Economical Optimization of Capacitor Placement for Large-Scale Practical Distorted Distribution Network

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Summary: This paper presents the optimization of large-scale practical distorted distribution network for maximum losses reduction and voltage profile improvement based on two-stage methodology for capacitor placement problem. In the first stage, a fuzzy expert system (FES) is used to find the optimal capacitor locations, and in the second stage, a selective particle swarm optimization (SPSO) is used to find the optimal capacitor sizing. The problem is posed as an optimization problem with objective to minimize the total cost of power and energy losses and capacitor banks including constraints for bus voltage and total harmonic distortion (THD) limits. Simulation results show the benefits from optimization and the effect of harmonics on optimal capacitor placement.

Key words: *Capacitor placement, distribution network, losses reduction, selective particle swarm optimization*

1. INTRODUCTION

Capacitors are widely used in distribution systems to achieve the power and energy losses reduction, power factor correction, system capacity release and to maintain a voltage profile within permissible limits. The capacitor placement problem involves determining the locations, sizes, and number of capacitors to be installed in a distribution system such that the maximum benefits are achieved while all operational constraints are satisfied at different load levels. This problem has been extensively researched over the past several decades. In the 50's, one of the early approaches based on analytical methods was presented [1]. In the 80's, more rigorous approaches were suggested by Grainger and Lee [2], Salama et al. [3], and Baran and Wu [4]. In the 90's and beyond, artificial intelligence-based algorithms were frequently introduced [5]–[9].

Nowadays, harmonic distortion level is steadily increasing in distribution systems due to proliferation of nonlinear loads. Capacitors significantly influence on the propagation of system harmonics and could cause harmonic resonance. Therefore, the optimal selection and placement of capacitor banks must be integrated with the estimation of harmonic levels to avoid excessive harmonic distortion.

In this paper, a two-stage approach based on fuzzy expert system (FES) and selective particle swarm optimization (SPSO) is proposed to select the optimal capacitor placement for practical distorted distribution network in the city of Mariupol, Ukraine. The objective function is formulated to minimize the total cost of losses and investments, with constraints including the bus voltage and total harmonic distortion (THD) limits.

2. PROBLEM FORMULATION

For present realistic distribution system, the objective is to minimize the total cost which is equivalent to maximization of profitability. The annual benefits are obtained from the savings produced by reduction of energy losses and avoiding the costs due to investment deferral in the expansion of network which can be verified as a reduction of maximum power provided that bus voltages and their corresponding THD will be maintained within prescribed values. Eqs. (1)-(4) represent the objective function whereas constraints are presented in (5)–(7).

$$\text{Minimize} \quad F = K^p P_{loss,1} + \sum_{l=1}^{L} K^e E_{loss,l} + K^c \tag{1}$$

$$P_{loss,l} = \sum_{h=1}^{H} \left(\sum_{j=1}^{m-1} P^h_{loss,l(j,j+1)} \right) \tag{2}$$

$$E_{loss,l} = T_l P_{loss,l} \tag{3}$$

$$K^c = \sum_{j=1}^{J} (k^c_f u^l_{fj} + k^c_s u^l_{sj}) \tag{4}$$

The first term in (1) corresponds to the cost of active power losses, where the power losses $P_{loss,1}$ are calculated at the maxi-mum load level by (2) considering $l=1$, and K^P represents the equivalent annual cost per unit of power losses. $P^h_{loss,l(j,j+1)}$ is the h-th harmonic power losses of the branch between buses j and $j+1$ at the l-th load level; m is the number of buses; H is the upper limit of considered harmonic order. The second term corresponds to the cost of energy per year, where the energy losses $E_{loss,l}$ are calculated at the different load levels l by (2) and (3), and K^e represents the equivalent annual cost per unit of energy losses. L is the number of load levels; T_l is the load duration at the l-th level. The third term represents the capacitors cost calculated by (4), where u^l_{fj} and u^l_{sj} are the sizes of fixed and switched type capacitors, respectively, which placed at the l-th load level and j-th bus. k^c_f and k^c_s are the standard capacitor costs for fixed and switched type capacitors, respectively; J is the set of candidate buses for capacitor placement.

For each load level, all the following constraints should be achieved.

1. Bus voltage constraint

$$U_{j\min} \le \left|U_j\right| \le U_{j\max} \quad (5)$$

where $U_{j\min}$ and $U_{j\max}$ are the minimum and maximum permissible rms voltages of bus j, respectively.

2. Harmonic constraint

$$THD_{jl} \le THD_{\max} \quad (6)$$

where THD_{jl} represents the total harmonic distortion for node j at load level l; $THD_{\max}$ is the maximum allowable total harmonic distortion.

3. VAR constraint

$$\sum_{j=1}^{J} Q_j^c \le Q_t \quad (7)$$

where Q_j^c represents the shunt capacitor size placed at bus j; Q_t is the total reactive power demand.

3. PROPOSED ALGORITHM

SPSO was presented in [10] as a simple modification of the binary PSO to search in a selected space. In basic PSO, the d-dimensional search space is modeled via position, velocity, best previous position for each particle (*i-th particle*) and best position for all particles that are represented by vectors described as $X_1=[x_{i1}, x_{i2}, \dots, x_{id}]$, $V_i=[\upsilon_{i1}, \upsilon_{i2}, \dots, \upsilon_{id}]$, $PB_i=[pb_{i1}, pb_{i2}, \dots, pb_{id}]$, and $GB_i=[gb_{i1}, gb_{i2}, \dots, gb_{id}]$, respectively. At iteration k the velocity and position for d-dimension of i-th particle are updated by (8) and (9) respectively:

$$\upsilon_{id}^{k+1} = w\upsilon_{id}^{k} + c_1 r_1 (pb_{id}^{k} - x_{id}^{k}) + c_2 r_2 (gb_{d}^{k} - x_{id}^{k}) \quad (8)$$

$$x_{id}^{k+1} = x_{id}^{k} + \upsilon_{id}^{k+1} \quad (9)$$

where $i=1, 2, \dots, n$; n is the set of particles in swarm (i.e. “population”) described as $pop=[X_1, X_2, \dots, X_n]$; w is the inertia weight; c_1 and c_2 are the acceleration constants; r_1 and r_2 are the two random values in range [0,1].

In SPSO, the sigmoid transformation is presented by (10), and the i-th coordinate of each particle’s position at a dimension d is a selective value, which updated by (11).

$$sigmoid(\upsilon_{id}^{k+1}) = dn \frac{1}{1+\exp(-\upsilon_{id}^{k+1})} \quad (10)$$

$$x_{id}^{k+1} = \begin{cases} s_{d1}, \text{ if } sigmoid(\upsilon_{id}^{k+1}) < 1 \\ s_{d2}, \text{ if } sigmoid(\upsilon_{id}^{k+1}) < 2 \\ s_{d3}, \text{ if } sigmoid(\upsilon_{id}^{k+1}) < 3 \\ \dots\dots\dots\dots \\ s_{dn}, \text{ if } sigmoid(\upsilon_{id}^{k+1}) < dn \end{cases} \quad (11)$$

where $s_{d1}, s_{d2}, s_{d3}, \dots, s_{dn}$ are the selected values in dimension d. Velocity values are restricted to some minimum and maximum values $[V_{\min}, V_{\max}]$ by means of (12). Equation (13) is used to avoid invariability of the value of i-th particle velocity in a d-dimension at maximum or minimum values and force each particle going through the search space.

$$\upsilon_{id}^{k+1} = \begin{cases} V_{\max}, \text{ if } \upsilon_{id}^{k+1} > V_{\max} \\ \upsilon_{id}^{k+1}, \text{ if } \left|\upsilon_{id}^{k+1}\right| \le V_{\max} \\ V_{\min}, \text{ if } \upsilon_{id}^{k+1} < V_{\min} \end{cases} \quad (12)$$

$$\upsilon_{id}^{k+1} = \begin{cases} rand \times \upsilon_{id}^{k+1}, \text{ if } \left|\upsilon_{id}^{k+1}\right| = \left|\upsilon_{id}^{k}\right| \\ \upsilon_{id}^{k+1} \quad \text{otherwise} \end{cases} \quad (13)$$

4. IMPLEMENTATION OF THE PROPOSED ALGORITHM

Based on the structure of SPSO, there are three main steps for proposed algorithm:

1) specifying the number of dimensions;
2) finding the search space for each dimension;
3) applying SPSO to find the optimal capacitor placement and sizing.

A. Specifying the Number of Dimensions

The number of dimensions equals the number of candidate buses for capacitor placement. In this paper, the candidate buses are identified by FES as given in [7] and [9]. In [7] Ng *et al*. applied FES to the capacitor placement problem by using fuzzy approximate reasoning. Voltage and power losses indices of the distribution system nodes were modeled by membership functions and FES containing a set of heuristic rules performs inferencing to determine a capacitor placement suitability index of each node. Capacitors are placed on the nodes with the highest suitability. In [9] the proposed in [7] method was adopted to determine the optimal capacitor locations. The method proposed in [7] and [9] can be summarized in the following steps.

1) Obtain the real and reactive power losses for the original system using load flow.
2) Compensate the total reactive load at every node of the distribution network.
3) Apply the load flow solution to the compensated system in step 2 to obtain the power losses reduction for every node.
4) The losses reductions are then linearly normalized into a [0, 1] range with the largest losses reduction having a value of 1 and the smallest one having a value of 0. Power losses index (PLI) value for n-th node can be obtained by following equation [9]:

$$PLI_{(n)} = \frac{Losses\ reduction_{(n)} - Losses\ reduction_{(\min)}}{Losses\ reduction_{(\max)} - Losses\ reduction_{(\min)}} \quad (14)$$

5) For determining the suitability of capacitor placement at particular node, a set of multiple-antecedent fuzzy rules have been established. The inputs to the rules are the p.u. nodal voltage and power losses reduction indices, and the output is the suitability of capacitor placement.

B. Finding the Search Space for Each Dimension

The search space for each dimension is a set of standard capacitor sizes.

C. Applying SPSO to Find the Optimal Capacitor Placement and Sizing

After specifying the number of dimensions and finding the search space for each dimension, SPSO would be used to find the optimal solution from the search space for each dimension by (8), (12), (13), (10) and (11), respectively.

5. SIMULATION RESULTS FOR PRACTICAL DISTRIBUTION SYSTEM

Fig. A1 shows the simplified single line diagram of 6 kV practical distribution network supplying the eastern part of city Mariupol, Ukraine. It consists of 37 feeders connected to three substations. These feeders contain 274 buses and 284 branches (273 of the branches are normally closed and 11 are normally opened). The line and load data are given in the Tables A1 and A2, respectively. For this practical network the optimal capacitor placement has been evaluated for two cases: harmonic frequencies are ignored (Case A) and harmonic frequencies are taken into consideration (Case B).

A. Main Data and Assumptions

Before starting network simulation, the main data and assumptions approved by measurements on four feeders with highest power losses and voltage drop can be summarized as follows.

1) The system is three-phase balanced system.
2) The base voltage is 6 kV whereas the three substations voltage magnitudes are set to 6.35 kV (about 1.06 p.u.).
3) There are three load levels represented as a percentage of the transformers ratings at each load point. These percentages are 26%, 49% and 91% for light, medium and heavy load levels, respectively.
4) The time period T representing the number of hours in one year (8760 hours) is divided into three intervals for three load levels. These intervals are 1752, 5256 and 1752 hours for light, medium and heavy load levels, respectively.
5) The power factor is equal to 0.85.
6) Harmonic generation is solely from the substation voltage supply.
7) The substation voltage contains 1.5%, 3.5% and 1.5% of 3-rd, 5-th and 7-th harmonics, respectively, resulting in the $THD = 3.9\%$. These are the average values of voltage harmonics that obtained by measurements at the substations.
8) The load is balanced.
9) The mutual coupling is ignored.
10) Smallest capacitor size is $Q_{c0} = 50$ kvar whereas maximum capacitor size is $Q_{c\max} = 800$ kvar .
11) The switched type capacitors are assumed to be with 50 kvar step.
12) The standard capacitor costs are presented in Table 1 (these prices assumed to be for fixed and switched type capacitors) while Table 2 shows the cost per kvar for capacitor sizes which used in the simulations. The data in Table 2 were received assuming the 10-years lifetime for capacitors.
13) The cost per unit of power losses $K^p = 168$ $/kW.
14) The cost per unit of energy losses $K^e = 0.035$ $/kW.
15) The dimensions for capacitor placement are obtained by FES as indicated in section IV-A. From FES the number of dimensions representing the candidate buses for capacitor placement is equal to 60. These candidate buses are [13 14 15 27 29 30 31 32 33 36 38 39 41 43 45 95 96 97 98 99 100 101 102 103 104 105 106 107 108 109 110 116 119 126 128 129 130 131 132 133 134 135 136 137 138 156 157 158 159 160 170 171 172 173 174 175 176 177 178 179 180 181 182 183 184 185 187 188 190 191 192 193 194 216 226 228 229 230 231 232 234 236 237 238 239 240 241 244 245 246 247 248 249 250 251].
16) The search space for each dimension is a set of standard capacitors as shown in Table 2.

B. Before Optimization

Based on the above data and assumptions, the total power, energy and losses at different load levels before optimization are shown in Table 3. Maximum demand for present network is equal to 62924.23 kW. About 91% of this demand is used to meet the needs of customers (in addition to power losses in the low-voltage networks and distribution transformers that

Table 1. Cost of standard capacitor sizes.

Size (kvar)	50	100	200	300	400	500	600	800
Cost ($)	1125	1775	2875	4000	4966	5938	6788	8700

Table 2. Cost per kvar for capacitor sizes.

Size (kvar)	50	100	150	200	250	300	350	400
Cost ($/kvar)	2.25	1.78	1.93	1.44	1.60	1.33	1.46	1.24
Size (kvar)	450	500	550	600	650	700	750	800
Cost ($/kvar)	1.35	1.19	1.28	1.13	1.22	1.22	1.29	1.09

Table 3. Total power, energy and losses for different load levels before optimization.

Load Level	Light	Medium	Heavy
Total power (kW)	17978.35	33882.28	62924.23
Load level period (hours/year)	1752	5256	1752
Total energy (kWh)	31498069	178085237	110243242
Total energy per year (kWh)	319826548.4		
Power losses (kW)	378.52	1344.43	4637
Energy losses (kWh)	663159.9	7066315.8	8123988.2
Total energy losses per year (kWh)	15853464		

Table 4. Simulation results.

Parameter	Base Case	Case A	Case B
Total capacitor sizes (kvar)	0	13000	12700
Minimum voltage (p.u.)	0.79	0.9	0.9
Maximum voltage (p.u.)	1.06	1.06	1.06
Maximum THD (%)	3.9	9	4.99
Power losses (kW)	4637	3804.59	3901.68
Energy losses per year (kWh)	15853464	13016471	13760130
Power losses (%)	7.4	6	6.2
Energy losses per year (%)	5	4	4.3
Power losses cost ($/year)	779012.57	639170.4	655481.7
Energy losses cost ($/year)	554871.24	455576.5	481604.5
Capacitor cost ($/year)	0	17213.3	15767.8
Total cost ($/year)	1333883.8	1111960.2	1152854
Benefits ($/year)	0	221923.6	181029.8
Benefits (%)	0	16.6	13.6
Investment cost for 10 years ($)		172133	157678
Power saving per year (kW)		832.39	735.3
Energy saving per year (kWh)		2836993	2093334
Power saving per year ($)		139841.5	123530.4
Energy saving per year ($)		99294.75	73266.69
Annual saving ($/year)		239136.25	196797.1
Payback period (year)		0.72	0.8
Remaining period (year)		9.28	9.2
Total saving for remaining period ($)		2219184.4	1810533.3

not incorporated into proposed SPSO algorithm) while 7.4% of this demand represents the power losses in conductors. On the other hand, the annual energy delivered to customers at three load levels is equal to 319826548.4 kWh while about 5% of this energy represents the energy losses in conductors. The minimum rms voltage is 0.79 p.u. and the maximum THD is 3.9%.

C. After Optimization

The simulation results for different cases are shown in Table 4 while Figs. 1 and 2 present the proposed capacitor sizes to be installed in the candidate buses at different load levels for Cases A and B. It can be seen that the total installed capacitor sizes in both cases are almost the same but the

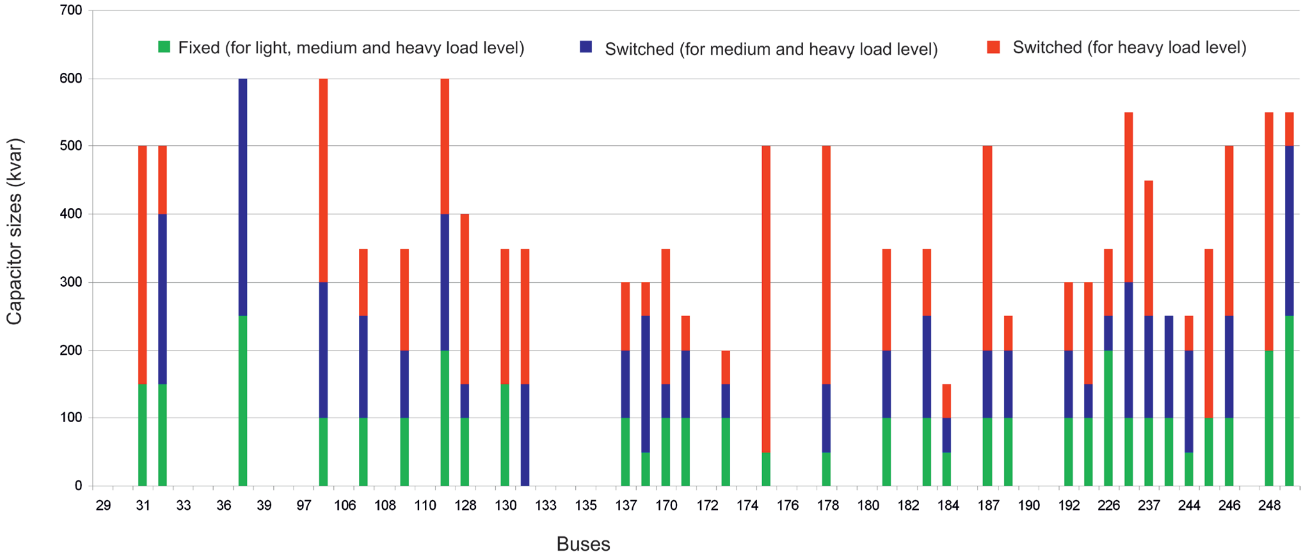

Figure 1. Optimal capacitor placement for Case A

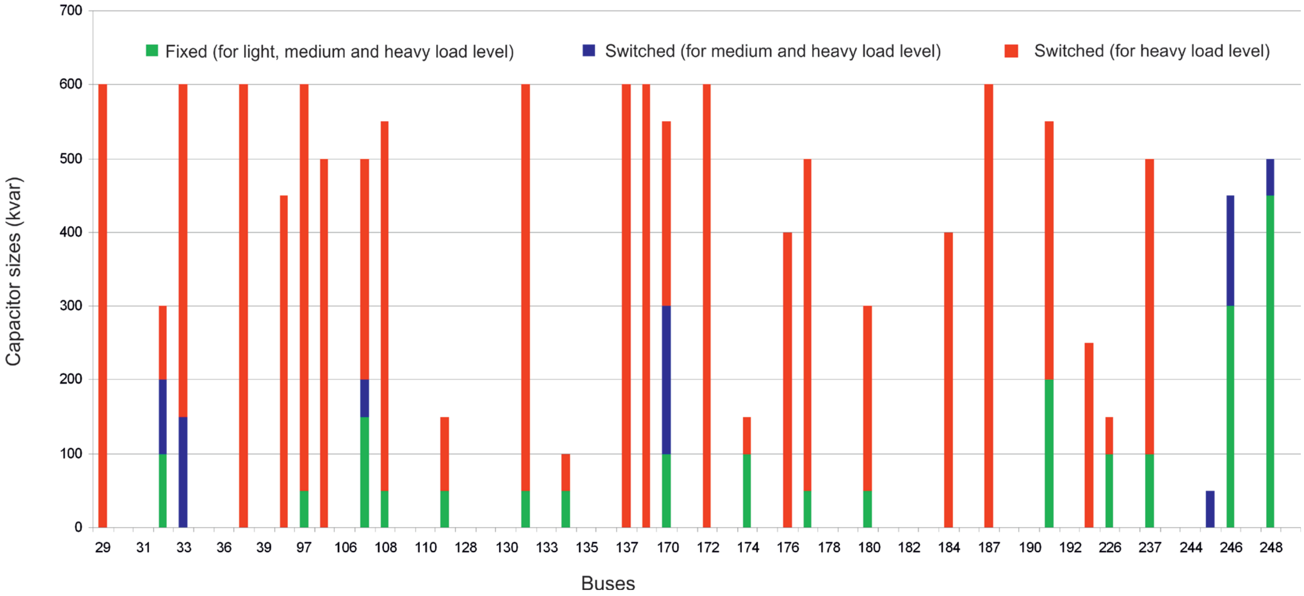

Figure 2. Optimal capacitor placement for Case B

distribution of capacitors among candidate buses at different load levels is not similar. In spite of FES chose 60 buses for capacitor placement, SPSO reduced these placements up to 33 and 29 for Cases A and B, respectively. Therefore SPSO not only find the optimal sizing but the optimal placement also. The minimum rms voltage improved in the two cases to acceptable level. The maximum THD in Case A raised over the limit (5%) so putting constraint to the maximum allowable THD in case B force the THD values for all buses to be under permissible limit at different load levels.

6. CONCLUSIONS

This paper combines SPSO with FES to the discrete optimization problem of fixed and switched shunt capacitor placement and sizing under harmonic conditions for large-scale practical distribution network. The solution method is divided to two stages. In the first stage, fuzzy approach has been used to reduce the search space by finding the most suitable buses for capacitor placement. Then, in the second stage, SPSO has been applied to these buses to find the sizes of the capacitors to be installed. The reduction of power and energy losses due to installed capacitors and cost of capacitors are used as objective function.

The effectiveness of the proposed algorithm has been illustrated by the positive economic response after simulation, in addition to keeping the maximum THD within prescribed level and improving the voltage profile. The payback period would be 0.8 year and the total saving for all project is equal to \$1810533.3 at the \$157678 investment cost and \$196797.1 annual saving when considering the cost of both power and energy losses reduction under harmonic conditions. Future work will be addressed to application of

SPSO in solving capacitor placement, reconductoring and reconfiguration problems of large-scale practical distorted distribution network simultaneously.

REFERENCES

1. Neagle N.M. and Samson D.R.: *Loss reduction from capacitors installd on primary feeders.* AIEE Trans. Power Apparatus and Systems, vol. 75, pp. 950-959, Oct. 1956.
2. Grainger J.J. and Lee S.H.: *Optimum size and location of shunt capacitors for reduction of losses on distribution feeders*. IEEE Trans. Power Apparatus and Systems, vol. 100, pp. 1105-1118, Mar. 1981.
3. Salama M.M.A., Mansour E.A.A., Chikhani A.Y., and Hackam R.: *Control of reactive power in distribution systems with an end-load and varying load conditions*. IEEE Trans. Power Apparatus and Systems, vol. 104, pp. 941-947, Apr. 1985.
4. Baran M.E. and Wu F.F.: *Optimal sizing of capacitors placed on a radial distribution system.* IEEE Trans. Power Delivery, vol. 4, pp. 735-743, Jan. 1989.
5. Sundhararajan S. and Pahwa A.: *Optimal selection of capacitors for radial distribution systems using a genetic algorithm*. IEEE Trans. Power Systems, vol. 9, pp. 1499-1507, Aug. 1994.
6. Gu Z. and Rizy D.T.: *Neural networks for combined control of capacitor banks and voltage regulators in distribution systems*. IEEE Trans. Power Delivery, vol. 11, pp. 1921-1928, Oct. 1996.
7. Ng H.N., Salama M.M.A., and Chikhani Y.A.: *Capacitor allocation by approximate reasoning: fuzzy capacitor placement*. IEEE Trans. Power Delivery, vol. 15, pp. 393-398, Jan. 2000.
8. Khalil T.M., Youssef H.K.M., and Abdel Aziz M.M.: *A binary particle swarm optimization for optimal placement and sizing of capacitor banks in radial distribution feeders with distorted substation voltages*. In Proc. 2006 Int. Conf. on Artificial Intelligence and Machine Learning, pp. 129-135.
9. Damodar Reddy M. and Veera Reddy V.C.: *Capacitor placement using fuzzy and particle swarm optimization method for maximum annual savings*. ARPN Journal of Engineering and Applied Sciences, vol. 3, pp. 25-30, June 2008.
10. Khalil T.M. and Gorpinich A.V.: *Selective particle swarm optimization.* International Journal of Multidisciplinary Sciences and Engineering, vol. 3, pp. 1-4, Apr. 2012.

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APPENDIX

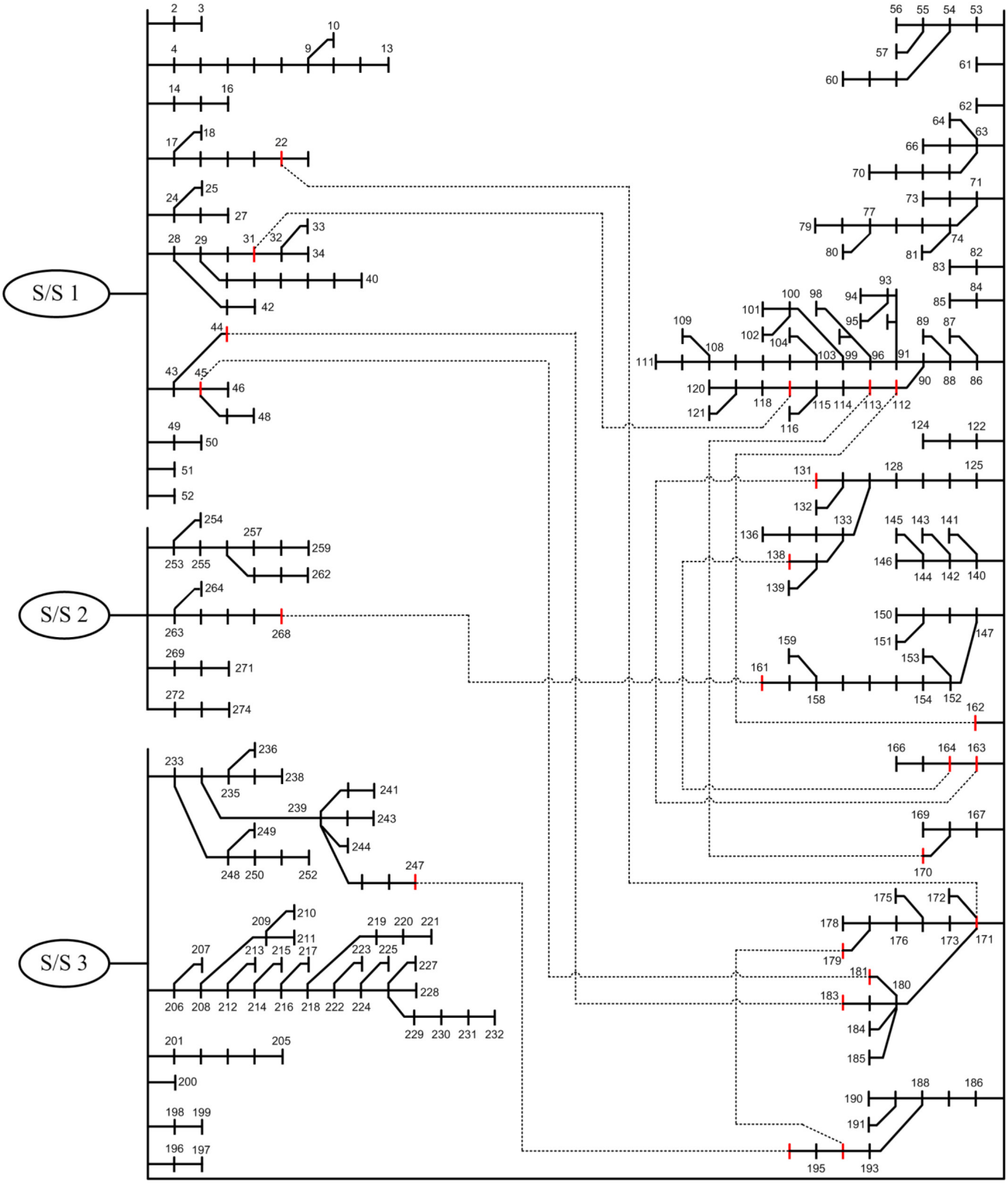

Figure A1. Simplified single line diagram of 6 kV practical distribution network supplying the eastern part of city Mariupol, Ukraine

TABLE A1. LINE DATA

Bus No.	From	To	Length (km)	Bus No.	From	To	Length (km)	Bus No.	From	To	Length (km)	Bus No.	From	To	Length (km)
1	1	2	2.2	72	72	73	0.15	143	142	144	0.25	214	214	215	0.25
2	2	3	0.734	73	71	74	0.4	144	144	145	0.003	215	214	216	0.8
3	1	4	2.2	74	74	75	0.47	145	144	146	1.1	216	216	217	0.3
4	4	5	0.17	75	75	76	0.1	146	1	147	1.7	217	216	218	0.05
5	5	6	0.345	76	76	77	0.429	147	147	148	0.17	218	218	219	0.15
6	6	7	0.35	77	77	78	0.45	148	148	149	0.17	219	219	220	0.4
7	7	8	0	78	78	79	1.515	149	149	150	0.52	220	220	221	0.05
8	8	9	0.42	79	77	80	0.3	150	149	151	0.39	221	218	222	1.35
9	9	10	0.32	80	74	81	0.47	151	147	152	1	222	222	223	0.1
10	9	11	0.38	81	1	82	1.4	152	152	153	0.09	223	222	224	0.65
11	11	12	0.25	82	82	83	0.29	153	152	154	0.36	224	224	225	0.05
12	12	13	0.35	83	1	84	1.4	154	154	155	0.3	225	224	226	1.05
13	1	14	2.37	84	84	85	0.74	155	155	156	0	226	226	227	0.5
14	14	15	0.185	85	1	86	0.68	156	156	157	0.53	227	226	228	0.38
15	15	16	0.37	86	86	87	0.062	157	157	158	0.3	228	226	229	0.8
16	1	17	0.3	87	86	88	0.4	158	158	159	1	229	229	230	0.85
17	17	18	0.98	88	88	89	0.95	159	158	160	0.56	230	230	231	0.2
18	17	19	0.145	89	88	90	0.4	160	160	161	0.45	231	231	232	0.45
19	19	20	0.285	90	90	91	1.3	161	1	162	1.4	232	1	233	3.3
20	20	21	0.96	91	91	92	0.15	162	1	163	2.71	233	233	234	0.63
21	21	22	1.2	92	91	93	0.15	163	163	164	1	234	234	235	1.406
22	22	23	0.55	93	93	94	0.1	164	164	165	0.65	235	235	236	0.22
23	1	24	0.3	94	93	95	0.15	165	165	166	0.53	236	235	237	0.539
24	24	25	0.98	95	91	96	0.15	166	1	167	0.95	237	237	238	0.22
25	24	26	0.58	96	96	97	0.35	167	167	168	0.25	238	234	239	0.68
26	26	27	0.285	97	96	98	0.06	168	168	169	0.03	239	239	240	0.76
27	1	28	2.11	98	96	99	0.15	169	168	170	0.675	240	240	241	0.9
28	28	29	0.4	99	99	100	0	170	1	171	4.05	241	239	242	0.55
29	29	30	0.55	100	100	101	0.195	171	171	172	1.52	242	242	243	0.98
30	30	31	0.47	101	100	102	1.04	172	171	173	1	243	239	244	2.5
31	31	32	1	102	99	103	0.15	173	173	174	0.15	244	239	245	0.792
32	32	33	0.05	103	103	104	0.1	174	174	175	0.35	245	245	246	0.505
33	32	34	0.22	104	103	105	0.5	175	174	176	0.35	246	246	247	0.51
34	29	35	0.05	105	105	106	0.1	176	176	177	0.15	247	233	248	0.414
35	35	36	0.178	106	106	107	0.15	177	177	178	0.45	248	248	249	0.16
36	36	37	0.17	107	107	108	0.65	178	177	179	0.76	249	248	250	0.616
37	37	38	0.05	108	108	109	0.04	179	171	180	1	250	250	251	1.138
38	38	39	1.3	109	108	110	0.45	180	180	181	1.354	251	251	252	0.29
39	39	40	0.55	110	110	111	0.5	181	180	182	0.25	252	1	253	1.5
40	28	41	1.63	111	90	112	0.35	182	182	183	0.3	253	253	254	0.74
41	41	42	0.14	112	112	113	0.2	183	180	184	2.5	254	253	255	0.8
42	1	43	2.58	113	113	114	0.045	184	180	185	0.61	255	255	256	0.215
43	43	44	0.43	114	114	115	0.54	185	1	186	3.9	256	256	257	0.215
44	43	45	0.49	115	115	116	0.65	186	186	187	0.1	257	257	258	0.35
45	45	46	0.54	116	115	117	0.185	187	187	188	0.35	258	258	259	0.15
46	45	47	0.25	117	117	118	0.9	188	188	189	0.35	259	256	260	0.35
47	47	48	0.38	118	118	119	0.26	189	189	190	0.3	260	260	261	0.1
48	1	49	1.8	119	119	120	1.3	190	189	191	0.7	261	261	262	0.35
49	49	50	0.35	120	119	121	0.28	191	187	192	0.55	262	1	263	1.5
50	1	51	2.5	121	1	122	0.85	192	192	193	0.11	263	263	264	0.74
51	1	52	2.5	122	122	123	0.7	193	193	194	0.36	264	263	265	0.8
52	1	53	1	123	123	124	0.15	194	194	195	0.26	265	265	266	0.16
53	53	54	0	124	1	125	0.115	195	1	196	2.61	266	266	267	0.16
54	54	55	0.19	125	125	126	1.1	196	196	197	0	267	267	268	0.23
55	55	56	0.165	126	126	127	0.55	197	1	198	2.61	268	1	269	1
56	55	57	0.1	127	127	128	0.6	198	198	199	0	269	269	270	0
57	54	58	1.24	128	128	129	0.578	199	1	200	2.89	270	270	271	1
58	58	59	0.5	129	129	130	0.25	200	1	201	1.11	271	1	272	2.2
59	59	60	0.238	130	130	131	0.44	201	201	202	0.08	272	272	273	0.12
60	1	61	1	131	130	132	1	202	202	203	0.45	273	273	274	0.08
61	1	62	0.075	132	128	133	0.65	203	203	204	1	274	162	112	0.8
62	1	63	1.685	133	133	134	0.4	204	204	205	0.27	275	170	113	0.422
63	63	64	0.75	134	134	135	0.523	205	1	206	1	276	117	31	1.6
64	63	65	1.7	135	135	136	0.406	206	206	207	0.15	277	171	22	1.1
65	65	66	0.15	136	133	137	0.228	207	206	208	0.55	278	181	45	0.3
66	63	67	0.115	137	137	138	0.25	208	208	209	0.6	279	183	44	0.15
67	67	68	0.335	138	137	139	0.38	209	209	210	0.1	280	193	179	0.4
68	68	69	0.62	139	1	140	0.8	210	209	211	0.1	281	247	195	0.4
69	69	70	0.2	140	140	141	0.625	211	208	212	0.45	282	164	138	0.5
70	1	71	1.685	141	140	142	0.3	212	212	213	0.1	283	131	163	1.29
71	71	72	1.7	142	142	143	0.076	213	212	214	0.15	284	268	161	0.65

TABLE A2. LOAD DATA (FOR HEAVY LOAD LEVEL)

Bus No.	P (kW)	Q (kvar)	Bus No.	P (kW)	Q (kvar)	Bus No.	P (kW)	Q (kvar)	Bus No.	P (kW)	Q (kvar)
1	0	0	70	309.4	191.83	139	309.4	191.83	208	0	0
2	487.31	302.13	71	309.4	191.83	140	487.31	302.13	209	0	0
3	487.31	302.13	72	309.4	191.83	141	0	0	210	193.38	119.89
4	0	0	73	487.31	302.13	142	0	0	211	309.4	191.83
5	487.31	302.13	74	309.4	191.83	143	309.4	191.83	212	0	0
6	309.4	191.83	75	309.4	191.83	144	0	0	213	193.38	119.89
7	487.31	302.13	76	247.52	153.46	145	139.23	86.323	214	0	0
8	309.4	191.83	77	309.4	191.83	146	193.38	119.89	215	193.38	119.89
9	487.31	302.13	78	247.52	153.46	147	0	0	216	0	0
10	487.31	302.13	79	243.65	151.06	148	247.52	153.46	217	309.4	191.83
11	487.31	302.13	80	139.23	86.323	149	0	0	218	0	0
12	487.31	302.13	81	247.52	153.46	150	0	0	219	243.65	151.06
13	487.31	302.13	82	773.5	479.57	151	193.38	119.89	220	773.5	479.57
14	487.31	302.13	83	309.4	191.83	152	309.4	191.83	221	193.38	119.89
15	487.31	302.13	84	0	0	153	487.31	302.13	222	0	0
16	487.31	302.13	85	487.31	302.13	154	309.4	191.83	223	139.23	86.323
17	487.31	302.13	86	0	0	155	193.38	119.89	224	0	0
18	487.31	302.13	87	433.16	268.56	156	247.52	153.46	225	154.7	95.914
19	487.31	302.13	88	0	0	157	309.4	191.83	226	0	0
20	123.76	76.731	89	773.5	479.57	158	487.31	302.13	227	243.65	151.06
21	309.4	191.83	90	0	0	159	309.4	191.83	228	0	0
22	309.4	191.83	91	0	0	160	309.4	191.83	229	773.5	479.57
23	309.4	191.83	92	123.76	76.731	161	309.4	191.83	230	193.38	119.89
24	487.31	302.13	93	0	0	162	309.4	191.83	231	123.76	76.731
25	487.31	302.13	94	123.76	76.731	163	309.4	191.83	232	139.23	86.323
26	487.31	302.13	95	773.5	479.57	164	309.4	191.83	233	487.31	302.13
27	487.31	302.13	96	0	0	165	123.76	76.731	234	123.76	76.731
28	247.52	153.46	97	0	0	166	193.38	119.89	235	193.38	119.89
29	139.23	86.323	98	193.38	119.89	167	773.5	479.57	236	773.5	479.57
30	243.65	151.06	99	0	0	168	0	0	237	193.38	119.89
31	309.4	191.83	100	0	0	169	193.38	119.89	238	309.4	191.83
32	309.4	191.83	101	123.76	76.731	170	309.4	191.83	239	247.52	153.46
33	487.31	302.13	102	193.38	119.89	171	247.52	153.46	240	247.52	153.46
34	193.38	119.89	103	123.76	76.731	172	309.4	191.83	241	227.41	140.99
35	193.38	119.89	104	123.76	76.731	173	193.38	119.89	242	309.4	191.83
36	193.38	119.89	105	139.23	86.323	174	0	0	243	139.23	86.323
37	487.31	302.13	106	139.23	86.323	175	773.5	479.57	244	0	0
38	193.38	119.89	107	193.38	119.89	176	193.38	119.89	245	309.4	191.83
39	123.76	76.731	108	0	0	177	0	0	246	487.31	302.13
40	309.4	191.83	109	309.4	191.83	178	193.38	119.89	247	193.38	119.89
41	123.76	76.731	110	193.38	119.89	179	309.4	191.83	248	487.31	302.13
42	309.4	191.83	111	123.76	76.731	180	0	0	249	487.31	302.13
43	309.4	191.83	112	0	0	181	487.31	302.13	250	487.31	302.13
44	487.31	302.13	113	0	0	182	309.4	191.83	251	193.38	119.89
45	309.4	191.83	114	154.7	95.914	183	487.31	302.13	252	247.52	153.46
46	773.5	479.57	115	0	0	184	309.4	191.83	253	0	0
47	309.4	191.83	116	193.38	119.89	185	154.7	95.914	254	309.4	191.83
48	0	0	117	487.31	302.13	186	309.4	191.83	255	309.4	191.83
49	618.8	383.66	118	139.23	86.323	187	0	0	256	193.38	119.89
50	974.61	604.26	119	0	0	188	309.4	191.83	257	309.4	191.83
51	0	0	120	309.4	191.83	189	309.4	191.83	258	309.4	191.83
52	0	0	121	193.38	119.89	190	123.76	76.731	259	123.76	76.731
53	243.65	151.06	122	0	0	191	309.4	191.83	260	309.4	191.83
54	247.52	153.46	123	0	0	192	193.38	119.89	261	309.4	191.83
55	0	0	124	309.4	191.83	193	309.4	191.83	262	123.76	76.731
56	193.38	119.89	125	193.38	119.89	194	154.7	95.914	263	0	0
57	487.31	302.13	126	309.4	191.83	195	309.4	191.83	264	309.4	191.83
58	193.38	119.89	127	309.4	191.83	196	309.4	191.83	265	123.76	76.731
59	123.76	76.731	128	0	0	197	193.38	119.89	266	247.52	153.46
60	487.31	302.13	129	247.52	153.46	198	309.4	191.83	267	309.4	191.83
61	139.23	86.323	130	193.38	119.89	199	193.38	119.89	268	243.65	151.06
62	309.4	191.83	131	487.31	302.13	200	0	0	269	487.31	302.13
63	0	0	132	139.23	86.323	201	193.38	119.89	270	773.5	479.57
64	247.52	153.46	133	0	0	202	309.4	191.83	271	193.38	119.89
65	309.4	191.83	134	193.38	119.89	203	247.52	153.46	272	193.38	119.89
66	487.31	302.13	135	154.7	95.914	204	123.76	76.731	273	139.23	86.323
67	487.31	302.13	136	247.52	153.46	205	243.65	151.06	274	309.4	191.83
68	243.65	151.06	137	193.38	119.89	206	0	0	—	—	—
69	243.65	151.06	138	243.65	151.06	207	139.23	86.323	—	—	—