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# CONSIDERATION OF SHEAR, ROTATIONAL INERTIA AND COMPRESSIVE FORCE DURING TRANSVERSE VIBRATIONS OF STRUCTURAL BEAM ELEMENTS

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Abstract

The procedure for taking into account shear and rotational inertia in the case of transverse vibrations of beam elements of material handling vehicles with different conditions of fastening (support) is considered. The dynamic model of the rod element is supplemented with compressive forces with a fixed line of action and monitors the angle of rotation of the rod. The Fourier method of separation of variables in the harmonic vibrations of beams is applied. This made it possible to obtain a differential equation, kinematic and static parameters in the amplitude state. The resulting differential equation is integrated, the fundamental functions are normalized, and the complete solution is presented in matrix form with initial parameters. Four cases of fundamental functions are revealed. For beam elements with different support conditions, the vibration frequency has been refined. With hinged support, the first 5 frequencies of this work coincide with the frequencies obtained by another approach.

Keywords: allowance forshear and rotational inertia, Fourier method of separation of variables, determination of refined vibration frequencies of structural beam elements

## 1. INTRODUCTION

Usually, the equations of transverse vibrations of beams do not take into account shear deformations and rotational inertia. Therefore, they quite well describe the transverse vibrations of a rod with a large ratio of length to cross-sectional height ($l/h$>10) and at low frequencies (characteristic operation of rod structural elements of material handling vehicles). However, for frame systems of foundations of heavy production equipment, and systems of material handling vehicles and similar machine-building structures, when $l_{sw}/nh < 6$, where $n$ – oscillation tone number; $h$ – characteristic size of the cross section; $l_{sw}$ – half-wave length of the elastic line of the rod, it is already necessary to take into account shear and rotational inertia [1, 2].

The problem of constructing more accurate solutions of transverse vibrations of a rod is also very relevant in the theory of stability in connection with the application of the dynamic method.

The differential equation of transverse vibrations of a rectilinear rod, taking into account shear deformations and rotational inertia, was derived by an outstanding scientist from Chernihiv Oblast, Ukraine, professor S.P. Tymoshenko [3]. His model has now established itself as the most accurate and is widely used in various tasks of structural mechanics. To apply the model of S.P. Tymoshenko, it needs to be supplemented with longitudinal force $F_x$ in stability tasks.

For this purpose, the rod compressed by consecutive force $F_1$ and force $F_2$ that has a fixed line of action is considered (Fig. 1).

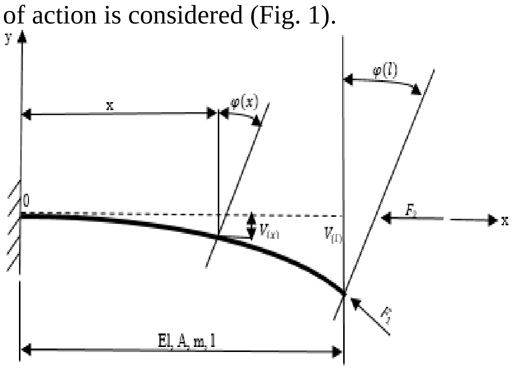


Fig. 1. Scheme of two following forces

## 2. ANALYTICAL LITERATURE REVIEW

Works [1-4] present mathematical models for accounting for shear and rotational inertia during

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harmonic vibrations of various beam systems. They do not take into account compressive forces and therefore cannot be used to solve stability problems by the dynamic method. This work corrects the mentioned shortcoming.

The finite element method can be used, but it will give an approximate value of the critical forces. The approach proposed by the authors of the article provides greater accuracy in the values of critical forces of non-conservative stability problems.

## 3. THE PURPOSE AND OBJECTIVES OF THE RESEARCH

The purpose of this scientific work is to build a mathematical model of harmonic vibrations of various beam structures, which takes into account shear, rotational inertia and compressive forces.

The following tasks are solved in this:

1. The model of Professor S.P. Tymoshenko is supplemented with compressive forces.
2. The variables in the dynamic model are separated by the Fourier method.
3. The refined differential equation is integrated, the fundamental functions are normalized, and the complete solution is given in matrix form with initial parameters.
4. Calculation of refined frequencies of natural oscillations of beams and their comparison with the results of other studies are completed.

## 4. FORMATION OF MATHEMATICAL MODELS

### 4.1. Application of the Fourier method

The expression for the bending moment follows from the geometric relations of the rod strained state

$$M\cdot(x,t)=EI\,\frac{\partial\psi\cdot(x,t)}{\partial x}+F_x\cdot y(x,t)-F_1\cdot y(l,t), \quad (1)$$

where $\psi_*(x,t)$ – the angle of inclination of the rod cross-section without taking into account shear;

$F_x=F_1+F_2$ –longitudinal force in the current section;

$y(x,t),\ y(l,t)$ – respectively, the deflection of the current and boundary points.

Here, the first derivative of the deflection in the rod curvature is not taken into account and it is assumed that as a result of small displacements $\cos(\partial y/\partial x)=1;\ \sin(\partial y/\partial x)=\partial y/\partial x$.

For force $F_2$ expression (1) is exact, for $F_1$– approximate. The full angle of rotation of the section is equal to sum (3)

$$\frac{\partial y(x,t)}{\partial x}=\psi(x,t)+\tau_*(x,t), \quad (2)$$

where $\tau_*(x,t)$–angle of transverse shear. Transverse force in the considered case will be an expression. The total angle of rotation is equal to the sum of the two terms.

$$Q\cdot(x,t)=-kAG\,\tau\cdot(x,t)-F\cdot\frac{\partial y(x,t)}{\partial x}+F_1\frac{\partial y(l,t)}{\partial x}, \quad (3)$$

where $AG$ – shear stiffness of the section; $k$ – coefficient that takes into account the effect of the cross-section shape on shear strain. Fundamentally, equations (1), (3) do not change if the rod is compressed by "dead" force. Next, the model of the strained state (1) - (3) is brought to the Cauchy problem. In this case, the starting equations are the equilibrium equations of the elementary part of the rod during its naturaloscillations:

sum of moments

$$Q\cdot(x,t)=-kAG\,\tau\cdot(x,t)-F\cdot\frac{\partial y(x,t)}{\partial x}+F_1\frac{\partial y(l,t)}{\partial x}, \quad (4)$$

sum of projection on the vertical axis

$$Q\cdot(x,t)=-kAG\,\tau\cdot(x,t)-F\cdot\frac{\partial y(x,t)}{\partial x}+F_1\frac{\partial y(l,t)}{\partial x}, \quad (5)$$

where $\rho=m/A$–density of the rod material; $A$ – cross-sectional area; $I$ – axial moment of inertia of the section; $m$ –evenly distributed mass; $q_*(x,t)$ – transverse dynamic load.

If function $\psi_*(x,t)$is excluded from equations (4), (5), then the equation of S.P. Tymoshenko, taking into account the action of longitudinal force $F_x$, will take on the form

$$EI\cdot\left(1+\frac{F_x}{\kappa AG}\right)\frac{\partial^4 y}{\partial x^4}+m\frac{\partial^2 y}{\partial t^2}-\left[\frac{EI}{\kappa G}\rho+\rho I\cdot\left(1+\frac{F_x}{\kappa AG}\right)\right]\frac{\partial^4 y}{\partial x^2\partial t^2}+$$
$$F_x\frac{\partial^2 y}{\partial x^2}+\frac{\rho^2 I}{\kappa G}+\frac{\partial^4 y}{\partial t^4}=q+\frac{\rho I}{\kappa AG}\cdot\frac{\partial^2 q}{\partial t^2}-\frac{EI}{\kappa AG}\cdot\frac{\partial^2 q}{\partial x^2}, \quad (6)$$

It is limited to the case of harmonic oscillations, which can be used to separate the linear and temporal coordinates according to the Fourier method, i.e.

$$y(x,t)=V(x)\,sin(at+\Psi_0);$$
$$q(x,t)=q(x)\sin(\omega t+\Psi_0);$$
$$\Psi(x,t)=\Psi(x)\sin(\omega t+\psi_0), \quad (7)$$

where $V(x), q(x), \psi(x)$–deflection amplitude, load and angle of inclination;

$\omega$ –frequency of natural oscillations;

$\psi_0$ –initial phase. Substituting (7) into (1) – (3), (6), the differential equation and the corresponding kinematic and static parameters in the amplitude state are obtained.

The authors of the work took into account only the amplitude components of deflection, load and angle of inclination.

$$V^{Iv}(x)+2r^2V''_{(x)}+S^4V(x)=q_y(x)/EI;$$
$$V(x);\ \varphi(x)=V'(x);$$
$$\frac{M(x)}{EI}=aV''(x)+a_2V_{(x)}+a_3q(x)-a_4V(l); \quad (8)$$
$$\frac{Q(x)}{EI}=\text{в}_1V^{\text{III}}(x)+\text{в}_2V^1+\text{в}_3q^1(x)-\text{в}_4\varphi(x),$$

where $\varphi(x)$, $M(x)$, $Q(x)$–amplitude full angle of rotation, bending moment and transverse force, coefficients and the right-hand side take the form

$$2r^2 = \frac{I\omega^2 m(E+\text{к}G)+(\rho I\omega^2+\text{к}AG)F_x}{EI(\text{к}AG+Fx)};$$

$$s^4 = \frac{\omega^2 m l^\rho I\omega^2-\text{к}AG}{EI(\text{к}AG+F_x)};$$

$$q_y(x) = \frac{\text{к}AG-\rho I\omega^2}{\text{к}AG+F_x}q(x)-\frac{EI}{\text{к}AG+F_x}q''(x); \quad (9)$$

$$a_1 = 1+\frac{F_x}{\text{к}AG};\ a_2 = \frac{m\omega^2}{\text{к}AG}+\frac{F_x}{EI};$$

$$a_3 = \frac{1}{\text{к}AG};\ a_4 = \frac{F_1}{EI};$$

$$\text{в}_1 = \frac{\text{к}AG+F_x}{\text{к}AG-\rho I\omega^2};$$

$$\text{в}_2 = \frac{I(Em+\text{к}AG\rho)\omega^2 t(\text{к}AG+\rho I\omega^2)F_x}{EI(\text{к}AG-\rho I\omega^2)};$$

$$\text{в}_3 = \frac{1}{\text{к}AG-\rho I\omega^2};\ \text{в}_4 = \frac{F_1\rho I\omega^2}{EI(\text{к}AG-\rho I\omega^2)}.$$

It is convenient to present the solution of equation (8) in matrix form after normalization of the fundamental functions

| | | | | | |
|---|---|---|---|---|---|
| *EIV*(x) | = | A11 | A12 | - A13 | - A14 |
| *EIφ*(x) | | A21 | A22 | - A23 | - A24 |
| *M*(x) | | - A31 | - A32 | A33 | A34 |
| *Q*(x) | | - A41 | - A42 | A43 | A44 |

(10)

| | | |
|---|---|---|
| *EIV*(0) | = | B11 |
| *EI* (0) | | B21 |
| *M* (0) | | - B31 |
| *Q* (0) | | - B41 |

where the "-" sign corresponds to the "down" direction of the *oy* axis. The appearance of the fundamental orthonormal functions depends on the roots of the characteristic equation. Four main cases of fundamental functions are presented.

Matrix equations (10) can be effectively used if the conditions of support of the beams are known. This is shown in the work as follows - when the supports are tightly clamped.

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | | | – A13 | – A14 |
| 2 | | | – A23 | – A24 |
| 3 | –1 | | A33 | A34 |
| 4 | | –1 | A43 | A44 |

| | | | |
|---|---|---|---|
| $EIV_{(0)}$=0; *M*(*l*) | – | $EIV_{(l)}$=0 | = 0. |
| $EI\varphi_{(0)}$ = 0 | | $EI\varphi_{(l)}$ = 0 | |
| *M*(0) | | *M*(*l*) | |
| *Q*(0) | | *Q*(*l*) | |

Columns 1 and 2 are zeroed, as they are equal to "0", i.e. $EIV_{(0)}$ and $EI\varphi_{(0)}$. *M*(*l*)is moved to the place of the zero strips of the matrix $X_{(0)}$ and the compensating coefficient *A*(1,3) = –1 appears. *Q*(*l*) is moved and the compensating coefficient *A*(2,4) = –1 appears.

Next, the matrix $A_*$ will take the form of a determinant. From which it follows that

$$|A_*(\omega)| = A_{13}\cdot A_{24}-A_{14}\cdot A_{23} = 0\cdot$$

This equation is a frequent equation for a rigidly clamped beam.

<u>1st case.</u> $(r^4-S^4)<0$.

The roots are complex

$$k_{1-4} = \pm\alpha\pm i\beta;\ \alpha = \sqrt{(s^2-r^2)/2};$$

$$\beta = \sqrt{(s^2+r^2)\,/\,2};$$

$$\Phi_1 = ch\,\alpha x\cdot sin\,\beta x;\ \Phi_2 = ch\,\alpha x\cdot cos\,\beta x;$$

$$\Phi_3 = sh\,\alpha x\cdot cos\,\beta x;\ \Phi_4 = sh\,\alpha x\cdot sin\,\beta x;$$

$$A_{11} = \Phi_2-\frac{a_2-a_1r^2}{2\alpha\beta a_1}\Phi_4;$$

$$A_{12} = \frac{\beta[B_2-B_1(\beta^2-3\alpha^2)]\Phi_3-\alpha[B_2+B_1(\alpha^2-3\beta^2)]\Phi_1}{2\alpha\beta S^2 B_1};$$

$$A_{13} = \frac{\Phi_4}{2\alpha\beta a_1};A_{14} = \frac{\alpha\Phi_1-\beta\Phi_3}{2\alpha\beta S^2 B_1};$$

$$A_{21} = \frac{\beta(a,s^2-a_2)\Phi_3-\alpha(a_1s^2+a_2)\Phi_1}{2\alpha\beta a_1};$$

$$A_{22} = \Phi_2\frac{\text{в}_2-\text{в}_1r^2}{2\alpha\beta a_1}\Phi_4;$$

$$A_{23} = \frac{\alpha\Phi_1+\beta\Phi_3}{2\alpha\beta a_1};\ A_{24} = \frac{\Phi_4}{2\alpha\beta\text{в}_1};$$

$$A_{31} = -\frac{(a_2-a_1r^2)+(2\,\alpha\,\beta\,a_1)^2}{2\alpha\beta a_1}\Phi_4;$$

$$A_{33} = \Phi_2+\frac{a_2-a_1r^2}{2\alpha\beta a_1}\Phi_4;$$

$$A_{32} = \frac{[\text{в}_2-\text{в}_1(\beta^2-3\alpha^2)](a_2-a_1r^2)}{2\alpha S^2\text{в}_1}\Phi_3 - \frac{2\alpha^2a_1[\text{в}_2+\text{в}_1(a^2-3\beta^2)]}{2\alpha S^2\text{в}_1}\Phi_3 - \frac{[\text{в}_2-\text{в}_1(a^2-3\beta^2)](a^2-a_1r^2)}{2\beta S^2\text{в}_1}\Phi_1 + \frac{2\beta^2a_1[\text{в}_2-\text{в}_1(\beta^2-3\alpha^2)]}{2\beta S^2\text{в}_1}\Phi_1;$$

(11)

$$A_{34} = \frac{[\alpha(a^2-a_1r^2)+2\alpha\beta^2a_1]\Phi_1}{2\alpha\beta S^2\text{в}_1} - \frac{[\beta(a_2-a_1r^2)-2\alpha^2\beta a_1]\Phi_3}{2\alpha\beta S^2\text{в}_1};$$

$$A_{41}=\frac{2\alpha^2 a_1[\text{в}_2+\text{в}_1(\alpha^2-\beta^2)]}{2\alpha a_1}\Phi_3-$$

$$-\frac{(a_2-a_1r^2)[\text{в}_2-\text{в}_1(\beta^2-3\alpha^2)]}{2\alpha a_1}\Phi_3-$$

$$-\frac{2\beta^2 a_1[\text{в}_2-\text{в}_1(\beta^2-3\alpha^2)]}{2\beta a_1}\Phi_1+$$

$$+\frac{(a_2-a_1r^2)[\text{в}_2+\text{в}_1(\alpha^2-3\beta^2)]}{2\beta a_1}\Phi_1;$$

$$A_{42}=-\frac{\alpha^2[\text{в}_2+\text{в}_1(\alpha^2-3\beta^2)]^2}{2\alpha\beta S^2\text{в}_1}\Phi_4-$$

$$-\frac{\beta^2[\text{в}_2-\text{в}_1(\beta^2-3\alpha^2)]^2}{2\alpha\beta S^2\text{в}_1}\Phi_4;$$

$$A_{43}=\frac{\alpha[\text{в}_2+\text{в}_1(\alpha^2-3\beta^2)]\Phi_1}{2\alpha\beta a_1}+$$

$$+\frac{\beta[\text{в}_2-\text{в}_1(\beta^2-3\alpha^2)]\Phi_3}{2\alpha\beta a_1};$$

$$A_{44}=\Phi_2+\frac{\text{в}_2-\text{в}_1 r}{2\alpha\beta\text{в}_1}\Phi_4.$$

<u>2[nd]case.</u> $\left(r^4-S^4\right)>0;\ S^4<0$.

The roots are real and imaginary

$$\text{к}_{1-2}=\pm\alpha;\ \text{к}_{3-4}=\pm i\beta;$$

$$\alpha=\sqrt{-r^2+\sqrt{r^4-s^4}};$$

$$\beta=\sqrt{r^2+\sqrt{r^4-s^4}};$$

$$A_{11}=\frac{-(a_2-a_1\beta^2)\,ch\,\alpha x+(a_2+a_1\alpha^2)\cos\beta x}{a_1(\alpha^2+\beta^2)};$$

$$A_{12}=\frac{-\beta(\text{в}_2-\text{в}_1\beta^2)\,\text{sh}\,\alpha x+\alpha(\text{в}_2+\text{в}_1\alpha^2)\sin\beta x}{\alpha\beta(\alpha^2+\beta^2)\text{в}_1};$$

$$A_{13}=\frac{ch\,\alpha x-\cos\beta x}{a_1(\alpha^2-\beta^2)};$$

$$A_{14}=\frac{\beta\,sh\,\alpha x-\alpha\sin\beta x}{\alpha\beta(\alpha^2+\beta^2)\text{в}_1};$$

$$A_{23}=\frac{\alpha\,\text{sh}\,\alpha x+\beta\sin\beta x}{(\alpha^2+\beta^2)a_1};$$

$$A_{21}=\frac{-(a_2-a_1\beta^2)\,ash\,\alpha\,x-\beta(a-a_1\alpha^2)\sin\beta x}{(\alpha^2+\beta^2)a_1};$$

$$A_{22}=\frac{-(\text{в}_2-\text{в}_1\beta^2)\,\text{ch}\,\alpha x+(\text{в}_2+\text{в}_1\alpha^2)\cos\beta x}{(\alpha^2+\beta^2)\text{в}_1};\ (12)$$

$$A_{24}=\frac{ch\,\alpha x-\cos\beta x}{(\alpha^2+\beta^2)\text{в}_1};$$

$$A_{31}=\frac{(a_2+a_1\alpha^2)(a_2-q\beta^2)(-\text{ch}\,\alpha x+\cos\beta x)}{a_1(\alpha^2+\beta^2)};$$

$$A_{32}=\frac{-\beta(a_2+a_1\alpha^2)(\text{в}_2-\text{в}_1\beta^2)\,sh\,\alpha x}{\alpha\beta(\alpha^2+\beta^2)\text{в}_1}+$$

$$+\frac{\alpha(a_2-a_1\beta^2)(\text{в}_2+\text{в}_1\alpha^2)}{\alpha\beta(\alpha^2+\beta^2)\text{в}_1};$$

$$A_{33}=\frac{(a_2+a_1\alpha^2)\,ch\,\alpha x-(a_2-a_1\beta^2)\cos\beta x}{(\alpha^2+\beta^2)a_1};$$

$$A_{34}=\frac{\beta(a_2+a_1\alpha^2)\,\text{sh}\,\alpha x-\alpha(a_2-a_1\beta^2)\sin\beta x}{\alpha\beta(\alpha^2+\beta^2)\text{в}_1};$$

$$A_{41}=\frac{-\alpha(a_2-a_1\beta^2)(\text{в}_2+\text{в}_1\alpha^2)\,sh\,\alpha x}{(\alpha^2+\beta^2)a_1}-$$

$$-\frac{\beta(a_2+a_1\alpha^2)(\text{в}_2-\text{в}_1\beta^2)\sin\beta x}{(\alpha^2+\beta^2)a_1};$$

$$A_{42}=\frac{(\text{в}_2+\text{в}_1\alpha^2)(\text{в}_2-\text{в}_1\beta^2)(-ch\,\alpha x+\cos\beta x)}{(\alpha^2+\beta^2)\text{в}_1};$$

$$A_{43}=\frac{\alpha(B_2+B_1\alpha^2)\,\text{sh}\,\alpha x+\beta(B_2-B_1\beta^2)\sin\beta x}{(\alpha_2+\beta^2)a_1};$$

$$A_{44}=\frac{(\text{в}_2+\text{в}_1\alpha^2)\,\text{ch}\,\alpha x(\text{в}_2-\text{в}_1\beta^2)\cos\beta x}{(\alpha^2+\beta^2)\text{в}_1};$$

<u>3[rd]case.</u> $\left(r^4-S^4\right)>0;\ S^4>0;\ r^2<0$.

A case of tensile force $(-F_x)$.

The roots are real

$$\text{к}_{1-2}=\pm\alpha;\ \text{к}_{3-4}=\pm\beta;$$

$$\alpha=\sqrt{-r^2+\sqrt{r^4-S^4}};$$

$$\beta=\sqrt{-r^2-\sqrt{r^4-S^4}};$$

$$A_{11}=\frac{(a_2+a_1\beta^2)\,ch\,\alpha x-(a_2+a_1\alpha^2)\,ch\,\beta x}{(\beta^2-\alpha^2)a_1};$$

$$A_{12}=\frac{\beta(\text{в}_2+\text{в}_1\beta^2)\,\text{sh}\,\alpha x-\alpha(\text{в}_2+\text{в}_1\alpha^2)\,\text{sh}\,\beta x}{\alpha\beta(\beta^2-\alpha^2)\text{в}_1};$$

$$A_{13}=\frac{-\text{ch}\,\alpha x+\text{ch}\,\beta x}{(\beta^2-\alpha^2)a_1};$$

$$A_{14}=\frac{-\beta\,\text{sh}\,\alpha x+\alpha\,\text{sh}\,\beta x}{\alpha\beta(\beta^2-\alpha^2)B_1};$$

$$A_{21}=\frac{\alpha(a_2+a_1\beta^2)\,sh\,\alpha x-\beta(a_2+a_1\alpha^2)\,sh\,\beta x}{(\beta^2-\alpha^2)a_1};$$

$$A_{22}=\frac{(\text{в}_2+\text{в}_1\beta^2)\,\text{ch}\,\alpha x-(\text{в}_2+\text{в}_1\alpha^2)\,\text{ch}\,\beta x}{(\beta^2-\alpha^2)\text{в}_1};$$

$$A_{23}=\frac{-\alpha\,sh\,\alpha x+\beta\,sh\,\beta x}{(\beta^2-\alpha^2)a_1};$$

$$A_{24}=\frac{-\text{ch}\,\alpha x+\text{ch}\,\beta x}{(\beta^2-\alpha^2)a_1};$$

$$A_{31}=\frac{(a_2+a_1\alpha^2)(a_2+a_1\beta^2)(\text{ch}\,\alpha x-\text{ch}\,\beta x)}{(\beta^2-\alpha^2)a_1};$$

(13)

$$A_{32}=\frac{\beta(a_2+a_1\alpha^2)(\text{в}_2+\text{в}_1\beta^2)\,sh\,\alpha x}{\alpha\beta(\beta^2-\alpha^2)\text{в}_1}-$$

$$-\frac{\alpha(a_2+a_1\beta^2)(\text{в}_2-\text{в}_1\alpha^2)\,sh\,\beta x}{\alpha\beta(\beta^2-\alpha^2)\text{в}_1};$$

$$A_{33}=\frac{-(a_2+a_1\alpha^2)\,ch\,\alpha x+(a_2+a_1\beta^2)\,ch\,\beta x}{(\beta^2-\alpha^2)a_1};$$

$$A_{34}=\frac{-\beta(a+a_1\alpha^2)\,\text{sh}\,\alpha x+\alpha(a_2+a_1\beta^2)\,\text{sh}\,\beta x}{\alpha\beta(\beta^2-\alpha^2)\text{в}_1};$$

$$A_{41}=\frac{\alpha(a_2+a_1\beta^2)(\text{в}_2+\text{в}_1\alpha^2)\,sh\,\alpha x}{(\beta^2-\alpha^2)a_1}-$$

$$-\frac{\beta(a_2+a_1\alpha^2)(\text{в}_2+\text{в}_1\beta^2)sh\beta x}{(\beta^2-\alpha^2)a_1};$$

$$A_{42}=\frac{(\text{в}_2+\text{в}_1\alpha^2)(\text{в}_2+\text{в}_1\beta^2)(ch\,\alpha x-ch\,\beta x)}{(\beta^2-\alpha^2)\text{в}_1};$$

$$A_{43}=\frac{-\alpha(\text{в}_2+\text{в}_1\alpha^2)\,\mathrm{sh}\,\alpha x+\beta(\text{в}_2+\text{в}_1\beta^2)\,\mathrm{sh}\,\beta x}{(\beta^2-\alpha^2)a_1};$$

$$A_{44}=\frac{-(\text{в}_2+\text{в}_1\alpha^2)\,\mathrm{ch}\,\alpha x+(\text{в}_2+\text{в}_1\beta^2)\,\mathrm{ch}\,\beta x}{(\beta^2-2^2)\text{в}_1}.$$

4[th]case $\left(r^4-S^4\right)>0;\ S^4>0;\ r^2>0$.

The roots are imaginary

$$\text{к}_{1-2}=\pm i\alpha;\ \text{к}_{3-4}=\pm i\beta;$$

$$\alpha=\sqrt{r^2-\sqrt{r^4-S^4}};$$

$$\beta=\sqrt{r^2+\sqrt{r^4-S^4}};$$

$$A_{11}=\frac{(a_2-a_1\beta^2)\cos\alpha x-(a_2-a_1\alpha^2)\cos\beta x}{(\alpha^2-\beta^2)a_1};$$

$$A_{12}=\frac{\beta(\text{в}_2-\text{в}_1\beta^2)\sin\alpha x-\alpha(\text{в}_2-\text{в}_1\alpha^2)\sin\beta x}{\alpha\beta(\alpha^2-\beta^2)B_1};$$

$$A_{13}=-\frac{\cos\alpha x+\cos\beta x}{(\alpha^2-\beta^2)a_1};$$

$$A_{14}=\frac{-\beta\sin\alpha x+\alpha\sin\beta x}{\alpha\beta(\alpha^2-\beta^2)\text{в}_1};$$

$$A_{21}=\frac{-\alpha(a_2-a_1\beta^2)\sin\alpha x+\beta(a_2-a_1\alpha^2)\sin\beta x}{(\alpha^2-\beta^2)a_1};$$

$$A_{22}=\frac{(\text{в}_2-\text{в}_1\beta^2)\cos\alpha^x-(\text{в}_2-\text{в}_1\alpha^2)\cos\beta x}{(\alpha^2-\beta^2)\text{в}_1};$$

$$A_{23}=\frac{\alpha\sin\alpha x-\beta\sin\beta x}{(\alpha^2-\beta^2)\text{в}_1};$$

$$A_{24}=\frac{-\cos\alpha x+\cos\beta x}{(\alpha^2-\beta^2)\text{в}_1};$$

(14)

$$A_{31}=\frac{(a_2-a_1\alpha^2)(a_2-a_1\beta^2)(\cos\alpha x-\cos\beta x)}{(\alpha^2-\beta^2)a_1};$$

$$A_{32}=\frac{\beta(a_2-a_1\alpha^2)(\text{в}_2-\text{в}_1\beta^2)\sin\alpha x}{\alpha\beta(\alpha^2-\beta^2)\text{в}_1}-$$

$$-\frac{\alpha(a_2-a_1\beta^2)(\text{в}_2-\text{в}_1\alpha^2)\sin\beta x}{\alpha\beta(\alpha^2-\beta^2)\text{в}_1};$$

$$A_{33}=\frac{-(a_2-a_1\alpha^2)\cos\alpha x+(a_2-a_1\beta^2)\cos\beta x}{(\alpha^2-\beta^2)a_1};$$

$$A_{34}=\frac{-\beta(a_2-a_1\alpha^2)\sin\alpha x+\alpha(a_2-a_1\beta^2)\sin\beta x}{\alpha\beta(\alpha^2-\beta^2)\text{в}_1};$$

$$A_{41}=\frac{-\alpha(a_2-a_1\beta^2)(\text{в}_2-\text{в}_1\alpha^2)\sin\alpha x}{(\alpha^2-\beta^2)a_1}+$$

$$+\frac{\beta(a_2-a_1\alpha^2)(\text{в}_1-\text{в}_2\beta^2)\sin\beta}{(\alpha^2-\beta^2)a_1};$$

$$A_{42}=\frac{(\text{в}_2-\text{в}_1\alpha^2)(\text{в}_2-\text{в}_1\beta^2)(\cos\alpha x-\cos\beta x)}{(\alpha^2-\beta^2)\text{в}_1};$$

$$A_{43}=\frac{\alpha(\text{в}_2-\text{в}_1\alpha^2)\sin\alpha x-\beta(\text{в}_2-\text{в}_1\beta^2)\sin\beta x}{(\alpha^2-\beta^2)a_1};$$

$$A_{44}=\frac{-(\text{в}_2-\text{в}_1\alpha^3)\cos\alpha x+(\text{в}_2-\text{в}_1\beta^2)\cos\beta x}{(\alpha^2-\beta^2)\text{в}_1};$$

The components depending on the external load and the limit parameters of the rod will take on the form

$$B_{11}=\int_0^x A_{14}(x-\xi)\,q_y(\xi)d\xi-a_3q(0)A_{13}(x)+a_4V(l)A_{13}(x)$$
$$-\text{в}_3q'(0)A_{14}(x)+\text{в}_4\varphi(l)A_{14}(x);$$

(15)

$$B_{21}=\int_0^x A_{24}(x-\xi)\,q_y(\xi)d\xi-a_3q(0)A_{23}(x)+a_4V(l)A_{23}(x)$$
$$-\text{в}_3q'(0)A_{24}(x)+\text{в}_4\varphi(l)A_{24}(x);$$

$$B_{31}=\int_0^x A_{34}(x-\xi)\,q_y(\xi)d\xi-a_3[q(0)A_{33}(x)-q(x)]$$
$$+a_4V(l)[A_{33}(x)-1]-\text{в}_3q'(0)A_{34}(x)$$
$$+\text{в}_4\varphi(l)A_{34}(x);$$

$$B_{41}=\int_0^x A_{44}(x-\xi)\,q_y(\xi)d\xi-a_3q(0)A_{43}(x)+a_4V(l)A_{43}(x)$$
$$-\text{в}_3[q'(0)A_{44}(x)-q'(x)]$$
$$+\text{в}_4\varphi(l)[A_{44}(x)-1].$$

### 4.2. Determination of frequencies of natural oscillations of beams

Integrating expressions (15) for any transverse load does not cause difficulties. The other cases of fundamental functions ($r^2=0;\ S^4=0;\quad r^4=S^4$) are of minor importance and are not given. Testing the solution of the Cauchy problem (10) is carried out on the problem of natural oscillations. In this case $F_x=0;\ q_y(x)=0$. The frequency equations of individual rods can be obtained when forming the boundary value problem. For example, in the case of rigid pinching of the boundary points, we will have

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | | | $-A_{13}$ | $-A_{14}$ |
| 2 | | | $-A_{23}$ | $-A_{24}$ |
| 3 | -1 | | $A_{33}$ | $A_{34}$ |
| 4 | | -1 | $A_{43}$ | $A_{44}$ |

| | | | | | |
|---|---|---|---|---|---|
| 1 | $EIV(\text{o})=0;$ $M(l)$ | | $EIV(l)=0$ | | |
| 2 | $EI\varphi(\text{o})=0;$ $Q(l)$ | - | $EI\varphi(l)=0$ | = | 0 → |
| 3 | M (o) | | M ($l$) | | |
| 4 | Q (o) | | Q ($l$) | | |

$$\rightarrow A_*(\omega)=A_{13}\cdot A_{24}-A_{14}\cdot A_{23}=0$$

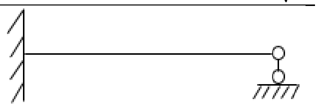
Table 1. Comparison of frequencies according to different methods of calculation

Dimensionless natural oscillation frequencies of individual rods $\omega = \omega l^2 \sqrt{r}$

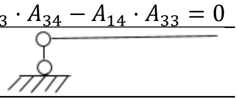
$D_1 = A_{13} \cdot A_{24} - A_{14} \cdot A_{23} = 0$

| Oscillation tone number | approximated | refined | error % |
|---|---|---|---|
| 1 | 22,3736 | 21,9260 | 2,04 |
| 2 | 61,6714 | 57,4781 | 7,30 |
| 3 | 120,9030 | 105,3970 | 14,71 |
| 4 | 199,8596 | 161,4254 | 23,81 |
| 5 | 298,5557 | 222,6361 | 34,10 |
| 6 | 416,9909 | 287,0476 | 45,27 |
| 7 | 555,1652 | 353,4163 | 57,09 |
| 8 | 713,0787 | 420,9801 | 69,39 |
| 9 | 890,7286 | 489,3061 | 82,04 |
| 10 | 1088,1239 | 558,2521 | 94,92 |

Dimensionless natural oscillation frequencies of individual rods $\omega = \omega l^2 \sqrt{r}$



$D_2 = A_{13} \cdot A_{34} - A_{14} \cdot A_{33} = 0$



| Oscillation tone number | approximated | refined | error % |
|---|---|---|---|
| 1 | 15,4184 | 15,1260 | 1,93 |
| 2 | 49,9652 | 46,7429 | 6,89 |
| 3 | 104,2480 | 91,2600 | 14,23 |
| 4 | 178,2700 | 144,5587 | 23,32 |
| 5 | 272,0311 | 203,5004 | 33,68 |
| 6 | 385,5317 | 265,9581 | 44,96 |
| 7 | 518,7714 | 330,5740 | 56,93 |
| 8 | 671,7503 | 396,4820 | 69,43 |
| 9 | 844,4094 | 463,1590 | 82,32 |
| 10 | 1036,8888 | 530,3037 | 95,53 |

Similarly, the frequency equations of any support conditions can be obtained. The highest frequency increase is determined by the method of linear search, when the initial value and the step for $\omega$ are specified. The results of the determinant calculation are output to a separate file. Viewing it allows detecting the change in the sign of the determinant and the rough value of the natural frequency. Further, it can be precisely refined during subsequent runs of the program with changed initial value and step $\omega$. Tables 1, 2 shows a comparison of frequencies according to the approximate solution of Acad. O.M. Krylov and the solution of the equation of S.P. Tymoshenko.

Frequencies were determined using the following initial values: Poisson's ratio

$$\mu = 0{,}3;$$
$$E = 2{,}1 \cdot 10^{11}\ Pa;$$
$$G = E/2(1+\mu) = 0{,}8077 \cdot 10^{11}\ Pa;$$
$$\rho = 7800\ kg/m^3;$$
$$l = 1{,}0\ \text{m};\ A = в \times h = 0{,}1 \times 0{,}1 = 0{,}01 m^2;$$
$$I = hB_3/12 = 8{,}3333 \cdot 10^{-6} m^4;$$
$$m = \rho A = 78{,}0\ kg/m;\ k = 5/6.$$

The absolute values of the frequencies led to a dimensionless form

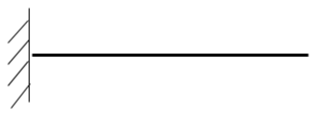
$$\omega_* = \omega l^2 \sqrt{m/EI}$$

It follows from Tables 1, 2 that the error of the approximate solution grows rapidly and reaches almost 100% at the 10th frequency at the ratio

$$l / h = 10.$$

Table 2. Comparison of frequencies according to different methods of calculation

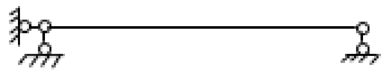
Dimensionless natural oscillation frequencies of individual rods $\omega = \omega l^2 \sqrt{r}$



$D_3 = A_{33} \cdot A_{44} - A_{34} \cdot A_{43} = 0$

| Oscillation tone number | approximated | refined | error % |
|---|---|---|---|
| 1 | 3,5161 | 3,5143 | 0,34 |
| 2 | 22,0348 | 21,3926 | 3,00 |
| 3 | 61,8633 | 56,8754 | 8,77 |
| 4 | 120,9023 | 103,9717 | 16,33 |
| 5 | 199,8596 | 158,8897 | 25,79 |
| 6 | 298,5557 | 218,8186 | 36,44 |
| 7 | 416,9909 | 281,7821 | 47,98 |
| 8 | 555,1656 | 346,5089 | 60,22 |
| 9 | 713,0787 | 412,1571 | 73,01 |
| 10 | 890,7310 | 478,1448 | 86,29 |

Dimensionless natural oscillation frequencies of individual rods $\omega = \omega l^2 \sqrt{r}$



$D_4 = A_{13} \cdot A_{34} - A_{14} \cdot A_{32} = 0$

| Oscillation tone number | approximated | refined | error % |
|---|---|---|---|
| 1 | 9,8699 | 9,7081 | 1,67 |
| 2 | 39,4786 | 37,0953 | 6,42 |
| 3 | 88,8265 | 78,1553 | 13,65 |
| 4 | 157,9138 | 128,6654 | 22,73 |
| 5 | 246,7403 | 185,3173 | 33,14 |
| 6 | 355,3060 | 245,8317 | 44,53 |
| 7 | 483,6108 | 308,7225 | 56,65 |
| 8 | 631,6547 | 373,0370 | 69,33 |
| 9 | 799,4384 | 438,1725 | 82,45 |
| 10 | 986,9472 | 503,7375 | 95,92 |

The accuracy of the frequencies of equation (10) can be judged by the fact that the first 5 frequencies of Tables 1, 2 at the hinge resistance coincide with 5 frequencies in work [4].

## 5. APPLICATION OF THE MODEL OF S.P. TYMOSHENKO IN M. BECK AND V.I. REUT'S PROBLEMS

More accurate solutions of differential equations open up new opportunities for solving various problems, including problems of stability. With regard to non-conservative problems of stability of a rectilinear rod, it can be noted that M. Beck and V. I. Reut's problems are quite well studied only on the basis of approximate solutions. The effort to clarify the existing results led to the appearance of works [5–9], where the model of S.P. Tymoshenko was used. In these works, only M. Beck's problem was investigated, and to an incomplete extent [10-14]. In this regard, a more complete and detailed solution of non-conservative problems, which will be considered in a combined form, is of scientific and practical interest (Fig. 2) [15-17].

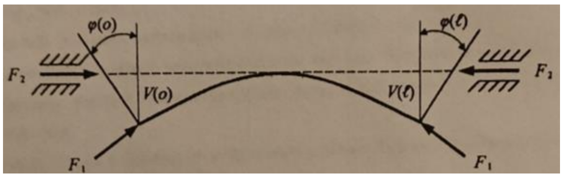



Fig. 2. Combined form of load

Simultaneous action of forces $F_1$, $F_2$. The linearized boundary conditions of the problem are very simple.

$$EIV(0) = EI\varphi(0) = 0;$$
$$M(\ell) = F_2 V(\ell);$$
$$Q(\ell) = F_2 \varphi(\ell).$$

Where x=ℓ and given boundary conditions, the matrix equation is brought to the form (B=0).

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | $-1+a_4A_{13}/EI$ | $b_4 A_{14}/EI$ | $A_{13}$ | $A_{14}$ |
| 2 | $a_4A_{23}/EI$ | $-1+b_4 A_{24}/EI$ | $A_{23}$ | $A_{24}$ |
| 3 | $[-F_2+a_4(A_{33}-1)]/EI$ | $b_4 A_{34}/EI$ | $A_{33}$ | $A_{34}$ |
| 4 | $a_4A_{43}/EI$ | $[-F_2+b_4(A_{44}-1)]/EI$ | $A_{43}$ | $A_{44}$ |

$$\begin{vmatrix} EIV(\ell) \\ EI\varphi(\ell) \\ M(0) \\ Q(0) \end{vmatrix} = 0 \qquad (16)$$

When $F_2$=0, the equation $|A(\omega, F_x)| = 0$ represents M. Beck's problem, when $F_1 = 0$ it represents V.I. Reut's problem based on the model of S.P. Tymoshenko, shear, rotational inertia and the strained state of the rod are additionally taken into account [18-21].

By determining the roots of this equation and the coordinates of the merge points of the first two frequencies by the method of linear search, it is possible to find the critical forces of various non-conservative stability problems. The results are shown in Table 3.

If the longitudinal forces ($F_1 = F_2 = 0$) are not taken into account in the coefficients $a_1$-$a_4$, $b_1$-$b_4$, then equation (16) will describe the model of a rigid rod, when the maximum deflections lie within (1/100- 1/1000)ℓ.

With large deflections, longitudinal forces $F_1$, $F_2$ affect the bending moment and transverse force. In this regard, Table 3 shows the critical forces according to two models of the rod – rigid (F) and conditionally flexible ($F_x$), also with different ratios of height and cross-section width. Cross-sectional area A = bh = 0.01m² did not change. The data in Table 3 allow drawing a number of interesting conclusions.

M. Beck's problem. Accounting for shear, rotational inertia, and the strained state of the rod slightly increases the critical force. According to the rigid model at ℓ/h=10, the refinement is 4.69%, according to the flexible model – 2.59%. Changing h/b ratio has little effect on the magnitude of the critical force [22-24].

V.I. Reut's problem. The flexible model leads to a significant reduction in the critical force (by 2.12 times) compared to the rigid model [25, 26]. Thus, a force with a fixed line of action is more dangerous than a force following the angle of rotation.

Two problems are considered in the article: the problem of M. Beck with force $F_1$ and the problem of V.I. Reut with force $F_2$.

Table 3. The value of the critical forces of M. Beck and V. I. Reut's problems

| Problems of stability | Coordinates of the merging points of the first two frequencies | The ratio of the height to the width of the section h/b; A=bh=0,01m² | | |
|---|---|---|---|---|
| | | 1,0 | 2,0 | 3,0 |
| M. Beck's $F_1$=F; $F_2$=0 | $\frac{F_1 l^2/EI}{\omega l^2\sqrt{m/EI}}$ | $\frac{20,57}{11,35}$ | $\frac{20,33}{11,05}$ | $\frac{20,25}{10,099}$ |
| | $\frac{F_2 l^2/EI}{\omega l^2\sqrt{m/EI}}$ | $\frac{20,99}{10,68}$ | $\frac{20,52}{10,95}$ | $\frac{20,37}{10,99}$ |
| V.I. Reut's $F_1$=0; $F_2$=F | $\frac{F_1 l^2/EI}{\omega l^2\sqrt{m/EI}}$ | $\frac{9,12}{16,02}$ | $\frac{9,31}{16,52}$ | $\frac{9,38}{16,66}$ |
| | $\frac{F_2 l^2/EI}{\omega l^2\sqrt{m/EI}}$ | $\frac{19,34}{12,02}$ | $\frac{19,72}{11,42}$ | $\frac{19,85}{11,34}$ |
| Combined $F_1$=F; $F_2$=F;$F_x$=2F | $\frac{F_1 l^2/EI}{\omega l^2\sqrt{m/EI}}$ | $\frac{12,62}{14,69}$ | $\frac{12,76}{15,11}$ | $\frac{12,81}{15,15}$ |

| Problems of stability | Coordinates of the merging points of the first two frequencies | The ratio of the height to the width of the section h/b; A=bh=0,01m² | | |
|---|---|---|---|---|
| | | 4,0 | 5,0 | 6,0 |
| M. Beck's $F_1$=F; $F_2$=0 | $\frac{F_1 l^2/EI}{\omega l^2\sqrt{m/EI}}$ | $\frac{20,21}{10,09}$ | $\frac{20,16}{11,05}$ | $\frac{20,15}{10,96}$ |
| | $\frac{F_2 l^2/EI}{\omega l^2\sqrt{m/EI}}$ | $\frac{20,30}{10,96}$ | $\frac{20,23}{11,05}$ | $\frac{20,21}{10,96}$ |
| V.I. Reut's $F_1$=0; $F_2$=F | $\frac{F_1 l^2/EI}{\omega l^2\sqrt{m/EI}}$ | $\frac{9,42}{16,70}$ | $\frac{9,42}{16,57}$ | $\frac{9,44}{16,68}$ |
| | $\frac{F_2 l^2/EI}{\omega l^2\sqrt{m/EI}}$ | $\frac{19,91}{11,22}$ | $\frac{19,92}{11,20}$ | $\frac{19,95}{11,12}$ |
| Combined $F_1$=F; $F_2$=F; $F_x$=2F | $\frac{F_1 l^2/EI}{\omega l^2\sqrt{m/EI}}$ | $\frac{12,83}{15,10}$ | $\frac{12,83}{15,08}$ | $\frac{12,84}{15,21}$ |

Combined problem. The joint action of forces $F_1$ and $F_2$ leads to a higher critical force than the case of the action of one force $F_2$, which is impossible with conservative compressive forces. In a rigid model, all frequencies individually tend to zero, a certain combination of non-conservative forces can lead to conservative problems [27-30].

2. The free rod is stressed at the boundary points by forces $F_1$ and $F_2$ (Fig. 2).

The boundary conditions of this problem

$$M(0) = F_2V(0);$$

$$Q(0) = F_2\varphi(0);$$

$$M(\ell) = F_2V(\ell);$$

$$Q(\ell) = F_2\varphi(\ell)$$

lead to the stability matrix.

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | $A_{11} - F_2A_{13}/EI$ | $A_{12} - F_2A_{14}/EI$ | $-1 - a_4A_{13}/EI$ | $b_2A_{14}/EI$ |
| 2 | $A_{21} - F_2A_{23}/EI$ | $A_{22} - F_2A_{24}/EI$ | $a_4A_{23}/EI$ | $-1+b_2A_{24}/EI$ |
| 3 | $-A_{31} - F_2A_{33}/EI$ | $-A_{32} - F_2A_{34}/EI$ | $[-F_2 + a_4(A_{33}-1)]/EI$ | $b_2A_{34}/EI$ |
| 4 | $-A_{41} - F_2A_{43}/EI$ | $-A_{42} - F_2A_{44}/EI$ | $a_4A_{43}/EI$ | $[-F_2 + b_2(A_{44}-1)]/EI$ |

To exclude zero leading elements of the matrix (rigid model), its rows should be rearranged in a new order, as shown by the numbers on the right [31-33]. The critical forces of this problem with a square section and ℓ/h=10 take the values

$$F_1 = 0; \; F_2 = F$$

$$F_x = 1{,}982EI/\ell^2 \quad \text{at}$$

$$\omega = 2{,}87\sqrt{EI/m}$$

$$F_x = 30{,}88EI/\ell^2 \quad \text{at} \qquad (17)$$

$$\omega = 9{,}48\sqrt{EI/m}$$

that is, for a free rod, the ratio of critical forces of rigid and flexible models increases sharply compared to a cantilever rod

$$F_1 = F_2 = F$$

$$F_x = 3{,}028EI/\ell^2 \text{ at}$$

$$\omega = 2{,}67$$

The other cases of action of compressive forces according to Fig. 2 lead to conservative problems.

## 6. CONCLUSIONS

The submitted material unequivocally proves that the work and tasks have been completed in full. Herewith:

1. The equation of S.P. Tymoshenko is supplemented with compressive force, which significantly increases the scope of application of his solution.
2. The variables in the equation of transverse vibrations of beams are separated by the Fourier method.
3. A more accurate differential equation is integrated, fundamental functions are normalized, and the complete solution is presented in a convenient matrix form with initial parameters.
4. Calculations of the frequencies of natural oscillations of the beams, taking into account shear and rotational inertia, have been made. For the hinged beam, the obtained refined frequencies coincided with the results of other authors.
5. The model presented in the work is the most accurate of those published. Therefore, it was possible to refine the critical forces of the non-conservative stability problems of M. Beck and V.I. Reut.

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