Implementation of the Exponential Smoothing Method for Forecasting of the Sales Volume of an Opencast Mine of Rock and Raw Materials

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Abstract

This article uses an exponential smoothing method to forecast the time series of temporary sales of an opencast rock and raw materials mine. Six models of foresight were developed: three seasonal additive models (with linear, exponential and fading trend) and three seasonal multiplicative models (with linear, exponential and fading trend). The exponential smoothing method can be used for current enterprise management, not just mining. This method can be used to make relevant decisions based on the verified forecasts.

It is designed for short-term forecasting-even several times during the day based on current changing data. This method is a useful tool for forecasting time series for not only sales. It can be used to forecast inventory, receivables, etc. However, despite the great progress in predictive methods of the future, which is particularly aided by computer technics, the forecast of the Economist is fraught with greater or lesser errors, and it is therefore necessary to verify developed models. The quality of the forecast should be determined by its relevance, which is determined by means of ex post errors (expired forecast errors). Furthermore, the quality should also be monitored and the forecasts should be corrected if necessary.

To develop forecasts in the six models mentioned, the STATISTICA program, which provides a transparent and quite rapid forecasting of the use of the exponential smoothing method in twelve possible variants. STATISTICA also allows to verify the developed model by drawing an adjustment chart of this model with actual time series, verifying ex post errors, and creating a histogram of the rest of the model.

This article also carried out verifications of the models developed by designating the errors of expired forecasts (ex post errors), as well as verification, on the basis of the histogram, whether the rest of the developed models have a normal distribution. For this purpose the Shapiro-Wilk test was used.

Keywords: exponential smoothing method, forecasting, seasonal additive model, seasonal multiplicative model

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Introduction

A group of exponential smoothing methods is used to determine a short-term forecasts that can be used in the current mine management. The characteristic feature of this group of methods is the constant update of forecasts along with the inflow of new information about the observed values of the predicted variable and the relevance of earlier predictions.

The exponential smoothing methods belong to a group of non-parametric time series models. This means that in their case there is no need to formulate any assumptions related to the properties of the analyzed series.

These methods are preferable if the needs dictate continuous forecasts – even several times a day. In such a situation, the continuous build-up of the correct statistical models is unenforceable in practice. The exponential smoothing methods, however, can be applied automatically, and without hindrance, "produce" the needed number of predictions [1, 5].

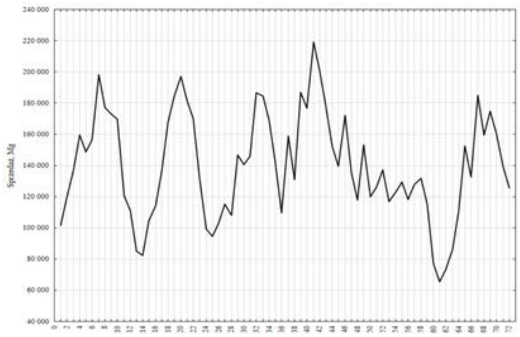
The researcher Robert Brown is considered to be the creator of the exponential smoothing method. He worked for the U.S. Navy in the 1950s, and co-created a system for the tracking and localization of submarine. The system was also used for forecasting demand for spare parts. Later, Charles C. Holt and Peter R. Winters developed this method.

The article foresees a time series of temporary sales of the quarries mine of the rock and raw materials. The predicted time series consists of 72 observations – monthly sales of the analyzed opencast mine of the rock raw materials. The graph of the analyzed time series illustrates the figure 1.

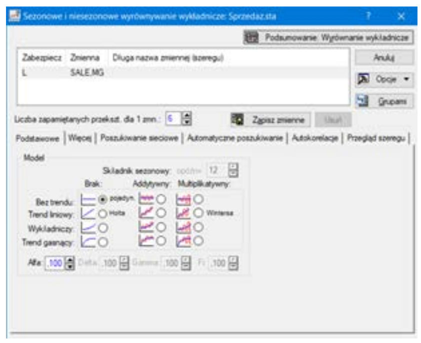
Exponential smoothing method – theoretical introduction

The future value of the variable, also in this analyzed case – time series of the sales of analyzed mine is determined by the exponential smoothing method on the basis of the weighted average of all previous observations. The weights of which are allocated to the individual observation decreases with their "age" and caused that, when the observation is older, the smaller its effect is to the value of forecast. The older observation received the exponential lower weight.

According to the recursive procedure, each new smoothed value is calculated as the weighted average of the current observation and the previous smoothed observation.



Rys. 1. Wykres analizowanego szeregu czasowego – sprzedaży kopalni x
Fig. 1. The chart of predicted time series – sale of mine x [Source: Own elaboration]



Rys. 2. Warianty wyrównywania wykładniczego w programie STATISTICA
Fig. 2. The variants of exponential smoothing in the STATISTICA program [Source: Own elaboration]

The previous smoothed observation was calculated again from the previous observed and smoothed value and in front of the previous observation, etc. Consequently, each smoothing value is the weighted average of the previous observations, with the weights decreasing exponentially depending on the value of the alpha (α) parameter. If alpha equals one, then the previous observations are completely ignored. If alpha is zero, the current observation is completely ignored, and the smoothed value consists entirely of the previous smoothed value (which is in turn calculated from the smooth observation that is in front of it, and so on, so all the smoothed values will be equal to the initial smoothed value [3, 4, 7].

$$y_{wt} = \propto \cdot y_t + (1-\alpha) \cdot y_{wt-1}$$

where:
y_t – means the observated values of the analyzed time series,
y_{wt} – smoothed values,
α – parameter of smoothing.

Exponential smoothing occurs in three variants [3]:

1. Simple exponential smoothing
2. Double exponential smoothing – Holt method
3. Triple smoothing exponential – Winters model

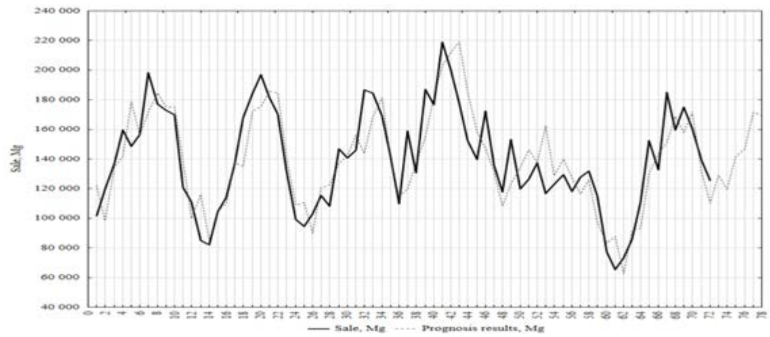
The STATISTICA program, used for forecasting in this article, has twelve options in the exponential smoothing method, which are applicable for the creation of predictions models. These variants are presented in Fig. 1.

The difference between models which are used by STATISTICA program is the trend and the seasonal component which are smoothed by using additional smoothing parameters.

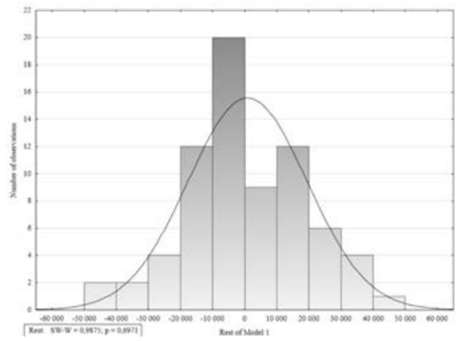
The simple exponential smoothing model is applied to non-trending time series. Holt's Model is suitable for time series with a clear trend. However, these models are not suitable for time series with seasonality modelling. For this purpose the Holt-Winters's model is used. This method requires an estimation of three parameters: alpha, delta, and gamma [6].

The STATISTICA program determines four parameters, which is used automatically network for searching them as follows [4, 5, 6]:

– the alpha (α) parameter - is a constant, parameter characterising the degree of smoothing, occurs in all exponential smoothing models. This parameter is treated as a "stiffness" parameter. The smaller the α is, the more smoother and stiffer the line is, i.e. that the smoothed line will not be so influenced by the random variability from observation to observation. The larger the α parameter is, the more flexible the smoothed line is, that means, the more it will respond for fluctuations in the observed values.



Rys. 3. Opracowany model prognostyczny 1 w porównaniu do źródłowego szeregu czasowego
Fig. 3. The developed prognosis Model 1 compared to the basic time series [Source: Own elaboration]



Rys. 4. Histogram reszt Modelu 1
Fig. 4. The histogram of the rest of Model 1 [Source: Own elaboration]

– the delta (δ) parameter is a constant, the seasonal smoothing parameter that is used for seasonal models. If δ is zero, the steady, stable component of seasonality is included in the calculation of the smoothed and predicted values. If δ is set to 1, then the seasonality component is converted from observation to observation.

– the gamma (γ) parameter – it is a parameter of trend smoothing, is used for linear and exponential trend models and for dumped trend models. If γ is zero, the constant slope is included in the calculation of the smoothed and predicted values. If γ is set to 1, then the slope is converted for each observation based on the directly previous smoothed value. It follows that the slope may vary from observation to observation as much as is necessary to ensure that observed values are well approximated.

– the fi (φ) parameter is a trend smoothing parameter applied to dumped trend models.

Applying exponential smoothing for forecasting of a time series of temporary sales of opencast mine X

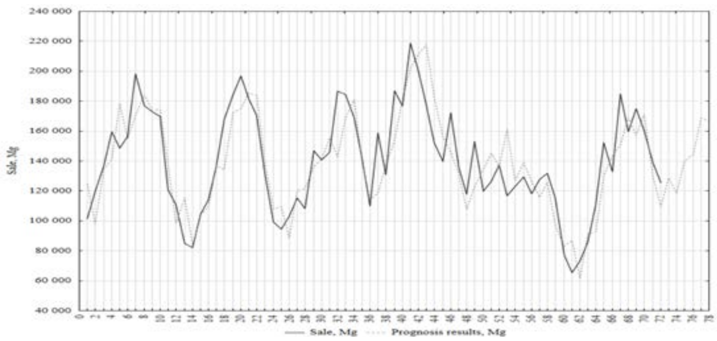
Many empirical time series include seasonal fluctuations, also the predicted time series in this article. The seasonal component in the exponential smoothing method is smoothed by using the additional parameter – delta (δ).

Seasonal components may be inherently additive or multiplicative. In the graphs of the time-series of the distinguishing feature of these two types of seasonal components, is that in the case of additively a time-series reveals monotonous seasonal fluctuations, regardless of the overall level of the series. In the case of multiplicative, the magnitude of seasonal fluctuations is variable, depending on the general level of the series [4].

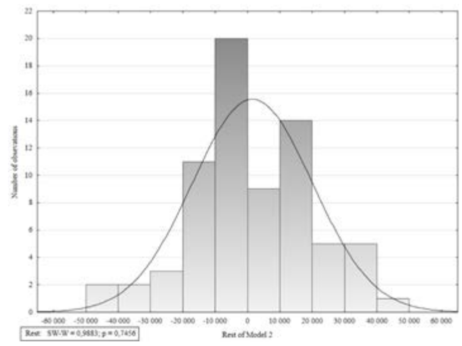
The subject of this article is to develop a forecast for the time-series of sales of opencast rock raw materials mine X - over a period of 72 months. On the basis of the time-series chart (Fig. 1), it can be stated, but not clearly, that this could be a variant of the additive model of the exponential smoothing method. In that case this article is also used for comparison – a variant of the multiplicative model of the exponential smoothing method which gave better adjustment parameters.

By using STATISTICA, the smoothing parameters of the forecast were estimated, by using network exploration. The results are presented in Table 1.

Then, forecasts was made for the three types of additive models and three types of multiplicative models and compared them in Table 2, which indicated the value of the predicted time-series, for the individual developed models. The specified number of forecast periods in the current study was 6 months. The verification of the models was carried out by making a histogram of the rest of the models and applying the Shapiro-Wilk test and by calculating the adjustment errors (Table 3). The article uses two variants (additive and multiplica-



Rys. 5. Opracowany model prognostyczny 2 w porównaniu do źródłowego szeregu czasowego
Fig. 5. The developed prognosis Model 2 compared to the basic time series [Source: Own elaboration]



Rys. 6. Histogram reszt Modelu 2
Fig. 6. The histogram of the rest of Model 2 [Source: Own elaboration]

tive) of seasonal exponential smoothing. It was developed; the additive model with a linear, exponential and fading trend, and a multiplicative variant with a linear, exponential and fading trend.

Seasonal additive model with linear trend – Model 1

Based on the graph (Fig. 3) obtained by using the seasonal exponential smoothing method with a linear trend, it can be concluded that model 1 is quite well adjusted to empirical data, the model captures upward and downward tendency.

The normal distribution of the rest of this model has been verified by a histogram and a Shapiro-Wilk test (Fig. 4). The value of the Shapiro-Wilk statistics is 0.9875 and the test probability value is 0.6971. Therefore, there are no reasons for rejecting the hypothesis of the normal distribution at the significance level $\alpha = 0.05$.

Seasonal additive Model with exponential trend – Model 2

Based on the graph (Fig. 5) obtained by using the seasonal exponential smoothing method with a exponential trend, it can be concluded that model 2 is quite well adjusted to empirical data, the model captures upward and downward tendency.

The normal distribution of the rest of this model has been verified by a histogram and a Shapiro-Wilk test (Fig. 6). The value of the Shapiro-Wilk statistics is 0.9883 and the test probability value is 0.7456. Therefore, there are no reasons for rejecting the hypothesis of the normal distribution at the significance level $\alpha = 0.05$.

Seasonal additive model with fading trend – Model 3

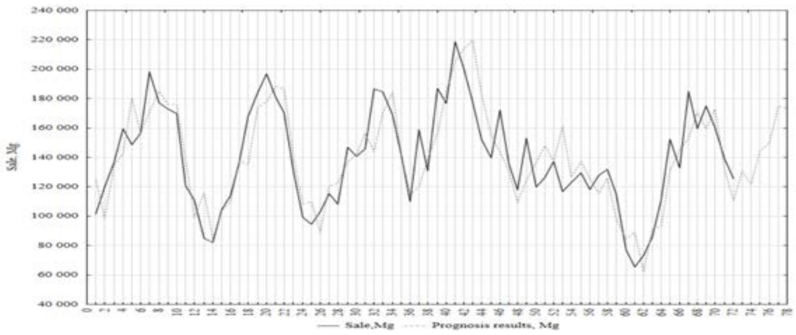
Based on the graph (Fig. 7) obtained by using the seasonal exponential smoothing method with a fading trend, it can be concluded that model 3 is quite well adjusted to empirical data, the model captures upward and downward tendency.

The normal distribution of the rest of this model has been verified by a histogram and a Shapiro-Wilk test (Fig. 8). The value of the Shapiro-Wilk statistics is 0.9889 and the test probability value is 0.7803. Therefore, there are no reasons for rejecting the hypothesis of the normal distribution at the significance level $\alpha = 0.05$.

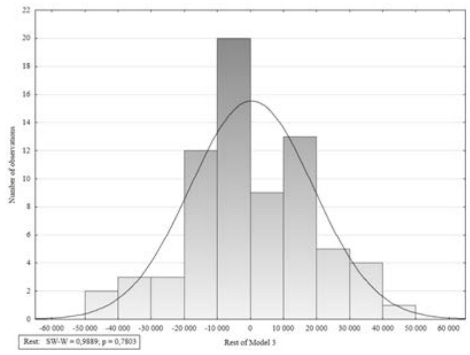
Seasonal multiplicative model with linear trend – Model 4

Based on the graph (Fig. 9) obtained by using the seasonal exponential smoothing method with a linear trend, it can be concluded that model 4 is quite well adjusted to empirical data, the model captures upward and downward tendency.

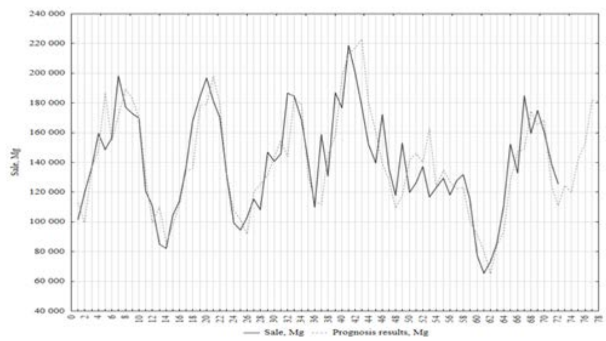
The normal distribution of the rest of this model has been verified by a histogram and a Shapiro-Wilk



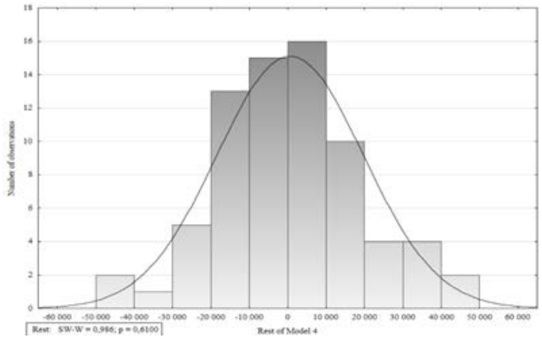
Rys. 7. Opracowany model prognostyczny 3 w porównaniu do źródłowego szeregu czasowego
Fig. 7. The developed prognosis Model 3 compared to the basic time series [Source: Own elaboration]



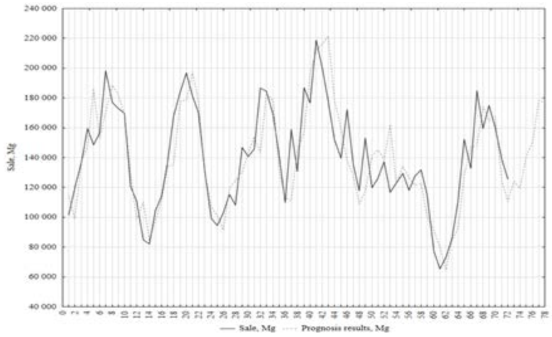
Rys. 8. Histogram reszt Modelu 3
Fig. 8. The histogram of the rest of Model 3 [Source: Own elaboration]



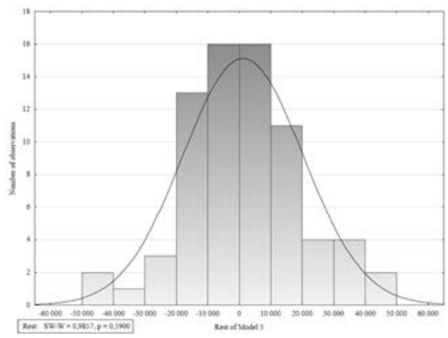
Rys. 9. Opracowany model prognostyczny 4 w porównaniu do źródłowego szeregu czasowego
Fig. 9. The developed prognosis Model 4 compared to the basic time series [Source: Own elaboration]



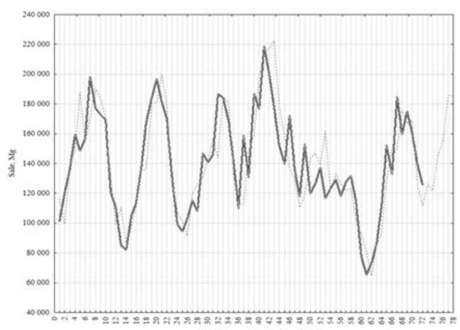
Rys. 10. Histogram reszt Modelu 4
Fig. 10. The histogram of the rest of Model 4 [Source: Own elaboration]



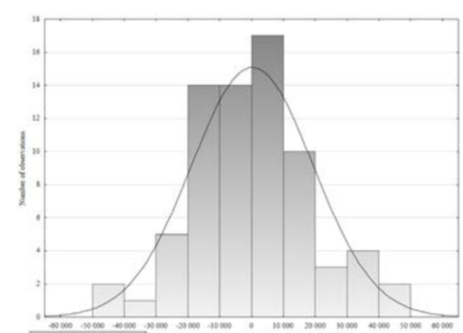
Rys. 11. Opracowany model prognostyczny 5 w porównaniu do źródłowego szeregu czasowego
Fig. 11. The developed prognosis Model 5 compared to the basic time series [Source: Own elaboration]



Rys.12. Histogram reszt Modelu 5
Fig. 12. The histogram of the rest of Model 5 [Source: Own elaboration]



Rys. 13. Opracowany model prognostyczny 6 w porównaniu do źródłowego szeregu czasowego
Fig. 13. The developed prognosis Model 6 compared to the basic time series [Source: Own elaboration]



Rys. 14. Histogram reszt Modelu 6
Fig. 14. The histogram of the rest of Model 6 [Source: Own elaboration]

Tab. 1. Stałe wygładzania wyznaczone przez program STATISTICA dla opracowanych modeli prognostycznych
Tab. 1. Smoothing factors of developed models designated by the STATISTICA program [Source: Own elaboration]

Stałe wygładzania	Alfa	Delta	Gamma	Fi
Model 1	0,7	0,1	0,1	-
Model 2	0,7	0,1	0,1	-
Model 3	0,4	0,1	-	0,3
Model 4	0,8	0,1	0,1	-
Model 5	0,8	0,8	0,1	-
Model 6	0,4	0,1	-	0,4

Tab. 2. Błędy prognozy opracowanych modeli prognostycznych
Tab. 2. The errors of expired forecasts of developed prognosis models [Source: Own elaboration]

Wyszczególnienie	Additive models			Multiplicative models		
	Seasonal additive Model with linear trend	**Seasonal additive Model with exponential trend**	**Seasonal additive Model with fading trend**	**Seasonal multiplicati ve Model with linear trend**	**Seasonal multiplicative Model with exponential trend**	**Seasonal multiplicativ e Model with fading trend**
Mean error (ME), Mg	559.86	1 192.50	102,69	308,95	868,45	-52,58
Mean absolute error (MAE), Mg	14 6710.54	14 632.01	14 766,59	14 731,10	14 742,91	14 782,91
Sum of squares error (SSE)	24 228 568 855.97	24 291 402 701.01	24 269 766 511,73	25 711 664 6 77,02	25 694 332 866,2 3	25 747 947 1 04,06
Square medium error (MSE)	336 507 90 0.78	337 380 593 .07	337 080 090,44	357 106 453, 85	356 865 734,25	357 610 376, 45
Average percentage error (MPE), %	-0.76	-0.28	-1,04	-0,62	-0,22	-0,85
Mean absolute percentage error (MAPE), %	**10.79**	**10.74**	**10,88**	**10,64**	**10,64**	**10,72**

test (Fig. 10). The value of the Shapiro-Wilk statistics is 0.986 and the test probability value is 0.6100. Therefore, there are no reasons for rejecting the hypothesis of the normal distribution at the significance level $\alpha = 0.05$.

Seasonal multiplicative model with exponential trend – Model 5

Based on the graph (Fig. 11) obtained by using the seasonal exponential smoothing method with a exponential trend, it can be concluded that model 5 is quite well adjusted to empirical data, the model captures upward and downward tendency.

The normal distribution of the rest of this model has been verified by a histogram and a Shapiro-Wilk test (Fig. 12). The value of the Shapiro-Wilk statistics is 0.9857 and the test probability value is 0.5900. Therefore, there are no reasons for rejecting the hypothesis of the normal distribution at the significance level $\alpha = 0.05$.

Seasonal multiplicative model with fading trend - Model 6

Based on the graph (Fig. 13) obtained by using the seasonal exponential smoothing method with a fading trend, it can be concluded that model 6 is quite well adjusted to empirical data, the model captures upward and downward tendency.

The normal distribution of the rest of this model has been verified by a histogram and a Shapiro-Wilk test (Fig. 14). The value of the Shapiro-Wilk statistics is 0.9856 and the test probability value is 0.5847. Therefore, there are no reasons for rejecting the hypothesis of the normal distribution at the significance level $\alpha = 0.05$.

Results and summary of developed prognosis models

In table 1, there are included the smoothing parameters which ware estimated by using the network searching. Their searching includes the execution of a series of computer experiments, consisting of the use of various combinations of smoothed parameters values, and then selecting the ones that minimizes the errors of expired forecasts [2].

Tab. 3. Wyniki prognozy dla sześciu opracowanych modeli prognostycznych
Tab. 3. Prognosis results of six developed foresight models [Source: Own elaboration]

Models - Forecast Results	Additive models			Multiplicative models		
	Seasonal additive Model with linear trend	Seasonal additive Model with exponential trend	Seasonal additive Model with fading trend	Seasonal multiplicative Model with linear trend	Seasonal multiplicative Model with exponential trend	Seasonal multiplicative Model with fading trend
Subsequent months	Mg	Mg	Mg	Mg	Mg	Mg
73	129 215,77	128 838,82	130 476,546	124 678,51	124 372,51	125 862,79
74	119 568,07	118 587,80	121 792,281	119 790,17	119 020,66	121 744,18
75	141 531,25	139 952,75	144 263,543	141 874,16	140 402,27	144 622,14
76	146 782,02	144 610,37	149 873,951	153 295,44	151 102,30	156 588,67
77	171 590,39	168 830,63	174 993,564	181 680,17	178 369,96	185 924,81
78	169 605,37	166 262,51	173 304,015	180 963,94	176 961,96	185 522,31

Table 2 shows the errors of forecasts of individual models developed. It turns out that the best adjusted models are in the case of multiplicative models with an exponential trend,

a linear trend and a fading trend. The forecast results for the six developed models are presented in Table 3. It may notice that the obtained results do not differ significantly.

On the basis of the statistical analysis of the rest of the developed models, it can be concluded that, taking into account the results of the Shapiro-Wilk test, there was no reasons for rejecting the hypothesis of the normal distribution of the remainder at the significance level $\alpha = 0.05$.

Based on the analysis of the assigned errors of expired forecasts, it was found that the best adjusted models were multiplicative models with an exponential trend, with a linear trend, and with a fading trend, then the additive model with an exponential, linear and fading trend.

It can be concluded that the forecast results obtained for each model that contains table 3 do not differ significantly. The value of the mean absolute percentage error accept values from 10.64% to 10.88%. These values indicate that the accuracy of the designated models was accurate enough.

This article has taken in to consideration a forecasting process using a group of methods of seasonal exponential smoothing. The prediction results was received by applying the STATISTICA program. Both the method and the use of the method in STATISTICA give the opportunity to forecast and quickly estimate and verify the predictions results. For the current management of the mine, but also any other company, when the decision making process needed the short-term prediction of time series this method is quite fast and gives precise results, which can be verified.

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Implementacja metody wyrównania wykładniczego do prognozowania wielkości sprzedaży kopalni odkrywkowej surowców skalnych

W artykule zastosowano metodę wyrównywania wykładniczego do prognozowania szeregu czasowego sprzedaży kopalni odkrywkowej surowców skalnych. Opracowano sześć modeli prognostycznych; trzy sezonowe modele addytywne (z trendem liniowym, wykładniczym oraz trendem gasnącym) oraz trzy sezonowe modele multiplikatywne (z trendem liniowym, wykładniczym oraz trendem gasnącym).

Metoda wyrównywania wykładniczego może być wykorzystywana w bieżącym zarządzaniu przedsiębiorstwem, nie tylko górniczym. Metoda ta może służyć do podejmowania trafnych decyzji opartych na opracowanych i zweryfikowanych prognozach. Przeznaczona jest do krótkookresowego tworzenia prognoz - nawet kilkukrotnie w ciągu dnia opartego na zmieniających się bieżących obserwacjach (danych). Metoda ta, jest to przydatne narzędzie do prognozowania szeregów czasowych dotyczących nie tylko sprzedaży. Można jej użyć do prognozowania zapasów, należności itp. Jednak pomimo dużego postępu w zakresie metod przewidywania przyszłości, szczególnie wspomaganego techniką komputerową, prognoza jaką posługuje się ekonomista jest obarczona większym lub mniejszym błędem, dlatego też potrzebna jest weryfikacja opracowanych modeli. Jakość prognozy powinna być określona poprzez jej trafność, którą określa się za pomocą błędów ex post (błędy prognoz wygasłych), powinna być również monitorowana i w razie potrzeby powinno się przeprowadzić korektę prognoz.

Do opracowania prognoz w sześciu wspomnianych modelach zastosowano program STATISTICA, który w sposób przejrzysty i dosyć szybki tworzy prognozy wykorzystując metodę wyrównywania wykładniczego w dwunastu możliwych do zastosowania wariantach wygładzania wykładniczego. Program STATISTICA umożliwia również szybką weryfikację opracowanego modelu poprzez sporządzenie wykresu dopasowania opracowanego modelu do rzeczywistego szeregu czasowego, weryfikację błędów ex post, jak również utworzenie histogramu reszt modelu.

W artykule przeprowadzono również weryfikację opracowanych modeli poprzez wyznaczenie błędów prognoz wygasłych (błędów ex post), jak również weryfikację, na podstawie histogramu, czy reszty opracowanych modeli mają rozkład normalny, do tego celu został wykorzystany test Shapiro-Wilka.

Słowa kluczowe: metoda wyrównywania wykładniczego, prognozowanie, model addytywny sezonowy, model multiplikatywny sezonowy