-2k_j L}\right)\cos k_j z\ .
\end{aligned}
\tag{31}
$$

Equations (28–31) make it possible to express all the constants B_j, C_j in terms of the constants E_m^1, E_m^2 and D_m^1, D_m^2. In order to find the desired relations, equations (28–31) are multiplied in succession by the functions $\cos k_j z$ ($j = 1, 2, \ldots$) and then integrated within the range ($0 \le z \le H$). This procedure leads to two systems of equations:

$$
\begin{aligned}
&x = b \\
&\sum_{n=1} JA_j^n \cdot \dot{A}_n + \sum_{m=1}\left(JE_j^m \cdot E_m^1 + KE_j^m \cdot E_m^2\right) + \\
&+ \sum_{m=1}\left(JD_j^m \cdot D_m^1 + KD_j^m \cdot D_m^2\right) = B_j, \quad j = 1, 2, \ldots
\end{aligned}
\tag{32}
$$

and

$$
\begin{aligned}
&x = -b \\
&\sum_{n=1} KA_j^n \cdot \dot{A}_n + \sum_{m=1}\left(-JE_j^m \cdot E_m^1 + KE_j^m \cdot E_m^2\right) + \\
&+ \sum_{m=1}\left(-JD_j^m \cdot D_m^1 + KD_j^m \cdot D_m^2\right) = -C_j\left(1 - e^{-2k_j h}\right), \quad j = 1, 2, \ldots,
\end{aligned}
\tag{33}
$$

where the following substitutions have been made:

$$
\begin{aligned}
JA_j^n &= \frac{2}{H}(-1)^n \left[\frac{1}{\sinh r_n d}\int_0^d \cosh r_n z \cdot \cos k_j z\, dz - \right. \\
&\quad \left. - \frac{1}{v_n}\int_c^H \left(e^{-r_n(z-c)} - e^{r_n(z-c-2h)}\right)\cdot \cos k_j z\, dz\right] = \\
&= \frac{2}{H}(-1)^n \frac{1}{(r_n)^2 + \left(k_j\right)^2}\left[r_n\left(\cos k_j d - \cos k_j c\right) + \right. \\
&\quad \left. + k_j\left(\frac{\sin k_j d}{\tanh r_n d} + \tanh r_n h \cdot \sin k_j c\right)\right], \quad j, n = 1, 2, \ldots,
\end{aligned}
\tag{34}
$$

$$JD_j^m = \frac{2}{H} k_m^* \tanh k_m^* b \int_c^H \cos k_m^*(z-c) \cdot \cos k_j z \, dz = \frac{2}{H} k_m^* \tanh k_m^* b \times$$
$$\times \begin{cases} \dfrac{k_j}{(k_m^*)^2 - \left(k_j\right)^2} \sin k_j c \quad \text{for} \quad k_m^* \neq k_j, \\ \dfrac{1}{2k_j} \left(k_j h \cdot \cos k_j c - \sin k_j c\right) \quad \text{for} \quad k_m^* = k_j, \end{cases} \qquad j, m = 1, 2, \ldots, \tag{35}$$

$$JE_j^m = \frac{2}{H} k_m \tanh k_m b \int_0^d \cos k_m z \cdot \cos k_j z \, dz = \frac{2}{H} k_m \tanh k_m b \times$$
$$\times \begin{cases} (-1)^m \dfrac{k_j \sin k_j d}{\left(k_j\right)^2 - (k_m)^2} \quad \text{for} \quad k_m \neq k_j, \\ \dfrac{d}{2} \quad \text{for} \quad k_m = k_j, \end{cases} \qquad j, m = 1, 2, \ldots, \tag{36}$$

and

$$\begin{aligned} KA_j^n &= \frac{2}{H} \frac{1}{\sinh r_n d} \int_0^d \cosh r_n z \cdot \cos k_j z \, dz - \\ &\quad - \frac{1}{v_n} \int_c^H \left[e^{-r_n(z-c)} - e^{r_n(z-c-2h)} \right] \cdot \cos k_j z \, dz = \\ &= \frac{2}{H} \frac{1}{(r_n)^2 + \left(k_j\right)^2} \left[r_n \left(\cos k_j d - \cos k_j c \right) + \right. \\ &\quad \left. + k_j \left(\frac{\sin k_j d}{\tanh r_n d} + \sin k_j c \cdot \tanh r_n h \right) \right], \quad j, n = 1, 2, \ldots, \end{aligned} \tag{37}$$

$$KD_j^m = \frac{2}{H} k_m^* \frac{1}{\tanh k_m^* b} \int_c^H \cos k_m^*(z-c) \cdot \cos k_j z \, dz = \frac{2}{H} k_m^* \frac{1}{\tanh k_m^* b} \times$$
$$\times \begin{cases} \left\{ \dfrac{k_j \sin k_j c}{(k_m^*)^2 - \left(k_j\right)^2} \quad \text{for} \quad k_m^* \neq k_j, \right. \\ \dfrac{1}{2k_m^*} \left(k_m^* h \cdot \cos k_j c - \sin k_j c\right) \quad \text{for} \quad k_m^* = k_j, \end{cases} \qquad j, m = 1, 2, \ldots, \tag{38}$$

$$KE_j^m = \frac{2}{H} k_m \frac{1}{\tanh k_m b} \int_0^d \cos k_m z \cdot \cos k_j z \, dz = \frac{2}{H} k_m \frac{1}{\tanh k_m b} \times$$
$$\times \begin{cases} (-1)^m \dfrac{k_j \sin k_j d}{(k_j)^2 - (k_m)^2} & \text{for} \quad k_m \neq k_j, \\ \dfrac{d}{2} \quad \text{for} \quad k_m = k_j, \end{cases} \qquad j, m = 1, 2, \ldots . \tag{39}$$

Formally, equations (32) and (33), together with relations (23–27), allow us to express all constants in terms of the constants $\dot{A}_n(t)$, $(n = 1, 2 \ldots)$. It should be stressed, however, that these coupled equations correspond to an infinite number of these constants $(m, n = 1, 2, \ldots, \infty)$, and therefore, in order to simplify our further discussion, we resort to a finite number of terms in the infinite series entering these equations. In this way, the resulting system of algebraic equations may be solved by a numerical procedure. Thus, in order to make our further discussion clear, let us denote by nd, ne, and nj the numbers of terms taken into account in the series corresponding to D_m^1, D_m^2, E_m^1, E_m^2, and B_j, C_j, respectively. For such a finite system, it is reasonable to resort to a matrix notation, which is more convenient in description of the phenomenon. To this end, equations (23–27) are written in the following matrix forms:

$$\begin{aligned} &E_0 = \boldsymbol{E}^T \cdot (\boldsymbol{B} + \boldsymbol{EX} \cdot \boldsymbol{C}), \\ &\boldsymbol{E1} + \boldsymbol{E2} = \boldsymbol{EB} \cdot \boldsymbol{B}, \\ &\boldsymbol{E1} - \boldsymbol{E2} = \boldsymbol{EB} \cdot \boldsymbol{EX} \cdot \boldsymbol{C}, \\ &\boldsymbol{D1} + \boldsymbol{D2} = \boldsymbol{DB} \cdot \boldsymbol{B}, \\ &\boldsymbol{D1} - \boldsymbol{D2} = \boldsymbol{DB} \cdot \boldsymbol{EX} \cdot \boldsymbol{C}, \end{aligned} \tag{40}$$

where $\boldsymbol{EX}$ is a square diagonal matrix

$$\boldsymbol{EX}_{diag} = (1 + e^{-2k_j L}), \quad j = 1, 2, \ldots, nj, \tag{41}$$

and $(1 \times nj)$ row matrix $\boldsymbol{E}^T$ is defined as follows:

$$\boldsymbol{E}^T = \left(-\frac{1}{2d} \frac{1}{(k_j)^2} \sin k_j d \right), \quad j = 1, 2, \ldots, nj. \tag{42}$$

Each of the vector matrices $\boldsymbol{B}$ and $\boldsymbol{C}$ in equations (40) has nj elements. At the same time, equations (23) and (24) lead to the $(ne \times nj)$ matrix $\boldsymbol{EB}$:

$$\boldsymbol{EB}_{m,j} = -\frac{2}{d} \left[\begin{array}{ll} \dfrac{(-1)^m \sin k_j d}{(k_j)^2 - (k_m)^2} & \text{for} \quad k_j \neq k_m, \\ \dfrac{d}{2k_j} \quad \text{for} \quad k_j = k_m. \end{array} \right. \tag{43}$$

Finally, the $(nd \times nj)$ matrix $\boldsymbol{DB}$ is defined by the formula

$$\boldsymbol{DB}_{m,j} = -\frac{2}{h}\begin{cases} \dfrac{\sin k_j c}{(k_m^*)^2 - (k_j)^2} & \text{for} \quad k_j \neq k_m^*, \\ \dfrac{1}{2(k_j)^2}\Big(k_j h \cdot \cos k_j c - \sin k_j c\Big) & \text{for} \quad k_j = k_m^*. \end{cases} \tag{44}$$

The sum and difference of the two equations (40) lead to the following relations:

$$\begin{aligned} 2\boldsymbol{E1} &= \boldsymbol{EB} \cdot (\boldsymbol{B} + \boldsymbol{EX} \cdot \boldsymbol{C}), \\ 2\boldsymbol{E2} &= \boldsymbol{EB} \cdot (\boldsymbol{B} - \boldsymbol{EX} \cdot \boldsymbol{C}), \\ 2\boldsymbol{D1} &= \boldsymbol{DB} \cdot (\boldsymbol{B} + \boldsymbol{EX} \cdot \boldsymbol{C}), \\ 2\boldsymbol{D2} &= \boldsymbol{DB} \cdot (\boldsymbol{B} - \boldsymbol{EX} \cdot \boldsymbol{C}). \end{aligned} \tag{45}$$

On the other hand, equations (32) and (33) give

$$\begin{aligned} &\boldsymbol{B} = \boldsymbol{JA} \cdot \dot{\boldsymbol{A}} + \boldsymbol{JE} \cdot \boldsymbol{E1} + \boldsymbol{KE} \cdot \boldsymbol{E2} + \boldsymbol{JD} \cdot \boldsymbol{D1} + \boldsymbol{KD} \cdot \boldsymbol{D2}, \\ &\boldsymbol{EY} \cdot \boldsymbol{C} = -\Big(\boldsymbol{KA} \cdot \dot{\boldsymbol{A}} - \boldsymbol{JE} \cdot \boldsymbol{E1} + \boldsymbol{KE} \cdot \boldsymbol{E2} - \boldsymbol{JD} \cdot \boldsymbol{D1} + \boldsymbol{KD} \cdot \boldsymbol{D2}\Big), \end{aligned} \tag{46}$$

where $\boldsymbol{EY}$ is a square diagonal matrix

$$\boldsymbol{EY}_{diag} = \Big(1 - e^{-2k_j L}\Big), \quad j = 1, 2, \ldots, nj. \tag{47}$$

Elements of the square matrices $\boldsymbol{JA}$, $\boldsymbol{JE}$, $\boldsymbol{JD}$, $\boldsymbol{KA}$, $\boldsymbol{KE}$ and $\boldsymbol{KD}$ are described by equations (34–39), respectively. The sum and difference of the two equations (46) yield the following relations:

$$\begin{aligned} \boldsymbol{B} + \boldsymbol{EY} \cdot \boldsymbol{C} &= (\boldsymbol{JA} - \boldsymbol{KA}) \cdot \dot{\boldsymbol{A}} + 2\boldsymbol{JE} \cdot \boldsymbol{E1} + 2\boldsymbol{JD} \cdot \boldsymbol{D1}, \\ \boldsymbol{B} - \boldsymbol{EY} \cdot \boldsymbol{C} &= (\boldsymbol{JA} + \boldsymbol{KA}) \cdot \dot{\boldsymbol{A}} + 2\boldsymbol{KE} \cdot \boldsymbol{E2} + 2\boldsymbol{KD} \cdot \boldsymbol{D2}. \end{aligned} \tag{48}$$

Now, from substitution of equations (45) into relations (48) one obtains

$$\begin{aligned} (\boldsymbol{I} - \boldsymbol{JW}) \cdot \boldsymbol{B} + (\boldsymbol{EY} - \boldsymbol{JW} \cdot \boldsymbol{EX}) \cdot \boldsymbol{C} &= (\boldsymbol{JA} - \boldsymbol{KA}) \cdot \dot{\boldsymbol{A}}, \\ (\boldsymbol{I} - \boldsymbol{KW}) \cdot \boldsymbol{B} - (\boldsymbol{EY} - \boldsymbol{KW} \cdot \boldsymbol{EX}) \cdot \boldsymbol{C} &= (\boldsymbol{JA} + \boldsymbol{KA}) \cdot \dot{\boldsymbol{A}}, \end{aligned} \tag{49}$$

where $\boldsymbol{I}$ is the $(nj \times nj)$ unit diagonal matrix, and

$$\begin{aligned} \boldsymbol{JW} &= \boldsymbol{JE} \cdot \boldsymbol{EB} + \boldsymbol{JD} \cdot \boldsymbol{DB}, \\ \boldsymbol{KW} &= \boldsymbol{KE} \cdot \boldsymbol{EB} + \boldsymbol{KD} \cdot \boldsymbol{DB}. \end{aligned} \tag{50}$$

The solutions of equations (49) may be expressed in the following form:

$$\begin{aligned} \boldsymbol{B} &= \boldsymbol{BA} \cdot \dot{\boldsymbol{A}}, \\ \boldsymbol{C} &= \boldsymbol{CA} \cdot \dot{\boldsymbol{A}}. \end{aligned} \tag{51}$$

Substitution of these relations into equations (45) gives

$$
\begin{aligned}
&E_0 = \boldsymbol{E}^T \cdot (\boldsymbol{BA} + \boldsymbol{EX} \cdot \boldsymbol{CA}) \cdot \dot{\boldsymbol{A}}, \\
&\boldsymbol{E1} = \frac{1}{2}\boldsymbol{EB} \cdot (\boldsymbol{BA} + \boldsymbol{EX} \cdot \boldsymbol{CA}) \cdot \dot{\boldsymbol{A}}, \quad \boldsymbol{E2} = \frac{1}{2}\boldsymbol{EB} \cdot (\boldsymbol{BA} - \boldsymbol{EX} \cdot \boldsymbol{CA}) \cdot \dot{\boldsymbol{A}}, \\
&\boldsymbol{D1} = \frac{1}{2}\boldsymbol{DB} \cdot (\boldsymbol{BA} + \boldsymbol{EX} \cdot \boldsymbol{CA}) \cdot \dot{\boldsymbol{A}}, \quad \boldsymbol{D2} = \frac{1}{2}\boldsymbol{DB} \cdot (\boldsymbol{BA} - \boldsymbol{EX} \cdot \boldsymbol{CA}) \cdot \dot{\boldsymbol{A}}.
\end{aligned}
\tag{52}
$$

With the above relations, all the unknown constants (parameters) of the problem considered are expressed in terms of independent parameters that correspond to the velocity of plate deflection. It is important to note that none of the matrices in these equations, i.e. $\boldsymbol{E}^T$, $\boldsymbol{EB}$, $\boldsymbol{DB}$, $\boldsymbol{EX}$, $\boldsymbol{BA}$, or $\boldsymbol{CA}$, depends on time.

4. Propagating Wave Solution

The solution presented in the preceding section corresponds to standing waves (the pressure at the free surface at $z = H$ is zero) associated with the motion of the plate. The second part of our solution (equation 15) corresponds to propagating waves generated by the piston-type generator (the rigid vertical wall OA in Fig. 1). In constructing a solution for the second part of the potential function, i.e. for Φ, a coupling of the two potentials at the free fluid surface should be taken into account (compare with the first equation 16). In the case of a small gap between the plate and the bottom, however, one may construct an approximate solution based on the assumption that $\partial\phi/\partial z|_{z=H}$ in that equation is so small that it may be neglected. Nevertheless, in what follows, a general case is considered in which this term is taken into account. On the other hand, as compared to problem formulation for the potential ϕ, in constructing a solution for Φ, a simply connected fluid domain is considered, which consists of a semi-infinite layer of fluid with a finite step in place of the plate. In a formal way, the fluid domain may be divided into three parts: two rectangular finite domains and a semi-infinite one, and the solution for the potential Φ may be expressed in the form of a sum of separate solutions constructed in these sub-domains. As compared to the solution for the potential ϕ, however, a time-dependent solution for the potential Φ becomes more complicated (the second time derivative of the potential emerges in the boundary condition at the free fluid surface). Therefore, in order to simplify the description of the problem considered, it is reasonable to resort to a discrete formulation for the second part of the potential function by the finite difference method (FDM). At the same time, continuous time is also replaced by a countable set of discrete time steps (time points). In the FD formulation, the continuous fluid domain is represented by a set of nodal points of the assumed grid. Thus, with respect to the geometry of the fluid domain, a rectangular spacing of points is considered in which Δx denotes the spacing of vertical lines and Δz is the spacing of horizontal lines. For a typical nodal point (i, j) in the fluid area, where i denotes a vertical line, and j denotes a horizontal one, the FD analogue of Laplace's equation for the velocity potential $\Phi(x, z, t)$ is

$$-\Phi_{i-1,j} - \varepsilon\ \Phi_{i,j-1} + K\Phi_{i,j} - \varepsilon\ \Phi_{i,j+1} - \Phi_{i+1,j} = 0, \tag{53}$$

where

$$\varepsilon = \left(\frac{\Delta x}{\Delta z}\right)^2, \quad K = 2(1+\varepsilon). \tag{54}$$

Equation (53) is used for all nodal points of the fluid domain, including boundary points. It is necessary to add, however, that with this discrete method, only a finite number of points may be taken into account, and thus, instead of the infinite fluid domain, only a finite part of the domain may be considered. And thus, instead of the infinite fluid layer in Fig. 1, a finite part of the layer, with a boundary at $x = L_R$, is taken into account. In this approach, it is necessary to formulate a transmitting boundary condition for this boundary. In a simple case of the harmonic monochromatic wave, this condition may be written in the following form:

$$\frac{\partial \Phi}{\partial t} + c_w \frac{\partial \Phi}{\partial x}\bigg|_{x=x_R} = 0, \tag{55}$$

where $c_w = \omega/k$ (with $\omega^2 = gk \tanh kh$) is the wave celerity and k is the wave number.

In the initial value problem considered here, the local wave number and wave velocity depend on time, and therefore, it is not possible to formulate the corresponding boundary condition in a closed analytical form. To this end, it is necessary to resort to a specific approximation dependent on the given problem. For instance, in the discrete integration of the problem in time, the associated transmitting condition may be subsequently modified at each time step. On the other hand, for a sufficiently large $x = L_R$ (the artificial boundary is assumed far off the generator-plate system) and a relatively small elapse of time from the start of wave generation (until then the system is at rest), it is reasonable to assume that the fluid at $x = L_R$ is at rest, and thus, in the numerical integration of the problem in time, it is admissible to assume that $\partial\Phi/\partial x|_{x=L_R} \approx 0$. This approach will be employed in our further discussion of the discrete integration of the problem equations in time. Accordingly, with respect to the above, the problem is reduced to unknown vales of the potential at nodal points of the finite fluid domain (interior and boundary points of the fluid domain). A solution of the system of FDM equations for these unknown values of the potential $\Phi_{i,j}$ depends not only on the generator amplitude, but also on the unknown potential $\phi(x, z)$, which in turn depends on the set of parameters $\dot{A}_n$ $(n = 1, 2, \cdots, na)$. In this way, the problem is transformed into a "new" coupled problem in which the solution for the potential $\Phi_{i,j}$ depends on an unknown set of parameters $\dot{A}_n$. Therefore, in order to find the solution of this coupled problem, a two-step procedure is employed. In the first step, the solution to the potential $\Phi(x, z)$ is expressed in the form of a linear combination of solutions, corresponding to unknown parameters $\dot{A}_n$ $(n = 1, 2, \cdots, na)$. And then, in the second step, these parameters are obtained by solving the plate equation. This, in turn, allows us to calculate the deflection of the plate, as well as the pressure field and the free-surface elevation.

With respect to the above, let us now consider the discrete formulation of the problem. Since the boundary condition at the free surface and the equation of the plate motion contain the time derivatives of the potential and deflection of the plate, it is necessary to implement a time integration of these functions. To this end, a discrete time integration by the Wilson θ method is employed, which is unconditionally stable. This method makes it possible to write FD equations for the unknown dependent values of the problem (the potential function and plate deflection) at a single level of time. It is based on a linear approximation of the second time derivative of a dependent variable at points of discrete time. To make our further discussion clear, some of the fundamental equations of the method are summarized here (for details see Bathe 1982). Assuming, that the solution of the problem is known at the moment t, i.e. $\Phi(t) = \Phi_{(1)},\ \dot{\Phi} = \dot{\Phi}_{(1)}$ and $\ddot{\Phi} = \ddot{\Phi}_{(1)}$, the standard equations of the method are

$$\ddot{\Phi}(\tau) = \ddot{\Phi}_{(1)} + \frac{\tau}{DT}\left(\ddot{\Phi}_{(3)} - \ddot{\Phi}_{(1)}\right), \tag{56}$$

and

$$\begin{aligned} \dot{\Phi}(\tau) &= \dot{\Phi}_{(1)} + \tau\ \ddot{\Phi}_{(1)} + \frac{\tau^2}{2DT}\left(\ddot{\Phi}_{(3)} - \ddot{\Phi}_{(1)}\right), \\ \Phi(\tau) &= \Phi_{(1)} + \tau\ \dot{\Phi}_{(1)} + \frac{\tau^2}{2}\ddot{\Phi}_{(1)} + \frac{\tau^3}{6DT}\left(\ddot{\Phi}_{(3)} - \ddot{\Phi}_{(1)}\right), \end{aligned} \tag{57}$$

where $\Phi_{(3)} = \Phi(t + DT),\ DT = \theta \cdot \Delta t,\ \theta = 1.47$, and $\tau$is measured from the first point ($\tau = 0$ for $\phi_{(1)}$), with Δt being the time step.
From these equations, one obtains

$$\begin{aligned} \dot{\Phi}_{(3)} &= \frac{3}{DT}\Phi_{(3)} - \frac{3}{DT}\left(\Phi_{(1)} + \frac{2}{3}DT\dot{\Phi}_{(1)} + \frac{DT^2}{6}\ddot{\Phi}_{(1)}\right) = \frac{3}{DT}\Phi_{(3)} - \frac{3}{DT}R^1_{(1)}, \\ \ddot{\Phi}_{(3)} &= \frac{6}{DT^2}\Phi_{(3)} - \frac{6}{DT^2}\left(\Phi_{(1)} + DT\dot{\Phi}_{(1)} + \frac{DT^2}{3}\ddot{\Phi}_{(1)}\right) = \frac{6}{DT^2}\Phi_{(3)} - \frac{6}{DT^2}R^2_{(1)}. \end{aligned} \tag{58}$$

In this procedure, all equations of the problem, together with boundary conditions, are written at a common level of time, i.e. at the time $t_{(3)} = t_{(1)} + DT$. Knowing the solution at this level of time, one can calculate the solution at the time level $t_{(2)} = t_{(1)} + \Delta t$:

$$\begin{aligned} \ddot{\Phi}(t + \Delta t) &= \ddot{\Phi}_{(1)} + \frac{\Delta t}{DT}\left(\ddot{\Phi}_{(3)} - \ddot{\Phi}_{(1)}\right), \\ \dot{\Phi}(t + \Delta t) &= \dot{\Phi}_{(1)} + \ddot{\Phi}_{(1)}\Delta t + \frac{(\Delta t)^2}{2DT}\left(\ddot{\Phi}_{(3)} - \ddot{\Phi}_{(1)}\right), \\ \Phi(t + \Delta t) &= \Phi_{(1)} + \dot{\Phi}_{(1)}\Delta t + \frac{(\Delta t)^2}{2}\ddot{\Phi}_{(1)} + \frac{(\Delta t)^3}{6DT}\left(\ddot{\Phi}_{(3)} - \ddot{\Phi}_{(1)}\right). \end{aligned} \tag{59}$$

For a nodal point 'N' at the free surface of the fluid (at $z = H$), the finite difference analogue of the first relation in equations (16) is

$$\frac{6}{DT^2}\Phi_N - \frac{6}{DT^2}R^2_{(1)} + \frac{g}{2b}\left(\Phi^+ - \Phi_{N-1}\right) + g\frac{\partial\phi}{\partial z}\bigg|_{z=h} = 0, \tag{60}$$

where $\Phi^+ = \Phi(H + \Delta z)$, and $\Phi_{N-1} = \Phi(H - \Delta z)$.
From the latter equation, it follows that

$$\Phi^+ = -\frac{12\Delta z}{gDT^2}\Phi_N + \frac{12\Delta z}{gDT^2}R^2_{(1)} + \Phi_{N-1} - 2\Delta z\,\frac{\partial\phi}{\partial z}\bigg|_{z=h}. \tag{61}$$

Substitution of this relation into equation (53) gives

$$-\Phi^L_N - 2\varepsilon\Phi_{N-1} + K^*\Phi_N - \Phi^R_N = \frac{12\Delta z\varepsilon}{gDT^2}R^2_{(1)} - 2\Delta z\varepsilon\,\frac{\partial\phi}{\partial z}\bigg|_{z=h}, \tag{62}$$

where

$$K^* = K + \frac{12\varepsilon\Delta z}{gDT^2}. \tag{63}$$

A remark is needed. The right-hand side of equation (62) contains the known value of $R^2_{(1)}$ at the moment $t_{(1)}$of time and the value of $\partial\phi/\partial z|_{z=h}$ at the level $t_{(3)} = t_{(1)} + DT$. The latter results from the explicit formulae

$$\frac{\partial\phi_{(3)}}{\partial z}\bigg|_{z=h} = \begin{cases} \displaystyle\sum_{j=1}^{nj}(-1)^{j+1}C_j(t_{(3)})\left(e^{-k_jx} + e^{-k_j(2L-x)}\right) \qquad \text{for} \quad 0 \le x \le L, \\ \displaystyle 2\sum_{n=1}^{na}\dot{A}_n(t_{(3)})\frac{1}{w_n}\exp(-r_nh)\sin r_n(x+b) + \\ \displaystyle +\sum_{m=1}^{nd}(-1)^m k_m\left[D^1_m(t_{(3)})\frac{\cosh(k_mx)}{\cosh(k_mh)} + D^2_m(t_{(3)})\frac{\sinh(k_mx)}{\sinh(k_mh)}\right] \\ \quad \text{for} \quad -b \le x \le b, \\ \text{and} \\ \displaystyle\sum_{j=1}^{nj}(-1)^{j+1}B_j(t_{(3)})\exp(-k_jx) \qquad \text{for} \quad 0 \le x \le \infty. \end{cases} \tag{64}$$

In the finite difference formulation, equations (53) are written for all nodal points, representing the finite fluid domain. For points of the free surface, relations (62) hold. In this way, the final system of nw algebraic equations for nodal values of the potential $\Phi_{(3)}(x_i, z_j) = \Phi_{(3)}(k = 1, 2, \ldots, nw)$ may be written in the following matrix form:

$$[\boldsymbol{RM}]\cdot(\Phi_{(3)}) = (\boldsymbol{PA}_{(3)}) + (\boldsymbol{PB}_{(1)}) + (\boldsymbol{PC}_{(3)}), \tag{65}$$

where $[\boldsymbol{RM}]$ is the $(nw \times nw)$ matrix of equations, $(\boldsymbol{PA}_{(3)})$ denotes the $(nw \times 1)$ vector associated with the generator velocity $\dot{x}_g(t_{(3)})$, $(\boldsymbol{PB}_{(1)})$ is the vector dependent on $R^2_{(1)}$, i.e. it depends on the known solution for points of the free surface at $t = t_{(1)}$, and $(\boldsymbol{PC}_{(3)})$ is calculated at points of the free surface by equations (64). Since the constants B_j, C_j and D^1_m, D^2_m in equation (64) depend on $\dot{A}_n$ (compare with equations (52)), the third vector in equation (65) may be written in the following form:

$$(\boldsymbol{PC}_{(3)}) = -2\Delta z \varepsilon \left[\left. \frac{\partial \phi}{\partial z} \right|_{z=h} \right] \cdot \left(\dot{\boldsymbol{A}}_{(3)} \right) = [\boldsymbol{F3}] \cdot \left(\dot{\boldsymbol{A}}_{(3)} \right), \tag{66}$$

where $[\boldsymbol{F3}]$ is the $(nw \times na)$ square matrix.

We are dealing with a linear problem, and therefore the solution of equation (65) is the sum of solutions corresponding to individual components of the right-hand side of this equation. And thus, we have

$$(\Phi_{(3)}) = \dot{x}_{g(3)} \cdot (\boldsymbol{FA}) + [\boldsymbol{FC}] \cdot \left(\dot{\boldsymbol{A}}_{(3)} \right) + (\boldsymbol{FB}_{(1)}). \tag{67}$$

The matrices $(\boldsymbol{FA})$ and $[\boldsymbol{FC}]$ in this equation do not depend on time. The matrix $\boldsymbol{FB}_{(1)}$ in this relation depends on time. It is defined by the known solution of the problem at the previous level of time, i.e. at $t = t_{(1)}$ (equation (67) corresponds to $t = t_{(3)} = t_{(1)} + DT$). The above equation demonstrates the main feature of the linear problem considered here, which is that the potential Φ may be treated as the sum of the three independent potentials:

$$\Phi_{(3)} = \Phi_{(3)}(\dot{x}_{g(3)}) + \Phi_{(3)} \left(\dot{\boldsymbol{A}}_{(3)} \right) + \Phi_{(3)} \left(R^2_{(1)} \right), \tag{68}$$

where arguments entering the right-hand side terms in this relation mean that the component potentials depend on their respective arguments.In order to find a final solution of this equation, it is necessary to resort to equation (3), describing the plate motion. Since the latter equation contains the fluid pressure (at the upper and lower surfaces of the plate), we have to complete our description by calculating these pressure fields. The pressure at the upper surface of the plate (at $z = c$ in Fig. 1) depends on the potentials Φ and ϕ. The pressure at the lower surface of this plate (at $z = d$ in Fig. 1) depends on the known potential φ (equation (20)) and the additional potential $\varphi_{add.}$, unknown at this moment. The latter should match the potential Φ at the boundaries (1–2) and (5–6) (see Fig. 1).

5. Additional Velocity Potential Beneath the Plate

With respect to the discussion in the preceding sections, let us consider now the solution problem for the potential $\varphi_{add.}(x, z, t)$, satisfying Laplace's equation within the rectangular fluid domain $(1, 5, 6, 2)$, and the following boundary conditions:

$$\left. \frac{\partial \varphi_{add.}}{\partial x} \right|_{(1-2)} = \left. \frac{\partial \varphi_{add.}}{\partial x} \right|_{(5-6)} = 0, \quad \left. \frac{\partial \varphi_{add.}}{\partial z} \right|_{(1-5)} = \left. \frac{\partial \varphi_{add.}}{\partial x} \right|_{(6-2)} = 0, \tag{69}$$

and

$$\varphi_{add.}|_{(1-2)} = \Phi|_{(1-2)}, \qquad \varphi_{add.}|_{(5-6)} = \Phi|_{(5-6)}. \tag{70}$$

Unfortunately, for these boundary conditions, there is no analytical solution of Laplace's equation obtained by the method of separation of variables. Therefore, in order to obtain a solution of this problem, we resort to the discrete FDM. As in the case of the potential Φ in the "external" fluid domain, equations of the form described by equations (53) are written for all nodal points of the assumed grid in the rectangular fluid domain $(1, 5, 6, 2$ in Fig. 1). The solution of these equations is expressed in the following form:

$$(\varphi_{add.}) = [\boldsymbol{FI}]\,(\Phi), \tag{71}$$

where $[\boldsymbol{FI}]$ is a rectangular matrix, and (Φ) is a vector matrix containing values of Φ at nodal points at boundaries (1–2) and (5–6). The values are taken from equation (63). With this formula, one may calculate the fluid pressure and solve the plate equation.

6. Equation of the Plate Vibrating in Fluid

For small deflections of the plate, its motion is described by equation (3). Knowing the potential functions in the respective fluid domains (at the upper and lower surfaces of the plate) and substituting equation (4) into the plate equation, one obtains

$$\beta\frac{\partial^2 w}{\partial t^2} + \frac{\partial^4 w}{\partial x^4} = \gamma\left[\frac{\partial \phi}{\partial t} + \frac{\partial \Phi}{\partial t} - \left(\frac{\partial \varphi}{\partial t} + \frac{\partial \varphi_{add.}}{\partial t}\right)\right], \tag{72}$$

where

$$\beta = \frac{m_{pl.}}{D^*}, \quad \gamma = \frac{\rho}{D^*}. \tag{73}$$

As mentioned above, the plate deflection may be expressed in terms of eigenfunctions of the plate vibrating in air. Thus, it is described by the following formula:

$$w(x,t) = \sum_n A_n(t)\sin r_n(x+b), \quad r_n = \frac{n\pi}{2b}, \quad n = 1, 2, \ldots\,. \tag{74}$$

Substitution of this relation and equations (19) and (20) into equation (72) and simple manipulations, give

$$\begin{aligned}
&\beta \sum_{n=1} \ddot{A}_n \sin r_n(x+b) + \sum_{n=1} A_n (r_n)^4 \sin r_n(x+b) = \\
&\quad = \gamma \Bigg\{ - \sum_{n=1} \ddot{A}_n(t) \frac{1}{r_n} \tanh r_n h \cdot \sin r_n(x+b) + \\
&\quad + \sum_{m=1} \left[\dot{D}^1_m \frac{\cosh(k^*_m x)}{\cosh(k^*_m b)} + \dot{D}^2_m \frac{\sinh(k^*_m x)}{\sinh(k^*_m b)} \right] + \\
&\quad - \sum_{n=1} \ddot{A}_n(t) \frac{1}{r_n} \frac{1}{\tanh r_n d} \cdot \sin r_n(x+b) - \dot{E}_0 + \\
&\quad - \sum_{m=1} (-1)^m \left[\dot{E}^1_m \frac{\cosh(k_m x)}{\cosh(k_m b)} + \dot{D}^2_m \frac{\sinh(k_m x)}{\sinh(k_m b)} \right] + \left(\dot{\Phi} - \dot{\varphi}_{add.} \right)_{plate} \Bigg\}, \\
&\quad r_n = \frac{n\pi}{2b}, \quad k_m = \frac{m\pi}{d}, \quad k^*_m = \frac{2m-1}{2h}\pi, \quad m = 1,2,\ldots, \quad n = 1,2,\ldots\, .
\end{aligned} \tag{75}$$

The partial time derivative of the potential $\varphi_{add.}$ in this equation is obtained by differentiation of equation (71):

$$(\dot{\varphi}_{add.}) = [\boldsymbol{FI}] \left(\dot{\Phi} \right). \tag{76}$$

At the same time, equation (67) is used to calculate the partial time derivative of the "external" potential Φ. Differentiation of this equation gives

$$\left(\dot{\Phi}_{(3)} \right) = \ddot{x}_{g(3)} \cdot (\boldsymbol{F}A) + [\boldsymbol{F}C] \cdot \left(\ddot{\boldsymbol{A}}_{(3)} \right) + \frac{\partial}{\partial t} (\boldsymbol{F}B_{(1)}). \tag{77}$$

In calculating the third term on the right-hand side of this equation, we may use relation (68) and the first formula in equations (58). Thus, in a formal way, one obtains

$$\frac{\partial}{\partial t} (\boldsymbol{FB}_{(1)}) = \dot{\Phi}_{(3)}(R^2_{(1)}) = \frac{3}{DT} \left[\Phi_{(3)}(R^2_{(1)}) - R^1_{(1)} \right]. \tag{78}$$

The remaining parameters entering equation (75), i.e. $\dot{D}^1_m, \dot{D}^2_m$ $(m = 1, 2\ldots)$ and $\dot{E}_0, \dot{E}^1_m, \dot{E}^2_m$ $(m = 1, 2, \ldots)$, may be easily obtained from the vector formulae (52). To this end, it is sufficient to introduce $\ddot{\boldsymbol{A}}$ in place of $\dot{\boldsymbol{A}}$ in the latter equations. With such manipulations, equation (75), describing the plate motion, is reduced to an equation containing only one set of parameters, namely $A_n(t)$ $(n = 1, 2, \ldots, na)$ together with their second time derivatives. Multiplication of this equation in succession by $\sin r_n(x + b)$, $n = 1, 2, \ldots, na$ and then integration of the ensuing result in the range $(-b \le x \le b)$ leads to the algebraic system of equations

$$[\boldsymbol{MA}] \cdot \left(\ddot{\boldsymbol{A}}_{(3)} \right) + [\boldsymbol{MB}] \cdot (\boldsymbol{A}_{(3)}) = (\boldsymbol{P}). \tag{79}$$

With respect to the second formula in equations (58), the second time derivative in this equation is expressed in the following form:

$$\left(\ddot{\boldsymbol{A}}_{(3)}\right) = \frac{6}{DT^2}\left(\boldsymbol{A}_{(3)}\right) - \frac{6}{DT^2}\left(\boldsymbol{A}_{(1)} + DT \cdot \dot{\boldsymbol{A}}_{(1)} + \frac{DT^2}{3}\ddot{\boldsymbol{A}}_{(1)}\right). \tag{80}$$

Substitution of this relation into equation (79) gives

$$[\boldsymbol{MA}^*] \cdot (\boldsymbol{A}_{(3)}) = (\boldsymbol{P}^*)\,, \tag{81}$$

where the matrices $[\boldsymbol{MA}^*]$ and $(\boldsymbol{P}^*)$ depend on the known solutions for $\boldsymbol{A}_n(t)$ and $\Phi(t)$ at the previous moment of time, i.e. at $t = t_{(1)}$. Substitution of the solution $\boldsymbol{A}_n(t = t_{(3)})$ of the above equation into equations (59) allows us to calculate a solution corresponding to the subsequent moment of time (at $t = t_{(2)} = t_{(1)} + \Delta t$). Knowing the solution for $\boldsymbol{A}_n(t = t_{(2)})$, it is a simple task to calculate the relevant potentials and thus the pressure and velocity field in the entire fluid domain. At the same time, a deflection of the plate may be obtained. In this way, it is possible to find solutions of the problem at subsequent points in time.

7. Numerical Experiments

In order to learn more about the time-dependent problem considered, and to illustrate procedures developed in the preceding sections, some numerical examples are presented below. The solution of the problem is applied to specific cases of water wave generation in fluid of constant depth. The problem considered corresponds directly to the generation of waves in a laboratory flume of finite length. For such a finite fluid domain and a limited elapse of time measured from the starting point, the solution in the finite fluid domain mimics the solution in the infinite fluid domain. Thus, let us consider wave generation in a fluid domain by a piston-type generator, which starts to move at a certain point in time. Three cases are considered, corresponding to surface waves of lengths $\lambda = 1.5$ m, 2.0 m and 2.5 m. For all the cases, the initial depth H of the fluid layer is the same and equal to 0.6 m. The submerged plate is placed at a distance of $L = 2.0$ m from the generator face. In each of the three cases, the same plate is installed at a specific distance from the fluid bottom. This steel plate of thickness $\delta = 0.004$ m and span $2b = 1.0$ m is simply supported at its ends. In order to assess the influence of the gap between the plate and the fluid bottom on the dynamics of the plate, a set of the gap widths is considered, which correspond to $c = 0.02, 0.03, \cdots, 0.11$ m in Fig. 1. An artificial boundary, which makes it possible to confine our attention to a finite fluid domain, is assumed at a distance of 11.0 m from the generator face. Waves reflected from this boundary are delayed in relation to the start of wave generation (generator motion). And therefore, for the elapse of time not exceeding ten seconds, measured from this starting point, the reflected waves do not disturb the plate motion (this elapse of time is too short for the reflected waves to travel that distance).

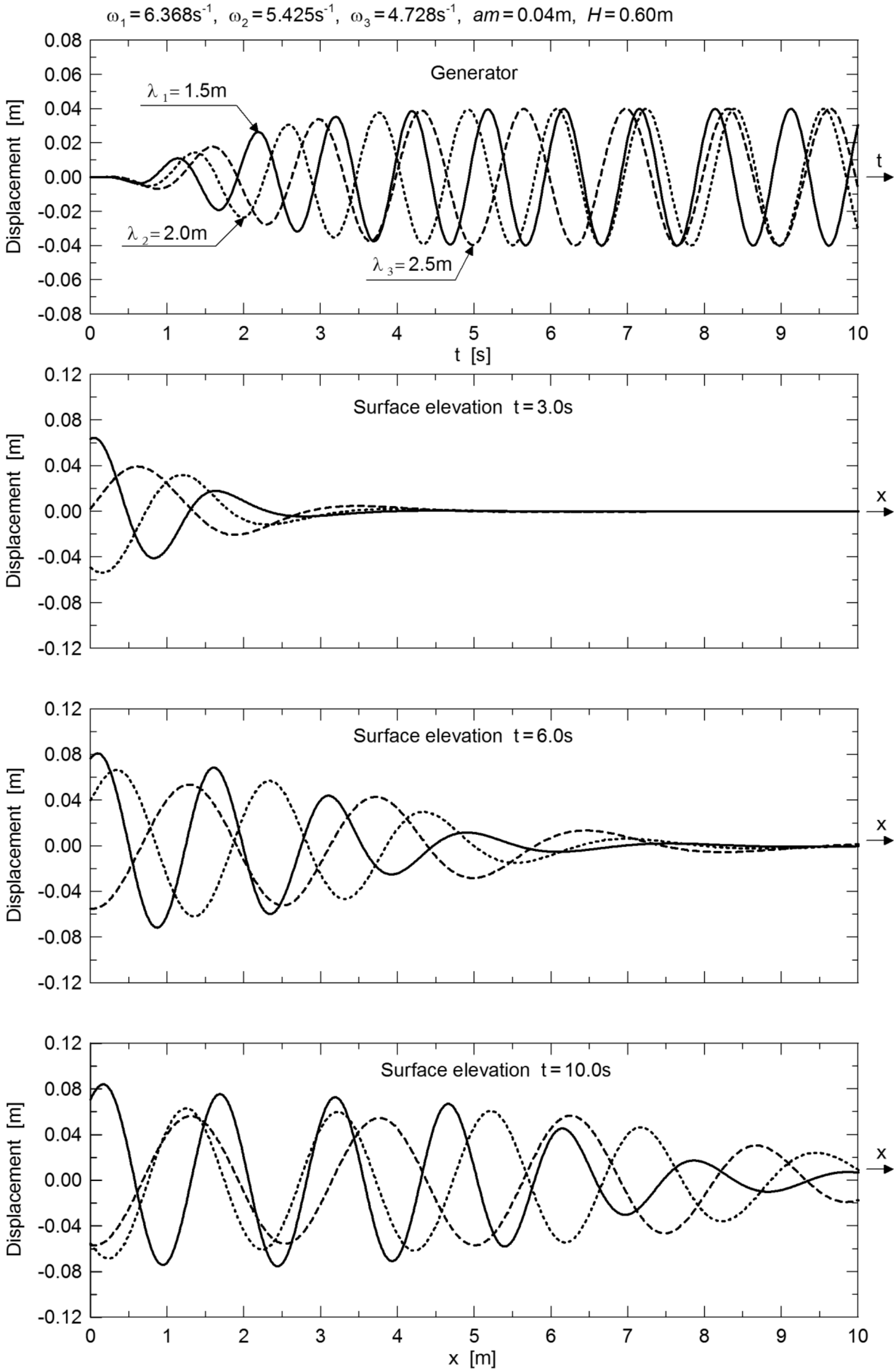

Fig. 2. Surface elevations at selected points in time

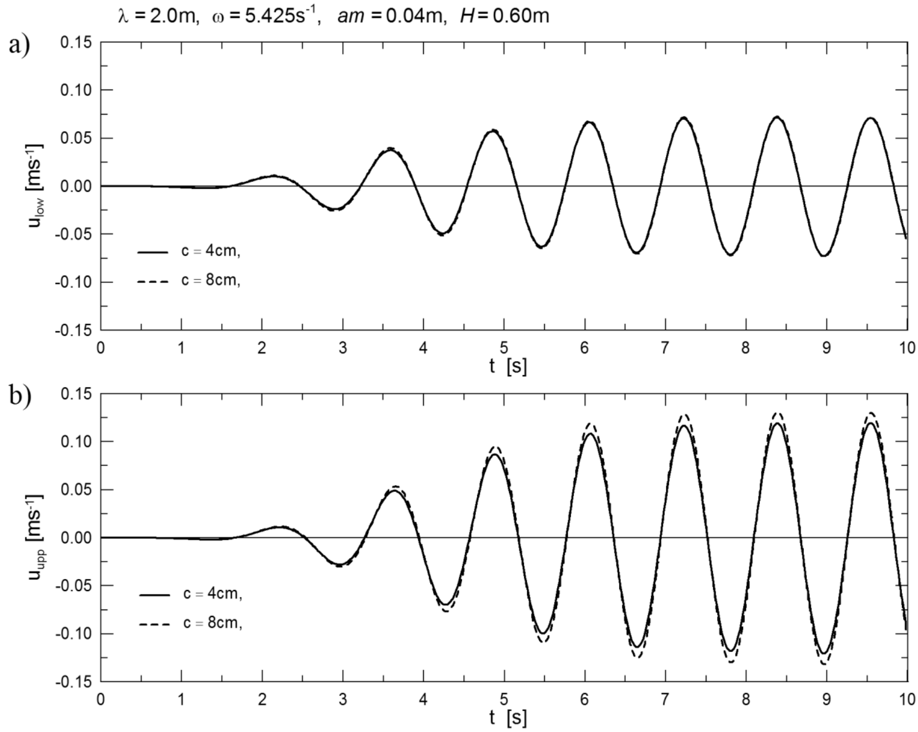

Fig. 3. Time distribution of the horizontal components of the fluid velocity at the lower (a) and upper (b) mid-span points of the plate

This feature may be observed in Fig. 2, where the plots represent the free surface elevation at selected points in time.

From these plots, it may be seen that the procedures described in the preceding sections leads to plausible results. Since the plate is placed relatively far off the free surface, and since its deflections are small, the free surface elevation is practically unaffected by the plate motion. In contrast, the width of the gap between the plate and the fluid bottom is important in calculating velocity and pressure fields in the vicinity of the plate, as well as the deflection of the plate. All these quantities depend on time and space, and therefore, in illustrating numerical results, we confine our attention to points in space corresponding to the mid-span point of the plate, which seem representative of the problem discussed. And therefore, in the first step, the horizontal components of the fluid velocity are calculated at the lower and upper mid-span points of the plate. The results obtained are illustrated in Fig. 3, where the plots show the distribution of these components in time for two gap widths (c = 0.04 m and c = 0.08 m in Fig. 1) and the generator frequency corresponding to the surface wave of length λ = 2.0 m.

Similar distributions hold for the remaining waves, i.e. for λ = 1.5 m and λ = 2.5 m. From these plots, it may be seen that there is a relatively big difference between velocities at points of the lower and upper surfaces of the plate. Another important characteristic of the problem is the fluid pressure at points of the plate surfaces. Since we are dealing with different widths of the gap between the plate and the fluid bottom, it is reasonable to normalize pressure at the lower and upper mid-span points of the plate by means of the hydrostatic pressure of the fluid at rest at the central point of the plate. Hence, the normalized pressure is

$$p(t) = p_{norm.} = \frac{1}{p_{stat}} p_{cal.} = \frac{1}{\rho g(H - z_{pl})} p_{cal.} \tag{82}$$

where $p_{cal.}$ denotes the pressure described by equation (4) and $z_{pl} = d + \delta/2$ (see Fig. 1).

For the two gap widths, the normalized pressure distribution in time is shown in Fig. 4. The plots in Fig. 4a and Fig. 4b show the pressure distribution at the lower and upper mid-span points of the plate, respectively. The resultant pressure force acting on the plate is shown in Fig. 4c, where the plots show the difference in the fluid pressure corresponding to the upper and lower points of the plate. It is important to note that, in the discussed cases, the dynamic pressure may exceed the hydrostatic one by 30%. From the plots in this figure, it is seen that the pressure load on the plate depends on the width of the gap between the plate and the fluid bottom. In order to illustrate the solution obtained, the deflection of the mid-span point of the plate is also calculated. The plots in Fig. 5 show the time distribution of the vertical displacement of the central mid-span point of the plate corresponding to selected gap widths.

The plots in Figures 3, 4, and 5 represent the time distribution of the above-mentioned parameters, i.e. the horizontal velocity, the pressure and the vertical displacement of the mid-span point of the plate. In order to compare solutions for selected gap widths, it is reasonable to resort to a single number which can characterize an important feature of the quantity distributed in time. Since the distributions correspond to a finite elapse of time, the maximum value of the given quantity within this range of time is chosen as representative of this quantity. And thus, such numbers are calculated for the resultant of fluid pressure and the displacement associated with the mid-span point of the plate. In order to obtain a better insight into the solution, the mid-span plate displacement is normalized by means of a static deflection corresponding to the own weight of the plate. Thus, the following relation is assumed:

$$w(t) = w_{norm.} = \frac{1}{w_{stat}} w_{cal.} = \frac{384 D^*}{5 q l^4} w_{cal.}, \tag{83}$$

where $q = g(\rho_s - \rho) \cdot \delta$ and $l = 1.0$ m.

Some results of computations are shown in Fig. 6 , where the maximum values of the pressure resultant and displacements, corresponding to the central point of the plate, are marked versus the gap widths.

From these graphs, it may be seen that, for a specific wave generation (length of water wave), one may expect significant changes in pressure forces, and thus in plate

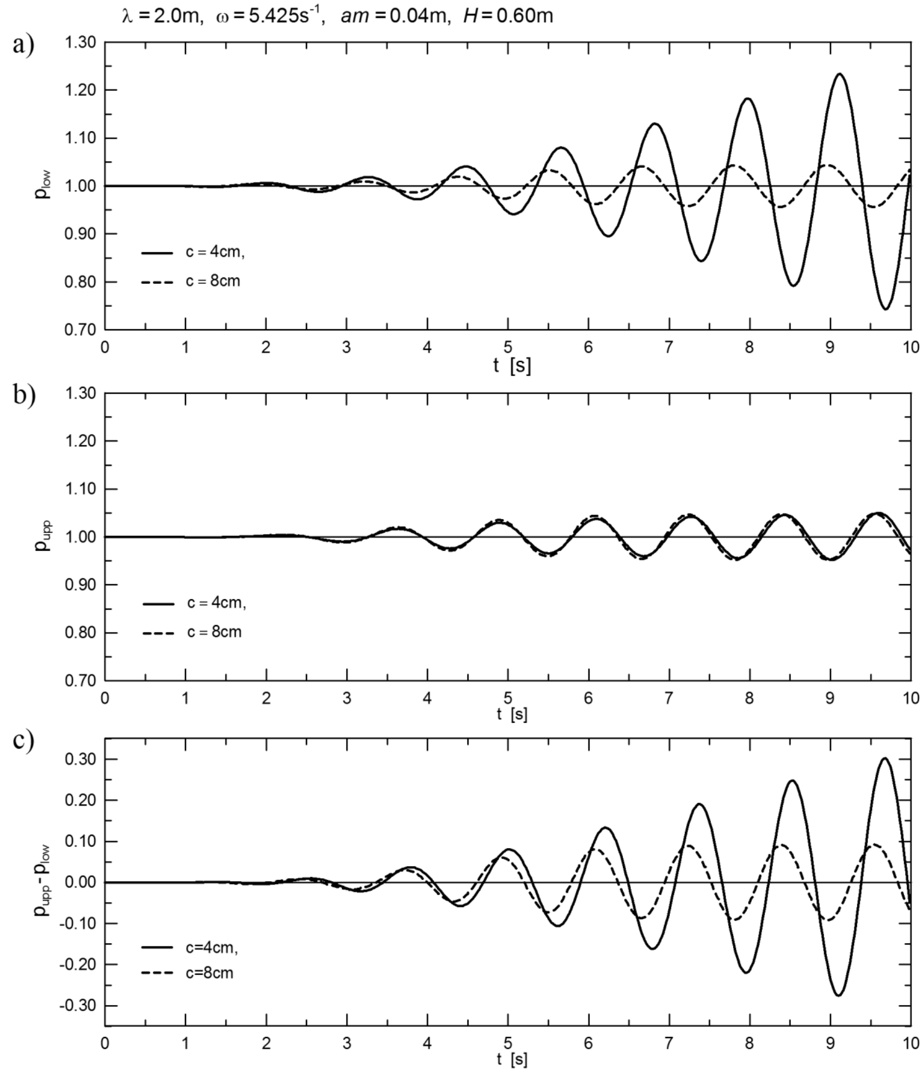

Fig. 4. Time distribution of the fluid pressure at the lower (a), upper (b), and mid-span points (c) of the plate and the resultant of pressure at these points

deflections, associated with a specific width of the gap between the plate and the fluid bottom.

8. Concluding Remarks

The formulation developed in this paper makes it possible to calculate pressure forces loading a horizontal elastic band plate submerged in fluid of constant depth. These forces result from gravitational waves propagating within the fluid and disturbances in fluid velocity related to the vibrations of the plate. With the standing wave solution,

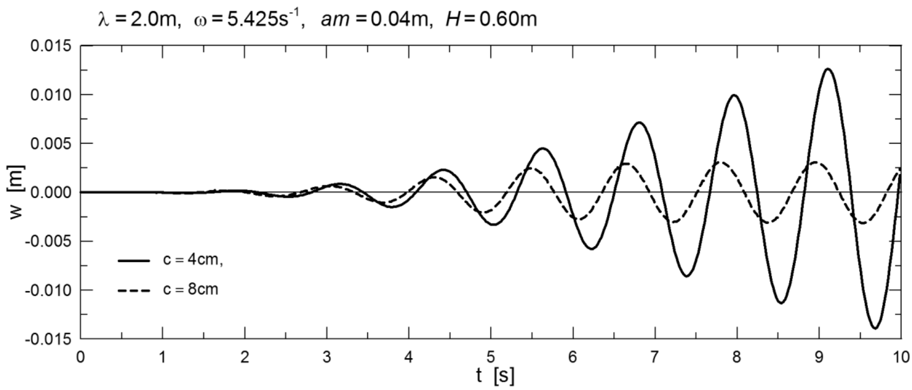

Fig. 5. Time distribution of plate deflection at the mid-span central point of the plate

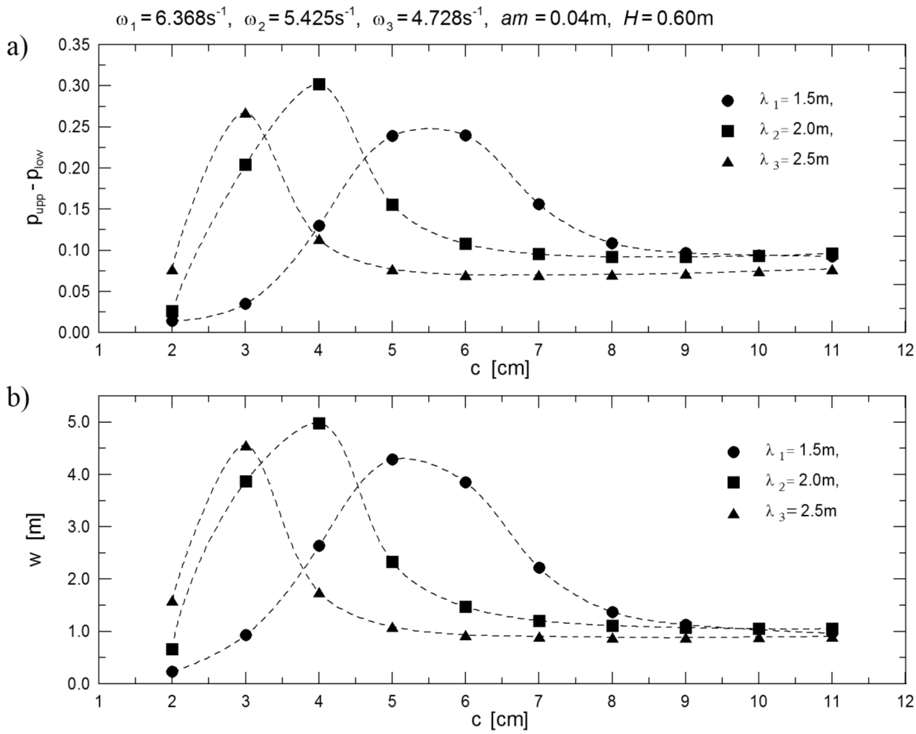

Fig. 6. Maximum resultant of the normalized pressure (a) and the normalized deflection (b) of the plate at its mid-span versus the gap width

developed in the third section of this study, it is possible to calculate a set of eigenfrequencies of this plate vibrating in fluid. Compared to vibrations of the plate in air, one can expect a significant reduction in these frequencies due to the co-vibrating mass of fluid. However, the most important result of investigations developed above is an assessment of the influence of the gap width on fluid forces loading this plate, and thus on its deflection. It was shown that, for a specific problem (length of the surface wave, span of the plate, and width of the gap between the plate and the fluid bottom), there exists a gap width which leads to extreme values of the pressure loading this plate and plate deflection. This theoretical result may be important in constructing plate-like engineering structures installed in offshore zones of seas.

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