



Assessing surface fatigue crack growth in railway wheelset axle

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ABSTRACT

Purpose: The aim of the proposed research is to create a calculation model of surface fatigue crack growth at the axle of railway wheelset working under operational loads.

Design/methodology/approach: The energy approach of the fracture mechanics was used to formulate the calculation model of fatigue crack propagation at the wheelset axle surface. The method of least squares was used to determine the investigated material mechanical constants that the kinetic equations of the calculation model contain. The system of differential equations of crack growth kinetics was solved numerically using the Runge-Kutta method.

Findings: On the basis of the energy approach of the fracture mechanics the calculation model of fatigue macrocrack growth in three-dimensional elastic-plastic body in case of a mixed-mode I+II+III macromechanism of fracture has been built. On the basis of the created calculation model, the kinetics of the growth of fatigue cracks was investigated both in the middle part of the wheelset axle and in the axle journal.

Research limitations/implications: The results obtained on laboratory specimens should be tested during a real railway wheelset axle investigation.

Practical implications: The created calculation model can be used in practice to formulate method of residual lifetime estimation of railway wheelset axle.

Originality/value: It was shown, that surface crack kinetics depends not only on the crack initial area but also significantly depends on the crack edge geometry and comparatively small crack-like defects at the wheelset axle surface can reach critical sizes in comparatively short run. It has been found that mechanical shear stresses caused by the weight of the loaded railway wagon in the cross section of the wheelset axle journal can significantly accelerate the growth of the transverse fatigue crack at the axle surface, reducing the period of crack subcritical growth by about 20%.

Keywords: Fatigue crack growth, Mixed mode, Railway axles, Stress intensity factor, Estimation model

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METHODOLOGY OF RESEARCH, ANALYSIS AND MODELLING**1. Introduction**

The problem of reliability and safe operation of the railway transport mechanical components, in particular wheelsets, at their long-term operation is an important and relevant scientific and technical task, since their failure can lead to catastrophic consequences [1-7]. That is why the wheelset is a safety critical component of the mechanical part of the railway rolling stock. However, cracklike defects often occur in its axle both in the manufacturing stage and in the course of operation.

The first incident caused by the fatal destruction of the locomotive axis occurred on May 8, 1842, on the line between Paris and Versailles [8], resulting in the deaths of more than 60 people. It had initiated a detailed study of the processes of fatigue and accumulation of damage in materials under the cyclic loading.

Despite considerable progress even nowadays the destruction of the railway axes caused by the fatigue damage still occurs (Fig. 1). So as mentioned in [9] the decommissioning in Russia in 1993 was 6800 axes because of defects found, which was about 0.3% of the total number of axes in operation. In recent years, there have also been reports of railway accidents caused by the wheel axles fracture, for example in Rickerscote (UK, 1996) [10], Floods (Canada, 2004) [11], Morrisburg (Canada, 2010) [12],

Petrovanky (Luhansk region, Ukraine, 2003) [13], Gorodische (Cherkasy region, Ukraine, 2014) [14].

To prevent possible accidents of such objects there is a problem of reliable prediction of their residual resource.

This problem is poorly understood and requires significant development especially in the theoretical direction [3, 15-17]. In the literature there is very little information about reliable methods of the residual life estimation which would guarantee the extension of long-operated mechanical elements service life with the desired reliability. The known methods of the residual life prediction of these elements are mainly based on statistical approaches and are not sufficiently effective because do not provide the desired reliability [18-21].

The wheelset axles of the railway rolling stock are components of long-term operation. But in spite of that, in practice their premature destruction due to cracklike defects (technological or operational) often takes place. Such defects growing by fatigue mechanism under operational loads may cause the complete destruction of the axle (Fig. 1).

Therefore, in order to prevent possible accidents, there is a need of reliable prediction of the residual service life of wheelset axles. To predict this fracture, one must be able to calculate the period of subcritical growth of the detected fatigue cracks in the axle, depending on their size and location.

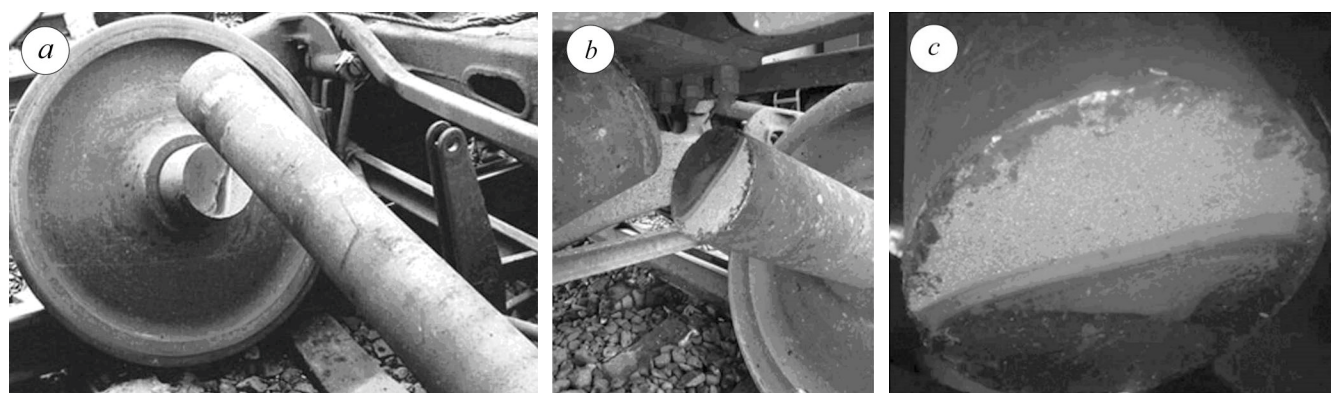


Fig. 1. Fatigue fracture of railway wheelset axle in the section between the wheels: (a) near the wheel internal surface (close to the joint of the axle with the wheel); (b, c) in the middle part of the axle

2. Model formulation

If the dependence of the fatigue crack growth rate V on its area S is known, then the period of subcritical crack growth N_{cr} from the initial size S_0 to the final critical size S^* is easily determined by the known relation [22]

$$N_{cr} = \int_{S_0}^{S^*} V^{-1}(S) dS \quad (1)$$

where $V \equiv dS/dN$; dS is the elementary increment of the crack area; dN is the elementary increment of the number of loading cycles.

Thus, in order to determine the value of N_{cr} , it is necessary to find the function of fatigue crack growth rate $V(S)$. For this we propose below a computational model based on the equation of energy balance in thermodynamics (the first law of thermodynamics) [23,24].

Let us consider an elastoplastic body weakened by a plane surface macrocrack of a depth l , with a front length L (Fig. 2). Let the stresses of tension p_1 , of shear p_2 and of torsion p_3 which vary cyclically with some constant load ratio $R = p_{(i)min}/p_{(i)max}$ act on the body externally, creating at the crack front mixed I+II+III stress-strain state (Fig. 2). The task is to determine the period of subcritical growth of such a fatigue crack.

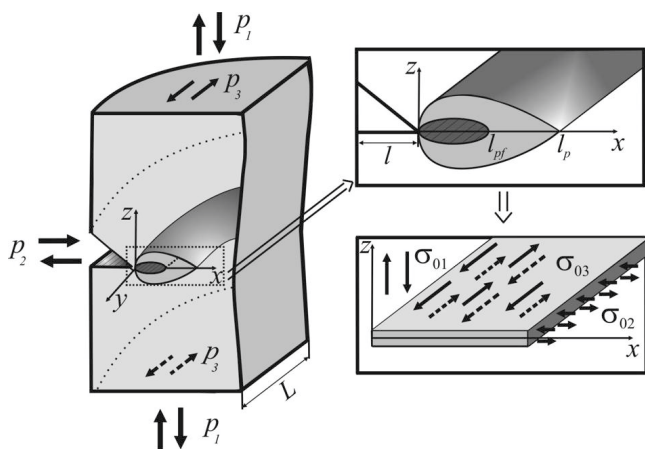


Fig. 2. Schematic diagram of loading of the solid body containing a crack and the plastic zone in the crack tip vicinity

The first law of thermodynamics [24] has been used to solve this task. According to the last one the following relation can be written for any volume of solid body and fatigue crack infinitesimal increment

$$dA + dQ = dK + dU + dU_0 \quad (2)$$

Here, dA is the work increment of the external surface and mass forces, dQ is the external inflow-outflow of heat; dU is the change in internal energy of the material, which in this case is determined by a change in the field of elastoplastic strains in its volume; dU_0 is the change in the surface energy in the body volume when new surfaces are formed during crack growth, which is equal to the work performed to move these surfaces and taken with the sign “minus”, as it is done against the action of internal cohesive forces in the material volume [23]; and dK is the change in the kinetic energy of the solid body.

In turn, the quantity dU can be given as the sum of two terms dU_1 and dU_2 . The first addend dU_1 , determines the irreversible part of the dissipated elastoplastic strain energy in the material volume that is spent for the accumulation of fatigue microdamages (microdefects) near the crack tip in the process of external cyclic loading and finally leads to the formation of new internal surfaces in the material (fatigue crack growth). The second term dU_2 specifies a part of the internal energy caused solely by the action of external factors, namely, by the changes in the work of external surface and mass forces dA and the external inflow-outflow of heat. Then the balance equation (2) can be rewritten in the form

$$dA + dQ - dU_2 = dK + dU_1 - u_0 dS \quad (3)$$

where u_0 is the density of fracture energy of the material [23,24].

Assuming that the heat exchange processes and inertia effects in the material volume are sufficiently small to set $dQ = dK = 0$, the Eq. (3) can be presented in the form

$$dS/dN = u_f / (u_0 - u_s) \quad (4)$$

where $u_f = dU_f/dN$ is the irreversible energy dissipation of elastoplastic strains spent for the formation of fatigue microdamages (microdefects) in a cyclic plastic zone per a cycle of the body loading; $u_s = dU_s/dS$ is the density of the irreversible energy dissipation of elastoplastic strains spent for the formation of fatigue microdamages (microdefects) in a static plastic zone (Fig. 2).

Assuming that the plastic zone along the fatigue crack front has a sufficiently elongated shape to be modelled as a thin layer, let us use known in fracture mechanics of materials Leonov-Panasyuk-Dugdale δ_c -model [25] to determine the energy components in the equation (4).

According to this model plastic zone near fatigue crack front is replaced by flat section with some averaged and uniformly distributed tensile σ_{01} and shear σ_{02} and σ_{03} stresses (Fig. 2). Then, based on it and results of [24, 26], the magnitudes of the energy components of the elastoplastic strains in the Eq. (4) can be expressed using the parameters of linear fracture mechanics, namely stress intensity factors (SIF), and finally presented in the form

$$\frac{dS}{dN} = \frac{\pi(1-R)^4}{64} \frac{[c_1 K_{I \max}^4 + c_2 K_{II \max}^4 + c_3 K_{III \max}^4]}{K_{Ic}^2 - [K_{I \max}^2 + K_{II \max}^2 + K_{III \max}^2]} \quad (5)$$

where $K_{I \max}$, $K_{II \max}$, and $K_{III \max}$ are the maximum SIF values for the fracture modes I, II and III respectively; K_{Ic} is the static crack growth resistance (fracture toughness) of the material; c_i ($i = 1, 2, 3$) are the mechanical parameters of the material which are determined from the experiment. Then the period N_{cr} of the fatigue macrocrack subcritical growth is determined on the basis of the Eqs. (1) and (5).

3. Problem formulation

As the practice shows, the most dangerous defects detected in the railway axles are similar in shape to a semielliptic crack (Fig. 3). Farther, task solution of subcritical period estimation of surface semielliptic transverse crack at the wheelset axle middle part is constructed (Fig. 4a).

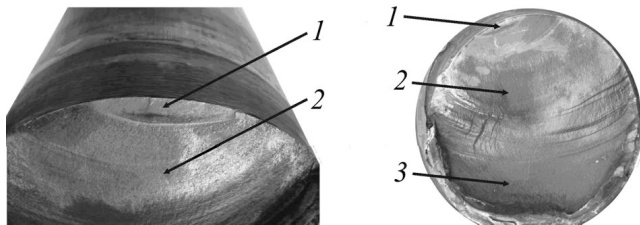


Fig. 3. Fracture surfaces of two corresponding parts of the destroyed wheelset axle: 1, 2 – the areas of initiation and propagation of the fatigue crack respectively; 3 – the area of the final fracture

Under the operational loading, the middle part of the wheelset axle is in conditions close to the pure bending (Fig. 4).

To determine the function of the fatigue crack growth rate V at an arbitrary point D of its semielliptic contour (Fig. 4b), we used the following equation which was

obtained on the basis of the mathematical model formulated above:

$$V_D(a, b) = a_1 \frac{\Delta K_{ID}^4(a, b) - a_2}{a_3 - (1-R)^{-2} \Delta K_{ID}^2(a, b)} \quad (6)$$

where ΔK_{ID} is the stress intensity factor range at the point D , R is the load ratio, a_i ($i = 1, 2, 3$) are some material constants which are determined from the fatigue experiment.

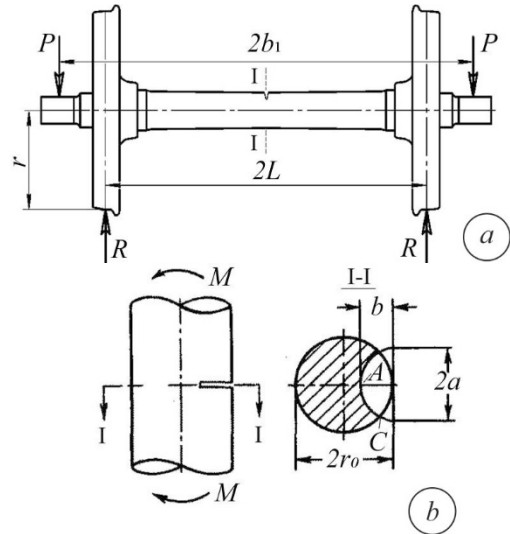


Fig. 4. Schematic diagram of loading of the wheelset (for a railway width of 1520 mm) containing (a) a semielliptic surface crack in the middle cross section II and (b) the geometry and location of the semielliptic transverse crack

The dependences of the lengths of semiaxes a and b of the semielliptic crack front on the number of loading cycles N will determine the kinetics of crack growth. Assuming that the contour of the fatigue crack during its growth maintains a semielliptic shape, these dependences can be obtained from the solution of the system of two ordinary differential equations

$$\begin{aligned} da/dN &= V_C(K_{IC \max}(a, b)), \\ db/dN &= V_A(K_{IA \max}(a, b)) \end{aligned} \quad (7)$$

with appropriate initial conditions

$$a(0) = a_0, \quad b(0) = b_0 \quad (8)$$

where $V_{C, A}$ are the functions of the growth rate of the semielliptic contour of the crack at points C and A

respectively (Fig. 4b); $K_{IC\max}$ and $K_{I\Delta\max}$ are the SIFs at these points (Fig. 4b).

In the work [21], a kinetic diagram of fatigue crack growth in a specimen of the wheelset axle steel was constructed (Fig. 5). Flat rectangular specimens with a central crack were used for cyclic tension tests at a load ratio $R = -1$. Based on these experimental results the constants a_i of the axle material (see Eq. 6) were determined by the method of least squares (R -square ≈ 0.93): $a_1 = 0.33 \cdot 10^{-9} \text{ MPa}^{-2} \text{ cycles}^{-1}$, $a_2 = 820 \text{ MPa}^4 \text{ m}^2$, $a_3 = 360 \text{ MPa}^2 \text{ m}$.

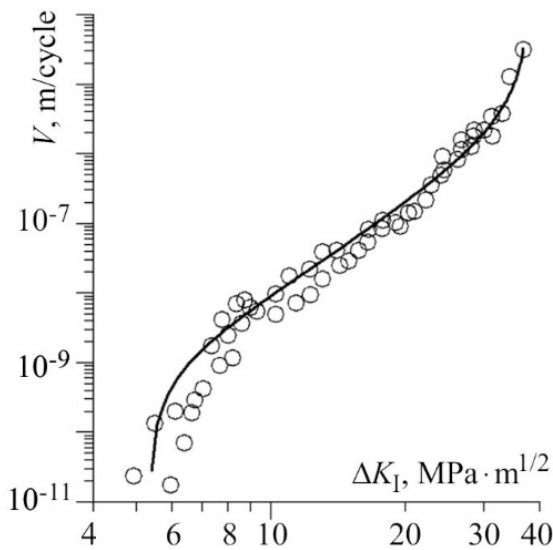


Fig. 5. Kinetic diagrams of fatigue crack growth in the steel of a wheelset axle: (○) experimental data [21], the solid line corresponds to the result of calculations according to Eq. (6)

It was assumed that the propagation of the surface crack at all points of its semielliptic contour corresponds to the kinetic diagram of fatigue crack growth in the material of the railway axle (Fig. 5).

The system of ordinary first order differential equations of the (7) was solved numerically by the Runge-Kutta method [27] taking into account formulas (6) and (8). It is known from the practice of the wheelset axles destruction that zone of final fracture during fatigue transverse crack propagation in the middle part of the axle takes about 35...40% of its total cross-sectional area (Fig. 3). Therefore, the critical condition for crack propagation on the axis was conventionally assumed as follows

$$b(N_{cr}) = r_0 \tag{9}$$

where N_c is the critical number of load-unload cycles.

The bending moment $M \approx 2.8 \cdot 10^{-2} \text{ MN} \cdot \text{m}$ in the middle part of the axle (Fig. 4a) was calculated in case of maximally loaded railway wagon using the known formula [28]

$$M = P \cdot (b_1 - L) \tag{10}$$

Since one complete turn of the wheelset axle during railway wagon motion corresponds to one load-unload cycle on the axle surface (Fig. 4a), then the number of load-unload cycles N_c can be easily converted into the residual run S_c expressed in kilometres. The results of the calculations are presented in the form of diagrams of the kinetics of fatigue crack propagation in order to determine the value of S_c (Fig. 6).

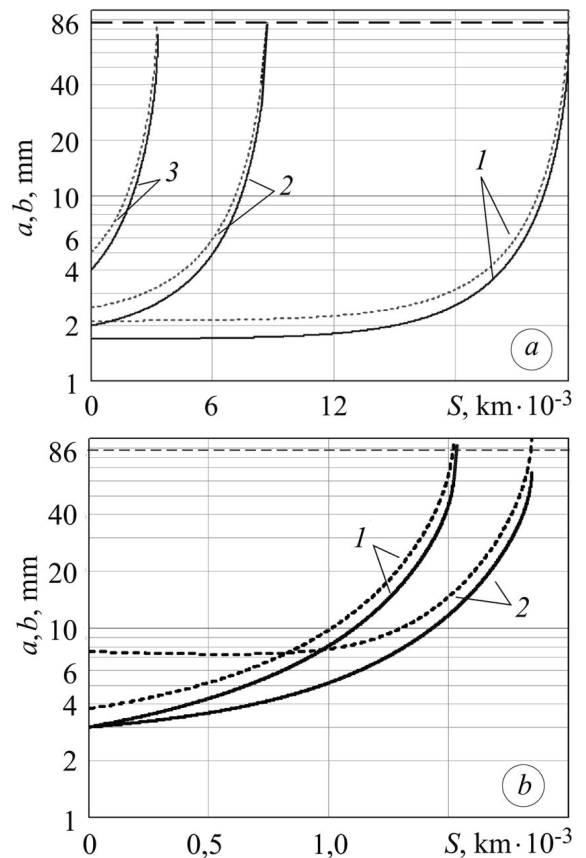


Fig. 6. Dependences of changes in the sizes of semi-axes of the semielliptic contour of a fatigue crack (the solid lines correspond to the semi-axis b and the dashed lines correspond to the semi-axis a) for various initial values: (a) for the ratio (for the ratio of the initial lengths) of the semi-axes $b_0/a_0 = 0.8$ [(1) $b_0 = 1.7 \text{ mm}$, (2) 2.0 , (3) 4.0]; (b) for the ratios (for the ratios of the initial lengths) of the semi-axes 0.8 (curves 1) and 0.4 (curves 2) and $b_0 = 3.0 \text{ mm}$

As it can be seen from the calculations the surface crack slowly grows up to the ~60% of the subcritical period and then rapidly propagates reaching half of the wheelset axle radius (Fig. 6). The subcritical period of the crack growth on the axle drops sharply with an increase in initial crack size (Fig. 6a). However, the initial crack semiaxes ratio significantly affects the fatigue crack growth kinetics and, if $b_0/a_0 \approx 0.8$, such cracks can reach critical sizes even faster than those having a significantly larger initial area but lower ratio b_0/a_0 (Fig. 6b), (Fig. 7).

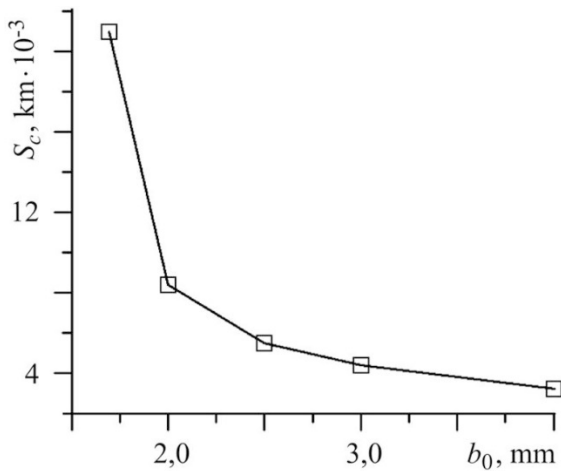


Fig. 7. Dependence of the residual run of a wheelset axle weakened by a semielliptic transverse crack on the initial length of the semi-axis b_0 of its contour for the ratio $b_0/a_0 = 0.8$

It happens often in practice of wheelset destruction that cracklike defects initiates and propagates in the axle journals. Therefore, we have considered the task of subcritical period estimation of surface crack growth on the wheelset axle journal (Fig. 8).

Under the operation load surface of the axle journal is subjected to cyclic tensile and shear loads.

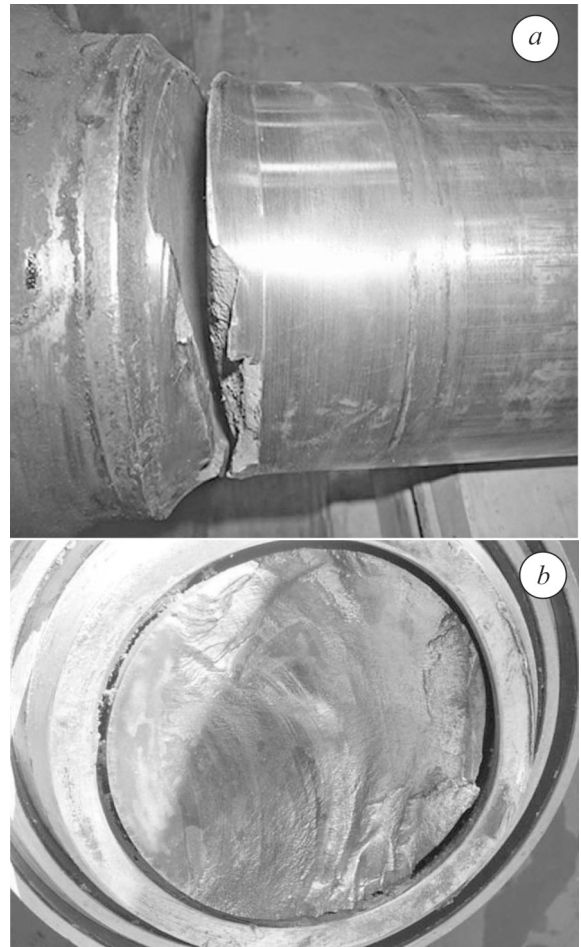


Fig. 8. Pictures of a destroyed wheelset axle: (a) the side view showing two separated parts of the axle; (b) the cross-sectional view of the fracture surface

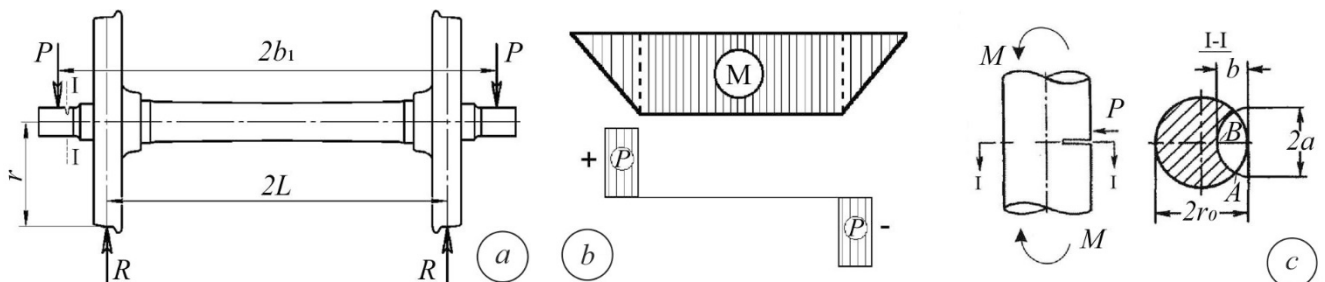


Fig. 9. Schematic diagram of loading of the wheelset containing (a) a surface crack in the cross section I-I of the axle journal, (b) the bending moment and cross-cutting force diagrams for the axle, and (c) the geometry and location of the semielliptic transverse crack

The function (11) of the fatigue crack growth rate V at an arbitrary point D of its semielliptic contour (Fig. 9c) is found on the basis of the mathematical model formulated above

$$V_D(a,b) = \frac{\pi(1-R)^4}{64} \frac{[c_1 K_{ID \max}^4(a,b) + c_2 K_{IID \max}^4(a,b)] - c_3^4}{K_{Ic}^2 - K_{ID \max}^2(a,b) + K_{IID \max}^2(a,b)} \quad (11)$$

where $K_{ID \max}$ and $K_{IID \max}$ are the maximum SIF values for the Mode I and Mode II fracture respectively at point D of the crack front.

Similarly to the previous case, the dependences of the lengths of semiaxes a and b on the number of load cycles N were determined from the solution of the system of two ordinary differential equations

$$\begin{aligned} da/dN &= V_A(K_{IA \max}, K_{IIA \max}), \\ db/dN &= V_B(K_{IB \max}, K_{IIB \max}) \end{aligned} \quad (12)$$

with appropriate initial conditions $a(0) = a_0$, $b(0) = b_0$.

As a critical condition for crack propagation on the axis journal the expression (9) was accepted.

Based on the results in the paper [29] the SIF values of K_I and K_{II} for the semielliptic crack at points A and B (Fig. 9b) can be given as

$$\begin{aligned} K_{IA,B} &= 4M \cdot \pi^{-1} r_0^{-3} \sqrt{\pi \cdot b} F_{IA,B}(b/r_0, b/a), \\ K_{IIA} &= \tau_0^\infty \sqrt{\pi \cdot b} F_{IIA}(b/2r_0, b/a) \end{aligned} \quad (13)$$

where M is the bending moment acting in the crack plane cross section; τ_0^∞ is the nominal shear stress in the crack plane cross section; $F_A = F_A(b/r_0, b/a)$, $F_{IIA} = F_{IIA}(b/2r_0, b/a)$, and $F_B = F_B(b/r_0, b/a)$ are some dimensionless correction functions taking into account the influence of the geometry of the wheelset axle with a crack on the SIFs at points A and B respectively.

In the paper [29], the method of finite elements was used to calculate the SIFs along the front of a semielliptic surface crack in a three-dimensional semi infinite body subjected to the mixed loading mode. In this paper, an analysis of the stressed field in the crack tip vicinity was carried out using numerical and analytical methods, and numerical data were obtained for functions $F_{IA,B}$ and $F_{IIB,B}$. For further calculations, we have approximated these data using the least squares method. The following approximation formulas obtained in this way are as follows

$$\begin{aligned} F_{IA}(\varepsilon, \eta) &= (-0.6302\varepsilon^2 + 1.0578\varepsilon + 0.1321) \cdot \eta^2 + \\ &+ (-1.8426\varepsilon^2 + 2.6992\varepsilon + 0.7177) \cdot \eta - 0.734\varepsilon^2 + \\ &+ 1.6815\varepsilon + 0.2215 \end{aligned} \quad (14)$$

$$\begin{aligned} F_{IB}(\varepsilon, \eta) &= (-1.0368\varepsilon^2 + 1.0839\varepsilon + 0.2840) \cdot \eta^2 + \\ &+ (1.1278\varepsilon^2 - 1.8659\varepsilon + 0.2680) \cdot \eta - 0.3918\varepsilon^2 + \\ &+ 0.6596\varepsilon + 0.3876 \end{aligned} \quad (15)$$

$$F_{IIA}(\chi, \varepsilon) = 0.671 \quad (16)$$

where $\varepsilon = b/a$, $\eta = b/r_0$, $\chi = b/2r_0$, the determination coefficient $R^2 \approx 0.9$.

The bending moment $M \approx 0.51 \cdot 10^{-2}$ MN·m and the shear stress $\tau_0^\infty \approx 8.9$ MPa in the axle journal (Fig. 9a) were calculated according to the known formulas of the structural mechanics for the case of the maximally loaded wagon.

The calculation results of the propagation kinetics of the semielliptic fatigue crack at the axle journal surface are graphically presented in Figure 10.

The calculations have shown that the shear fracture macromechanisms can have a significant effect on the propagation kinetics of the fatigue crack in the wheelset axle journal, reducing subcritical period of crack growth up to 20%.

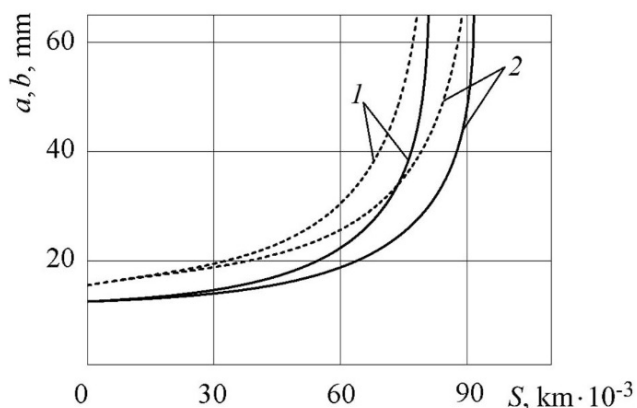


Fig. 10. Dependences of changes in the sizes of semiaxes of the semielliptic contour of a fatigue crack (the dashed lines correspond to the semiaxis a and the solid lines correspond to the semiaxis b) for the initial ratio of the semiaxes $b_0/a_0 = 0.8$ ($b_0 = 12.5$ mm): (1) the cross-cutting force and bending moment are taken into account; (2) only the bending moment is taken into account

4. Conclusions

1. It is shown that cracklike defects of relatively small sizes initiated on the surface of a wheelset axle can reach a critical size for a short run. The kinetics of the fatigue

surface crack growth depends not only on the area of the initial defect, but also significantly depends on the initial configuration, that is, the semi-minor to semi-major axis ratio for its semielliptic contour.

2. Defects with the ratio of the initial lengths of the semiaxes b_0/a_0 close to 0.8 are the most dangerous, since they at relatively small initial depths ($b_0 < 2$ mm) can reach a critical size approximately after 20,000 km run of the railway wagon, and with the increase in the initial size of such cracks the subcritical period of their growth is significantly reduced.
3. It is shown that the mechanical shear stresses induced by the weight of the loaded railway wagon in the cross section of the wheelset axle journal can significantly accelerate the growth of the transverse fatigue crack initiated on the axle surface, reducing the subcritical period of crack growth by about 20%.

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