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# THE INFLUENCE OF FRICTION FORCES ON LONGITUDINAL WAVE PROPAGATION IN A STUCK DRILL STRING IN A BOREHOLE

## WPŁYW SIŁ TARCIA NA PROPAGACJE FAL WZDŁUŻNYCH W ZABLOKOWANEJ W ODWIERCIE KOLUMNIE RUR WIERTNICZYCH

**Key words**: drill string, release of a stuck drill string, propagation of longitudinal waves, dry friction.

**Abstract** A mathematical model and the computer software for the analysis of dynamic processes occurring in the drilling pipes in the borehole under stuck drill string release by means of an impact mechanism (a jerking device) or a pulse-wave installation, equipped with electric linear pulse motor are presented. The drill string with an impact mechanism, which is inserted over the stuck section after failure, is detected and is activated by lowering and taking the non-stuck upper part of the string by means of the drilling rig drive and is considered as a discrete-continuous mechanical system. As a result of the impact of the hammer on the body of the impact mechanism, wave processes are formed in the drill string, which helps to release the stuck drill string. The influence of friction forces on propagation of longitudinal waves in the drill pipe string is investigated. Practical recommendations are developed regarding the above-mentioned efficiency of drilling for oil and gas.

**Słowa kluczowe:** kolumna rur wiertniczych, oswabadzanie od zablokowania w odwiercie, propagacja fal wzdłużnych, tarcie suche.

**Streszczenie** W artykule przedstawiono model matematyczny i opracowano program komputerowy do analizy procesów dynamicznych, zachodzących w zablokowanej w odwiercie kolumnie rur wiertniczych podczas oswabadzania kolumny za pomocą mechanizmu udarowego (szarpaka) lub urządzenia impulsowo-falowego, wyposażonego w elektryczny liniowy silnik impulsowy. Kolumna rur wiertniczych z mechanizmem udarowym, który wstawia się nad zablokowanym odcinkiem po ujawnieniu awarii i uruchamia się poprzez opuszczanie i podejmowanie górnej części kolumny za pomocą układu napędowego wiertnicy, rozpatruje się jako układ mechaniczny dyskretno-ciągły. Wskutek uderzenia bijaka w korpus mechanizmu udarowego w kolumnie wiertniczej powstają procesy falowe, sprzyjające oswobodzeniu zablokowanej kolumny. Zbadano wpływ sił tarcia na propagacje fal wzdłużnych w kolumnie rur wiertniczych. Opracowano rekomendacje praktyczne, dotyczące podwyższenia efektywności wiercenia otworów na ropę i gaz.

## INTRODUCTION

Provision of the world community with raw materials and fuel-power resources is related with a continuous extension of the drilling activity, especially for gas and oil, thus enabling the improvement of the drilling technology, technical means for geology survey and oil extracting plants. Nowadays, the character of such activity undergoes significant changes because of increase of the average depth of boreholes. Therefore, the improvement of the technical level of the drilling tool, its efficiency, reliability, and life time are very important.

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Modern drilling units are complicated complexes of equipment and constructions that differ significantly both in construction and technical features. Operation of the elements of the drilling complexes is accompanied by the intensive mechanical vibrations, caused by frequent starts and shut downs of the gear rigs, changes of the drill strings resistance forces to movement, the non-uniform flow of washing liquid in a borehole, the non-equilibrium state of the rotating elements and units, the variability of the elastic-dissipative and inertia properties of separate sections, and construction errors in mounting of the equipment and carrying structures. Therefore, a comprehensive study of the drilling machines dynamics is a necessary condition of their rational design and effective operation.

Special attention is paid to the investigation of the strength, stability and oscillations of drilling pipes [**L. 1, 2**] that have a size that may exceed 6000 m and sometime even 10000 m. Static and dynamic interaction of the string and a borehole [**L. 3, 4**] and also the interaction of the drill tool and a stall are widely investigated [**L. 2, 5, 6**]. The influence of the drill string curvature on the stress state of the drilling pipes and boring cases and also on the friction forces of a string with a borehole is analysed [**L. 3, 4**]. The strength and reliability of a threaded joint of the drilling pipes and well-casing are investigated [**L. 7, 8**].

In paper [**L. 1**], a dynamic model of the spatial oscillations of the drill string, using the finite element method, is constructed. Components of the string (drilling pipes, couplings, locks) are considered as a set of finite elements each of which has 12 degrees of freedom. The model considers gyroscopic effects, the interaction of inertia forces arising under torsion and bending, and also the action of gravitational forces. Modal analysis of the drill string is done and peculiarities of the time-free vibrations are illustrated. The string and a well wall interaction have not been considered in this paper. A similar mathematical model for the investigation of the forced vibrations and semi-vibrations of the drill string is presented in paper [**L. 2**]. Investigations of the semi-vibration processes excited by the interaction of a drilling bit and a rock are carried out.

Fundamental paper [**L. 3**] proposes an integrated mathematical model of spatial oscillations of the drill string for the analysis of dynamic phenomena that are observed in reality on technical objects of the gas and oil industry. To write the equations of movement, 6 continuous and at the same time independent degrees of freedom of the cross-sections are used, thus enabling the description of the longitudinal, rotary, and transverse oscillations of the string both in the rectilinear and space curved borehole. Interaction of the drilling tool and a rock is described by the non-linear characteristic of dry friction. The mathematical model is used to investigate the dynamic stability of the drill string in a vertical borehole, taking into account the effect of the axial, angular, and transversal excitations.

In paper [**L. 5**], based on the use of the discrete calculation model with three degrees of freedom, the rotary and translation vibrations of the drill string heavy-weight bottom and also rotary vibrations of the rotor drilling turntable are investigated. Peculiarities of the bit-rock interaction of the rotary and translation vibrations of the heavy-weight part of the string and also vibration damping due to the string and washing liquid interaction as well as the use of the active vibration insulation system are studied. It is shown that, by increasing damping by the drilling mud and by a rational choice of the damping coefficients, it is possible to provide the effective drilling regime and to decrease the rotor table vibration amplitude.

Investigations of the self-vibration phenomena that occur in the drill string, based on the continuum representation of the mechanical system, are presented in paper [**L. 6**]. Boundary conditions at the string bottom part are constructed bit-rock friction contact. Forces that the drilling tool takes during operation excite in the drill string the interrelated longitudinal and torsion vibrations. Discretization of the equations in partial derivatives, describing dynamic processes, is done by the finite element method. Comparative analysis of the calculation results and the results obtained with application of the simplified dynamic models with a finite number of degrees of freedom is carried out. It is found that friction vibrations of the mechanical system occur at the drill string's own frequencies, which are different from the main one.

In paper [**L. 9**], a certain generalization of the popular methods of mathematical modelling of the longitudinal, transverse, and rotary vibrations of the drill string are presented. Models of the bit-rock friction are analysed. Examples of the mathematical models of rotation and longitudinal vibrations of the drill string taking account of the drilling pipes and borehole interaction by the viscous friction forces are presented.

Interaction of the pumping-compressive pipes of the gas underground storage and the well wall by the dry friction forces is represented in detail in the mathematical model of oscillation processes in paper [**L. 4**]. The proposed model is based on the finite element method and considers an insignificant space curving of the borehole axis. The drill string is considered as a rod that has in the non-deformed state a curvilinear and in the operation state – somewhat curved axis due to the borehole axis curvature. Elemental sections of the pumping-compressive pipes under action of internal forces can move in the transverse direction from the location that is coaxial with the borehole only within the borders of a gap between the casing and pumping-compressive pipes. As a result of the pumping-compressive and casing pipe contact, an elastic contact appears at which normal and dry friction forces arise.

The drilling tool should be designed taking account of the specific features of their operation in emergency conditions resulting from the sudden sticking or release of the stuck drill string. To remove the mentioned accidents, the impact and impact-vibration mechanisms as well as the pulse-wave methods are widely used. As a result, the elements of the lifting system experience intensive dynamic loadings. Since the sticking of the drill string is the most wide spread and the most severe type of accidents on the drilling assembly, the development of the effective technical means for the release of the drill strings and the substantiation of their operation conditions is a very important problem in geological survey and oil and gas production industries.

The analysis of the wave phenomena that appear in the drill string under operation is directly related with investigation of the solutions of equations in partial hyperbolic-type derivatives. In book [**L. 10**], a general approach to the mathematical modelling and numerical analysis of the dynamic processes within the hyperbolic boundaries, which in a general case consist of the components involved in the 3-D system which deformation occurs due to wave propagation along the mentioned components, is considered. Bridge constructions, frame structures, drill strings, carbon nanopipes, and vessel stent systems, main transport systems, information networks, and so on can be an example of hyperbolic networks.

The structure of a hyperbolic network is represented by a topographic graph. The analysis of dynamic processes in hyperbolic systems is reduced to solving the non-linear wave equations under sufficiently complex boundary conditions. In this case, both analytical and numerical methods of mathematical analysis, namely, the finite difference method and also the finite element method, are used.

In paper [**L. 11**], the investigations of the resonance oscillations of an elastic rod under effect of harmonic loading taking account of the material relaxation properties and also the environment resistance are carried out with the application of general theoretical approaches. Differential equations of the rod movement were obtained, using the rheological models of Kelvin-Voigt and Maxwell [**L. 12**]. Applying numerical methods, the influence of the relaxation characteristics of material and environment parameters on the character of the system resonance oscillations is explained.

In paper [**L. 13**], the Euler finite-difference model of non-linear longitudinal waves in conical elastic rods is developed using the complex disturbance theory. It is shown that numerical realization of the proposed model allows us to reproduce the evolution of the wave processes and impact phenomena and also to represent typical non-linear properties of the oscillation system with sufficient accuracy.

In paper [**L. 14**], the drill string wave phenomena that can be caused by longitudinal impact of the drill string on the borehole bottom during its lowering-in or a bit jumping in a stall under drilling process are shown. Natural values and natural functions of the mechanical system are studied. The stresses that appear in the drilling pipes due to longitudinal impact are evaluated.

The results of experimental and theoretical investigations of longitudinal oscillations of the drill string of length 8000 m with a compensator under a borehole drilling in a shelf are considered in paper [**L. 15**]. The mathematical model is constructed taking account of the viscous friction in the pipe material. Significant influence of the compensator on the oscillation processes frequency in the drill string under operation is noted.

Paper [**L. 16**] presents the investigations of the loading transfer from the drill string to the bit by exciting the longitudinal and torsion oscillations of the string. A mathematical model of oscillation phenomena realized by the finite element method is constructed. Results of modelling indicate that application of vibrations for the loading transfer allows us to avoid the tool adhesion in a stall and to provide a fluent borehole drilling process. Among the considered methods of oscillation excitations the best effect concerning the decrease of friction and stabilization of the tool operation was obtained in the case of the application of the axial oscillations of the drill string lower part.

In paper [**L. 17**], the investigations of the longitudinal and torsion oscillations of the drill string coupled through the bit-rock interaction were carried out by the continual calculation models of the drill strings. A minimum number of parameters that characterize linearized axial-torsion dynamics of the string as a system with distributed parameters are established. Stability maps in dimensionless parameters that cover a wide spectrum of real parameters of the investigated system are constructed. Recommendations on ensuring the drill strings movement stability that are of practical interest and can be used both at the stage of design and operation are substantiated.

As shown in the articles [**L. 18–20**], by increasing the accuracy of the approximation of unknown functions by which deformations and displacement of elastic systems are described, the efficiency of the analysis of wave processes in longitudinal structures by the finite element method can be considerably improved. Nonlinear finite elements of variable sizes have been built in the mentioned studies, based on which mathematical models and calculation algorithms for dynamic calculations of systems with moving limits or loads are developed, as well as for studying the interaction of the drill column (pipeline) with the environment that could be transported.

Obviously, the problems of dynamics and strength of drill strings include a rather large field of scientific investigations and are of interest to numerous researchers and scientific teams. Despite the fact that significant researches have been done in this field of science, the coupling of a drill string and a borehole walls requires further study and the methods of mathematical modelling of oscillations of the drill string inserted into a borehole – the improvement and clarification.

In this paper, the non-linear continual-discrete mathematical model of dynamic processes in a stuck drill string that appear under drill string release by the pulse-action installations is considered and the influence of dry and viscous friction on longitudinal waves propagation in the string is investigated.

## MATHEMATICAL MODEL OF DYNAMIC PROCESSES IN A STUCK DRILL STRING

The calculation model of a stuck drill string is presented in **Fig. 1a**, where $l$ is a string length; $m_1$ and $m_2$ are masses of drilling tool and a part of the equipment for stuck release, coupled with a string, that are considered as solids; $x$ is a stable longitudinal coordinate with an origin at point $O$, that is a string cross-section centre when no longitudinal deformations of the drilling pipes are present; $u(x, t)$ is movement of a certain transverse cross-section of the string in the direction of coordinate $x$, being a function of this coordinate and time.

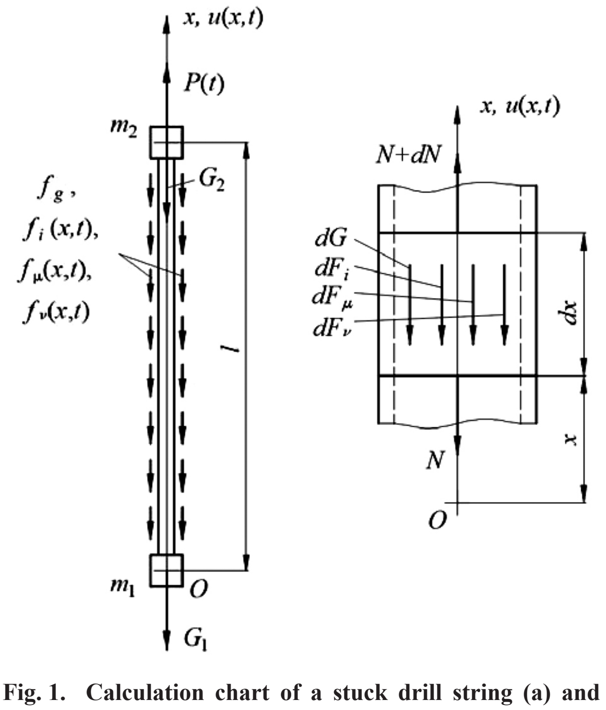


**Fig. 1. Calculation chart of a stuck drill string (a) and loading mode of a string infinitesimal element (b)**

Rys. 1. Schemat obliczeniowy zablokowanej w odwiercie kolumny rur wiertniczych (a) oraz schemat obciążenia nieskończenie małego odcinka kolumny (b)

In the case of the stuck release by a deep-well impact tool (for example, hydraulic impact mechanism), the stuck drill string part is rigidly coupled by the upper part with the installation case. When an on-land installation for sticking release equipped with an electric linear pulse engine is used, the stuck drill string is coupled by the upper part with an installation anchor.

Bearing in mind the relaxation of the environment of the drilling pipes coupling through the borehole walls, let us consider that a string in the state of rest is in equilibrium under action of the distributed forces of own weight and expulsive forces of a string of intensity $f_g$, concentrated weight forces coupled through the string of solids $G_1$ and $G_2$, and also static reaction of a bearing unit $P_s$. During sticking release, the upper part of the drill string undergoes the action, except the static loading of the pulse axial loading $P_d(t)$, as a result of which the wave processes arise in the string accompanied by the action of inertia forces of intensity $f_i(x, t)$, and also by the forces of dry and viscous friction of intensity $f_\mu(x, t)$ and $f_\nu(x, t)$, correspondingly, distributed along the string length. In a general case, the force applied to the upper part of the string is designed as $P(t)$.

The intensity of the distribution of its own weight and expulsive forces acting on the string is expressed by the following dependence:

$$f_g = \rho A g \left(1 - \frac{\rho_0}{\rho}\sin^2\alpha\right)\cos\alpha \qquad (1)$$

where ρ and $\rho_0$ are the density of drilling pipes material and the washing liquid density; $A$ is the string cross-section area; and, α is an averaged angle of the string axis inclination to the vertical.

The intensity of the inertia forces and forces of viscous friction distribution is written as follows:

$$f_i = \rho A \frac{\partial^2 u}{\partial t^2}; \qquad f_\nu = \nu \frac{\partial u}{\partial t} \qquad (2)$$

where $t$ is time; ν is coefficient of viscous friction, numerical value of which is equal to the force of the string viscous resistance to movement in a borehole, that corresponds to the string of a unit length at a unit movement rate.

The intensity of the dry friction forces distribution is determined by the Coulomb. Law, according to which the maximum value of intensity is calculated as follows:

$$f_{\mu\,max} = \frac{\mu|N|}{R} + f_\tau \qquad (3)$$

where μ is coefficient between the string and the borehole wall dry friction; $N$ is the drill string force tension; $R$ is an averaged curvature radius of the borehole axis; $f_\tau$ is the intensity of distribution of boundary dynamic force of the string displacement in a viscous-plastic liquid.

Assuming that the borehole material obeys the rheological model of Kelvin-Voigt, the longitudinal force in the string is written as follows:

$$N = EA\frac{\partial u}{\partial x} + \eta A\frac{\partial^2 u}{\partial x \partial t} \qquad (4)$$

where $E$ and η is the elasticity modulus and the coefficient of internal viscous friction of the borehole material.

Substituting Dependence (4) in Formula (3), produces

$$f_{\mu\ max} = \frac{\mu A}{R}\left|E\frac{\partial u}{\partial x} + \eta\frac{\partial^2 u}{\partial x \partial t}\right| \qquad (5)$$

Let us consider an infinitely small element of the drill string (**Fig. 1b**) that is in a dynamic equilibrium under the elementary external forces action

$$dG = f_g dx\,;\; dF_i = f_i dx\,;\; dF_\mu = f_\mu dx\,;\; dF_\nu = f_\nu dx \qquad (6)$$

in this case,

$$dF_{\mu\ max} = f_{\mu\ max} dx \qquad (7)$$

and also internal forces $N$ and $N+dN$. According to Equation (4), the increment of the longitudinal force is determined as follows:

$$dN = EA\frac{\partial^2 u}{\partial x^2}dx + \eta A\frac{\partial^3 u}{\partial x^2 \partial t}dx \qquad (8)$$

The equation of dynamic equilibrium of an infinitely small string element is written as follows:

$$dN - dF_i = dG + dF_\mu + dF_\nu \qquad (9)$$

in this case,

$$\begin{aligned} &dF_\mu = dN - dF_i - dG\,, \\ &\text{if } \frac{\partial u}{\partial t} = 0\,, \quad |dN - dF_i - dG| \le dF_{\mu\ max}\,; \\ &dF_\mu = dF_{\mu\ max}\operatorname{sign}(dN - dF_i - dG)\,, \\ &\text{if } \frac{\partial u}{\partial t} = 0\,, \quad |dN - dF_i - dG| > dF_{\mu\ max}\,; \\ &dF_\mu = dF_{\mu\ max}\operatorname{sign}\left(\frac{\partial u}{\partial t}\right), \quad \text{if } \frac{\partial u}{\partial t} \ne 0 \end{aligned} \qquad (10)$$

Taking account of correlations (1)–(3), (5), (6), (8) reduce equation (9) to the form

$$a^2\frac{\partial^2 u}{\partial x^2} + b^2\frac{\partial^3 u}{\partial x^2 \partial t} - \frac{\partial^2 u}{\partial t^2} = g^* + f_\mu^* + f_\nu^* \qquad (11)$$

where

$$a^2 = \frac{E}{\rho}\,, \quad b^2 = \frac{\eta}{\rho}\,, \quad g^* = g\left(1 - \frac{\rho_0}{\rho}\sin^2\alpha\right)\cos\alpha\,, \qquad f_\mu^* = \frac{f_\mu}{\rho A}\,, \quad f_\nu^* = \frac{f_\nu}{\rho A} \qquad (12)$$

From correlations (1)–(3), (5), (6)–(8), (10), we obtain the intensity of dry friction forces distribution along the drill string length at arbitrary instant of time

$$\begin{aligned} &f_\mu = EA\frac{\partial^2 u}{\partial x^2} + \eta A\frac{\partial^3 u}{\partial x^2 \partial t} - \rho A\frac{\partial^2 u}{\partial t^2} - f_g\,, \\ &\text{if } \frac{\partial u}{\partial t} = 0\,, \left|EA\frac{\partial^2 u}{\partial x^2} + \eta A\frac{\partial^3 u}{\partial x^2 \partial t} - \rho A\frac{\partial^2 u}{\partial t^2} - f_g\right| \le f_{\mu\ max}\,; \\ &f_\mu = f_{\mu\ max}\operatorname{sign}\left(EA\frac{\partial^2 u}{\partial x^2} + \eta A\frac{\partial^3 u}{\partial x^2 \partial t} - \rho A\frac{\partial^2 u}{\partial t^2} - f_g\right), \\ &\text{if } \frac{\partial u}{\partial t} = 0\,, \left|EA\frac{\partial^2 u}{\partial x^2} + \eta A\frac{\partial^3 u}{\partial x^2 \partial t} - \rho A\frac{\partial^2 u}{\partial t^2} - f_g\right| > f_{\mu\ max}\,; \\ &f_\mu = f_{\mu\ max}\operatorname{sign}\left(\frac{\partial u}{\partial t}\right), \text{ if } \frac{\partial u}{\partial t} \ne 0 \end{aligned} \qquad (13)$$

The boundary condition of the integration of the equation in partial derivatives (11), which should be fulfilled at the lower end of the drill string, is written as

$$\begin{aligned} &EA\left.\frac{\partial u}{\partial x}\right|_{x=0} + \eta A\left.\frac{\partial^2 u}{\partial x \partial t}\right|_{x=0} - m_1\frac{\partial^2 u}{\partial t^2} = G_1\left(1 - \frac{\rho_0}{\rho}\right) - \\ &- Al\rho_0 g\cos\alpha + F_{\mu 1} + \nu_1\frac{\partial u(0,t)}{dt} \end{aligned} \qquad (14)$$

where

$$\begin{aligned} &F_{\mu 1} = EA\left.\frac{\partial u}{\partial x}\right|_{x=0} + \eta A\left.\frac{\partial^2 u}{\partial x \partial t}\right|_{x=0} - m_1\frac{\partial^2 u}{\partial t^2} + \\ &-G_1\left(1 - \frac{\rho_0}{\rho}\right) + Al\rho_0 g\cos\alpha\,, \\ &\text{if } \frac{\partial u(0,t)}{\partial t} = 0\,, \left|EA\left.\frac{\partial u}{\partial x}\right|_{x=0} + \eta A\left.\frac{\partial^2 u}{\partial x \partial t}\right|_{x=0} - m_1\frac{\partial^2 u}{\partial t^2} + \right. \\ &\left. -\nu_1\rho g\left(1 - \frac{\rho_0}{\rho}\right) + Al\rho_0 g\cos\alpha\right| \le F_{\mu 1\ max}\,; \\ &F_{\mu 1} = F_{\mu 1\ max}\operatorname{sign}\left(EA\left.\frac{\partial u}{\partial x}\right|_{x=0} + \eta A\left.\frac{\partial^2 u}{\partial x \partial t}\right|_{x=0} + \right. \\ &\left. -m_1\frac{\partial^2 u}{\partial t^2} - G_1\left(1 - \frac{\rho_0}{\rho}\right) + Al\rho_0 g\cos\alpha\right), \\ &\text{if } \frac{\partial u(0,t)}{\partial t} = 0\,, \left|EA\left.\frac{\partial u}{\partial x}\right|_{x=0} + \eta A\left.\frac{\partial^2 u}{\partial x \partial t}\right|_{x=0} - m_1\frac{\partial^2 u}{\partial t^2} + \right. \\ &\left. -\nu_1\rho g\left(1 - \frac{\rho_0}{\rho}\right) + Al\rho_0 g\cos\alpha\right| > F_{\mu 1\ max}\,; \\ &F_{\mu 1} = F_{\mu 1\ max}\operatorname{sign}\left(\frac{\partial u(0,t)}{\partial t}\right), \text{ if } \frac{\partial u(0,t)}{\partial t} \ne 0 \end{aligned} \qquad (15)$$

Similarly, we can write the boundary condition for the upper end of the drill string in the following way:

$$EA\left.\frac{\partial u}{\partial x}\right|_{x=l} + \eta A\left.\frac{\partial^2 u}{\partial x \partial t}\right|_{x=l} + m_2\frac{\partial^2 u(l,t)}{\partial t^2} = P(t) - G_2 \qquad (16)$$

where

$$P(t) = P_s + P_d(t) \tag{17}$$

in this case,

$$P_d = P_{d\,max}\text{, if } t \le \Delta t\text{; } P_d = 0\text{, if } t > \Delta t \tag{18}$$

So, the analysis of the wave phenomena in a stuck drill string consist in solving the equation in partial derivatives (11) for boundary conditions (14), and (16) taking account of analytical correlations (12), (13), (15), and (17). The pulse loading of a unit of mass $m_2$ present in form (18). The formulated problem is solved in the following sequence: at first discretization of Equation (11) where the space coordinate is used, owing to this, the mathematical model of dynamic process is written in the form of a non-linear system of normal differential equations; after that, the numerical integration of the obtained system is done applying widely used software. As initial conditions use the original values of displacements of the drill string transverse cross-sections $u(x, 0)$ that are easily determined based on the analysis of static equilibrium of an elastic system and also initial velocities of the mentioned cross-sections $\partial u(x, t)/\partial t$, that at $t = 0$ are assumed to be equal zero.

## INVESTIGATION OF THE INFLUENCE OF FRICTION FORCES ON WAVE PROCESSES IN A DRILL STRING

Let us analyse the influence of the internal and external friction forces on wave processes, excited in the drill string by pulse loading on the example of long-length constructions, mounted of pipes with a passage diameter of 114 mm, wall thickness $\delta$ = 10 mm and length $l$ = 2000 m. The internal and external diameters and the area of the pipe transverse cross-section are $D$ = 114.3 mm, $d$ = 94.30 mm, and $A = 3.277 \cdot 10^{-3}$ m$^2$. Mechanical characteristics of the pipe material are $E = 2.1 \cdot 10^{11}$ Pa; $\eta = 7.0 \cdot 10^{7}$ Pa·s; $\rho = 7800$ kg/m$^3$.

Note that drill muds refer mainly to the viscous-plastic liquids, rheological properties of which are described by different models [**L. 21–23**]. In investigations, let us consider that the drilling mud obeys the Bingham equation [**L. 24**]; as a result, the drill string is subjected from the side of mud to the influence of viscous and dry friction forces.

The intensity of the distribution of viscous friction forces is determined by the second relation (2). In this case, the coefficient of viscous friction $\nu = 12\pi\eta_0$, where $\eta_0$ is plastic viscosity of the washing liquid. The value of this parameter is within the range of 1.0 … 60.0 Pa·s.

The maximum value of the intensity of dry friction between the drill string and the drill mud is calculated as component $f_\tau$ in Formula (3), in this case, $f_\tau = 4\pi d\tau_0$, where $\tau_0$ is the boundary dynamic shear force with the washing liquid value ranging from 2.0 to 20.0 Pa. The coefficient of dry friction between the drill string and the borehole wall μ can change within the range 0.05 …0.2. The drill mud density $\rho_0$ is assumed to be 1300.0 kg/m$^3$.

In the case when the stuck drill string is released by means of a pulse-wave installation, the masses of the drilling tool $m_1$ and the linear pulse motor anchor $m_2$ are assumed to be 40 kg and 750 kg, respectively. The averaged values of the inclination angle of the drill string axis to the vertical and the curvature radius of the borehole axis are set as $\alpha = 2°$; $R$ = 200 … 10000 m.

Pulse loading of the mechanical system is determined according to (17) and (18). In this case, $P_s$ = 425.26 kN; $P_{d\,\max}$ = 400.00 kN; and, $\Delta t$ = 0.02 s.

Wave process excited in the drill string under action of the minimum friction forces ($\eta = 7.0 \cdot 10^7$ Pa·s; $\eta_0$ = 1.0 Pa·s; $\tau_0$ = 2 Pa; $\mu$ = 0.05) is illustrated in **Figs. 2–4**.

A stepwise character of the plots of the motion of the clamped string transverse cross-sections (**Fig. 2**) is explained by the fact that the dynamic process is affected by not only a direct deformation wave caused by alternate reflection of the excited wave from this or that edge of the long-length construction. Steps on the curves of motion are formed under movement of the direct or reflected deformation wave through the given transverse cross-section of the string. So, to provide a translation motion of the stuck drill string, it is required that the excited deformation wave moves along the whole drill string.

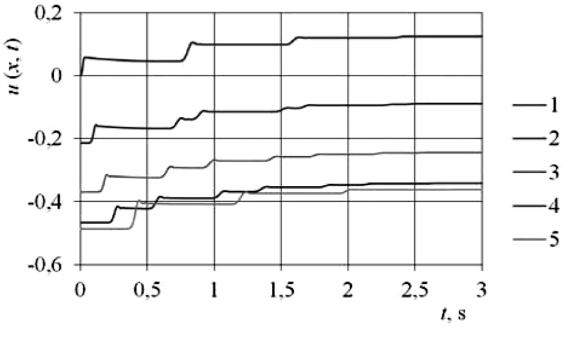


**Fig. 2. Dependences of the drill string transverse cross-sections movement on time: 1 – $x$ = 0; 2 – $x$ = 800 m; 3 – $x$ = 1200 m; 4 – $x$ = 1600 m; 5 – $x$ = 2000 m**

Rys. 2. Zależności przemieszczeń przekrojów poprzecznych kolumny rur wiertniczych od czasu: 1 – $x$ = 0; 2 – $x$ = 800 m; 3 – $x$ = 1200 m; 4 – $x$ = 1600 m; 5 – $x$ = 2000 m

Wave phenomena in the drill string are represented also on the plots of movement rates of the drill string transverse cross-sections (**Fig. 3**). The influence of the friction forces on dynamic processes conditions the clearly expressed oscillations damping. The deformation wave passes the intermediate cross-section

(**Fig. 3b**) twice quicker than it appears on this or that end of the string (**Figs. 3a, c**). The amplitude value of the final cross-section movement rate can exceed the corresponding value of the intermediate cross-section propagation rate due to the direct and the backward deformation wave superpositions at the ends of the long-length construction.

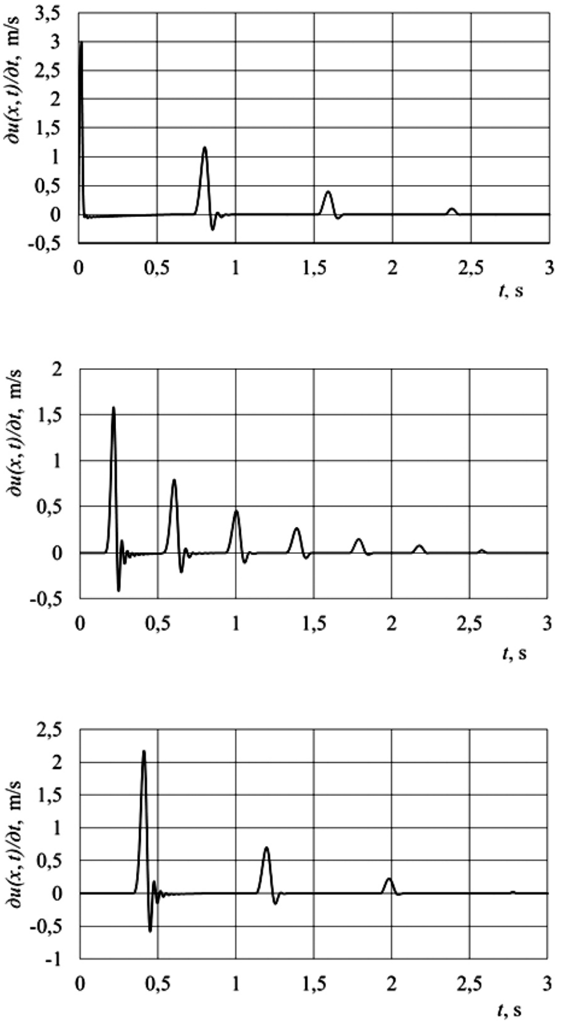


**Fig. 3. Dependence of the drill string transverse cross-section movement rate on time: *a* – *x* = 0; *b* – *x* = 1000 m; *c* – *x* = 2000 m**

Rys. 3. Zależności prędkości ruchu przekrojów poprzecznych kolumny rur wiertniczych od czasu: *a* – *x* = 0; *b* – *x* = 1000 m; *c* – *x* = 2000 m

The maximum longitudinal force that appears in the upper end of the cross-section ($x = l$) of the drill string is 818.0 kN (**Fig. 4**). Its static component is equal to 412.9 kN and a dynamic one – 405.1 kN, i.e. it is close to the pulse loading amplitude. When the deformation wave moves from the upper end of the drill string to the lower part the amplitude of the longitudinal force decreases significantly because of the oscillation energy dissipation. For steels that are used for the manufacture of the drill strings the yield strength is 380, 500, 550, 650, and 750 MPa, while the boundary longitudinal force of the pipe with a passage diameter of 114 mm and a wall strength of 10 mm is 1245, 1638, 1802, 2130, and 2458 kN, respectively. This proves the expediency of the use in engineering practice of the pipes, the material of which has the yield strength not less than 650 MPa, thus enabling an increase in the drill string pulse loading during mitigation of accidents.

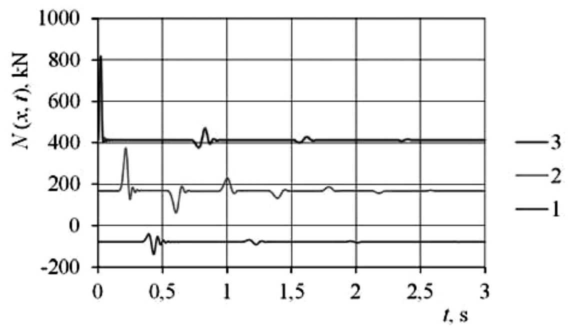


**Fig. 4. Dependence of longitudinal forces in the drill string transverse cross-sections on time: 1 – *x* = 0; 2 – *x* = 1000 m; 3 – *x* = 2000 m**

Rys. 4. Zależności sił wzdłużnych w przekrojach poprzecznych kolumny rur wiertniczych od czasu: 1 – *x* = 0; 2 – *x* = 1000 m; 3 – *x* = 2000 m

As seen from Dependence (3), the intensity of the distribution of the dry friction maximum forces, caused by the drill string coupling through the borehole wall is directly proportional to $\mu/R$. With this ratio increase within practically instantiated limits, the movement of the string cross-section significantly decreases as well as the distance at which the elastic deformation wave, excited by the axial pulse loadings, propagates (**Table 1**). In this case, the dynamic force component in the transverse cross-section located immediately at a point of pulse loading application, decreases by an

**Table 1. Movement of transverse cross-sections of the drill strings (in mm) as a function of $\mu/R$**

Tabela 1. Przemieszczenia przekrojów poprzecznych kolumny wiertniczej (w mm) jako funkcji stosunku $\mu/R$

| **μ/R, m$^{-1}$** | ***x*, m** | | | | | |
|---|---|---|---|---|---|---|
| | **0** | **400** | **800** | **1200** | **1600** | **2000** |
| $0.5\cdot10^{-5}$ | 125.1 | 125.0 | 125.1 | 125.1 | 125.1 | 125.1 |
| $0.2\cdot10^{-4}$ | 112.9 | 113.1 | 113.4 | 113.5 | 113.6 | 113.9 |
| $0.5\cdot10^{-4}$ | 94.50 | 94.81 | 95.32 | 95.69 | 95.80 | 96.11 |
| $0.1\cdot10^{-3}$ | 71.93 | 72.58 | 73.71 | 75.37 | 78.44 | 82.01 |
| $0.2\cdot10^{-3}$ | 54.98 | 52.87 | 54.26 | 55.79 | 57.26 | 58.61 |
| $0.3\cdot10^{-3}$ | 41.47 | 40.14 | 41.55 | 43.21 | 46.14 | 55.63 |
| $0.4\cdot10^{-3}$ | 30.14 | 29.64 | 30.99 | 32.72 | 41.23 | 54.78 |
| $0.5\cdot10^{-3}$ | 20.35 | 20.12 | 21.62 | 24.74 | 37.65 | 53.98 |
| $0.6\cdot10^{-3}$ | 11.89 | 11.73 | 13.23 | 19.97 | 34.29 | 53.20 |
| $0.7\cdot10^{-3}$ | 5.017 | 5.217 | 7.464 | 15.96 | 31.11 | 52.44 |
| $0.8\cdot10^{-3}$ | 0.356 | 1.559 | 4.215 | 12.37 | 28.14 | 51.71 |
| $0.9\cdot10^{-3}$ | 0.000 | 0.000 | 1.705 | 9.223 | 25.33 | 51.00 |
| $1.0\cdot10^{-3}$ | 0.000 | 0.000 | 0.083 | 6.502 | 22.71 | 50.30 |

insignificant value. However, with the increase in the distance from the mentioned point to the transverse cross-section, the decrease of the dynamic component of the longitudinal force is more pronounced with the increase of $\mu/R$ (**Table 2**). In the cross-sections that are not reached by the dynamic component of the longitudinal force is equal to zero.

**Table 2. Maximum longitudinal forces in the transverse cross-sections of the drill string (in kN) as a function of $\mu/R$**

Tabela 2. Maksymalne siły wzdłużne w przekrojach poprzecznych kolumny wiertniczej (w kN) jako funkcji stosunku $\mu/R$

| $\mu/R$, $m^{-1}$ | *x*, m | | | | | |
|---|---|---|---|---|---|---|
| | 0 | 400 | 800 | 1200 | 1600 | 2000 |
| $0.5\cdot10^{-5}$ | 37.76 | 164.6 | 190.1 | 224.5 | 275.6 | 405.1 |
| $0.2\cdot10^{-4}$ | 37.21 | 160.5 | 186.4 | 221.5 | 273.7 | 405.0 |
| $0.5\cdot10^{-4}$ | 36.14 | 152.5 | 179.0 | 215.5 | 270.1 | 404.9 |
| $0.1\cdot10^{-3}$ | 34.25 | 139.7 | 167.1 | 205.7 | 264.0 | 404.6 |
| $0.2\cdot10^{-3}$ | 30.02 | 115.9 | 144.3 | 186.7 | 251.9 | 403.9 |
| $0.3\cdot10^{-3}$ | 25.58 | 94.16 | 123.1 | 168.3 | 240.0 | 403.1 |
| $0.4\cdot10^{-3}$ | 21.69 | 74.22 | 102.9 | 150.5 | 228.2 | 402.4 |
| $0.5\cdot10^{-3}$ | 18.62 | 56.03 | 83.97 | 133.4 | 216.7 | 401.6 |
| $0.6\cdot10^{-3}$ | 15.57 | 39.31 | 66.08 | 116.9 | 205.2 | 400.7 |
| $0.7\cdot10^{-3}$ | 11.59 | 24.03 | 48.95 | 100.9 | 193.9 | 399.9 |
| $0.8\cdot10^{-3}$ | 4.501 | 10.41 | 32.68 | 85.32 | 182.8 | 399.0 |
| $0.9\cdot10^{-3}$ | 0.000 | 0.000 | 17.10 | 70.10 | 171.7 | 398.1 |
| $1.0\cdot10^{-3}$ | 0.000 | 0.000 | 2.858 | 55.27 | 160.8 | 397.2 |

Rheological characteristics of the drill mud have significant influence on the wave processes occurring in the drill string (**Tables 3** and **4**). With the increase in the plastic viscosity of the washing liquid $\eta_0$ and in the limiting dynamic shear stress $\tau_{0,}$ both the movement of the transverse cross-sections of the string (**Table 3**) and dynamic components of longitudinal forces at the transverse cross-sections remote from the point of dynamic loading application, decrease significantly (**Table 4**).

**Table 3. Movement of transverse cross-section of the drill string (in mm) as a function of the rheological characteristics of the drill mud**

Tabela 3. Przemieszczenia przekrojów poprzecznych kolumny wiertniczej (w mm) jako funkcji charakterystyk reologicznych płuczki wiertniczej

| $\eta_0$, Pa·s | $\tau_0$, Pa | *x*, m | | | | | |
|---|---|---|---|---|---|---|---|
| | | 0 | 400 | 800 | 1200 | 1600 | 2000 |
| 1 | 2.0 | 125.1 | 125.0 | 125.1 | 125.1 | 125.1 | 125.1 |
| 5 | 2.75 | 26.59 | 25.31 | 25.29 | 32.73 | 43.51 | 55.16 |
| 10 | 3.5 | 8.521 | 9.084 | 10.65 | 17.82 | 30.92 | 51.76 |
| 15 | 5.25 | 2.068 | 2.897 | 5.216 | 10.03 | 22.40 | 48.84 |
| 20 | 7.0 | 0.000 | 0.3418 | 2.244 | 6.197 | 16.54 | 46.30 |
| 25 | 8.75 | 0.000 | 0.000 | 0.8353 | 4.135 | 12.42 | 44.04 |
| 30 | 10.5 | 0.000 | 0.000 | 0.1700 | 2.711 | 9.471 | 42.01 |
| 35 | 12.25 | 0.000 | 0.000 | 0.000 | 1.726 | 7.334 | 40.18 |
| 40 | 14.0 | 0.000 | 0.000 | 0.000 | 1.035 | 6.062 | 14.60 |

**Table 4. Maximum longitudinal forces in the transverse cross-sections of the drill string (in kN) as a function of the rheological characteristics of the drill mud**

Tabela 4. Maksymalne siły wzdłużne w przekrojach poprzecznych kolumny wiertniczej (w kN) jako funkcji charakterystyk reologicznych płuczki wiertniczej

| $\eta_0$, Pa·s | $\tau_0$, Pa | *x*, m | | | | | |
|---|---|---|---|---|---|---|---|
| | | 0 | 400 | 800 | 1200 | 1600 | 2000 |
| 1 | 2.0 | 37.76 | 164.6 | 190.1 | 224.5 | 275.6 | 405.1 |
| 5 | 2.75 | 14.01 | 66.47 | 95.80 | 141.17 | 216.77 | 396.9 |
| 10 | 3.5 | 4.282 | 22.79 | 42.74 | 81.63 | 162.9 | 387.1 |
| 15 | 5.25 | 1.607 | 7.831 | 19.72 | 48.56 | 124.0 | 377.8 |
| 20 | 7.0 | 0.000 | 2.777 | 9.204 | 29.62 | 95.62 | 368.9 |
| 25 | 8.75 | 0.000 | 0.000 | 4.100 | 18.43 | 74.70 | 360.4 |
| 30 | 10.5 | 0.000 | 0.000 | 1.730 | 11.63 | 58.98 | 352.3 |
| 35 | 12.25 | 0.000 | 0.000 | 0.108 | 7.361 | 47.09 | 344.5 |
| 40 | 14.0 | 0.000 | 0.000 | 0.000 | 5.004 | 37.95 | 337.1 |

## CONCLUSIONS

1. A non-linear continual-discrete mathematical model of dynamic processes that appear under the stuck drill string release, using pulse installations with a detailed account of the internal and external friction in mechanical system, is developed. The algorithm of the problem solution is that, at the initial stage, the equation discretization in partial derivatives by the space coordinate is done, thus reducing the mathematical model to the non-linear system of natural differential equations, after that numerical integration of the obtained system is done using the widely used software.
2. As the investigation results indicate the multi-component friction that arises in the material and joints of the drill string and also in the areas of immediate contact of the string and the borehole wall as well as on the surfaces of the drill string and the drill mud interaction has a significant influence on the longitudinal waves propagation in the drill string and significantly decreases the efficiency of activities on accident elimination.
3. The investigation results showed the effectiveness of application in engineering practice of the pipes which material has the yield strength not less than 650 MPa that will enable the increase in the pulse loading of the drill string during accidents elimination. The effectiveness of the stuck drill string release can be improved by a rational choice of hydraulic impact mechanisms and pulse-wave installations during the stuck drill string release.

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