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Fuzzy Alliance and Coalitions that Can Be Formed by Alliance Agents

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***Abstract*—The paper deals with alliances and coalitions that can be formed by agents or entities. It is assumed that alliance agents cooperate and form coalitions for performing the tasks or missions. It is considered that alliance agents are unselfish. That is, they are more interested in achieving the common goal(s) than in getting personal benefits. In the paper, the concept of fuzzy alliance was introduced. A fuzzy alliance is considered as generalization of traditional alliance allowing agents to decide on the capabilities that their agents can and wanted deliver to coalition. Coalitions that can be formed by fuzzy alliance agents were considered. The definition of the "best" coalition was explained. The method of how to find the "best" coalition among all possible coalitions was suggested and verified by computer simulation.**

***Keywords*—alliance; coalition; multi-agent systems; modeling and simulation**

I. Introduction

THE concepts of alliance and coalition are usually considered in context of multi-agent systems (MAS). Multi-agent systems are subject of research in different fields, such as computer science, economic, business, transportation, social fields, etc. In most cases, an agent is perceived as computer system that is able to perform some actions in unpredictable and open environment. Currently, in many research, the term agent is used to present different entities ranging from low-level implementation (e.g., transport unit [1], computer processor [2]) to high-level (e.g., non-governmental organizations, army troops, etc. [3], [4], [5]) that can cooperate and coordinate their actions for achieving the common goal.

Agent can be considered in a broader sense as autonomous agent in MAS and as entity in complex social systems. Usually, agents cooperate and coordinate their activities in large and complex systems. In the given case, agents join together and form a group to fulfil the tasks (or mission) in a more efficient way.

There are a number of facets characterizing a group of agents:

- duration of cooperation of agents in a group (long-time, short-time);
- the level of agent's responsibility for achieving the goal of mission;
- the extent to which the agents are interested in their own benefits and in successful achievement of the common group goal(s);
- the level of independence of agents to act;
- distribution of roles among agents;
- interoperability and information sharing among agents; etc.

The variety of facets predetermines the variety of possible groups of agents. M. Tamber and D. Pynodath [6] considered a team of agents in which each member behaves responsibly towards other members of team. The team is provided with a proxy capable of general teamwork reasoning. As a result, the team using such proxies can achieve its goal robustly, with agents automatically covering for failed teammates, supplying key information to help each other. L. Camarinha and H. Afsarmanesh [7] introduced the cluster of enterprises which represents a group or pool of enterprises and related and supporting institutions that have the potential, and the will, to cooperate with each other through the establishment of a long-term cooperation agreement. In [3] M. Pechoucek et al. considered a group which they called as alliance. They defined the alliance as a set of agents that agree to share some of their private information and cooperate eventually.

In the paper, we consider alliance as a group of agents (entities) that agree to cooperate and work together when task or problem arises. In order to perform the tasks, agents form coalition and deliver their capabilities to this coalition. Agents do not expect to gain some benefits (i.e., they carry out the tasks without regards for personal payoff) from their participation in coalition. Agents of an alliance can have preferences. Therefore, they can refuse to cooperate with particular alliance agents, although giving their agreement to carry out the tasks that may come from in-need entity. Alliance which agents can have such refusals was called restricted alliance and was considered in [8], [9]. Agent of restricted alliance either entirely refuse to cooperate with another alliance agent or cooperate with him without any restrictions. Although the cooperation between agents of alliance can be graduated (from entirely to partially). In the paper, we call alliance which includes such agents as fuzzy alliance.

When task or problem arises a coalition is formed by the agents of alliance. Depending on the tasks which should be executed and on the agents preferences each agent makes decision on capabilities (services, resources) it can (want) deliver to coalition.

We considered coalition as a set of agents which agreed to fulfil a single, well-specified goal. Coalition members committed themselves to cooperate on the within-coalition-

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shared goal. A coalition, unlike an alliance, is usually regarded as a short-term agreement among collaborative agents.

In order to avoid disunity in understanding the tasks by the agents, in order to evaluate possible coalitions and in order to prevent the leaking information some additional agents can be added to alliance. For example, interface agent, filter agents and arbiter [4].

Being faced with the problem how to reduce the dependence of coalition formation on the correct functionality of each single agent, on the reluctance of agents to provide information to the central authority and on possible faulty situations stimulate the researchers tend to distribute the roles of interface agent, filter agent and arbiter among the active agents.

Several formal description techniques, methods and tools are used for alliance and coalition formation. Here we list only some of them, particularly: LoTOS [10], SDL [11], language Z [12], agent UML [13], Petri Nets [14], finite state machines [15]. In this paper, we are going to consider how to find the "best" coalition that can be formed by agents of fuzzy alliance. Under the term "best coalition" we mean the coalition with minimal number of agent and at the same time this coalition can fulfil the coalition task. Coalition can fulfil the task when it has capabilities greater or equal to the required amount of capabilities.

II. Fuzzy Alliance

We use the following table (see Table 1) to present the willingness to cooperate among any two agents. We suggest quantifying this willingness by using real numbers that lie in interval [0.1]. This quantifying or mapping very resemble membership function of fuzzy sets where an element belongs to a fuzzy set with some degree [16], [17].
For simplicity reason the table is designed for the fuzzy alliance with five agents.

TABLE I
FUZZY ALLIANCE WITH FIVE AGENTS

$\mu(a_i, a_j)$	a_1	a_2	a_3	a_4	a_5
a_1	-	0.7	0.3	0.4	0.3
a_2	0.1	-	0.2	0.6	0.9
a_3	0.8	0.4	-	0.9	0.5
a_4	0.6	0.5	0.9	-	1.0
a_5	0.7	0.2	1.0	0.9	-

In the table, a_i denotes the agent *i*. $\mu(a_i,a_j)$ denotes the degree of cooperation that agent a_i is going to adhere. Another words, the value $\mu(a_i,a_j)$ shows which part of its capability agent a_i is going to deliver for cooperation with agent a_j [18], [19]. It is worth noting that $\mu(a_i,a_j)$ can be different from $\mu(a_j,a_i)$. That is, the table is not symmetric. Evidently that in case of restricted alliance values $\mu(a_i,a_j)$ can take only values equal to either 1 or 0. Thus, fuzzy alliance can be considered as generalization of restricted alliance.

Agents' capabilities are denoted as Ca_i , $i=1,\dots,n$. Where n is the number of alliance agents. In the paper, we represent these capabilities via integer numbers. It should be noted that this restriction can be further omitted.

Let us consider the extension of cooperation of agents on more than two agents. In Fig. 1, the example with three agents (a_1, a_2 and a_3) is depicted.

Let us consider cooperation of agent a_3 with agent a_1 (i.e., $\mu(a_3,a_1)$) and with agent a_2 (i.e., $\mu(a_3,a_2)$). Similarly to fuzzy sets (e.g., set A and set B with elements a_1, a_2 and a_3), we can consider cooperation of agent a_3 the same way as belonging element a_3 to fuzzy set A (i.e., $\mu_A(a_3)$) and to fuzzy set B (i.e., $\mu_B(a_3)$) (see Fig. 1). Intersection of fuzzy sets A and B (i.e., A ∩ B) corresponds to cooperation of agent a_3 with agent a_1 and with agent a_2. From this it can be derived that agent a_3 is going to deliver to coalition with agents a_1 and a_2 the part of its capability which is denoted as $\mu_{coal}(a_3)$. In this research, we assume that

$$\mu_{coal}(a_3) = min\{\mu(a_3,a_1), \mu(a_3,a_2)\} \tag{1}$$

Taking into account the capability of agent a_3 (i.e., Ca_3), it is possible to determine the capability that agent a_3 will deliver to coalition, $C_{a3}\mu_{coal}(a_3)$. Similarly, it is possible to determine the capabilities of agents a_1 and a_2 which they can and want deliver to coalition, i.e., $C_{a1}\mu_{coal}(a_1)$ and $C_{a2}\mu_{coal}(a_2)$.

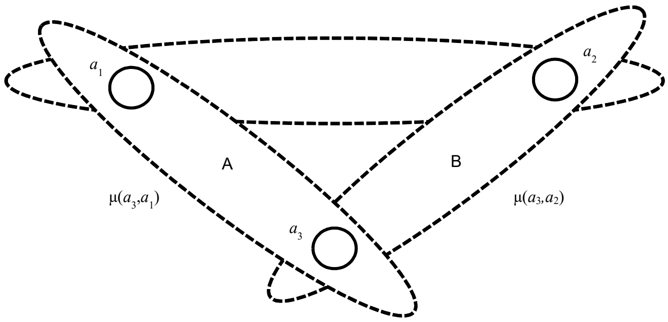

Fig. 1. Example of alliance with three agents (a_1, a_2 and a_3)

Thus, the total coalition (agents a_1, a_2, a_3) will have capability

$$C_{coal} = \sum_{i=1}^{n} C_{a_i}\, \mu_{coal}(a_i) \tag{2}$$

Coalition can perform the arising task if amount of its capability C_{coal} is greater or equal to amount of capability required to fulfil the task, C_R. That is, if $C_{coal} \geq C_R$.

There is a probability that one of the alliance agents will be able to fulfil the arising task alone. In the given case, no coalition will be formed. In case when such agent is not found, there is an option to find the pair of agents which have summary capability greater or equal to C_R. If such pair of agents is not found, the procedure of finding the coalition capable of fulfilling the arising task continues. This is done by way of considering coalitions with number of agents increased every time by one. We assume that coalition is the "best" if it is able to fulfil the task by minimal number of agents. This approach to finding the "best coalition" required to consider all possible combinations of agents (i.e., coalitions) for each particular number of agents in coalition, n.

Thus, the complexity of such searching procedure, θ (i.e., the total number of coalitions that should be considered) at the worth case is equal to

$$\theta = \sum_{i=1}^{n} C_i^n \tag{2}$$

where i is equal to the number of agents at Step i of the searching procedure.

When number of agents in fuzzy alliance is great, finding the "best" coalition is very complex (i.e., NP-hard problem). In view of this, we suggest another approach to finding the "best" coalition which is considered in next Section.

III. COALITIONS THAT CAN BE FORMED BY AGENTS OF FUZZY ALLIANCE

The proposed approach uses the reverse direction of searching. That is, it starts searching by examining the coalition with maximum number of agents and continues with coalitions with number of agents reduced every time (at each Step) by one. At the first Step, the coalition with number of agents that is equal to the number of agents of fuzzy alliance is examined. For the example of fuzzy alliance which presented with the help of Table 1 the number of coalition agents is equal to five. Thus, the first considered coalition has number of agents, n=5. Considering or examining consists in calculating coalition capability C_{coal} according to (2). For the example under consideration, we extend Table 1 by adding one new column. This column contains the values $C_{ai}\mu_{coal}(a_i)$, i=1,..,n. Below the table (at margin-bottom of table) the total coalition capability C_{coal} is displayed (see Table 2). We assume that capability of agents can be mapped onto integer numbers. For simplicity reason, we set capability of each agent equal to 10. Obviously, this setting can be changed and arbitrary values of capabilities can be considered.

TABLE II
COALITION CAPABILITY FOR n=5

$\mu(a_i, a_j)$	a_1	a_2	a_3	a_4	a_5	$Ca_i\mu_{coal}(a_i)$
a_1	-	0.7	0.3	0.4	0.3	3
a_2	0.1	-	0.2	0.6	0.9	1
a_3	0.8	0.4	-	0.9	0.5	4
a_4	0.6	0.5	0.9	-	1.0	5
a_5	0.7	0.2	1.0	0.9	-	2
						C_{coal}=15

Let's assume that for the fulfilling the task by coalition it is required the capability which is equal to 23. It means that in case when all five agents of fuzzy alliance form the coalition, the task cannot be fulfilled because $C_{coal} < C_R$.

Next Step of searching procedure consists in consideration of all possible coalitions which can be obtained from the coalition with n=5 by way of removing exactly one agent from it. Every time the new (different) agent should be removed. Thus, five different coalitions will be obtained that are subject for examination. These coalitions are depicted with the help of Tables 3÷7.

TABLE III
COALITION $\{a_2, a_3, a_4, a_5\}$

$\mu(a_i, a_j)$	a_2	a_3	a_4	a_5	$Ca_i\mu_{coal}(a_i)$
a_2	-	0.2	0.6	0.9	2
a_3	0.4	-	0.9	0.5	4
a_4	0.5	0.9	-	1.0	5
a_5	0.2	1.0	0.9	-	2
					C_{coal}=13

TABLE IV
COALITION $\{a_1, a_3, a_4, a_5\}$

$\mu(a_i, a_j)$	a_1	a_3	a_4	a_5	$Ca_i\mu_{coal}(a_i)$
a_1	-	0.3	0.4	0.3	3
a_3	0.8	-	0.9	0.5	5
a_4	0.6	0.9	-	1.0	6
a_5	0.7	1.0	0.9	-	7
					C_{coal}=21

TABLE V
COALITION $\{a_1, a_2, a_4, a_5\}$

$\mu(a_i, a_j)$	a_1	a_2	a_4	a_5	$Ca_i\mu_{coal}(a_i)$
a_1	-	0.7	0.4	0.3	3
a_2	0.1	-	0.6	0.9	1
a_4	0.6	0.5	-	1.0	5
a_5	0.7	0.2	0.9	-	2
					C_{coal}=11

TABLE VI
COALITION $\{a_1, a_2, a_3, a_5\}$

$\mu(a_i, a_j)$	a_1	a_2	a_3	a_5	$Ca_i\mu_{coal}(a_i)$
a_1	-	0.7	0.3	0.3	3
a_2	0.1	-	0.2	0.9	1
a_3	0.8	0.4	-	0.5	4
a_5	0.7	0.2	1.0	-	2
					C_{coal}=10

TABLE VII
COALITION $\{a_1, a_2, a_3, a_4\}$

$\mu(a_i, a_j)$	a_1	a_2	a_3	a_4	$Ca_i\mu_{coal}(a_i)$
a_1	-	0.7	0.3	0.4	3
a_2	0.1	-	0.2	0.6	1
a_3	0.8	0.4	-	0.9	4
a_4	0.6	0.5	0.9	-	5
					C_{coal}=13

It is worth noting that values $\mu_{coal}(a_i)$, $i=1,..,n$ depend on concrete constitution of coalition (i.e., depend on which particular agents constitute the coalition).

Comparing capabilities of coalition with four agents allows finding the coalition which has the greater value (amount) of capability C_{coal} at this Step of searching procedure. Coalition presented by Table 4 (i.e., coalition $\{a_1, a_3, a_4, a_5\}$) has the greatest capability equal to 21. This value is lesser than value C_R and therefore the searching procedure continues.

We suggest to choose for the next Step of searching procedure only coalition $\{a_1, a_3, a_4, a_5\}$ that had max capability at the current Step. It will allow reducing considerably the complexity of searching procedure. Our proposition is based on the assumption that the "best" coalition, if such exist, is derived from the coalition which at the current Step of searching procedure had max capability. For the example under consideration the next Step of searching procedure consists in examination of all possible coalitions that can be obtained from coalition $\{a_1, a_3, a_4, a_5\}$ by way of removing exactly one agent from it. Every time different agent is removed. As a result of such removing of agents the following four coalitions are obtained (see Tables 8÷11).

TABLE VIII
COALITION $\{a_3, a_4, a_5\}$

$\mu(a_i, a_j)$	a_3	a_4	a_5	$Ca_i\mu_{coal}(a_i)$
a_3	-	0.9	0.5	5
a_4	0.9	-	1.0	9
a_5	1.0	0.9	-	9
				C_{coal}=23

TABLE IX
COALITION $\{a_1, a_4, a_5\}$

$\mu(a_i, a_j)$	a_1	a_4	a_5	$Ca_i\mu_{coal}(a_i)$
a_1	-	0.4	0.3	3
a_4	0.6	-	1.0	6
a_5	0.7	0.9	-	7
				C_{coal}=16

TABLE X
COALITION $\{a_1, a_3, a_4\}$

$\mu(a_i, a_j)$	a_1	a_3	a_4	$Ca_i\mu_{coal}(a_i)$
a_1	-	0.3	0.4	3
a_3	0.8	-	0.9	8
a_4	0.6	0.9	-	6
				C_{coal}=17

TABLE XI
COALITION $\{a_1, a_3, a_5\}$

$\mu(a_i, a_j)$	a_1	a_3	a_5	$Ca_i\mu_{coal}(a_i)$
a_1	-	0.3	0.3	3
a_3	0.8	-	0.5	5
a_5	0.7	1.0	-	7
				C_{coal}=15

Coalition shown in Table 8 has the max capability at this Step of searching procedure. This coalition $\{a_3, a_4, a_5\}$ has capability equal to required capability, i.e., $C_{coal(a3, a4, a5)} = C_R$. Therefore, we preliminary call this coalition as “good” and classify this coalition as candidate for the “best” coalition. Then, the searching procedure proceeds with the next Step.

Next Step of the searching procedure consists in examinations of all possible coalitions that can be obtained from the coalition $\{a_3, a_4, a_5\}$ by way of removing agents from it. Every time exactly one agent is removed. As a result, the following three coalitions are obtained (see Tables 12, 13, 14).

TABLE XII
COALITION $\{a_4, a_5\}$

$\mu(a_i, a_j)$	a_4	a_5	$Ca_i\mu_{coal}(a_i)$
a_4	-	1.0	10
a_5	0.9	-	9
			C_{coal}=19

TABLE XIII
COALITION $\{a_3, a_5\}$

$\mu(a_i, a_j)$	a_3	a_5	$Ca_i\mu_{coal}(a_i)$
a_3	-	0.5	5
a_5	1.0	-	10
			C_{coal}=15

TABLE XI
COALITION $\{a_3, a_4\}$

$\mu(a_i, a_j)$	a_3	a_4	$Ca_i\mu_{coal}(a_i)$
a_3	-	0.9	9
a_4	0.9	-	9
			C_{coal}=18

At this Step of searching procedure, coalition shown with the help of Table 12 has max capability (equal to 19). Nevertheless, this max value of capability is not enough for fulfilling the coalition task. Formally the searching procedure may proceed with the next Step. At this final Step, coalitions will include only one agent. To be precise in definition, one agent cannot be considered as coalition. Therefore, we do not perform this final Step and searching procedure is over.

To make conclusion on the example under consideration, the coalition that was preliminary classified as candidate for the “best” coalition (i.e., coalition $\{a_3, a_4, a_5\}$) becomes actually the “best” coalition.

It is possible to present the searching procedure in the form of directed acyclic graph. Nodes of this graph correspond to the tables that, in its turn, present the coalitions considered in the example. Edges of the graph correspond to the connections among coalitions (or tables). Connection is understood as the fact that one coalition can be derived from another coalition by way of removing exactly one agent from it. At the top of the graph, the node that corresponds to the initial table is placed. This table is denoted as T_0. In just considered example, Table 2 acts as table T_0. At the first Step, table T_0 is examined. At the second Step, the tables which can be derived from table T_0 are examined. They are denoted as T_1^1, .., T_1^5. Tables 3÷7 act as tables T_1^1, .., T_1^5. As a result of examination of these tables, table T_1^2 that has max value C_{coal} is chosen for processing. At the third Step, the tables that can be derived from table T_1^2 (correspond to Table 4) are examined. In the graph, these tables are denoted as T_2^1,.., T_2^4. Table T_2^1 (corresponds to table 8) that has max value C_{coal} is processed. As a result of such processing, tables T_3^1, T_3^2 and T_3^3 are obtained. Then these tables are examined. This was the final Step which ends the searching procedure.

Above described directed acyclic graph is depicted in Fig. 2. Generalization of searching procedure for arbitrary number of agents of fuzzy alliance gives the following results.

Total number of Steps, k

$$k=n-1$$

Number of examined coalitions, M

$$M = 1 + \sum_{i=1}^{n-3}(n-i)$$

It should be noted that at each Step of searching procedure only one coalition, particularly the coalition that has max capability compared to capabilities of other coalitions considered at this Step, is processed. Note also that coalitions examined at Step j, $j=1,..,k$ include $(n+1)-j$ agents.

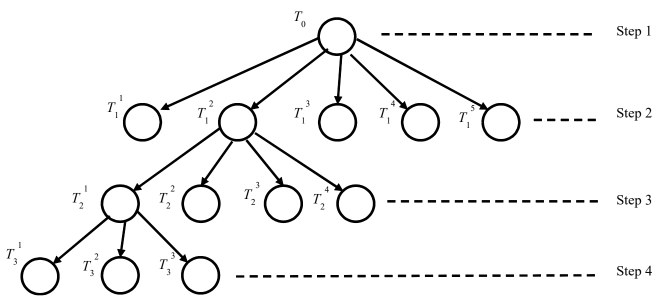

Fig. 2. Graph of searching procedure

We differentiate between the concepts of "coalition examination" and "coalition processing". Term "examined" means that only coalition's capability is calculated and no other operations on this coalition are performed. "Processed" means that sequence operations on coalition are performed. Operations consist in removing agents from this coalition. As a result of coalition processing, new coalitions will be obtained.

If at Step 2 all the coalitions (depicted by tables T_1^1, .., T_1^5) will be processed (see Fig. 2) then at Step 3 ten different coalitions each of which includes 3 agents will be presented. In the given case, all these ten coalitions should be examined and the coalition that has max capability should be determined. According to the above made assumption, this coalition (i.e., coalition with max capability) should be derived from the coalition depicted with the help of table T_1^2. Therefore, processing coalitions T_1^1, T_1^3, T_1^4 and T_1^5 is redundant. Above reasoning concerns also the next Steps of searching procedure.

In order to confirm the made assumption, the computer simulation was performed. Data for simulation, such as capabilities of agents, Ca_i, and values $\mu\ (a_i\ ,\ a_j)$, $i,j=1,..,n$, were generated randomly. Interval for values Ca_i, $i=1,..,n$, was chosen from 1 to 100. Simulation was performed for number of fuzzy alliance agents, n, $1 \leq n < 11$.

Pseudocode [19] used for simulation follows below.

<u>Algorithm for verifying main assumption</u>:
Input: n – number of fuzzy alliance agents.
1. Generate $C_{ai} \leftarrow$ random (1,100) for $i=1,..,n$
 if $i \neq j$ then $\mu\ (a_i\ ,\ a_j) \leftarrow$ random (0,1) for $i,j=1,..,n$
 else $\mu\ (a_i\ ,\ a_j) \leftarrow 10$
2. $\mu_{coal}(a_i) \leftarrow$ min { $\mu\ (a_i\ ,\ a_j)$}, $i,j=1,..,n$
3. $S_0 \leftarrow$ sum ($C_{ai}\mu_{coal}(a_i)$), $i=1,..,n$
4. Form $T_0[n]$
5. While $k \leq n$
6. k← 1
7. For $i.j=1,..,n$ and $i,j \neq k$
8. $\mu_{coal}(a_i) \leftarrow$ min { $\mu\ (a_i\ ,\ a_j)$}
9. $S_k \leftarrow$ sum($C_{ai}\mu_{coal}(a_i)$)
10. Form $T_k[n-1]$
11. $k \leftarrow k+1$
12. $E_1 \leftarrow$ max {S_k}, $k=1,..,n$
13. $l_1 \leftarrow k$, if $S_k = E_1$, $k=1,..,n$
14. For $T_r[n-1]$, $r=1,..,n$
15. While $k \leq (n-1)$
16. $k \leftarrow 1$
17. For $i.j=1,..,$n and $i.j \neq k$
 if a_i, a_j in $T_r[n-1]$
18. $\mu_{coal}(a_i) \leftarrow$ min { $\mu\ (a_i\ ,\ a_j)$}
19. $S_k^r \leftarrow$ sum($C_{ai}\mu_{coal}(a_i)$)
20. Form $T_k^r[n-2]$
21. $k \leftarrow k+1$
22. $Z \leftarrow$ max{S_k^r}, $r=1,..,n$, $k=1,..,n-1$
23. $l_2 \leftarrow k$, $q \leftarrow r$
 if $S_k^r = Z$, $r=1,..,n$, $k=1,..,n-1$
24. If $q = l$, V=1 Else $V = 0$
Output: V (value 1 means positive result)

<u>Description and explanation of algorithm</u>

From Item 1 to Item 4, the initial table has been formed. Initial table has n rows and n columns which correspond to n agents of fuzzy alliance that formed coalition.

From Item 5 to Item 13, the tables of size $(n\text{-}1) \times (n\text{-}1)$ has been formed. $(n\text{-}1)$ rows and $(n\text{-}1)$ columns of these tables correspond to $(n\text{-}1)$ agents of coalition. Each coalition has the set of agents different from the sets of agents which have other coalitions.

In Items 12 and 13, the coalition with $(n\text{-}1)$ agents that has max capability as compared to capabilities of other coalitions has been determined.

From Item 14 to Item 21, from each table of size $(n\text{-}1) \times (n\text{-}1)$ there were derived $(n\text{-}1)$ tables of size (n-2) × (n-2). Totally, $n(n\text{-}1)$ tables were derived. It should be noted, that some tables were repeated. Total number of different tables is equal to C_{n-2}^n.

In Items 22 and 23, the coalition with $(n\text{-}2)$ agents that has max capability as compared to capabilities of other coalition (that has the same number of agents) has been determined.

Item 24 allows to determine the table (i.e., coalition) from which the table of size $(n\text{-}2)$, which corresponds to coalition with max capability, was derived. For this purpose the algorithm remembers (save) index l_1 which belongs to coalition of size $(n\text{-}1)$ with max capability.

If the condition (Item 24) is satisfied, the main assumption is considered as confirmed.

Hundreds runs were carried out and only in 5% cases (on average in relation to n) the main assumption was not confirmed. Negative result is possible when very rare combination of values Ca_i , $i=1,\ldots,n$, and $\mu(a_i,a_j)$, $i,j=1,..,n$, takes place.

When coalition that has enough capability to fulfil the task was not found, it is possible to proceed with the searching procedure. In the given case, coalition which has second in order value of capability could be processed the same way as it was described above. This additional processing allows reducing considerably the percent of runs when main assumption is not confirmed. The percent of runs when main assumption is not confirmed reduces also when dispersion of values of capabilities of alliance agents increases. When number of fuzzy alliance agents, n, increases the percent of negative results reduces. However, these claims should be confirmed by the computer simulation which will allow gathering more data for analysis.

It should be noted, that each coalition agent a_i, $i=1,..,n$, can decide on how to compute the value $\mu_{coal}(a_i)$. It has several options, such as for example

$$\mu_{coal}(a_i)=min\ \{\ \mu(a_i,a_j)/a_i,\ a_j \subset Coal\ \}$$
$$\mu_{coal}(a_i)=max\ \{\ \mu(a_i,a_j)/a_i,\ a_j \subset Coal\ \}$$
$$\mu_{coal}(a_i)=mean\ \{\ \mu(a_i,a_j)/a_i,\ a_j \subset Coal\ \}$$

In the paper, only the first option was considered. We left the considering of other options for the future research.

IV. Conclusion

In the paper, we did not consider the problem of how a coalition is forming (i.e., coalition formation). We only determined the possible coalitions that can be formed by fuzzy alliance agents (entities).

We did not aim to completely consider all problems related to alliances and coalitions. We only introduced the concept of fuzzy alliance which is generalization of existing concept of alliance.

Generally, the number of coalitions that can be formed by the agents of fuzzy alliance is very large. The task arises to choose among these coalitions the one that will fulfil the task coming from in-need agent (entity). In the paper, we proposed the term "best coalition" which means the coalition capable of fulfilling the tasks with minimal number of agents (i.e., by involving minimal number of agents).

Searching the "best" coalition is a difficult problem. Direct approach to searching this coalition based on consideration of all possible coalitions has large complexity. In view of this, we suggest approximate method for searching procedure which has considerably lesser complexity.

The suggested method was verified for certain setting. We used confined range for values Ca_i , $i=1,\dots,n$, and $\mu(a_i,a_j)$, $i,j=1,..,n$. The proposed searching procedure doesn't guarantee finding the "best" coalition (when such coalition exists). Nevertheless, in vast majority of cases, the "best" coalition is found.

Such an approximate method can be further studied in more detail, evaluated and improved. In this article, we just try to delineate a research area related to coalitions that can be formed by fuzzy alliance agents. We can mention the following application areas for the alliances and coalitions considered in this article. These are rescue operations, non-combat military operations, non-combat evacuation operations, Internet services, etc.

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