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THE POSSIBILITIES OF USING ANISOTROPIC MODELS OF ROCK MASS TO DESCRIBE DEFORMATIONS OF THE MINING AREA SURFACE

MOŻLIWOŚCI ZASTOSOWANIA MODELI ANIZOTROPOWYCH GÓROTWORU DO OPISU DEFORMACJI POWIERZCHNI TERENU GÓRNICZEGO

This paper presents the analysis of numerical modeling results of the influence of mining exploitation influence on the deformations of the area surface, with the use of an anisotropic elasto-plastic ubiquitous joint model and the transversely isotropic elastic model. The comparison of computer modeling results and geodetic measurements shows that with the use of transversely isotropic elastic model and ubiquitous joint model there is a possibility of the simultaneous description of both perpendicular and horizontal displacements of the area surface, caused by mining exploitation.

Keywords: rock mass, calculation model, subsidence trough, area deformations

Jak dotąd zasadniczym problemem związanym z modelowaniem numerycznym było zbudowanie modelu, który możliwie dokładnie opisywałby wszystkie etapy procesu deformacji górotworu. Niecki osiadania, uzyskiwane w wyniku modelowania numerycznego metodą elementów skończonych dla górotworu traktowanego jako ośrodek liniowo sprężysty izotropowy, znacznie różniły się od tych wyznaczanych z pomiarów geodezyjnych. Niecki te okazywały się zbyt rozległe i charakteryzowały się znacznie mniejszym (nawet 2,5-krotnie) nachyleniem zboczy w stosunku do niecek rzeczywistych.

W celu wykazania, że stosowanie modeli numerycznych pozwala uzyskać w miarę dokładny opis rzeczywistych deformacji powierzchni terenu górniczego, przetestowano wiele modeli matematycznych, opisujących własności mechaniczne górotworu pod kątem zgodności z pomiarami geodezyjnymi. Testy te udowodniły, że w przypadku modelu liniowo sprężystego o transwersalnie izotropowej budowie warstw możliwe jest uzyskanie z obliczeń numerycznych w miarę dokładnego przybliżenia rzeczywistych deformacji powierzchni terenu górniczego (Tajduś, 2007; Białek et al., 2002; Mielimąka, 2009; Wesołowski et al., 2010). Pomimo poprawnego opisu deformacji powierzchni terenu stosowanie modelu transwersalnie izotropowego budzi wiele kontrowersji z uwagi na konieczność przyjęcia bardzo zróżnicowanych parametrów ośrodka w kierunku pionowym i poziomym.

Pewne cechy podobieństwa do ośrodka transwersalnie izotropowego wykazuje model ubiquitous joint (Rys. 1). Odpowiednikiem kierunku prostopadłego do płaszczyzn izotropii modelu transwersalnie izotropowego może być tu kierunek prostopadły do płaszczyzn osłabienia. Model ubiquitous joint jest

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zatem anizotropowym modelem plastycznym zawierającym płaszczyzny osłabienia określonej orientacji. W modelu tym zaimplementowany został warunek wytrzymałościowy (uplastycznienia) Coulomba-Mohra. Do procesu modelowania numerycznego deformacji terenu górniczego, przeprowadzonego na potrzeby niniejszej pracy, wykorzystano program różnic skończonych *FLAC* (Itasca Consulting Group, Inc. 1992).

Celem przeprowadzenia symulowanej komputerowej eksploatacji górniczej i określenia jej wpływu na deformacje powierzchni terenu zbudowano płaski model o wymiarach 2200 m × 913 m. Na głębokości 600 m (głębokość spągu pokładu) zamodelowano przeznaczony do eksploatacji pokład węgla o grubości 2 m. Schemat geometryczny modelu przedstawiono na Rys. 2.

Wyniki symulacji komputerowej w zakresie opisu deformacji terenu górniczego porównane zostaną z przykładowymi pomiarami geodezyjnymi prowadzonymi na linii nr 100 podczas eksploatacji ścianowej pokładu 338/2 w KWK „Budryk" (Rys. 3).

Parametry wytrzymałościowe oraz odkształceniowe warstw przyjęte zostały na podstawie literatury (Kidybiński, 1982; Prusek & Bock, 2008). Określając wartości parametrów płaszczyzn osłabienia posłużono się przypadkiem opisanym w pracy (Sainsbury et al., 2008). Zakres zmienności parametrów materiałowych warstw skalnych modelu ubiquitous joint oraz modelu transwersalnie izotropowego przyjęte do obliczeń przestawiono w tabelach 1 i 2.

W pracy porównane zostały możliwości stosowania modelu transwersalnie izotropowego oraz modelu ubiquitous joint pod kątem zgodności opisu deformacji terenu górniczego wywołanych prowadzoną eksploatacją górniczą. W oparciu o wymienione powyżej modele górotworu przeprowadzona została symulacja komputerowa eksploatacji górniczej. Wyniki tych symulacji wykazały, że:

1. Przeprowadzony w ramach pracy cykl symulacji komputerowych wykazał, że zarówno dla modelu ubiquitous joint oraz modelu transwersalnie izotropowego istnieje możliwość jednoczesnego opisu zarówno pionowych, jak i poziomych ruchów górotworu, wywołanych eksploatacją górniczą. Uzyskanie bliskiego rzeczywistości opisu ruchów poziomych wymagało wprowadzenia płaszczyzn kontaktu (*Interface*) do modelowania połączeń międzywarstwowych.
2. Profil asymptotycznej niecki obniżeniowej ściany jest dla rozpatrywanych modeli asymetryczny względem wybranego pola. W profilach tych wartość maksymalnego nachylenia w rejonie krawędzi rozpoczynającej eksploatację jest nawet o kilkadziesiąt procent większa niż wartość maksymalnego nachylenia w rejonie krawędzi kończącej eksploatację. Podobne zależności dotyczą odkształceń poziomych.
3. Przedstawione wyniki symulacji komputerowych wskazują na to, że przy wykorzystaniu odpowiedniego ośrodka istnieje możliwość opisu kolejnych etapów deformacji terenu górniczego, w tym również wpływu kierunku prowadzenia eksploatacji na kształt profilu niecki obniżeniowej, kształtującej się nad postępującym frontem ścianowym.
4. Niewątpliwą zaletą modelu bazującego na ośrodku transwersalnie izotropowym jest stosunkowo mała ilość parametrów odkształceniowych koniecznych do obliczeń oraz możliwość łatwego dostosowania wyników obliczeń do wyników obserwacji geodezyjnych (Wesołowski, 2013).

Uzupełnieniem prowadzonej analizy są rysunki 8 i 9 przedstawiające zasięg stref uplastycznienia rozpatrywanych układów modelu górotworu.

Przeprowadzone w ramach pracy obliczenia komputerowe pokazały, że przy zastosowaniu numerycznych modeli górotworu opierającego się na sprężystym ośrodku transwersalnie izotropowym oraz anizotropowym modelu ubiquitous joint możliwy jest opis deformacji terenu górniczego jakościowo i ilościowo zgodny z obserwacjami geodezyjnymi.

Słowa kluczowe: górotwór, model obliczeniowy, niecka obniżeń, deformacje terenu

1. Introduction

The main problem connected with using numerical models of continuous medium to describe deformations of the mining area surface is the correct presentation of all deformation indexes. A considerable part of the works, in which the isotropic models were used, dealt only with trying to make a qualitative description of the deformations in the mining area. As stated in (Tajduś, 2007, 2008; Białek & Wesołowski, 2011), the use of linear isotropic models causes that the profiles of calculated subsidence troughs are of too shallow a profile in relation to the real ones. Consideration of rock mass plastic properties in this model do not improve the description of the mining area surface deformation. However the author's own studies (Wesołowski, 2001) show, that the use of a transversally isotropic medium in the modeling process may assure the qualitatively and quantitatively good description of the surface subsidence in the mining area.

Despite a correct description of the area surface deformation presented in the following papers (Białek et al., 2002; Mielimąka, 2009; Wesołowski et al., 2010) the use of the transversally isotropic model brings on many controversies because of the fact that it is necessary to consider very differentiated parameters describing the medium's properties in a vertical and horizontal direction. Therefore the following article is an attempt to describe the deformation of the surface of the mining area with the use of elasto-plastic ubiquitous joint model. Because of the implemented strength conditions this model may serve to imitate the most essential features of the rock mass. The results of computer simulation describing the deformation of the mining area will be compared with the exemplary geodetic measurements led on the line number 100 during the longwall exploitation of seam 338/2 in the "Budryk" coal mine. In the modeling process the finite differences *FLAC* program (Itasca Consulting Group, Inc. 1992) was used in which both the analyzed models of medium are implemented.

2. The characteristics of analyzed models

The key characteristics of the majority of rocks is anisotropy of properties, which means that they present different properties depending on the direction of a study (Wesołowski et al., 2010). One of the factors that cause anisotropies is the stratified composition of sedimentary rocks. Sedimentary rocks in planes parallel to the bedding behave as an isotropic mediums. In the perpendicular direction to the bedding they show distinct features, often considerably different from their properties in bedding planes. This special case of anisotropy within the rocks of which the rock mass is composed, describes well the transversally isotropic medium (Fig. 1a).

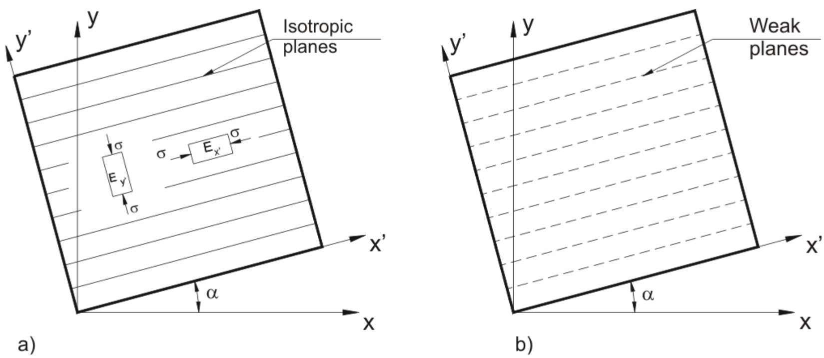

Fig. 1. The comparison of the transversally isotropic model and the ubiquitous joint model

Some similarity to the transversally isotropic medium shows the ubiquitous joint model (Fig. 1b). Here, the equivalent of the perpendicular direction to the isotropy planes in the transversally isotropic model can be perpendicular in the direction to the weakness planes. The ubiquitous joint model is therefore the anisotropic plastic model which includes specifically oriented weakness planes evenly disposed in the whole volume. In this model the Coulomb – Mohr strength (plasticity) condition has been implemented. Plasticity may occur in both weakness planes and rock mass. The isotropy planes and weakness planes can be sloped at any angle f to the horizontal plane described in the coordinate system by *X-Y* axes.

The following part of the paper compares the possibilities of using both models to describe the mining area deformations caused by the ongoing mining exploitation.

To perform a computer simulated mining exploitation and determine its influence on deformations of the area surface, the flat model with the dimensions of 2200 m × 913 m has been developed. At the depth of 600 m (the depth of the seam floor) the coal seam aimed for exploitation has been modeled with the depth of 2 metres. The assumed length of exploitation field equals to 1000 m. The geometrical scheme of the model is shown in Fig. 2.

Because it is necessary to select many parameters of the medium, for simplification purposes the flat model of the rock mass has been considered. In real conditions the flat state of deformation appears when the edges of the exploitation field are at considerably greater distance than the exploitation depth from the model's plane. This means that in two dimensional case the results of simulation can be compared with the following cases:

1. When the measuring line is situated along the panel length of a very shallow longwall,
2. When the measuring line is situated perpendicularly to the panels length of longwalls situated on medium and large depths.

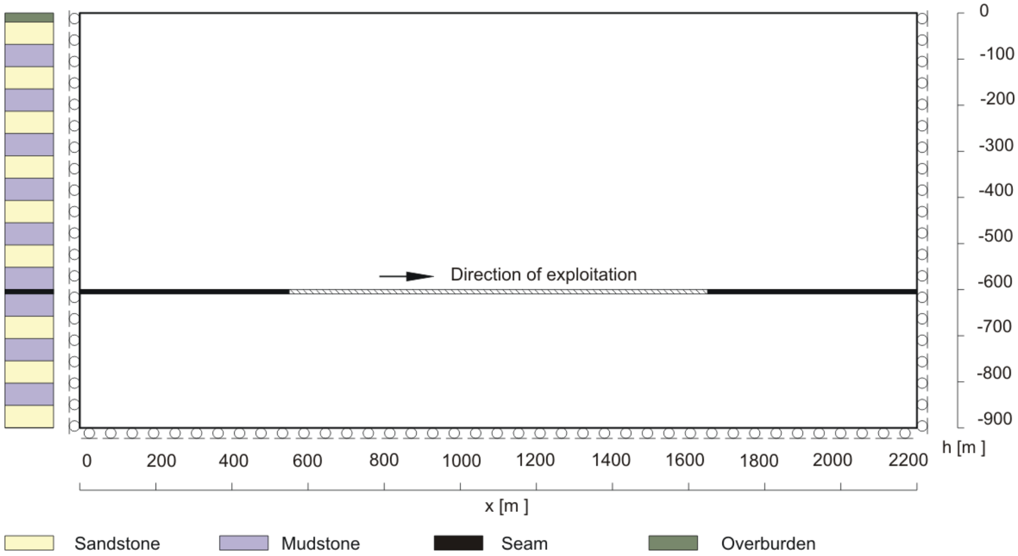

Fig. 2. The structural scheme of rock mass model

The comparison of the measured and calculated deformations was based on the second case with the use of measurements conducted on line 100 in "Budryk" coal mine (Fig. 3).

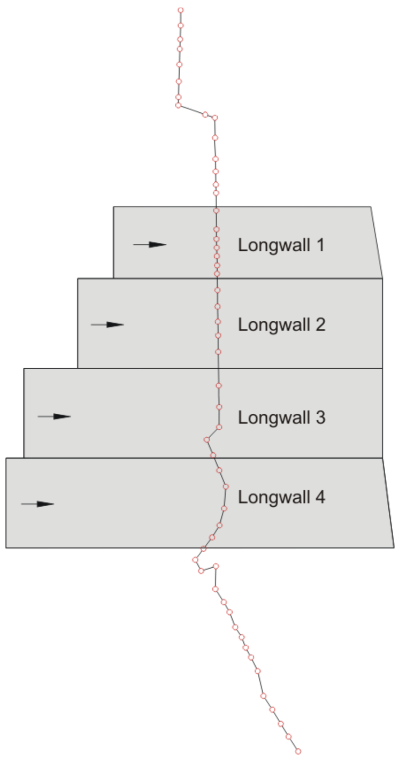

Fig. 3. The position of the measuring lines points number 100 in relation to the exploitation edge

The size of the exploitation field and the exploitation depth of the analyzed rock mass model is approximately the same as the exploitation conditions of seam 338/2 in the “Budryk” coal mine in the area of observational line number 100.

The rock mass model was divided into horizontal layers with different strain and strength parameters. The rock mass above the seam was described with the use of 12 layers with the thickness of about 50 m. The last subsoil layer of about 10 m thickness will make up the overburden (of soil and quaternary sands). In the seam floor 6 layers of about 50 m thickness were modeled.

The layers' strength and strain parameters were assumed on the basis of literature (Kidybiński, 1982; Prusek & Bock 2008; Tajduś, 2010). The base of the defining weakness planes parameters was the case described in (Sainsbury et al. 2008). The changeability range of the ubiquitous joint model rock layers material parameters, which are a base of calculations, is shown in table 1.

TABLE 1

Strain and strength parameters of the ubiquitous joint model layers assumed for calculations

		Overburden	**Mudstone**	**Sandstone**	**Seam**
Parameters of rock mass					
Coefficient of shear elasticity *G*	[MPa]	114	2680	4350	864
Coefficient of volumetric elasticity *K*	[MPa]	247	2630	4760	1830
Cohesion *c*	[MPa]	0,46	10,7	20,75	6,26
Angle of internal friction ϕ	[deg]	24	24	35	25
Tensile strength *Rr*	[MPa]	0,1	5,25	9,50	1,6
Volumetric density ρ	[kg/m^3]	1950	2610	2450	1400
Parameters of weakness planes					
Cohesion *c*	[MPa]	0,0046	0,1	0,2	0,06
Angle of internal friction ϕ	[deg]	24	24	24	24
Tensile strength *Rr*	[MPa]	0,001	0,05	0,1	0,01

For the model based mainly on the transversally isotropic medium, the main part of the rock mass, to the depth of 550 m, was attributed with the strain parameters shown in table 2. They enabled imitating the subsidence trough with the inclinations (for the beginning of exploitation edge) close to the ubiquitous joint model. The selection of strain parameters scheme is described in greater detail in (Kołodziejczyk & Wesołowski, 2010; Wesołowski et al., 2010).

The coal seam, as well as roof and floor layers were described by the elasto-plastic medium, in which the plasticity was determined by the Coulomb-Mohr plasticity condition (Wesołowski et al., 2010). The use of the isotropic elasto-plastic medium allows for modeling permanent plastic changes, which appear during the mining exploitation. The parameters of these layers are identical to those in table 1 (with the exclusion of weakness planes parameters).

TABLE 2

Strain parameters of the transversally isotropic model layers assumed for calculations

		Overburden	**Mudstone**	**Sandstone**	**Seam**
Parameters of rock mass					
Coefficient of linear elasticity $E_x = E_y$	[MPa]	296	6000	10000	2200
Poisson coefficient $v_x = v_y$	[-]	0,3	0,12	0,15	0,3
Coefficient of shear elasticity G_{xy}	[MPa]	6,02	142	236	51,9
Volumetric density ρ	[kg/m^3]	1950	2610	2450	1400

The model of the contact plane was situated between the distinguished layers (Fig. 4), thus simulating the interlayer connection. As stated in (Wesołowski et al., 2010) providing for the stratified composition of the rock mass and the use of contact elements as connections between the stratified layers is the prerequisite for obtaining the correct description of deformations in the model's horizontal surfaces. The values of interlayer contact planes parameters were assumed as parameters of the weaker layer creating the given contact and they are identical for the analyzed models. In the pre-plastic state the contact elements behave identically as the elements which describe the rock layers. After getting into a plastic state they have a possibility of considerably larger deformation than the traditional zones of the finite differences mesh which digitize the continuous medium.

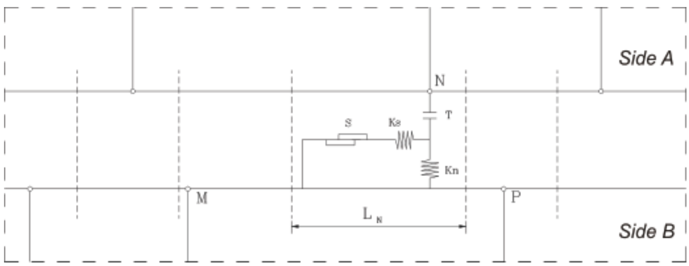

Fig. 4. The scheme of the contact plane (Itasca Consulting Group, Inc. 1992)

When building the differences mesh, it was assumed that the nodal points, present on vertical side edges of the shield, can freely move only in a vertical direction. The nodes on the base of the model shield can freely move only in a horizontal direction. Remaining nodal points belonging to the model have the possibility of moving freely in any direction in the *X-Y* plane.

Regarding the extremely small influence of the support on deformations of the area surface, the longwall workspace and the support of the mine working were not taken into consideration during the modeling of the abandoned workings. The simulation of exploitation will be conducted by a cyclic removal of the finite differences mesh of individual zones in order corresponding with the drawing of the longwalls. At the same time, between the roof and floor layers, contact elements were introduced (a contact plane), which prevents interpenetration of the roof and floor of the seam. This method of mining exploitation's simulation eliminates the need for introducing any additional parameters assigned to caving zones. The simulation of rock mass displacement conforms to the quasi-static model, in which the influences caused by exploitation (removal) of the next element of the seam show themselves immediately. There are no viscous elements here and the concept of time does not exist. On the other hand, because of the plastic properties of the medium, the order of removing (exploiting) the individual zones of mesh is important.

Upon defining the boundary conditions, it was assumed that the value of initial stress in the rock mass comes from the gravitational forces only. In the case of tectonically undisturbed rocks such assumptions are sufficient to determine the initial conditions of simulated exploitation (Wesołowski, 2001).

3. The results of computer simulations

After the full computational cycle, which included the simulated exploitation of the seam on the panel length of 1000 m, dislocations of nodal points on the model's surface were analyzed. Because of the quasi-static properties of the medium model the obtained results conform to the asymptotic subsidence trough. The analysis of numerical calculations results allowed for formulating the following remarks:

The simulated mining exploitation of a coal seam has shown that for both models the full subsidence trough appeared (Fig. 4), for which the maximum subsidence value is equal to $w_{max} = 1{,}6$ m. Regarding the thickness of the exploited seam the received value of exploitation coefficient $a = 0{,}8$. The subsidence of the point located over the edge of exploitation beginning ($x = 600$) equals to about 0,33 w_{max}, but over the edge of exploitation end ($x = 1600$) 0,28 w_{max} for the ubiquitous joint model and 0,32 w_{max} for the transversally isotropic model. Analogous values obtained from geodetic measurements are in the range of (0,19-0,5)w_{max} (Wesołowski, 2001).

In both the beginning and end of the exploitation edges there was a shift in the location of a point with subsidence of 0,5w_{max} in the direction of the excavated space. The value of this shift in the case of the ubiquitous joint model is practically equal for the beginning of the exploitation edge and the end of the exploitation edge and equals 0,12 h. In the transversally isotropic model the subsidence of 0,5 w_{max} appears in the distance of 0,11 h for the beginning of the exploitation edge and 0,12 h for the end of the exploitation edge. For comparison, the value of such shift observed in Polish conditions is in the range of $d = 0\text{-}0{,}15\ h$, while the average value of this shift in English conditions, for the first exploitation, equals to about 0,14 h (Wesołowski, 2001). The above account shows that the shift values for the point of 0,5 w_{max} subsidence are for both models within the limits of values obtained via geodetic measurements. It may be a result of simulating the exploitation with the use of the flat model.

The profile of the asymptotic subsidence trough along the panel length of a longwall, obtained via numerical calculations, is clearly asymmetric in relation to the excavated field (Fig. 5). The value of maximum inclination in the area of the beginning of exploitation edge is equal to 2,62 w_{max}/h for the ubiquitous joint model and 2,73 w_{max}/h for the transversally isotropic model (Fig. 6). In the area of the end of exploitation edge the maximum inclination of the trough profile

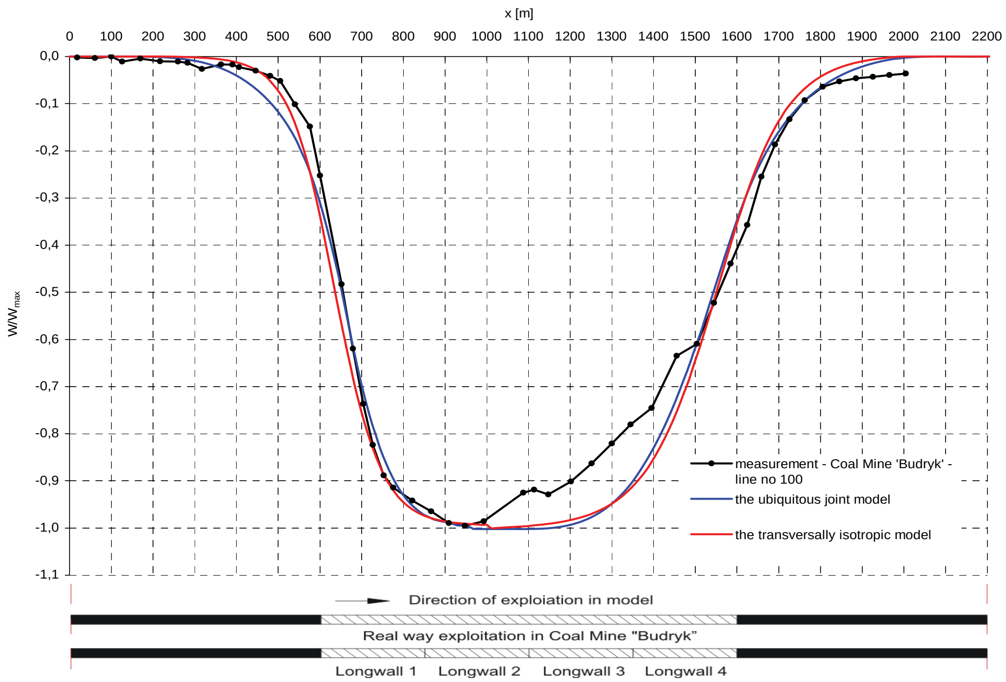

Fig. 5. Subsidence determined with the use of numerical model

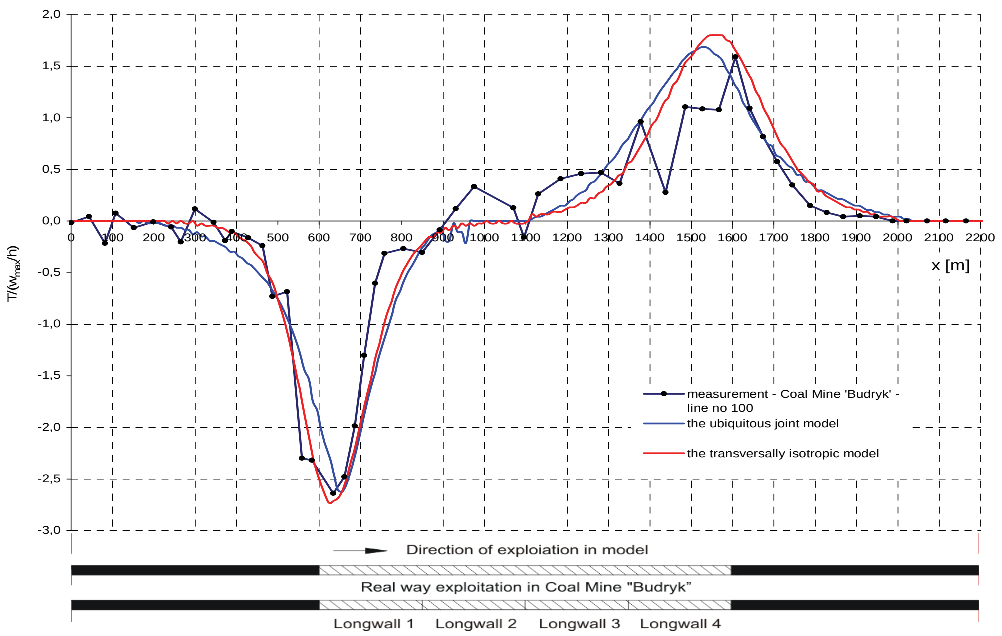

Fig. 6. Inclinations of the subsidence trough profile

is equal to 1,67 w_{max}/h and 1,8 w_{max}/h respectively. The results presented above show that the difference of inclinations between the beginning and the end of exploitation edges is equal to 35%.

The slight shortcoming of the numerical models analyzed is the flat bottom of the subsidence trough profile. In the floor parts of the real subsidence troughs profiles the differentiation of subsidence and the shallowing of the floor from the end of exploitation edge side is visible (Mielimąka, 2009; Białek, 2013). Most probably, the observed disorders of subsidence trough floor part shape are the result of rapid increments of subsidence after exploring subsequent longwalls, as well as plastic and brittle deformations inside a rock mass which accompany them.

Similarly as in the case of subsidence, the attribute of horizontal strains is clear asymmetry (Fig. 7), resulting from the influence of the direction of mining exploitation. The extreme values of horizontal strains, both positive[1] and negative[2], are about 50% larger by the start-up edge of a longwall then strains by the end of exploitation edge. In the beginning of mining exploitation edge area the maximum value of positive strains is equal to 1,62 w_{max}/h for the ubiquitous joint model and 1,10 w_{max}/h for the transversally isotropic model, and near the end of exploitation edge the analogous values of positive strains are equal to 0,78 w_{max}/h as well as 0,64 w_{max}/h respectively.

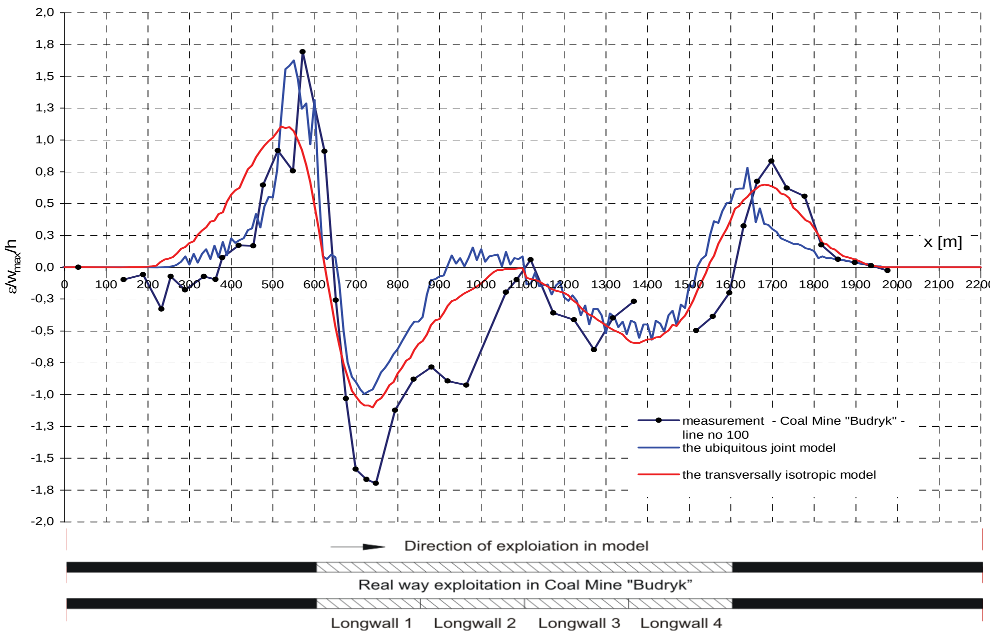

Fig. 7. Horizontal strains of the subsidence trough profile

The value of the extreme negative strains calculated for the ubiquitous joint model equals to –1,00 w_{max}/h for the start-up of exploitation edge and –0,55 w_{max}/h for the end of exploitation edge. In the case of the transversally isotropic model the analogous values of negative strains are equal to –1,10 w_{max}/h and –0,59 w_{max}/h. For comparison, the extreme values of the horizontal strains

[1] Positive strain – strain caused by tensile

[2] Negative strain – strain caused by compression

calculated with the use of the original formulas of Budryk-Knothe theory (Budryk & Knothe, 1950) are equal to ±1,21 w_{max}/h. Taking into account the works of E. Popiołek and J. Ostrowski (Popiołek & Ostrowski, 1980) one can assume that the observed values of horizontal strains are averagely equal to ±0,97 w_{max}/h.

The uneven distribution of the model's surface horizontal deformations is the result of differences in the states of stresses and strains that occur during the formation of the subsidence trough, because the process of formation of the trough's slope over the beginning of the exploitation edge was subject to a significant influence of close exploitation (resulting from the vicinity of both exploitation edges).

The results of computer simulations show that in the case of the ubiquitous joint model, the largest zones of plasticity appeared in layers immediately surrounding the exploited coal seam (Fig. 8). This applies to both the roof and the floor layer. The plasticity zones appeared in the start-up edge of mining exploitation, practically reaching the surface of the model. Plasticity of this part of the model is the reason of larger inclinations within the subsidence trough profile. In subsoil layers, the plasticity of rock layers is caused by strains overrunning the strength parameters of the weakness planes. The plasticity appeared also in the overburden. These zones make up the main cause of unequal distribution of the horizontal strains in the model surface.

A much greater range of plasticity zones, occurring in the region of the longwall 1 and 2 areas, results from the interaction of influences of both exploitation edges. This interaction disappears with the progression of mining exploitation. This is especially visible for longwalls 3 and 4. In their area, plasticity zones are the result of stresses arising in the vicinity of one (moving) exploitation edge.

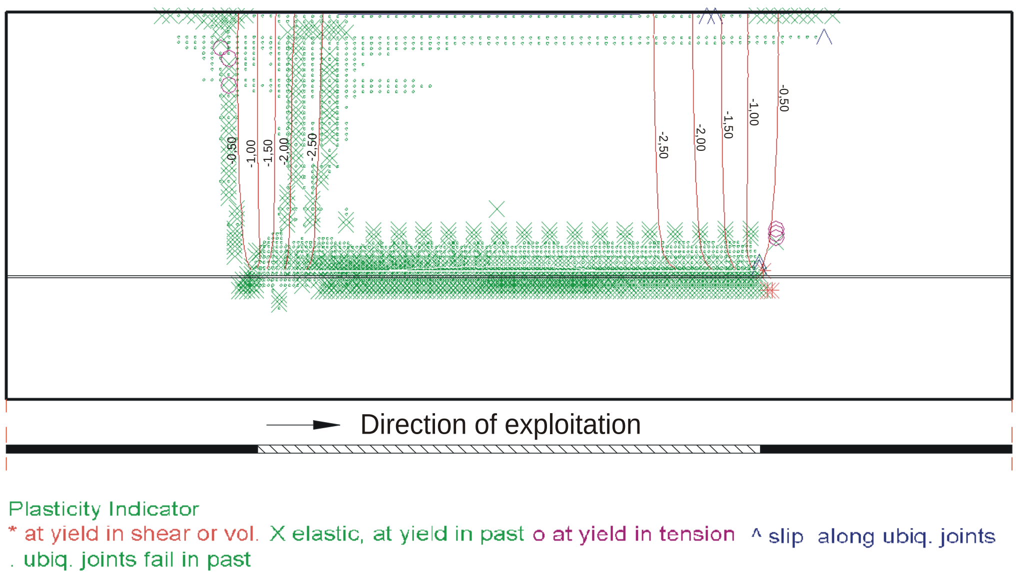

Fig. 8. Subsidence and plasticity zones in the ubiquitous joint model

In the transversally isotropic model, plasticity appeared first of all in the roof layer of the exploited coal seam (Fig. 9). Similarly as in the ubiquitous joint model, the largest plasticity occurred in the start-up of mining exploitation edge area. During the advance of longwall working, the height of plasticity zones undergoes gradual reduction. This allowed for imitating the asymmetry effect of the subsidence trough over the beginning and the end of mining exploitation edges.

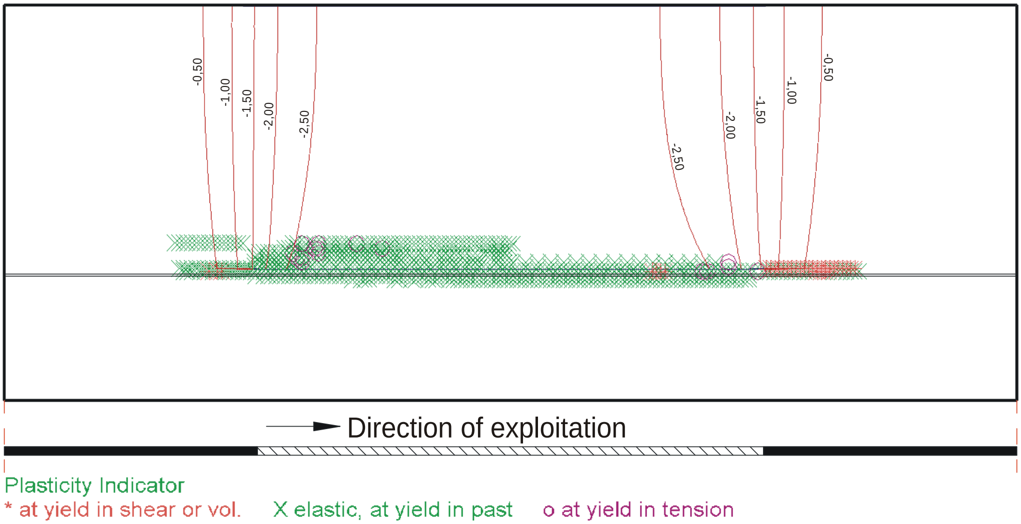

Fig. 9. the subsidence and plasticity zones in the transversally isotropic model

4. Summary and final conclusions

This paper presents the numerical modeling results of mining exploitation influence on the deformations of the mining area surface, with the use of an anisotropic elasto-plastic ubiquitous joint model and the model based on transversely isotropic and plastic isotropic medium. The results of conducted analyzes led to the following conclusions:

The cycle of computer simulations conducted in frames of the work has shown, that both for the ubiquitous joint model and the transversally isotropic model there is a possibility of simultaneous description of both the vertical and horizontal movements of rock mass, caused by mining exploitation. The obtainment of close to reality description of horizontal movements required the introduction of contact planes (*Interface*) for modeling interlayer connections.

The profile of the longwall's asymptotic subsidence trough is, for the discussed models, asymmetric in relation to the excavated field. In these profiles the value of maximum inclination in the beginning of the exploitation edge area is even about several dozens of percent larger than the value of maximum inclination in the end of the exploitation edge area. Similar dependences refer to horizontal strains.

The results of computer simulations show that the use of suitable medium allows for describing the following stages of mining area deformations, including the influence of the mining exploitation direction on the profile of the subsidence trough, which forms over the advancing longwall front.

The unquestionable advantage of the model based on the transversally isotropic medium is the relatively small quantity of strain parameters necessary for calculations and the possibility of easy adaptation of calculation results to the results of geodetic observations (Wesołowski, 2013).

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